

Synthesizing Programmatic Reinforcement Learning Policies with Memory-Based Decision Trees

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Abstract

1 Programmatic reinforcement learning (PRL) has been explored for representing policies
2 through programs as a means to achieve interpretability and generalization, meaning
3 involving higher order constructs such as control loops. Despite promising outcomes,
4 current state-of-the-art PRL methods are hindered by sample inefficiency and very little
5 is known on the theoretical front about programmatic RL. A burning question is
6 studying the trade-offs between sizes of programmatic policies and their performances.
7 Hence, the motivation of this work is to construct programmatic policies with the short-
8 est paths to the target region, ensuring near optimal behavior. Alongside this, we also
9 investigate how we can reason with programmatic policies by using two learning sys-
10 tems: Reward Prediction Error (RPE) and Action Prediction Error (APE). How can we
11 learn programmatic policies that can generalize better? The goal of this paper is to give
12 first answers to these questions, initiating a theoretical study of programmatic RL. Our
13 main contributions are construction a near optimal policy using memory-based decision
14 trees, and studying the generalizability and size performance.

15 1 Introduction

16 Reinforcement Learning (RL) is a very popular and successful field of machine learning where
17 the agent learns a policy in an unknown environment through numerical rewards, modelled as a
18 Markov decision process (MDP). In the tabular setting, the environment is given explicitly, which
19 implies that typically policies are also represented explicitly, meaning as functions mapping each
20 state to an action (or distribution of actions). Such a representation becomes quickly intractable
21 when the environment is large and makes it hard to compose policies or reason about them. In
22 the general setting, the typical assumption is that the environment can be simulated as a black-box.
23 Deep reinforcement learning algorithms which learn policies in the form of large neural networks
24 have been scaled to achieve expert-level performance in complex board and video games (Silver et
25 al. 2018; Vinyals et al. 2019). However, they suffer from the same drawbacks as neural networks
26 which means that the learned policies are vulnerable to adversarial attacks (Qu et al. 2021) and do
27 not generalize to novel scenarios (Sunderhauf et al. 2018). Moreover, big neural networks are very
28 hard to interpret and their verification is computationally infeasible.

29 On the other hand, Deep reinforcement learning (DRL) has had a massive impact on the field of
30 machine learning and has led to remarkable successes in the solution of many challenging tasks
31 (Mnih et al., 2015; Silver et al., 2016; 2017). While neural networks have been shown to be very
32 effective in learning good policies, the expressivity of these models makes them difficult to interpret
33 or to be checked for consistency for some desired properties, and casts a cloud over the use of such
34 representations in safety-critical applications.

35 To alleviate these pitfalls, a growing body of work has emerged which aims to learn policies in the
36 form of programs (Verma et al. 2018; Bastani, Pu, and Solar-Lezama 2018; Verma et al. 2019;
37 Zhu et al. 2019; Inala et al. 2020; Landajuela et al. 2021; Trivedi et al. 2021; Qiu and Zhu 2022;

Andriushchenko et al. 2022; Liang et al. 2023), under the name “programmatic reinforcement learning”. Programmatic policies can provide concise representations of policies which are easier to read, interpret, and verify. Furthermore, their short size compared to neural networks would mean that they can also generalize well to out-of-training situations while also smoothing out erratic behaviors. The goal of the line of work we initiate here is to lay new paradigms for programmatic reinforcement learning.

In this work we focus on developing a short-optimal policy using memories. We form, represent, and save each event and experience with a memory represented in a decision tree as a node. An interesting concept rise when knowing when exactly to form a memory, meaning when does the agent form a memory and how? We introduce the concept of the memory being “significant”. We also seek to study how can we enable agents to reason with programmatic policies by integrating two separate learning systems: Reward Prediction Error (RPE), and Action Prediction Error (APE), which we believe can represent the agent’s reasoning abilities. This is an in-progress work and we leave experimental results for the future.

2 Algorithm

We are now ready to state our main theorem. It asserts that our algorithm, is a programmatic policy algorithm, ignoring lower order terms. In other words, after this many episodes interacting with the MDP. **Algorithm overview.** Our algorithm proceeds with following high level steps:

1. Form a set of memories m which allow us to visit all “significant” states with reasonable probability.
2. collect a sufficient amount of data and store each significant memory in the decision tree.
3. compute the empirical transition matrix \hat{P} and the programmatic policy using the collected data.
4. for each reward function r , find a near-optimal policy by invoking a planning algorithm with transitions \hat{P} and reward r .

The first two steps are performed in the exploration phase, while the latter two steps are performed in the planning phase.

2.1 Exploration Phase

The goal of exploration is to visit all possible states so that the agent can gather sufficient information in order to find the optimal policy eventually. However, rather different from the bandit setting where agent can select an arbitrary arm to pull, it is possible that certain state in the MDP is very difficult to reach no matter what policy the agent is taking. Therefore, we first introduce the concept of the state being “significant”.

Definition 3.2. A memory m in step h is δ -significant if there exists a policy π , so that the probability to reach m following policy π is greater than δ . In symbol: $\max P_h^\pi(m) \geq \delta$. Intuitively, with limited budget of samples and runtime, one can only hope to store all significant memories. On the other hand, since insignificant memories can be rarely stored no matter what policy is used, they will not significantly change the value from the initial states. Thus, for the sake of finding near-optimal policies, it is sufficient to visit all significant states with proper significance level ϵ .

Theorem 2.1 There exists absolute constant $c > 0$ such that for any $\epsilon > 0$ and $p \in (0; 1)$, if we set $N_0 \geq cS^2AH^4l_0^3\delta$ where $l_0 := \log(SAH = (p\delta))$, then with probability at least $1p$, that Algorithm returns a dataset D consisting of N trajectories $Z_{n=1}^N$, which are i.i.d sampled from a distribution μ

83 satisfying:

$$\forall \delta - \text{significant}(s, h), \max \frac{P_h^\pi(s, a)}{\mu_h(s, a)} \leq 2SHA \quad (1)$$

84 Theorem 2.1 claims that using Algorithm 2, we can collect data from a underlying distribution
 85 μ , which ensures that for policy π , the ratio $P_h^\pi(s; a) = \mu_h(s; a)$ will be upper bounded for any
 86 significant state and action. That is, all significant state and action will be visited by distribution μ
 87 with reasonable amount of probability. Notice as δ becomes smaller, there will be more significant
 88 states and the condition (1) becomes stronger. As a result we need to take larger N_0 . As we will see
 89 later, the δ we take eventually will be $\epsilon = (2SH^2)$, where ϵ is the suboptimality of the policy we
 90 find in the planning phase.

91 Concretely, for each state s at step h , our algorithm first creates a reward function r that is always
 92 zero except for the state s at step h . Then we can simulate a standard MDP by properly feeding this
 93 designed reward r when an agent interacts with the environment. It is easy to verify that the optimal
 94 policy for the MDP with this reward r is precisely the policy that maximizes the probability to reach
 95 $(s; h)$.

96 2.2 Reasoning Learning Systems: RPE and APE

97 We rely on two separate learning systems: one for evaluating outcomes and another for reinforcing
 98 repeated actions. Known as reward prediction error (PRE) and action Prediction Error (APE), these
 99 systems help the agent to reason about his own actions and policy. While RPE helps the agent learn
 100 from outcomes. APE strengthens behaviors we repeat often, enabling more efficient multitasking by
 101 freeing up cognitive resources. In summary, Action Prediction Error (APE) allows habits to form by
 102 reinforcing frequent actions.

103 2.3 Constructing Programmatic Policies

104 What remains to be done is derive from the tree a programmatic policy; one could say ‘‘compress’’
 105 paths, or find regularity. This is the purpose of Algorithm 1. that provides the main idea for the
 106 construction of the policy synthesis procedure.

Algorithm 1 Synthesizing a programmatic policy

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1: Input A branch in the tree  $([a_1, b_1], \dots, [a_p, b_p])$  and the corresponding sequence of edges
    $(e_1, \dots, e_p)$  and regions  $(R_1, \dots, R_{(p1)})$  VisitedEdges =  $\phi$ 
2: for  $i=1$  to  $p-1$  do
3:   if  $e_i \notin \text{VisitedEdges}$  then
4:     add  $e_i$  to VisitedEdges
5:     start new Do Until block with local goal  $e_i$ 
6:   else
7:     add to the top of the instruction From  $e_i$  a new target  $[a_{i+1}, b_{i+1}]$  with preference  $a_{i+1}$ 
8:   end if
9: end for
10: End Procedure

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107 3 Conclusion

108 This work is a step to study and learn programmatic reinforcement learning, and more specifically a
 109 step to address the question of designing domain-specific languages for policies which are optimal
 110 and shorter, investigating the size of programmatic and how it changes and affects the performance.
 111 We introduced the concept of ‘‘memories’’ as weighted nodes in a decision tree. We also integrated
 112 two learning systems RPE and APE as a reasoning mechanism for the agent. We leave empirical
 113 experiments and results for future study.

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