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Conference Paper · October 2010

DOI: 10.1109/BTAS.2010.5634521 · Source: IEEE Xplore

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A Novel Similarity Measurement for Minutiae-based Fingerprint Verification

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***Abstract*—Similarity measurement construction is a key to fingerprint verification system. Highly accurate decision of genuine and impostor is directly inferred from a similarity score given by a sophisticated similarity measurement in the fingerprint matching process. This paper proposes a novel similarity measurement, which derives the likelihood ratio between two fingerprints with the consideration of the individual error rate instead of the average error rate of fingerprints. The proposed algorithm is also theoretically proved to be optimal for the minutiae-based fingerprint verification in terms of system error rates. Experimental results show that the proposed method has more efficient performance on separating genuine and impostor pairs.**

I. Introduction

Fingerprint verification is the most widely used method for personal identification because of the uniqueness and invariance properties of fingerprints [1]. The distinctive feature of fingerprints can be represented by the local ridge structure (minutiae), which refer to the ridge endings and bifurcations. Most classical algorithms verify a person's claimed identity by measuring the features such as the coordinates and directions of minutiae between two fingerprints [2], which consist of two stages: alignment and matching. The alignment stage employs a special point-pattern matching approach to achieve the best alignment between two feature sets. The matching stage compares the minutiae sets under the estimated transformation parameters and returns a similarity score using a constructed similarity measurement. If the similarity score is larger than an acceptance threshold, the two fingerprints are recognized as a genuine pair, otherwise the claimed identity is rejected.

Associating with the similarity threshold, there are two error rates: False Match Rate (FMR) and False Non-match Rate (FNMR). FMR denotes the probability that the score of an impostor pair is larger than the threshold. FNMR denotes the probability that the score of a genuine pair is less than the threshold. The overall FMR and FNMR for a set of fingers are the integration or average of the FMR and FNMR for all individual fingers in the data set. Conventional methods construct the similarity measurement with simple decisions [3] [4] or multi-decisions based on fusing the similarity scores of different features [5] [6], which use one unified threshold for all fingers to make the final decision. Their similarity thresholds are experimentally determined to assure that the average error rates are lower than a required level, while the individual error rates of some fingers are higher than this required level although the average error rates for all fingers are sufficient. The difficulty of constructing the similarity measurement is that the threshold which balances the tradeoff between the overall FMR and FNMR may not be optimal for each individual finger and thus not optimal for the overall FMR and FNMR of all fingers.

This paper proposes a novel similarity measurement based on flexible-length feature of minutiae by deriving the likelihood ratio between two fingerprints. The proposed likelihood ratio is calculated from two kinds of probabilities to handle the individual error rates: the Accidental Coincident Probability (ACP), which denotes the probability that two fingerprints originated from different fingers are accidentally matched; the Accidental Inconsistent Probability (AIP), which denotes the probability that two fingerprints belong to the same finger are accidentally not matched. To our knowledge, this is the first time that AIP is calculated.

The rest of this paper is organized as follows. In section II, fingerprint alignment derives the transformation parameters between two fingerprints. In section III, fingerprint matching presents the estimation of ACP under the assumption that two fingerprints have no correlation and AIP under the assumption that two fingerprints are highly correlated. In section IV, the likelihood ratio between two fingerprints is achieved as a similarity measurement from the calculated probabilities, and is proved theoretically to be optimal for the minutiae-based fingerprint verification. Section V conducts several experiments to evaluate the proposed method.

II. Fingerprint Alignment

Generally, two fingerprints should be aligned before matching. Our alignment estimates the transformation parameters such as translation, rotation and scaling between two minutiae sets with a Hough transform-based point pattern matching method [7], which discretizes the parameter space and accumulates evidence in the discretized space by deriving transformation parameters that relate two sets of points using a substructure of the feature matching technique. After accumulation, the generated peak position, corresponding to the maximum mated minutiae number, is regarded as the best alignment.

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III. FINGERPRINT MATCHING

The proposed minutiae-based likelihood ratio similarity measurement is presented here. While calculating the probabilities, we assume that a reasonable alignment is obtained in section II. To simplify the calculation, we consider the minutia position only in the following of this paper although several kinds of features can be extracted by feature extraction.

A. Definitions

As shown in Fig.1, if T and I represent the template and input fingerprints, given the estimated transformation parameters, we denote S as the overlapped area of T and I. In the overlapped area S, there are M minutiae from T, N minutiae from I. Square area S_0 is denoted as the tolerance area of minutiae spatial distance. A minutia from T is considered to be mated with a minutia from I if and only if their positional difference is equal or smaller than the tolerance threshold r_0, where $sd = \max(\max(|x_i - x_j''|, |y_i - y_j''|) \leqslant r_0$. There are K pairs of corresponding minutiae between fingerprint T and I. Let event X represent a state that formed by the above parameters. The derivation of ACP and AIP between T and I is as follows.

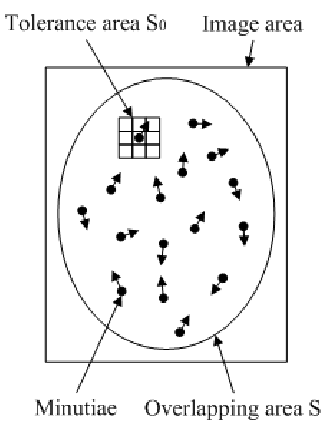

Fig. 1. The distribution of minutiae positions after reasonable alignment

B. Accidental Coincident Probability

The calculation of ACP is under the assumption that fingerprint T and I are originated from different fingers and have no correlation between each other. If the result probability is large enough, the hypothesis is accepted, the two fingerprints are impostor pair. From the viewpoint of fingerprint individuality, the ACP can be calculated from two aspects: mated minutiae and unmated minutiae [3] [8].

1) Mated Minutiae: The probability that a randomly distributed minutia from fingerprint T corresponds with one of the N minutiae from fingerprint I in the overlapped area S can be estimated by:

$$P_{MM}(1) = \frac{N \times S_0}{S} = \frac{N}{E} \qquad (E = \frac{S}{S_0}) \tag{1}$$

If there are $i-1$ minutiae from T arbitrarily located in S, and all of which are mated with minutiae from I, the rest overlapped area can be represented with $S-(i-1)S_0$. The unmated randomly distributed minutiae number of I in S is $N-(i-1)$. Therefore, the probability that the $i-th$ randomly distributed minutia from T in S corresponds to one of the $N-(i-1)$ minutiae from I can be denoted with:

$$P_{MM}(i) = \frac{N-(i-1)}{E-(i-1)} \qquad (i = 1 \dots K) \tag{2}$$

2) Un-mated Minutiae: Since the number of corresponding minutia pairs between T and I under the estimated transformation parameters is K, the rest ACP can be calculated as unmated minutiae. Assuming the minutiae from fingerprint I are sparse and the tolerance area of each minutia does not overlap with that of another minutia's, the probability that the $(K+1)-th$ randomly distributed minutia from T does not correspond to any minutia from I in the overlapped area S can be represented by:

$$P_{UM}(1) = \frac{E-N}{E-K} \tag{3}$$

The probability that the $(K+j)-th$ minutia from T is randomly distributed in the rest overlapped area $S-(K+j)S_0$ and does not correspond to any minutia from I in S can be calculated with:

$$P_{UM}(j) = \frac{E-(N+j)}{E-(K+j)} \qquad (j = 1 \dots M-K) \tag{4}$$

3) ACP Between T and I: Therefore, the ACP between fingerprint T and I under the assumption that T and I have no correlation can be given as:

$$P_{ACP}(X|I \neq T) = C_M^K \prod_{i=1}^{K} P_{MM}(i) \prod_{j=1}^{M-K} P_{UM}(j) \tag{5}$$

C. Accidental Inconsistent Probability

The calculation of AIP is under the assumption that fingerprint T and I come from the same finger and have high correlation between each other. If the result probability is large enough, the hypothesis is accepted, the two fingerprints are genuine pair. The AIP between T and I is also calculated from aspects: ground truth minutiae and spurious minutiae, respectively.

1) Condition Analysis: With the definitions in subsection III-A, we have the Given row in Table I, and condition $(1) K \leqslant \min(M, N)$ is naturally satisfied.

As shown in the Assumption row Table I, considering that the poor quality fingerprints and incorrect ridge structures detected during fingerprint acquisition and feature extraction may cause some ground truth minutiae to be missing or spurious minutiae to be detected, we assume the ground truth minutiae from fingerprint T and I in the overlapped area S are m and n, respectively. Conditions $(2) m \in [0, M]$ and $(3) n \in [0, N]$ are naturally satisfied. Thus, the spurious minutiae counts in fingerprint T and I are $M-m$ and $N-n$. For the ground truth minutiae between T and I, there

TABLE I
CONDITION ANALYSIS FOR AIP

	Minutiae Count	Fingerprint T	Fingerprint I	Conditions
Given	Total	M	N	$(1) K \leqslant \min(M, N)$
	Mated	K		
Assumption	Ground Truth	m	n	$(2) m \in [0, M]$ $(3) n \in [0, N]$ $(4) a = \min(m, n)$ $(5) b = \min(K, m, n)$ $(4) a = \min(m, n)$ $(5) b = \min(K, m, n)$ $(6) a \geqslant b$ $(7) h \in [0, b]$ $(8) g \in [0, a-h]$ $(9) g + h \leqslant a$ $(10) K - h \leqslant \min(M-m, N-n)$
	Mated Ground Truth	h		
	Un-mated Ground Truth	g		
	Spurious	M-m	N-m	
	Mated Spurious	K-h		
Overlapped	Identical Ground Truth	IGTM=m+n-(h+g)		$(11) IGTM \geqslant \max(m, n)$ $(12) ISM \geqslant \max(M-m, N-n)$
	Identical Spurious	ISM=(M-m)+(N-n)+(K-h)		
	Total	M+N-K-g		

should be some one to one correspondence between each other. But due to the existence of finger deformation, minutiae position change and minutiae missing, there are position gaps between the corresponding minutiae of two fingerprints even for genuine pairs. The position gaps of the missing ground truth minutiae are treated as ∞. We assume that the ground truth minutiae, which located inside the tolerance threshold r_0 are h and the ground truth minutiae, which located outside r_0 are g. If we denote conditions $(4) a = \min(m, n)$ and $(5) b = \min(K, m, n)$, then conditions $(6) a \geqslant b$, $(7) h \in [0, b]$, $(8) g \in [0, a-h]$ and $(9) g + h \leqslant a$ are naturally satisfied. For the spurious minutiae from T and I in S, there may happen that some spurious minutiae of T located inside the tolerance area of some of those in I. Since the number of corresponding minutia pair between T and I is K, the mated spurious minutiae can be represented by $K - h$. Condition $(10) K - h \leqslant \min(M-m, N-n)$ should be satisfied before the calculation of AIP.

Consider all the minutiae count in the overlapped area S, the identical ground truth minutiae can be calculated as $IGTM = m + n - (h + g)$, where condition $(11) IGTM \geqslant \max(m, n)$ is naturally satisfied; the identical spurious minutiae can be calculated as $ISM = (M-m) + (N-n) + (K-h)$, where condition $(12) ISM \geqslant \max(M-m, N-n)$ should be satisfied before the calculation of AIP. In practice, if condition (10) is satisfied, then condition (12) is also fulfilled. The total minutiae count in S is thus calculated with $M + N - K - g$, as shown in the Overlapped row Table I.

2) Ground Truth Minutiae: Since the Probability Distribution Function (PDF) of the positional differences in corresponding minutiae extracted from mated fingerprints is similar to a bivariate Gaussian distribution [3] [8]. We calculate the probability that a pair of mated minutiae occurs as:

$$P_{PD}(sd \leqslant r_0) = \int_0^{r_0} G(r)dr \tag{6}$$

where $G(r)$ is the PDF of position difference for mated minutiae. The probability that the position difference with respect to the corresponding minutiae exceeds the tolerance threshold r_0 can be represented with:

$$P_{PD}(sd > r_0) = 1 - \int_0^{r_0} G(r)dr \tag{7}$$

Therefore, the probability that h ground truth minutiae are located inside r_0 and g ground truth minutiae are located outside r_0 is calculated by:

$$P_{GTM} = C_{h+g}^{h} P_{PD}(sd \leqslant r_0)^h P_{PD}(sd > r_0)^g \tag{8}$$

3) Spurious Minutiae: For the spurious minutiae, since there is no one to one correspondence between each other, the AIP calculation can be accomplished by similar equations in subsection III-B.

For the mated spurious minutiae, equation (1) and (2) can be applied, where M is replaced by $M - m$, N is replaced by $N - n$ and S is replaced by $S + [h - (m+n)]S_0$. Therefore, the probability that the $i - th$ randomly distributed spurious minutia of $M - m$ from T in $S + [h - (m+n)]S_0$ corresponds to one of the $(N-n) - (i-1)$ spurious minutiae from I is denoted with:

$$P'_{MM}(i) = \frac{(N-n) - (i-1)}{E + [h - (m+n)] - (i-1)} \tag{9}$$
$$(i = 1 \ldots K - h)$$

For the un-mated spurious minutiae, equations (3) and (4) can be applied, where M is replaced by $M - m$, N is replaced by $N - n$, S is replaced by $S + [h - (m+n)]S_0$, and K is replaced by $K - h$. The probability that the $(K - h + j) - th$ spurious minutia of $M - m$ from T is randomly distributed in the rest overlapped area $S + [h - (m+n)]S_0 - (K - h + j)S_0$ and does not correspond to any spurious minutia of $N - n$ from I in $S + [h - (m+n)]S_0$ is derived by:

$$P'_{UM}(j) = \frac{E + [h - (m+n)] - [(N-n) + j]}{E + [h - (m+n)] - [(K-h) + j]} \tag{10}$$
$$(j = 1 \ldots [(M-m) - (K-h)])$$

Therefore, the probability that $K-h$ spurious minutiae are mated and $(M-m)-(K-h)$ spurious minutiae are un-mated between $M-m$ and $N-n$ spurious minutiae from T and I is calculated as:

$$P_{SM} = C_{M-m}^{k-h} \prod_{i=1}^{K-h} P'_{MM}(i) \prod_{j=1}^{(M-m)-(K-h)} P'_{UM}(j) \quad (11)$$

4) AIP Between T and I: The AIP between fingerprint T and I under the assumption that T and I are highly correlated is given by:

$$P_{AIP}(X|I=T) = \sum_{m=0}^{M} \sum_{n=0}^{N} \sum_{h=0}^{b} \sum_{g=0}^{a-h} p(m,n,h,g) \quad (12)$$

$$p = \begin{cases} C_{IGTM+ISM}^{IGTM}(C_{IGTM}^{h+g} P_{GTM}) \times (C_{ISM}^{K-h} P_{SM}) \\ \qquad\qquad\qquad \text{if condition (10) is true;} \\ 0 \\ \qquad\qquad\qquad \text{else.} \end{cases} \quad (13)$$

Where $IGTM$ and ISM is given in the Overlapped row Table I.

IV. MINUTIAE-BASED LIKELIHOOD RATIO SIMILARITY MEASUREMENT AND ITS OPTIMALITY

A. Likelihood Ratio Similarity Measurement

In detection theory, it has long since been known that the use of likelihood ratio is asymptotically optimal [9]. In detection, a given feature has to be classified as originating from a predefined situation or not. Since the detection problem is in some sense equivalent to the verification problem here, we expect the using of likelihood ratio is optimal as well.

With the calculated ACP and AIP, the minutiae-based likelihood ratio similarity measurement between fingerprint T and I is derived as:

$$L(I,T) = \frac{P_{AIP}(X|I=T)}{P_{ACP}(X|I \neq T)} \quad (14)$$

If the similarity score exceeds a threshold $TH \in (0, \infty)$, the claimed identity of fingerprint I is accepted as a genuine of T.

In the following of this section, we prove theoretically that the proposed similarity measurement is optimal for the minutiae-based fingerprint verification in terms of average error rates. Optimality here is defined as the lowest FMR for a given FNMR and vice versa.

B. Average Error Rates Expressions

For a specific finger F_i and a given threshold TH, the acceptance region $A(TH, F_i)$ and reject region $R(TH, F_i)$ can be defined in the similarity score space S as:

$$A(TH, F_i) = \{L(I,T) \in S | L(I,T) \geqslant TH\} \quad (15)$$

$$R(TH, F_i) = \{L(I,T) \in S | L(I,T) < TH\} \quad (16)$$

Using the proposed measurement as a function of the PDF of the fingerprints, the FMR measures the probability that an input I is accepted as a genuine of T, although they are from different fingers can be represented as:

$$FMR(TH, F_i) = P(L(I,T) \in A(TH) | I \neq T) \quad (17)$$
$$= \int_{A(TH,F_i)} P_{ACP} dL$$

The average FMR(TH) can be achieved by summing over all fingers:

$$FMR(TH) = \sum_{i=1}^{n} FMR(TH, F_i) \times P(F_i) \quad (18)$$

Where F denotes the whole finger set, n is the number of fingers in F and $P(F_i)$ is the probability density on the appearance of F_i when evaluating the error rates.

Similarly, for a specific finger F_i and a given threshold TH, the FNMR measures the probability that an input I is rejected to be a genuine of T, although they originate from the same finger can be expressed as:

$$FNMR(TH, F_i) = P(L(I,T) \in R(TH) | I = T) \quad (19)$$
$$= \int_{R(TH,F_i)} P_{AIP} dL$$

The average FNMR(TH) can be derived by:

$$FNMR(TH) = \sum_{i=1}^{n} FNMR(TH, F_i) \times P(F_i) \quad (20)$$

The dependence of both error rates on the threshold can be visualized in a plot of FMR against FNMR for varying threshold values, which is called the receiver operating characteristic (ROC). The expression that describes the tradeoff between FMR and FNMR for the likelihood ratio similarity measurement-based fingerprint verification system can be derived by:

$$\frac{dFNMR(TH, F_i)}{dFMR(TH, F_i)} = -TH \quad (21)$$

C. Optimality of the Proposed Method

To prove the optimality of the proposed method, we define $V_i(L|F - F_i)$ as the PDF of the similarity measurement for an input that is not taken from finger F_i and $V_i(L|F_i)$ as the PDF of the similarity measurement for an input I taken from finger F_i. The well-known relation between these PDFs can be described with [9]:

$$V_i(L|F_i) = L \times V_i(L|F - F_i) \quad (22)$$

As a function of the threshold TH, the error rates FMR and FNMR for finger F_i are given by:

$$FMR(TH, F_i) = \int_{TH}^{\infty} V_i(L|F - F_i) dL \quad (23)$$

$$FNMR(TH, F_i) = \int_{0}^{TH} V_i(L|F_i) dL \quad (24)$$

The expressions of the average $\overline{FMR}$ and $\overline{FNMR}$ with a finger dependent threshold $TH(F_i)$ can be represented as:

$$\overline{FMR} = \int_F P(F_i) \int_{TH(F_i)}^{\infty} V_i(L|F - F_i) dL dF_i \quad (25)$$

$$\overline{FNMR} = \int_F P(F_i) \int_0^{TH(F_i)} V_i(L|F_i) dL dF_i \quad (26)$$

To optimize the performance of the minutiae-based fingerprint verification, we need to choose a suitable threshold TH as a function of F_i which can result in the minimum ROC. This problem is solved by Lagrange optimization [10]. To minimize FMNR under the condition of a constant FMR, the threshold can be chosen as $TH = TH_{opt}(F_i) + \varepsilon f(F_i)$, where $TH_{opt}(F_i)$ is the optimal threshold, $f(F_i)$ is a function of F_i, ε is a small constant and a specific value for FMR is chosen as additional condition. Then the following equation can be achieved:

$$H = \int_F P(F_i) \int_0^{TH(F_i)+\varepsilon f(F_i)} V_i(L|F_i) dL dF_i$$

$$+a[\int_F P(F_i) \int_{TH(F_i)+\varepsilon f(F_i)}^{\infty} V_i(L|F-F_i) dL dF_i - FMR(TH)] \quad (27)$$

By setting the derivative with respect to ε to zero, H can be minimized as:

$$\int_F P(F_i) V_i(TH_{opt}(F_i) + \varepsilon f(F_i)|F_i) f(F_i) dF_i$$

$$-a \int_F P(F_i) V_i(TH_{opt}(F_i) + \varepsilon f(F_i)|F - F_i) f(F_i) dF_i = 0 \quad (28)$$

It can be noticed, that this expression must hold for any $f(F_i)$, thus the integrals over all F_i can be omitted. Furthermore, since TH_{opt} is optimal, ε is equal to zero, the expression can be simplified as:

$$V_i(TH_{opt}(F_i)|F_i) - a V_i(TH_{opt}(F_i)|F - F_i) = 0 \quad (29)$$

After applying equation (22), we have:

$$TH_{opt}(F_i) V_i(TH_{opt}(F_i)|F-F_i) - a V_i(TH_{opt}(F_i)|F-F_i) = 0 \quad (30)$$

By dividing both sides by $V_i(TH_{opt}(F_i)|F - F_i)$ and rearranging the expression, we achieve:

$$TH_{opt}(F_i) = a \quad (31)$$

Since a is a constant, the optimal threshold $TH_{opt}(F_i)$ is constant too, independent of F_i. Therefore, the optimal verification results can be achieved by using a constant likelihood ratio threshold when averaging over all fingers.

Similar result is derived by Bazen et. al [11] based on fixed length feature vectors.

V. EXPERIMENTAL RESULTS

This section conducts experiments on the standard database FVC2004 DB2 to demonstrate the improvement of the proposed method. There are 2 sets of fingerprints in FVC2004 DB2: A set contains 100 fingers for verification evaluation; B set contains 10 fingers for parameter estimation. Each finger in the 2 sets has 8 impressions. The images were captured with an optical sensor at 500 dpi, resulting in the image size of 328×364.

We use B set to estimate the spatial distance tolerance threshold r_0. All the minutiae in the genuine pairs are manually extracted and the alignment results are adapted by human inspection. The position difference between to ground truth minutiae is calculated by $\max(|x_i - x_j''|, |y_i - y_j''|)$ and the distribution of spatial distance in mated minutiae of genuine pairs is similar to a bivariate Gaussian distribution as shown in Fig. 2. We choose the smallest r_0 for $P(\max(|x_i - x_j''|, |y_i - y_j''|) \leqslant r_0) \leqslant 97.5\%$, which results $r_0 = 16$ pixels under 500 dpi.

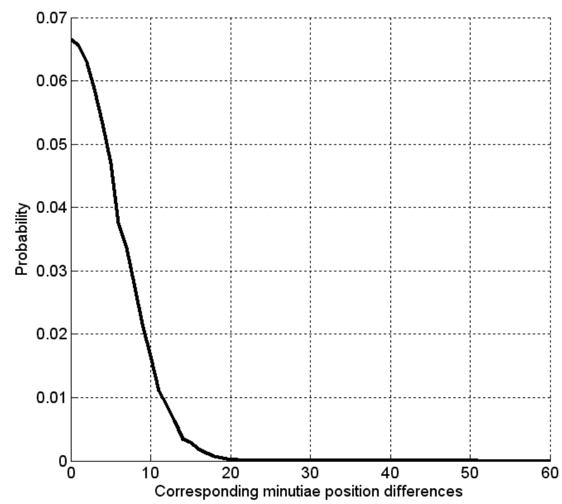

Fig. 2. The distribution of positional differences in corresponding minutiae

We compare the proposed approach with two existing methods [4] and [5]. The three methods are implemented into a same minutiae-based fingerprint verification system. During the fingerprint verification, they share the same feature extraction and minutiae alignment. We use A set to construct the evaluation, in which there are total number of 2800 genuine and 316,800 impostor matches. The performances of different methods are shown in a representation of the ROC curves, which are plotted as FMR against FNMR, as shown in Fig.3.

From the ROC curves, it can be observed that the proposed algorithm causes the most improvement. With a given FMR, the proposed approach can help the system to obtain the lowest FNMR. Statistically, compared with the other two systems, the proposed algorithm can reduce the system FNMR from 79.1% and 65.3% to 36.4% when FMR=0.01%.

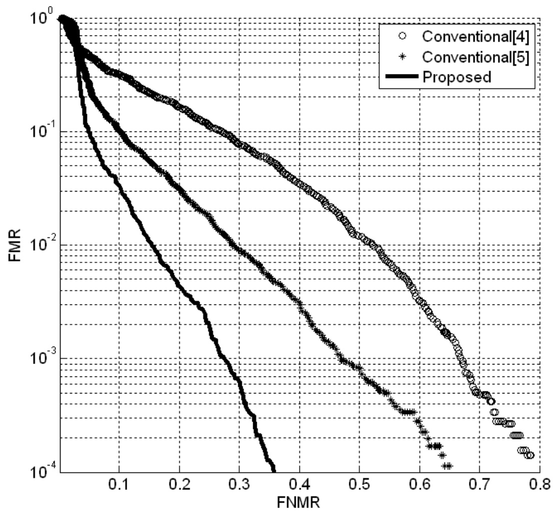

Fig. 3. System performance comparison on FVC2004 DB2

VI. CONCLUSIONS

This paper proposes a novel likelihood ratio similarity measurement for minutiae-based fingerprint verification system. Comparing with other methods, the proposed method can obtain the best performance for separating the genuine and impostor, which benefits from the utilization of ACP and AIP to construct the likelihood ratio. It is capable to improve the fingerprint verification system EER three times than the conventional methods.

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