

Контрольна робота 3
Студента групи НМІ-24
Копина Ілля

$$1) \quad \begin{matrix} \dot{x}_1 \\ \dot{x}_2 \end{matrix} = \begin{pmatrix} x_1 \\ 4x_1 + 5x_2 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 \\ 4 & 5-\lambda \end{vmatrix} = \lambda^2 - 6\lambda + 5 = (\lambda - 1)(\lambda - 5)$$

$$\lambda_1 = 1, \quad \lambda_2 = 5$$

$$\lambda_1: \begin{pmatrix} 0 & 0 \\ 4 & 4 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0$$

$$\alpha = -\beta$$

$$h_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\lambda_2: \begin{pmatrix} -4 & 0 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0$$

$$-4\alpha = 0$$

$$\alpha = 0$$

$$h_2 = \begin{pmatrix} 0 \\ \beta \end{pmatrix}$$

тип особливих точок — вузол (H1)

$$\lambda_1 = 1 \quad | \quad h_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = 5 \quad | \quad h_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x_1 = 0 \quad \downarrow$$

$$x_2 = -\frac{4}{5} x_1 \quad \Rightarrow$$

Курсові ізоміи:

~~тут~~

$$2) \quad \begin{cases} \dot{x} = 1 \\ \dot{y} = 2x \end{cases}$$

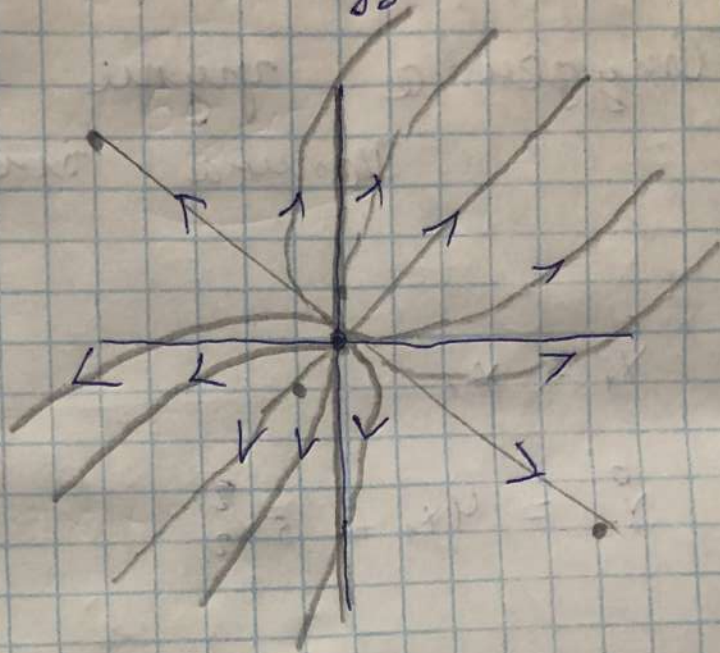
$$\frac{dx}{1} = \frac{dy}{2x}$$

$$2x \cdot dx = 1 \cdot dy$$

$$x^2 - y = C_1$$

$$d(x^2 - y) = 0$$

$$u(x, y) = x^2 - y$$



$$3) a) \begin{cases} \dot{x} = (x-1)^2 + y^2 - 4 \\ \dot{y} = (x+1)^2 + y^2 - 4 \end{cases}$$

$$\begin{cases} (x-1)^2 + y^2 - 4 = 0 \\ (x+1)^2 + y^2 - 4 = 0 \end{cases} \quad y^2 = 4 - (x+1)^2$$

$$(x-1)^2 + 4 - (x+1)^2 - 4 = 0$$

$$\cancel{x^2} - 2x + 1 - \cancel{x^2} + 2x - 1 = 0$$

$$-4x = 0$$

$$x = 0$$

$$y^2 = 3, \quad y_1 = \sqrt{3} \\ y_2 = -\sqrt{3}$$

Ocađruba mona:

$$(0; \sqrt{3})$$

$$(0; -\sqrt{3})$$

Dai

$$b) \begin{cases} \dot{x} = x - 3y - 2 \\ \dot{y} = 6y - 2x + 4 \end{cases}$$

$$\begin{cases} x - 3y - 2 = 0 \\ 6y - 2x + 4 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x - 3y - 2 = 0 \\ -2x + 6y + 4 = 0 \end{cases}$$

$$0 = 0$$

$$(2+3y, y)$$

~~forall~~

$$y \in \mathbb{R}$$

Безліч

$$b) \begin{cases} \dot{x} = x - y \\ \dot{y} = xy - 2x + 1 \end{cases}$$

$$\begin{cases} x - y = 0 \\ xy - 2x + 1 = 0 \end{cases}$$

$$\begin{cases} x = y \\ xy - 2x + 1 = 0 \end{cases}$$

$$y^2 - 2y + 1 = 0$$

$$(y - 1)^2 = 0$$

$$y = 1$$

$$x = 1$$

одна особлива
точка

$$(1; 1)$$

$$b) \begin{cases} x = (x-1)(y-2) \\ y = (x-2)(y-1) \end{cases}$$

$$\begin{cases} (x-1)(y-2) = 0 \\ (x-2)(y-1) = 0 \end{cases}$$

$$(1; 2)$$

$$(2; 1)$$

$$(x-1)(y-2) = f_1(x_1, x_2) = xy - x \cdot 2 - y + 2$$

$$(x-2)(y-1) = f_2(x_1, x_2) = xy - x - 2y + 2$$

$$\frac{\partial f_1}{\partial x_1} = (y-2)$$

$$\frac{\partial f_1}{\partial x_2} = (x-1)$$

$$\frac{\partial f_2}{\partial x_1} = (y-1)$$

$$\frac{\partial f_2}{\partial x_2} = (x-2)$$

$$A_1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

~~Handwritten scribbles~~

$$A_1: \begin{pmatrix} -1-\lambda & 0 \\ 0 & -1-\lambda \end{pmatrix} = (-1-\lambda)^2 = 0$$

$$\lambda_1 = \lambda_2 = -1$$

⇒ критическим значением
было

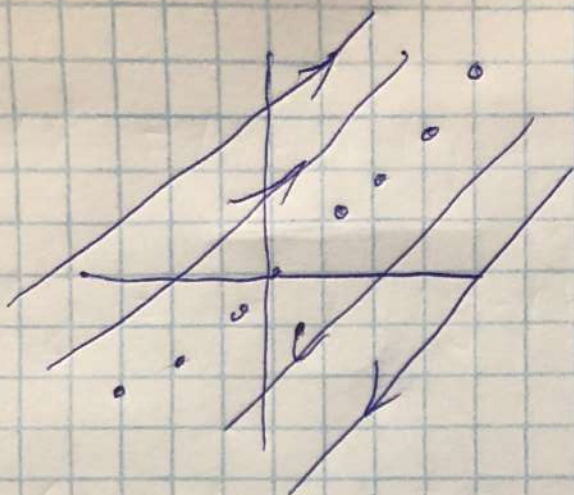
$$A_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A_2: \begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix} = \lambda^2 - 1 = 0$$

$$\lambda_1 = 1 \quad \lambda_2 = -1$$

⇒ критическим значением

5)



- C5

$$\begin{pmatrix} \lambda & \varepsilon \\ 0 & \lambda \end{pmatrix}$$

$$\lambda = 0$$

$$\varepsilon = 1$$

a) $\dot{x}_1 = 3x_2$
 $\dot{x}_2 = 0$

$$A = \begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}$$

$$\lambda^2 = \lambda^2$$

$\lambda_1 = 0$

all

~~$\dot{x}_1 = 3x_2$
 $\dot{x}_2 = 0$~~

$\lambda_1:$

$$\begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}$$

$$\varepsilon \neq 0 \neq 1$$

5)

$$\begin{cases} \dot{x}_1 = x_1 - 3x_2 \\ \dot{x}_2 = 2x_1 - 6x_2 \end{cases}$$

$$A = \begin{pmatrix} 1 & -3 \\ 2 & -6 \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda & -3 \\ 2 & -6-\lambda \end{pmatrix} = \lambda^2 + 5\lambda = \lambda(\lambda+5)$$

$$\lambda_1 = 0 \quad \lambda_2 = -5 \rightarrow \text{не регулярное}$$

$$b) \begin{cases} \dot{x}_1 = -x_1 + 3x_2 \\ \dot{x}_2 = -2x_1 + 6x_2 \end{cases}$$

$$A = \begin{pmatrix} -1 & 3 \\ -2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} -1-\lambda & 3 \\ -2 & 6-\lambda \end{pmatrix} = \lambda^2 - 5\lambda$$

не регулярное

$$\lambda_1 = 0 \quad \lambda_2 = 5 \rightarrow$$

$$2) \begin{cases} \dot{x}_1 = -2x_1 \\ \dot{x}_2 = -2x_2 \end{cases}$$

$$A = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

не регулярное

$$\begin{pmatrix} -2-\lambda & 0 \\ 0 & -2-\lambda \end{pmatrix} = \lambda^2 + 4\lambda + 4$$

$$\lambda_1 = -2$$

$$\begin{cases} \dot{x}_1 = -2x_1 \\ \dot{x}_2 = -2x_2 \end{cases}$$