DUALITY ANALYSIS

TOYOCO assembles three types of toys-trains, trucks and cars using three operations. The daily limits on the available times for the three operations are 430,460 and 420 minutes respectively and the revenues per unit of toy train, truck and cars are $3, $2 and $5 respectively. The assembly time at the three operations per train are (1, 3, 1) minutes, per truck are (2, 0, 4) minutes and per car are (1, 2, 0) minutes.

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 4x_2 \leq 420$$

The optimal table is given as

Basic	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z	4	0	0	1	2	0	1350
x_2	$\frac{-1}{4}$	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
s_3	2	0	0	-2	1	1	20

<u>Dual Prices:</u>

Operation 1 = $1/min

Operation 2 = $2/min

Operation 3 = $0/min

There are two methods to determine the optimal solution of dual problem:

Method 1.

$$\begin{pmatrix}\text{Optimal value of} \\ \textit{dual} \text{ variable } y_i\end{pmatrix} = \begin{pmatrix}\text{Optimal primal } z\text{-coefficient of } \textit{starting} \text{ variable } x_i \\ + \\ \textit{Original} \text{ objective coefficient of } x_i\end{pmatrix}$$

Method 2.

$$\begin{pmatrix}\text{Optimal values} \\ \text{of } \textit{dual} \text{ variables}\end{pmatrix} = \begin{pmatrix}\text{Row vector of} \\ \text{original objective coefficients} \\ \text{of optimal } \textit{primal} \text{ basic variables}\end{pmatrix} \times \begin{pmatrix}\text{Optimal } \textit{primal} \\ \text{inverse}\end{pmatrix}$$

Changes Affecting Feasibility

(a) Change in the right hand side

Question 1: Suppose that TOYOCA wants to expand its assembly lines by increasing the daily capacity of operation 1, operation 2 and operation 3 by 40% to 602, 644 and 588 minutes respectively. How would this change effect the total revenue?

Question 2: Another proposal was made to shift the slake capacity of operation 3 to operation 1. How would his change effect the optimum solution?

Basic	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z	4	0	0	1	2	0	
x_2	$\frac{-1}{4}$	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	0	
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	
s_3	2	0	0	-2	1	1	

(b) Addition of a new constraint

Question 1: Suppose that TOYOCA is changing the design of its toys and that the change will require the addition of a fourth operation in the assembly lines. The daily capacity of the new operation is 500 minutes and the times per unit for the three products on this operation are 3, 1 and 1 minutes respectively. Study the effect of this new optimum n the operation.
Question 2: Suppose instead that TOYOCO unit times on the fourth operation are changed to 3, 3 and 1 minutes respectively. All the reaming data of the model remain same. Will the optimum solute change?

Basic	x_1	x_2	x_3	s_1	s_2	s_3		Solution
z	4	0	0	1	2	0		
x_2	$\frac{-1}{4}$	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	0		
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0		
s_3	2	0	0	-2	1	1		

Changes Affecting optimality

(a) Change in the original objective coefficient

Question 1: In the TOYOCO model suppose the new pricing policy to meet the competition. The unit revenues under the new policy are $2, $3, and $4 for train truck and car toys respectively. How the optimal solution is effected?

Question 2: suppose the TOYOCO objective function is changed to **Maximize $z = 6x_1 + 3x_2 + 4x_3$**. Will the optimum solution changed?

Basic	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z							
x_2	$\frac{-1}{4}$	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
s_3	2	0	0	-2	1	1	20

(b) **<u>Addition of new economic activity to the model</u>**

Question 1: TOYOCO recognizes that the toy rain are not currently in production because they are not profitable. The company wants to replace the toy train with a new product a toy fire engine to be assemble on the existing facilities. TOYOCO estimates the revenue per toy fire engine to be $4 and the assembly times per unit to be 1 minute on operation 1, 1 minute on operation 2 and 2 minutes on operation 3. How would this change impact the solution?

Basic	x_1	x_2	x_3		s_1	s_2	s_3	Solution
z	4	0	0		1	2	0	1350
x_2	$\frac{-1}{4}$	1	0		$\frac{1}{2}$	$\frac{-1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1		0	$\frac{1}{2}$	0	230
s_3	2	0	0		-2	1	1	20