

# Joint taxation of income and wealth<sup>\*</sup>

Mehmet Ayaz<sup>†</sup>

Dominik Sachs<sup>‡</sup>

March 2024

## Abstract

We study the integration of income and wealth taxation, exploring efficiency gains and social implications. Utilizing a two-period model, we uncover the efficiency benefits of conditioning wealth taxation on income. Our theoretical examination of elementary joint tax reforms highlights tagging benefits, leveraging wealth as a tag for income taxation to improve efficiency. However, this approach also introduces distortion costs, impacting wealth accumulation decisions. Our numerical analysis based on U.S. data reveals that joint tax reforms may reduce the marginal excess burden compared to standalone income and wealth tax reforms, their effectiveness varies based on the income-wealth correlation. Importantly, joint tax reforms can enhance social welfare even beyond the capabilities of income and wealth tax reforms.

## JEL classification:

**Keywords:** Wealth taxation, Income taxation, Tax reforms, Tagging

---

<sup>\*</sup>We acknowledge valuable comments and suggestions from the participants of EEA Annual Congress 2021, 77th Annual Congress of the IIPF, VfS Annual Conference 2021, 8th Mannheim Taxation Conference, and ZEW Public Finance Conference 2022. Special thanks to Pierre Boyer, Antoine Ferey, Clemens Fuest, Yena Park, and Nicholas Werquin for their insightful feedback.

<sup>†</sup>**Ayaz:** LMU Munich (ayaz@econ.lmu.de).

<sup>‡</sup>**Sachs:** University of St. Gallen (dominik.sachs@unisg.ch).

# 1 Introduction

Taxes are core elements of fiscal policy, providing the necessary revenue to fund public goods and provide essential services. They play a crucial role in resource allocation, economic behavior, income and wealth distribution, and overall social welfare. At their core, taxes are a tool for governments to achieve various economic, social, and fiscal objectives.

Traditionally, taxation has focused primarily on income, with progressive income tax systems designed to redistribute wealth from higher-income individuals to lower-income individuals through a system of tax brackets and deductions. However, wealth taxation has received increasing attention in recent years as a means of addressing the concentration of wealth among the richest members of society, largely sparked by the influential work of [Piketty \(2014\)](#).

We aim to uncover how and why combining wealth taxation with income taxation can be beneficial. In particular, we investigate the efficiency gains and social implications associated with integrating income and wealth taxation systems. By examining theoretical frameworks and empirical evidence, we provide insights into the potential advantages and challenges of joint tax reforms. Additionally, our analysis considers the distributional effects of such reforms and their implications for social welfare.

Firstly, we examine a simplified two-period model where labor income is uncertain. This model strikes a balance between analytical clarity and substantive implications, providing a foundational framework to explore the impact of conditioning wealth taxation on labor income. In scenarios of uncertain labor income, the veil of ignorance argument suggests a governmental incentive to act as an insurer, allocating greater resources to individuals with lower labor income ([Farhi and Werning, 2013](#)). We demonstrate that implementing state-dependent saving taxes based on income levels leads to varying levels of distortion in saving behavior. This prompts governments to adjust saving taxes beyond the typical insurance motive. Our findings indicate that lowering state-dependent saving taxes for individuals with lower incomes and raising them for those with higher incomes enhance social welfare through more efficient taxation.

Secondly, we study the fiscal and social impacts of joint tax reforms. Building upon the elementary tax reforms proposed by [Golosov et al. \(2014\)](#), we investigate the implications of conditioning the marginal tax rate of income on wealth, and vice versa. Such conditioning offers tagging benefits if income and wealth are correlated ([Cremer et al., 2010](#)); however, an additional distortion arises due to the endogeneity of the tag provided by joint taxation.

Consider, for instance, an income tax increase that is conditioned on having a certain

level of net worth. As the income distribution conditional on higher net wealth has a fatter right tail, given a positive correlation between income and wealth, using high net worth as a tag for income taxation can increase its efficiency. However, this approach also creates an incentive for individuals to decrease their net worth, thereby introducing additional distortion costs.

We derive formulas for the marginal excess burden and welfare effects of joint tax reforms, expressing them in terms of the marginal excess burdens of separate income and wealth tax reforms. We then decompose these formulas into a tagging component, capturing the fiscal gains from better targeting, and a distortion component, capturing the secondary distortion of such reforms.

As an important benchmark, we consider the scenario where income and wealth are uncorrelated, demonstrating that while there is no benefit from tagging, the secondary distortion still exists. In this case, we show that the marginal excess burden of joint reforms is the sum of the excess burdens of separate income and wealth tax reforms.

Subsequently, we apply our theoretical framework to quantify the fiscal and social impacts of integrating income and wealth taxation within the U.S. personal tax schedule. Our analysis reveals that joint tax reforms may result in a lower marginal excess burden compared to standalone income and wealth tax reforms, depending on the local correlation between income and wealth. We identify regions where joint tax reforms outperform separate income and wealth tax reforms in terms of their fiscal impact. Notably, we find that the tagging benefit of joint tax reforms is most pronounced in the middle of income and wealth distributions. Conversely, distortion costs are most pronounced for tax reforms targeting the lower end of income and wealth distributions, distorting labor supply or wealth accumulation decisions.

Lastly, we address how joint tax reforms affect social welfare without adopting a normative viewpoint on society's preference for redistribution. Instead, we assume that the separate income and tax schedules we observe are individually optimal. Utilizing the inverse-optimum approach introduced by [Bourguignon and Spadaro \(2012\)](#), we estimate the marginal social welfare weights along the income and wealth distributions separately. Combining these welfare weights, we demonstrate that joint tax reforms enhance welfare, even though separate tax reforms on income and wealth are not inherently able to do so by construction.

**Related literature.** The optimal income taxation problem has been extensively explored in the literature, with [Mirrlees \(1971\)](#) pioneering foundational work in this area. His work has since been refined and expanded upon, as evidenced by subsequent studies such as [Diamond \(1998\)](#) and [Saez \(2001\)](#), which have further clarified and improved

upon Mirrlees’ framework. Moreover, the analysis has been extended to encompass various dimensions of adjustments, including the choice of whether to work at all at the extensive margin (Kleven and Kreiner, 2006; Jacquet et al., 2013; Hansen, 2021), general equilibrium effects (Stiglitz, 1982; Rothschild and Scheuer, 2013; Sachs et al., 2020), couples taxation (Kleven et al., 2009), and considerations of fiscal pressure, defined as the need to generate additional revenue (Heathcote and Tsujiyama, 2021; Ayaz et al., 2023). Regarding capital income taxation, despite initial theoretical arguments against it in the long run, (Judd, 1985; Chamley, 1986), subsequent studies have presented compelling reasons for positive capital taxation (Conesa et al., 2009; Piketty and Saez, 2013; Saez and Stantcheva, 2018; Straub and Werning, 2020; Ferey et al., 2023).

Wealth taxation has gained traction as a means to counter rising wealth inequality, particularly influenced by Piketty (2014). His work highlights historical wealth trends and proposes progressive wealth taxation to address inequality. Saez and Zucman (2019) further advocate for progressive wealth taxation, suggesting policies to tax various forms of wealth more effectively. Recent literature has explored the middle and long-run effects of wealth taxation (Seim, 2017; Jakobsen et al., 2020; Brühlhart et al., 2022; Zoutman, 2018; Advani and Tarrant, 2021). For a comprehensive review of global wealth taxation policies and a survey of arguments for and against wealth taxation, see Scheuer and Slemrod (2021). Additionally, in the realm of politics, Senators Elizabeth Warren and Bernie Sanders, two prominent contenders in the 2020 U.S. presidential election, included wealth taxation as a key component of their campaign platforms.

We aim to bridge the gap between the strands of literature on income and wealth taxation by investigating the consequences of their integrated implementation. While Golosov et al. (2003) and Albanesi and Sleet (2006) approach this issue through a mechanism-design lens, Golosov et al. (2014) introduce a variational approach that underpins our analysis of joint tax reforms. While other studies have investigated the optimal taxation of multiple variables (Jacquet and Lehmann, 2021; Spiritus et al., 2022), we focus on the intuitive understanding of the underlying implications of joint taxation of income and wealth.

Moreover, we add to the discourse on the concept of tagging. Akerlof (1978) first highlighted the advantages of incorporating correlated observables into optimal income taxation, a notion expanded upon by subsequent studies (Cremer et al., 2003; Boadway and Pestieau, 2006). While tagging variables such as height (Mankiw and Weinzierl, 2010), gender (Alesina et al., 2011), and age (Bastani et al., 2013) have been explored, our contribution lies in examining wealth as an endogenous tag for income taxation, offering a novel perspective to this strand of literature.

The remainder of this chapter is structured as follows. Section 2 focuses on the foundational two-period model and highlights the efficiency benefits of joint taxation. Then, in Section 3, the theoretical framework for the analysis of joint taxation reforms is provided. Following that, Section 4 combines the theoretical results with U.S. data. Finally, Section 5 concludes the chapter.

## 2 State-Dependent Saving Taxation

In this section, we present a simple two-period model to demonstrate the effects of implementing joint taxation. The model is intentionally kept simple to streamline the analysis, but it can be expanded to incorporate additional factors, such as endogenous labor supply, time preference, or interest.

### 2.1 Individual Optimization

All individuals receive the same endowment in the first period. They consume a portion of their endowment in the first period and save the remainder for the second period. The labor productivity in the second period is stochastic, with individuals experiencing either lower or higher productivity levels. After learning their labor productivities, individuals provide an exogenous unit of labor to earn labor income. Their savings from the first period are taxed in the second period depending on the productivity shock individuals receive. If individuals have low labor productivity, their savings are taxed at a rate of  $\tau_l$ . If they have the higher labor productivity, then the tax rate on savings is  $\tau_h$ . The expected lifetime utility maximization problem of individuals can be formulated as

$$\begin{aligned} \max_a \quad & U = u(c_1) + \mathbb{E}[u(c_{2i})] \\ \text{s. t.} \quad & c_1 = I - a \\ & c_{2i} = \begin{cases} a(1 - \tau_l) + \theta_l, & \text{if } i = l \\ a(1 - \tau_h) + \theta_h, & \text{if } i = h \end{cases} \end{aligned} \tag{1}$$

where  $u(c_1)$  and  $u(c_{2i})$  are increasing and concave functions that denote the utility derived from consumption in the first and second periods, respectively. The consumption in the second period depends on the productivity shock individuals receive for two reasons. Firstly, individuals with varying levels of productivity earn different amounts of labor income. Secondly, they face different tax rates based on their labor productivity. Here,  $I$  represents the heterogeneous endowments individuals receive in the first

period, while  $a$  represents the chosen level of savings. Parameters  $\theta_l$  and  $\theta_h$  capture labor productivity in the event of a low-productivity shock and a high-productivity shock, respectively.

In this simplified context, the only decision individuals make is the consumption-saving choice in the first period. They determine how much to save for the second period considering the uncertainty in their future productivity. The optimality conditions for this problem are expressed as follows.

$$\phi(a) = -u'(c_1) + \sum_{i=l,h} p_i u'(c_{2i})(1 - \tau_i) = 0 \quad (2)$$

$$\psi(a) = u''(c_1) + \sum_{i=l,h} p_i u''(c_{2i})(1 - \tau_i)^2 < 0 \quad (3)$$

where,  $p_l$  denotes the probability of receiving the low-productivity shock, and  $p_h = 1 - p_l$  represents the probability of the high-productivity shock. The function  $\phi(a)$  represents the first-order condition, while  $\psi(a)$  captures the second-order condition. The first-order condition highlights the trade-offs associated with saving an additional unit for the second period. Choosing to save one additional unit results in a decrease in utility from first-period consumption but an increase in expected utility in the second period. Concerning the second-order condition, the concavity of the consumption utility function ( $u''(c) < 0$ ) ensures its satisfaction.

To measure how individuals adjust their choices in response to changes in taxation, we compute the elasticity of saving concerning both the low-state net-of-tax rate ( $1 - \tau_l$ ) and the high-state net-of-tax rate ( $1 - \tau_h$ ). This is achieved by applying the implicit function theorem to the first-order condition of individuals. The elasticity of savings with respect to either of the net-of-tax rates is given by

$$\varepsilon_{a,1-\tau_i} = \frac{p_i(1 - \tau_i)u'(c_{2i})}{-\psi(a)a} \left( 1 + \frac{u''(c_{2i})}{u'(c_{2i})} a(1 - \tau_i) \right), \quad i = l, h \quad (4)$$

Adjusting tax rates has two opposing effects on individual saving levels, each represented by an additive term in brackets. The first effect, stems from changes in the relative prices of consumption between the two periods. If either of the state-dependent net-of-tax rates increases, first-period consumption becomes relatively more expensive compared to second-period consumption. Consequently, individuals allocate more consumption to the second period by increasing their savings. This effect is referred to as the substitution effect. The second effect, captured by  $\frac{u''(c_{2i})}{u'(c_{2i})} a(1 - \tau_i)$ , arises due to changes in lifetime resources. An increase in net-of-tax rates expands the available resources in the second period. To smooth this increase over their lifetime, individuals

reduce their savings and consume some of these extra resources in the first period already. This effect is known as the income effect.

For sufficiently high levels of elasticity of intertemporal substitution, the substitution effect outweighs the income effect. In the case where individuals exhibit constant elasticity of intertemporal substitution, the condition becomes  $\sigma > 1 - \frac{\theta_i}{c_{2i}}$ , where  $\sigma$  is the elasticity of intertemporal substitution of individuals. This indicates that for the logarithmic utility case ( $\sigma = 1$ ), which is widely assumed in the literature, the substitution effect prevails over the income effect, leading to a negative distortion of savings in response to an increase in state-dependent saving taxation. If the elasticity of intertemporal substitution is higher than the threshold, an increase in saving taxes discourages saving.

The ratio of the saving elasticity with respect to the high-state net-of-tax rate to that with respect to the low-state net-of-tax rate is a crucial measure that captures how individuals respond to state-dependent taxation. This ratio, pivotal for later analysis, is given by

$$\frac{\varepsilon_{a,1-\tau_h}}{\varepsilon_{a,1-\tau_l}} = \frac{p_h(1-\tau_h)}{p_l(1-\tau_l)} \cdot \frac{u'_{2h}}{u'_{2l}} \cdot \frac{1 + \frac{u''(c_{2h})}{u'(c_{2h})}a(1-\tau_h)}{1 + \frac{u''(c_{2l})}{u'(c_{2l})}a(1-\tau_l)} \quad (5)$$

The intertemporal elasticity of substitution also affects the relationship between the two elasticities. Suppose that the probability of receiving each shock is equal to each other, as well as the saving tax rate applied after receiving each shock. If labor income in the second period constitutes a significant portion of individuals' consumption in that period, the saving elasticity with respect to the low-state net-of-tax rate is higher than that with respect to the high-state net-of-tax rate. In the case of constant elasticity of intertemporal substitution, this condition translates to

$$\sigma > \frac{a(1-\tau_i)}{\theta_i} \quad (6)$$

where  $\sigma$  denotes the elasticity of intertemporal substitution.

## 2.2 Optimal State-Dependent Tax Rates

Similar to the analysis of individual behavior, the welfare analysis is also streamlined to highlight the insights of state-dependent taxation. The government's objective is to maximize social welfare while simultaneously meeting an exogenously determined revenue requirement in the second period. This is achieved through the imposition of state-dependent saving taxation. That is, individuals' savings are taxed based on their labor productivity in the second period. The government's welfare maximization

problem reads as

$$\begin{aligned} \max_{\tau_l, \tau_h} \quad & u(c_1) + \sum_{i=h,l} p_i u(c_{2i}) \\ \text{s. t.} \quad & G \leq (p_l \tau_l + p_h \tau_h) a \end{aligned} \quad (7)$$

where  $G$  is the exogenous revenue requirement. The law of large numbers ensures that the ratio of individuals experiencing low or high-productivity shocks in the second period equals the probability of receiving each shock.

**Proposition 1.** *The optimal tax rates that solve the optimization problem the government are given by*

$$\frac{\tau_l}{1 - \tau_l} = \frac{1}{\varepsilon_{a,1-\tau_l}} \cdot \left( 1 - \frac{u'(c_{2l})}{\lambda} \right) \cdot \frac{p_l}{p_l + p_h \frac{\tau_h}{\tau_l}} \quad (8)$$

$$\frac{\tau_h}{1 - \tau_h} = \frac{1}{\varepsilon_{a,1-\tau_h}} \cdot \left( 1 - \frac{u'(c_{2h})}{\lambda} \right) \cdot \frac{p_h}{p_l \frac{\tau_l}{\tau_h} + p_h} \quad (9)$$

where  $\lambda$  captures the marginal value of public funds.

*Proof.* See Appendix A. □

The optimal state-dependent saving tax rates are determined by three key factors.<sup>1</sup> The first determinant of the optimal tax rates is the saving elasticity. Greater responsiveness to saving taxation in one state leads to a larger decrease in tax revenue due to behavioral changes in response to increased state-dependent taxes. Consequently, optimal tax rates decrease as saving elasticity increases. Another determinant is the impact of tax rates on expected utility in terms of public funds. An increase in a state-dependent saving tax rate decreases expected utility, thereby limiting the optimal tax rate. Lastly, the balance between these effects determines optimal tax rates. The numerator of the third term captures the proportion of individuals directly affected by the respective state-dependent tax rate, while the denominator reflects the change in the effective tax base resulting from a unit change in savings.

The impact on the optimal tax rates due to changes in expected utility can be conceptualized through an insurance motive of the government.<sup>2</sup> Since individuals are identical a priori, the government is motivated to act as an insurer by regulating their consumption levels after experiencing different productivity shocks. If the marginal utility from consumption decreases with the level of consumption, this creates an

---

<sup>1</sup>The optimal tax formulas provided in Equations (8) and (9) bear resemblance to the optimal income taxation formulas outlined by [Diamond \(1998\)](#). Indeed, the three effects determining the optimal tax rates in this study are analogous to those in his paper.

<sup>2</sup>For a more detailed analysis, see [Farhi and Werning \(2013\)](#).



incentive to increase the tax rate for individuals experiencing high-productivity shocks and decrease the tax rate for those experiencing low-productivity shocks.

For further analysis, it is possible to eliminate the impact of the insurance motive on optimal tax rates. Assuming that the marginal value of public funds is significantly greater than any of the individual marginal utilities results in a solution where the government's primary concern is tax revenue alone. Consequently, the insurance motive is not effective on optimal tax rates anymore.

The other effect stemming from the variability in saving elasticity is particularly intriguing. Each tax rate introduces a distortion in individual savings; however, the magnitude of this distortion differs between the tax rates applied in the high-productivity shock scenario and the low-productivity shock scenario. If individuals exhibit differing responses to changes in the tax rate in different scenarios, the government must take that into account.

This variability in saving elasticity highlights a crucial distinction between two interpretations of marginal tax rates. Firstly, marginal tax rates on savings determine the extent to which government tax revenue changes when individuals adjust their savings. This interpretation is independent of individual utilities but depends on the relative prevalence of each state in the economy. Secondly, marginal tax rates distort individual decision-making and influence the optimal level of saving for individuals. This interpretation, however, relies on the well-being of individuals in different states, particularly the marginal effects on their well-being in each scenario. These two interpretations diverge in scenarios where marginal utilities are not constant and there is uncertainty in how consumption evolves over time, as is the case within the framework of this section.

Two optimal tax rate formulas outlined in Equations (8) and (9) can be combined to provide further insight into how the two tax rates should be set optimally in relation to each other.

$$\frac{1 - \frac{u'_{2h}}{\lambda}}{1 - \frac{u'_{2l}}{\lambda}} = \frac{\varepsilon_{a,1-\tau_h} p_l(1 - \tau_l)}{\varepsilon_{a,1-\tau_l} p_h(1 - \tau_h)} \quad (10)$$

**Corollary 1.** *Suppose that the condition outlined in Equation (6) holds. Then, not utilizing the state-dependency of saving taxation, despite its availability, leads to a sub-optimal solution.*

*Proof.* Setting  $\tau_l = \tau_h = \tau$ , the right-hand side of Equation (10) is smaller than one. However, the diminishing marginal utility implies that the left-hand side is larger than one. Therefore,  $\tau_l = \tau_h$  cannot be optimal.  $\square$

Corollary 1 provides an insight into the advantages of leveraging state-dependency of saving taxation. Not utilizing state-dependent saving taxation still yields sub-optimal

outcomes. This result can be attributed to the insurance and efficiency channels on the optimal tax rates discussed earlier. Setting the two tax rates equal to each other implies that the government does not provide any insurance for the productivity shock. Moreover, given that one of the state-dependent saving taxes results in a larger distortion, the government must consider this factor when setting the tax rates optimally.

**Corollary 2.** *Suppose that the condition outlined in Equation (6) holds and the government is not utilizing the state-dependency of saving taxation. Then, increasing the tax rate applied in the high-productivity shock scenario and decreasing the one applied in the low-productivity shock scenario such that the immediate effects on tax revenue cancel each other out, lead to an improvement. This result holds even if the government's sole concern is tax revenue.*

*Proof.* See Appendix B □

Corollary 2 builds upon the previous result, highlighting that the efficiency gains arising from heterogeneous saving elasticities provide a valuable channel to increase tax revenues. This stems from the divergence between the additional revenue effect of state-dependent saving tax rates and their distortion on individual decision-making. Despite both tax rates in different states generating the same amount of additional tax revenue,<sup>3</sup> the tax rate applied in the low-shock scenario induces greater distortion. Consequently, marginally replacing the more distortive tax rate with the less distortive one enhances taxation efficiency. Moreover, this finding holds true regardless of whether the government aims to provide insurance for productivity shocks.

### 3 Theoretical Framework: Joint Tax Reforms

In this section, our analysis shifts to another perspective, where we integrate income and wealth taxation. We examine a static model incorporating a joint distribution of income and wealth, employing a reduced-form approach to study taxation. We focus on the joint tax functions of income and wealth, characterized by bivariate tax payment functions. Last but not least, we rely on elasticities of income and wealth with respect to net-of-tax rates, which serve as sufficient statistics for measuring the impacts of tax reforms.

#### 3.1 Bivariate Tax Payment Functions

The bivariate tax payment functions we study are mathematical functions that determine the total tax liability owed by individuals based on two variables: income and

---

<sup>3</sup>This statement overlooks the impact of the prevalence of different shocks, which influences the relative distortion in the same proportion.

wealth, denoted by  $T(y, a)$  where  $y$  represents taxable income and  $a$  represents net wealth. These functions take both income and wealth as inputs and yield the corresponding tax liability.

Marginal tax rates of a bivariate tax payment function are calculated as the partial derivative of the function with respect to either of its inputs. The marginal income tax rate measures how the tax liability changes when taxable income increases by one unit. Similarly, the marginal wealth tax rate captures the change in tax liability resulting from a unit change in net wealth.

$$\begin{aligned}\tau_Y(y, a) &\equiv \frac{\partial T(y, a)}{\partial y} \\ \tau_A(y, a) &\equiv \frac{\partial T(y, a)}{\partial a}\end{aligned}$$

For instance, a bivariate tax payment function may represent a tax system where income and wealth are taxed separately. In this scenario, we can express the bivariate tax payment as the sum of two univariate tax payment functions: one for income taxation and one for wealth taxation. In this case, the bivariate tax payment function is given by

$$T(y, a) = T_Y(y) + T_A(a)$$

where  $T_Y(y)$  and  $T_A(a)$  represent the univariate tax payment functions for income and wealth, respectively. With such bivariate tax payment functions, the marginal income and wealth tax rates are solely determined by the respective univariate tax functions.

$$\begin{aligned}\tau_Y(y) &= \frac{\partial T_Y(y)}{\partial y} \\ \tau_A(a) &= \frac{\partial T_A(a)}{\partial a}\end{aligned}$$

Furthermore, if the bivariate tax function is simply the summation of two univariate tax functions, the cross-marginal rates are zero. This implies that the marginal tax rate of one argument (e.g., income) does not depend on the other argument (e.g., wealth). Mathematically, this condition is expressed as

$$\frac{\partial^2 T(y, a)}{\partial y \partial a} = 0$$

Alternatively, a bivariate tax function could represent a system where income and wealth taxation are interdependent. In such a scenario, the bivariate tax payment function cannot be expressed as the simple summation of univariate tax functions. Crucially,

in this context, the cross-marginal tax rates are not zero everywhere, indicating that the marginal tax rate of one argument (e.g., income) may depend on the other argument (e.g., wealth), and vice versa.

### 3.2 Elementary Tax Reforms

The utilization of elementary tax perturbations is a widely adopted approach to analyze the effects of univariate tax reforms. Saez (2001) popularized this method by employing it to study optimal income taxation relying on income elasticity, which serves as a sufficient statistic for measuring the effects of income tax reforms. In this context, an income tax perturbation refers to an infinitesimal increase in the marginal income tax rate within a small interval along the income distribution. Perturbation analysis yields insightful results as any differentiable tax function can be reconstructed through a linear combination of these reforms at various points on the income distribution, along with a lump-sum adjustment that modifies the tax liability at any given point on the income distribution.

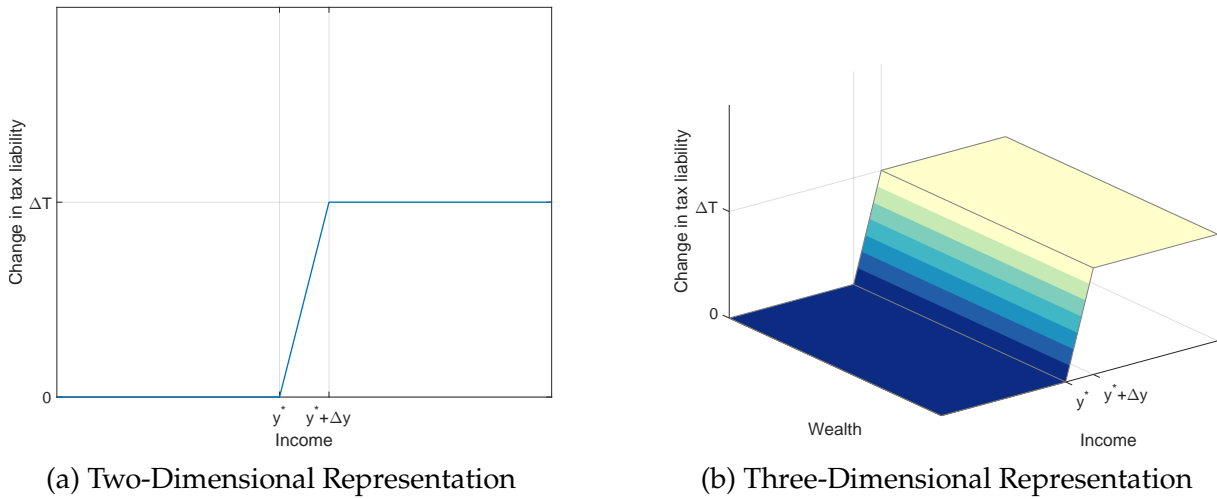


Figure 1: Elementary Income Tax Reform

*Notes:* The left panel illustrates the change in tax liability along the income distribution after an elementary income tax reform. It is represented in a 2D graph, where the x-axis represents taxable income and the y-axis represents the change in tax liability. The right panel presents the same information in a 3D format. Here, the x-axis represents taxable income, the y-axis represents net wealth, and the z-axis denotes the change in tax liability.

Figure 1 depicts the impact of an elementary income tax reform on tax liability. The income distribution is divided into three distinct regions, each experiencing varying effects from the reform.

In the first region, comprising incomes below the threshold at which the reform is implemented, individuals are unaffected by the reform and thus do not change their

behavior. This is based on the assumption that they are rational optimizers.

The second region encompasses a small interval immediately above the threshold income level. Here, individuals face higher marginal tax rates post-reform, leading to a decrease in labor supply. This response is captured by the income elasticity. The decrease in individuals' labor supply results in a reduction in tax revenue, referred to as efficiency costs (denoted by  $\Delta R^E$ ). Importantly, assuming the change in the marginal tax rate is infinitesimal, their utility remains unchanged due to the envelope theorem.

The third and final region includes incomes exceeding the threshold level. Individuals in this category experience an increase in tax liability, resulting in a reduction in utility. The higher tax liabilities of these individuals increase tax revenue (denoted by  $\Delta R^M$ ). However, without income effects, individuals' labor supply remains unchanged from pre-reform levels.

The marginal excess burden of elementary income tax reforms, defined as the loss in tax revenue due to individuals' response to taxation per unit of extra tax revenue, can be calculated. It represents the ratio of the loss in tax revenue due to individuals in the second region decreasing their labor supply to the increase in tax liability of individuals in the third region. Mathematically, it can be expressed as

$$MEB_Y(y^*) = -\frac{\Delta R^E(y^*)}{\Delta R^M(y^*)} = \varepsilon_{y,1-T'_y}(y^*) \frac{\tau_Y(y^*)}{1 - \tau_Y(y^*)} \frac{y^* f_Y(y^*)}{1 - F_Y(y^*)} \quad (11)$$

Here,  $y^*$  represents the threshold at which the tax reform is applied.  $\varepsilon_{y,1-T'_y}$  captures the elasticity of taxable income with respect to the net-of-income tax rate.  $F_Y(y)$  is the cumulative distribution function of the income distribution, and  $f_Y(y)$  is the density function.

Attaching a normative average welfare weight to individuals whose tax liability increases post-reform allows for assessing the welfare effects of elementary income tax reforms. Mathematically, the total welfare effect per unit of tax revenue becomes

$$\Delta W_Y(y^*) = \frac{(1 - \bar{g}_Y(y^*))\Delta R^M(y^*) + \Delta R^E(y^*)}{\Delta R^M(y^*)} \quad (12)$$

where  $\bar{g}_Y(y^*)$  represents the average welfare weight of individuals whose tax liability increases in terms of public funds.<sup>4</sup>

A similar elementary tax reform, focusing on wealth taxation, can be defined in parallel to the one applied to income taxation.<sup>5</sup> Notably, [Saez and Stantcheva \(2018\)](#)

<sup>4</sup>Essentially, the weights reflect how the welfare of the population segment above a specific income is valued relative to the average welfare of the entire population.

<sup>5</sup>These reforms yield analogous results to income tax reforms regarding their marginal excess burden and welfare effect.

utilize such reforms to investigate optimal capital income taxation.

Unlike univariate tax functions, linear combinations of elementary tax reforms for income and wealth, along with a lump-sum adjustment, are insufficient for reproducing any given bivariate tax payment function. Specifically, these combinations fail to generate cross-marginal tax rates.<sup>6</sup> Another elementary reform that leverages the bivariate nature of the tax function is necessary for the complete reconstruction of any given function.

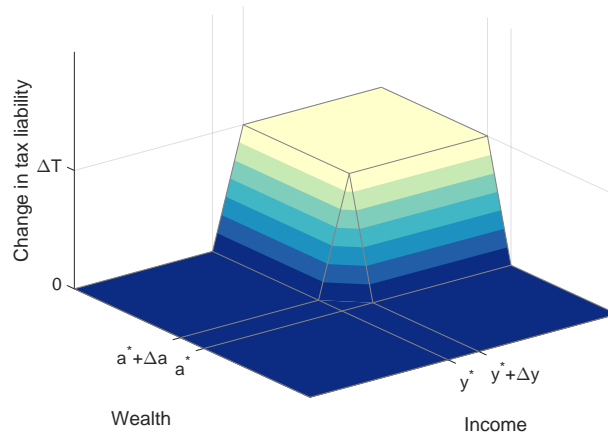


Figure 2: Elementary Joint Tax Reform

*Notes:* The figure illustrates the change in tax liability after an elementary joint tax reform. The x-axis represents taxable income, the y-axis represents net wealth, and the z-axis denotes the change in tax liability.

Figure 2 depicts the impact of an elementary joint tax reform on tax liability. Golosov et al. (2014) utilize similar reforms to analyze multivariate tax functions. This reform segments the joint distribution of income and wealth into four distinct regions.

The first region encompasses points along the joint distribution where either income or wealth is smaller than their respective threshold point at which the joint reform occurs. Similar to univariate tax reforms, individuals in this region neither face a different marginal tax rate nor experience an increase in their tax liability post-reform.

The second region comprises points exactly at the income threshold but larger than the wealth threshold. Here, individuals face an increase in the marginal income tax rate, leading them to decrease their labor supply. The resulting efficiency cost in tax revenue due to income distortion is denoted by  $\Delta R_Y^E$ .

The third region mirrors the second, including points exactly at the wealth threshold but larger than the income threshold. Individuals in this region face higher wealth taxes after the reform and decrease their wealth accumulation. This distortion causes another

---

<sup>6</sup>Formally, the impact of univariate elementary reforms is limited to altering the diagonal variables of the Hessian matrix of the tax payment function.

efficiency cost, denoted by  $\Delta R_A^E$ .

The impact on individuals located precisely at both the income and wealth thresholds is of second order and therefore negligible. This analysis is carried out considering the limit  $(\Delta y, \Delta a) \rightarrow (0, 0)$ , implying that the region where both thresholds intersect diminishes at a much faster rate compared to other regions.

We opt to abstract from cross-elasticities, assuming that a change in the marginal income tax rate does not affect wealth accumulation, and vice versa. We have two primary reasons for this approach. Firstly, we aim to maintain simplicity in our expressions while conveying the fundamental insights of joint taxation of income and wealth. Secondly, accurately measuring cross-elasticities presents a challenge, and the empirical literature lacks consensus on their magnitude. Hence, by disregarding cross-elasticities, we focus on the core dynamics of income and wealth taxation interactions without introducing unnecessary complexity.

Lastly, the fourth region consists of points that exceed both the income and wealth thresholds. While individuals in this region will pay more taxes after the reform, their behavior in terms of labor supply and wealth accumulation remains unchanged from pre-reform levels. The additional tax revenue collected from these individuals is denoted by  $\Delta R^M$ .

This family of joint reforms can also be referred to as double-progressive tax reforms. This is because, following these reforms, income taxation becomes more progressive when wealth is high, and conversely, wealth taxation becomes more progressive when income is high.

The marginal excess burden of a joint reform can be defined similarly to that of univariate reforms. It entails weighing the additional revenue generated from individuals in the fourth region, whose tax liability increases post-reform, against the loss in tax revenue resulting from distortions on labor supply in the second region and on wealth accumulation in the third region.

$$MEB_{\text{joint}}(y^*, a^*) = -\frac{\Delta R_Y^E(y^*, a^*) + \Delta R_A^E(y^*, a^*)}{\Delta R^M(y^*, a^*)} \quad (13)$$

**Proposition 2.** *Suppose the initial bivariate tax payment function is separable, and the elasticities of income and wealth remain constant. In this case, the marginal excess burden of an elementary joint reform can be expressed as a weighted sum of the marginal excess burdens of the univariate reforms that occur at the same levels of income and wealth. Specifically,*

$$MEB_{\text{joint}}(y^*, a^*) = w_Y(y^*, a^*)MEB_Y(y^*) + w_A(y^*, a^*)MEB_A(a^*) \quad (14)$$

where  $w_Y(y, a)$  and  $w_A(y, a)$  denote the weights determined by the joint distribution of income and wealth, respectively. They are defined as follows.

$$w_Y(y^*, a^*) = \frac{f_Y(y^*|a > a^*)}{1 - F_Y(y^*|a > a^*)} \frac{1 - F_Y(y^*)}{f_Y(y^*)} \quad (15)$$

$$w_A(y^*, a^*) = \frac{f_A(a^*|y > y^*)}{1 - F_A(a^*|y > y^*)} \frac{1 - F_A(a^*)}{f_A(a^*)} \quad (16)$$

where  $F_Y(y|a > a^*)$  is the conditional cumulative income distribution function given wealth exceeds the threshold  $a^*$ . Similarly,  $F_A(a|y > y^*)$  is the conditional cumulative wealth distribution function given income exceeds the threshold  $y^*$ .

*Proof.* See Appendix C. □

Proposition 2 addresses a scenario wherein the bivariate tax function comprises two separate univariate tax functions. In this context, the marginal excess burden of joint reforms can be computed using the marginal excess burdens of univariate reforms, along with weights that solely depend on the joint distribution of income and wealth. Importantly, these weights remain independent of the initial tax schedule.

The hazard rate, which measures the density at a specific value relative to the probability of the random variable exceeding that value, emerges as a crucial parameter for assessing the impact of tax reforms (Diamond, 1998). This statistic is pivotal because it identifies the number of individuals who face higher marginal tax rates and those whose tax liability increases.

The weights that scale the effects of univariate marginal excess burdens precisely quantify how the hazard rates of unconditional and conditional distributions interrelate. For instance,  $w_Y(y^*, a^*)$  quantifies the hazard rate of the conditional income distribution relative to the unconditional case. This comparison helps assess the efficiency costs of joint reforms in relation to the additional tax revenue they generate.<sup>7</sup>

Another approach to conceptualizing elementary joint reforms involves starting with an elementary univariate tax reform and extending it with a threshold. For example, consider an elementary wealth tax reform. Setting an income limit above which this reform applies creates the elementary joint reform. To maintain the continuity of the tax payment function, the marginal income tax rate must increase within a small interval around the income limit. Limiting the wealth tax reform based on income gives rise to two additional effects beyond the effects of the univariate wealth tax reform.

---

<sup>7</sup>An alternative method of defining the weights in Proposition 2 involves using conditional probabilities. The weight for the marginal excess burden of income tax reforms can be expressed equivalently as  $w_Y(y^*, a^*) = \frac{\Pr(a > a^*|y=y^*)}{\Pr(a > a^*|y > y^*)}$ . This expression reflects the scaling of additional tax revenue relative to the scaling of efficiency costs.



Firstly, the income limit restricts the number of individuals whose tax liability increases and those who face higher marginal wealth tax rates. Individuals who exceed the wealth threshold but remain below the income limit do not experience a change in their liability. Similarly, individuals who are at the original wealth threshold but remain below the income limit do not face higher marginal tax rates either. Reducing the number of people who pay more taxes post-reform contributes to a higher marginal excess burden. However, reducing distortion by not increasing the marginal wealth tax rate acts as a counterforce towards a lower marginal excess burden. If wealth and income are positively correlated, income acts as a tag, and the positive effect via lower distortion overcomes the negative effect via lower tax revenue (Akerlof, 1978). Therefore, the overall effect becomes positive. We refer to this channel as the "*tagging benefit*."

Secondly, as income is an endogenous variable, using it as a tag and creating higher marginal income tax rates for a portion of the population comes at a cost. Individuals who face higher marginal income tax rates as a result of the joint reform respond to the reform by reducing their labor supply. This creates another source of distortion for joint tax reforms, which increases the marginal excess burden. We refer to this channel as the "*distortion cost*."

The identity provided in Equation (14) can be rewritten to reflect this alternative conceptualization of joint reforms explained above.

$$MEB_{\text{joint}}(y^*, a^*) = MEB_A(a^*) \underbrace{-(1 - w_A(y^*, a^*))}_{\text{Tagging benefit}} MEB_A(a^*) + \underbrace{w_Y(y^*, a^*)}_{\text{Distortion cost}} MEB_Y(y^*) \quad (17)$$

If wealth and income are positively correlated, the hazard rate of the unconditional wealth distribution is larger than that of the conditional wealth distribution. This implies that  $w_A(y^*, a^*)$  is less than one. Consequently, the tagging benefit is negative, decreasing the excess burden. On the other hand,  $w_Y(y^*, a^*)$  is larger than zero unless wealth and income are perfectly correlated. Therefore, the distortion cost is positive, increasing the excess burden.

If wealth and income are positively correlated, the hazard rate of the unconditional wealth distribution is greater than that of the conditional wealth distribution. Consequently,  $w_A(y^*, a^*)$  is less than one. This implies tagging benefit term is negative, which decreases the excess burden. On the other hand,  $w_Y(y^*, a^*)$  is greater than zero unless wealth and income are perfectly correlated. Therefore, the distortion cost is positive, resulting in an increased excess burden.

This conceptualization does not have to start with a univariate wealth tax reform. In fact, a mirroring argument can be made stating with a univariate income tax reform,

extended with a wealth limit.

In the scenario of perfect correlation between wealth and income, one of the weights equals zero while the other equals one. The determination of which weight equals one depends on the location of the joint reform. If  $w_A(y^*, a^*) = 1$  and  $w_Y(y^*, a^*) = 0$ , the tagging benefit vanishes, and there is no distortion cost. Conversely, if  $w_A(y^*, a^*) = 0$  and  $w_Y(y^*, a^*) = 1$ , the tagging benefit perfectly offsets the initial distortion, and the distortion cost is equivalent to the marginal excess burden of the income reform.

It is worthwhile to consider another case where the distributions of income and wealth are independent, meaning that the joint distribution of income and wealth equals the product of the marginal income and wealth distributions. Mathematically, this is expressed as  $f(y, a) = f_Y(y) \cdot f_A(a)$ . In this scenario, the conditional and unconditional distributions are identical. Consequently, both weights provided in Equations (15) and (16) are equal to one. The marginal excess burden of joint reforms is then given by the unweighted sum of marginal excess burdens of univariate reforms.

$$MEB_{\text{joint}}(y^*, a^*) = MEB_Y(y^*) + MEB_A(a^*) \quad (18)$$

Intuitively, this result suggests that in the absence of a correlation between wealth and income, there is no tagging benefit. The distortion cost is equivalent to the marginal excess burden of the univariate income tax reform. Consequently, the excess burden of joint reforms always exceeds that of either separable reform alone.<sup>8</sup>

This finding is consistent with the conclusions reached by [Albanesi and Sleet \(2006\)](#), who suggest that cross-marginal tax rates should be negative in situations where income and wealth are uncorrelated. In the context of this study, elementary joint reforms lead to an increase in cross-marginal tax rates. In scenarios characterized by uncorrelated distributions, these reforms are less favorable compared to univariate tax reforms in terms of revenue effects.

Similar to univariate tax reforms, the total welfare effect of joint tax reforms can be computed with the inclusion of normative welfare weights. The equation is formulated as

$$\Delta W_{\text{joint}}(y^*, a^*) = \frac{(1 - \bar{g}(y^*, a^*))\Delta R^M(y^*, a^*) + \Delta R_Y^E(y^*, a^*) + \Delta R_A^E(y^*, a^*)}{\Delta R^M(y^*, a^*)} \quad (19)$$

where  $\bar{g}(y^*, a^*)$  represents the average welfare weight of individuals who exceed both thresholds at which the joint tax reform is implemented.

---

<sup>8</sup>Unless one of the marginal burdens of univariate tax reforms is negative, which would indicate that a Pareto improvement is possible by decreasing the univariate tax rates.

## 4 Numerical Analysis

In the previous section, we derived formulas to assess the impact of integrating jointness into the tax system, which initially consists of two separate univariate tax functions. We demonstrated that the marginal excess burden of elementary joint reforms depends on both the marginal excess burden of univariate tax reforms and the joint distribution of income and wealth. Furthermore, we showed that joint reforms are less favorable than univariate reforms when the distributions of income and wealth are independent.

In this section, we apply our theoretical findings to real-world data and evaluate the effects of elementary joint reforms. To accomplish this, we utilize the *Survey of Consumer Finances* (SCF) provided by the Federal Reserve. This triennial cross-sectional household survey offers a representative sample of the entire U.S. population. The SCF dataset is particularly valuable for our analysis because it includes household income and net wealth, allowing for the estimation of the joint distribution of income and wealth, which is essential for our study.

### 4.1 Joint Distribution of Income and Wealth

The SCF provides comprehensive data on household income and net wealth breakdowns. In our analysis, we define income as the total of wages, salaries, business income, and transfers. Notably, we exclude capital gains we use long-term capital gains taxation to construct our baseline wealth taxation system, as will be elaborated shortly. Wealth is defined as the sum of financial and non-financial assets, excluding retirement savings, minus total household debt. This measure represents a household's net worth.

	Income	Wealth
Mean	\$107,700	\$831,400
Median	\$62,100	\$161,700
Gini coefficient	0.550	0.821
Top-10% share	44.0%	73.9%
p90/p50 ratio	3.23	8.26

Table 1: Descriptive Statistics of Income and Wealth Distributions

*Notes:* The table summarizes the key statistics of the income and wealth distributions using the 2019 wave of the Survey of Consumer Finances. The top panel provides the mean and median variables of each distribution, while the bottom panel presents three measures of inequality.

Table 1 shows that wealth exhibits significantly greater inequality compared to income. The Gini coefficient of the wealth distribution exceeds that of income by 27.1 percentage points. Similarly, households within the top 10% of the wealth distribution

possess nearly 74% of all wealth owned by U.S. households, whereas those within the top 10% of the income distribution garner less than half of the total income.

		Wealth groups		
		Bottom 50%	Mid 40%	Top 10%
Income groups	Bottom 50%	35.6	14.0	0.4
	Mid 40%	14.1	22.3	3.6
	Top 10%	0.3	3.7	6.0

Table 2: Joint Distribution of Income and Wealth

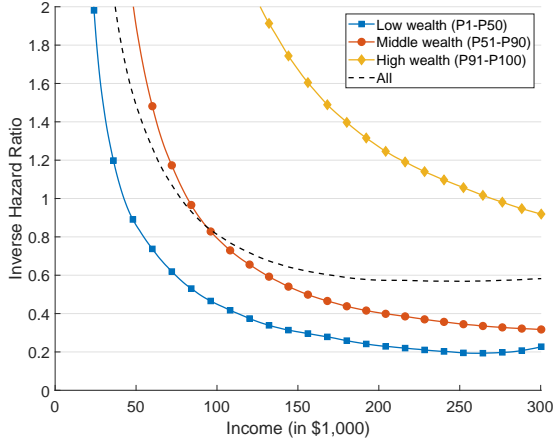
*Notes:* The table summarizes the joint distribution of income and wealth from the 2019 wave of the Survey of Consumer Finances. The income distribution is divided into three bins. The bottom 50% represents households up to the 50th percentile, the mid 40% represents households between the 50th and 90th percentile. The top 10% represents households above the 90th percentile of the income distribution. The same division is applied to the wealth distribution as well. The numbers in each cell represent the percentage of households in the respective joint bin.

Table 2 provides insight into the joint distribution of income and wealth, highlighting a strong correlation between the two variables in U.S. households. For instance, conditional on being within the top 10 percent of the income distribution, there is a 60% likelihood that a household also falls within the top 10 percent of the wealth distribution, while only a 4% chance exists that they are in the bottom 50%. Similarly, over 70% of households within the bottom 50% of the income distribution are also within the bottom 50% of the wealth distribution, with a mere 0.8% presence within the top 10% of the wealth distribution. The total correlation between household income and wealth, as measured by Kendall’s  $\tau$  coefficient,<sup>9</sup> stands notably high at 0.617.

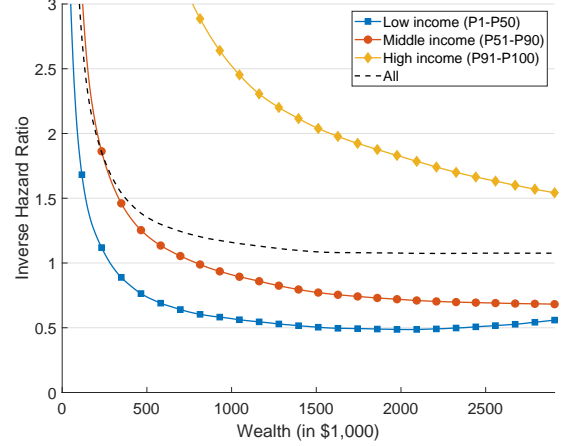
To estimate the joint distribution of income and wealth, we utilize the kernel density estimation method. This non-parametric technique is employed to estimate the probability density functions of one or more random variables. Its non-parametric nature allows us to refrain from making assumptions about any underlying distributions. Additionally, given that the correlation between income and wealth is pivotal for the analysis, not assuming any specific functional form of correlation is a more robust approach.

Cremer et al. (2010) demonstrate the significant role played by the scaled inverse hazard rate in shaping optimal marginal tax rates. Specifically, they highlight the positive gains from tagging when hazard rates differ across subgroups. Figure 3 presents these metrics for both unconditional and conditional income and wealth distributions. Conditioning the income distribution on being within the top 10% of the wealth dis-

<sup>9</sup>It is a statistic used to measure the ordinal association between two measured quantities. Mathematically, it is given by  $\tau = \frac{2}{n(n-1)} \sum_{i < j} \text{sgn}(y_i - y_j) \text{sgn}(a_i - a_j)$  where  $\text{sgn}$  represents the signum function.



(a) Across Income Distribution



(b) Across Wealth Distribution

Figure 3: Conditional Inverse Hazard Rates

*Notes:* The left panel illustrates the scaled inverse hazard rate of the income distribution, denoted by  $\frac{1-F_Y(y)}{yf_Y(y)}$ , for both the unconditional income distribution and income distributions conditional on three distinct wealth groups. The right panel illustrates the scaled inverse hazard rate of the wealth distribution, expressed as  $\frac{1-F_A(a)}{af_A(a)}$ , for both the unconditional wealth distribution and wealth distributions conditional on three different income groups.

tribution results in substantially higher inverse hazard rates across all income levels compared to the unconditional scenario. These elevated inverse hazard rates suggest a need for higher marginal tax rates for the high-wealth group in the absence of distortion costs. Conversely, focusing solely on the lower wealth group reduces the inverse hazard rates across all income levels, thereby lowering optimal marginal tax rates for this demographic. A similar pattern emerges when examining the wealth distribution: the higher the income level within a subpopulation, the greater the inverse hazard rates across all wealth levels.<sup>10</sup>

## 4.2 Baseline Tax System

Determining the baseline tax system over which a tax reform is applied is crucial in assessing the fiscal effects of the reform. This is because the efficiency costs resulting from changes in household behavior heavily depend on the baseline tax system. When there is a decrease in labor supply or wealth accumulation, the resulting decrease in tax revenue is directly influenced by the marginal tax rates.

To measure the impact of the joint reforms we study, we assume that the U.S. household taxation system consists of two separable univariate tax functions: one

<sup>10</sup>The non-crossing nature of hazard rates across different subgroups is also significant. In the absence of additional distortion costs, the subgroup with lower inverse hazard rates benefits from tagging, whereas the group with higher rates experiences losses (Cremers et al., 2010).

depending on income, excluding capital gains, and the other depending on wealth.

Currently, there is no wealth taxation in the U.S. Therefore, we substitute capital gains taxation with wealth taxation, assuming a constant yearly return on capital. This substitution relies on the assumption that the returns to capital are homogeneous. In such a scenario, the substitution is given by

$$\tau_A = \frac{r\tau_K}{1+r}$$

Combining the 20% long-term capital gains tax rate with an average annual return of 6.3% (Jordà et al., 2019) is equivalent to an annual wealth tax rate of 1.19%. This serves as the baseline wealth taxation system in the analysis.

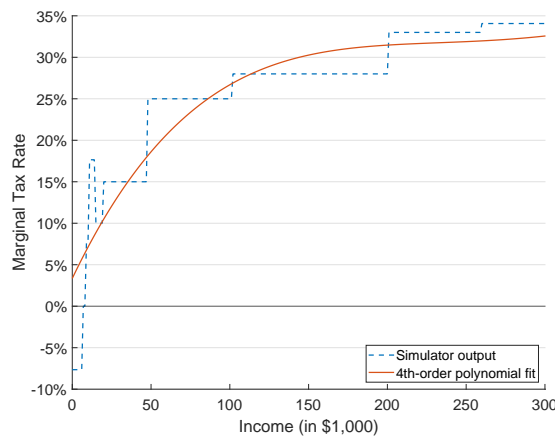


Figure 4: Estimated Marginal Income Tax Rates

*Notes:* The figure illustrates the estimation process of the baseline marginal income tax rates. The blue dashed line represents the output from the NBER’s tax liability simulator, while the red line depicts the 4th-order polynomial fit.

To estimate the income taxation system, we rely on TAXSIM, the National Bureau of Economic Research’s (NBER) tax liability simulator (Feenberg and Coutts, 1993).<sup>11</sup> Through the simulator, the federal marginal income tax rate is computed at various points along the income distribution. A 4th-order polynomial fit is then employed to derive the baseline income taxation for the analysis. Figure 4 illustrates the simulator output as well as the result of the polynomial fit we utilize in the analysis.

### 4.3 Marginal Excess Burden

Once we’ve estimated the joint distribution of income and wealth and established the baseline taxation system, we can apply our formulas from Equations (14), (15), and

<sup>11</sup>The version of the simulator we use, TAXSIM v32, is accessible at <http://www.nber.org/~taxsim/taxsim.html>.

(16) to compute the marginal excess burden of any univariate elementary income or wealth tax reform, as well as any elementary joint tax reform. We derive the income and wealth elasticities from existing literature. Specifically, we assume the elasticity of taxable wealth to be 0.50, in line with findings from [Chetty et al. \(2011\)](#). Furthermore, we adopt the baseline 4-year elasticity of taxable wealth reported by [Brülhart et al. \(2022\)](#), which amounts to 34.

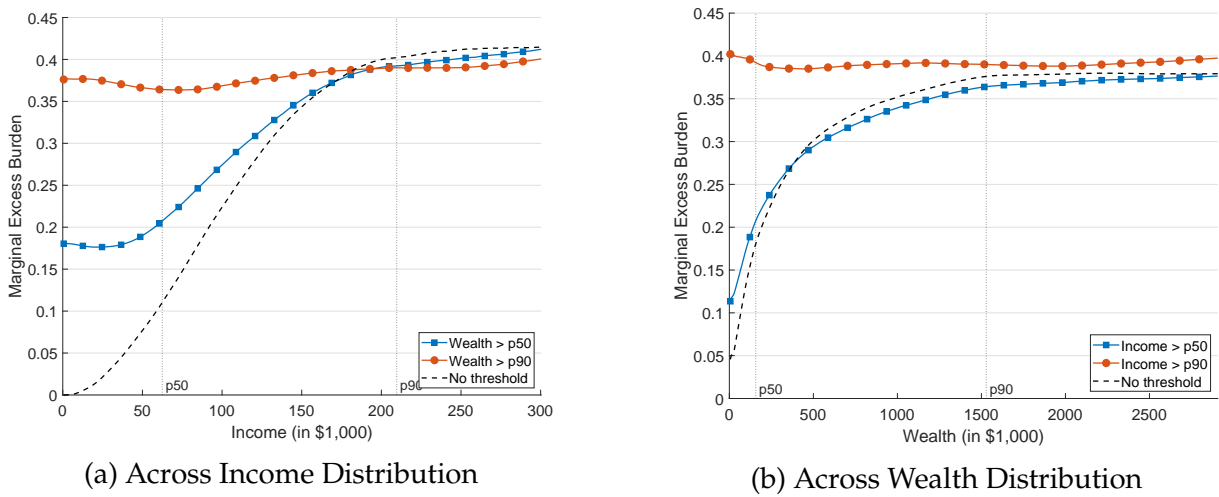


Figure 5: Marginal Excess Burden of Elementary Joint Tax Reforms

*Notes:* The left panel illustrates the marginal excess burden of tax reforms carried out at different income levels. The black dashed line represents univariate income tax reforms. Blue and red lines represent joint reforms with wealth thresholds set at the 50th and 90th percentiles. The right panel illustrates the marginal excess burden of tax reforms carried out at different wealth levels. A black dashed line represents univariate wealth tax reforms. Blue and red lines represent joint reforms with income thresholds set at the 50th and 90th percentiles.

Figure 5 summarizes our results in terms of the fiscal effects of joint and univariate tax reforms. It should be noted that the same joint tax reform may appear in both Figure 5a and 5b. For instance, consider a joint tax reform implemented at the 50th percentile of the income distribution and the 90th percentile of the wealth distribution. In the left panel, this reform is represented by the red line (corresponding to the 90th percentile of the wealth distribution). Observing this line's value at the 50th percentile of the income distribution reveals that the marginal excess burden of this reform is 36 cents per dollar of tax revenue. Similarly, in the right panel, reading the value of the blue line (corresponding to the 50th percentile of the income distribution) at the 90th percentile of the wealth distribution yields the same result of 36 cents.

We find that, depending on the location of the reform, joint reforms may have smaller or larger efficiency costs than univariate tax reforms. For instance, a univariate income tax reform at the 50th percentile of the income distribution incurs an efficiency cost of 11 cents per dollar of tax revenue. Appending this reform with a wealth threshold at



the 50th percentile of the wealth distribution increases the cost to 21 cents per dollar of tax revenue. This increase is due to the larger distortion cost around the wealth threshold outweighing the tagging benefit.

Conversely, setting the same wealth threshold at the 50th percentile reduces the efficiency costs from 40 cents to 38 cents per dollar of tax revenue if the income threshold is placed at the 90th percentile of the income distribution. For the top 10% of the income distribution, an elementary joint reform incurs lower efficiency costs than a univariate income tax reform because the positive tagging benefit can counteract the distortion cost.

The right panel of Figure 5 offers a similar story. Limiting a wealth tax reform to the upper half of the income distribution increases the efficiency cost from 18 to 21 cents per dollar if the wealth tax reform is implemented at the 50th percentile of the wealth distribution. However, the same limitation decreases efficiency costs from 37 to 36 cents if the wealth tax reform is implemented at the 90th percentile.

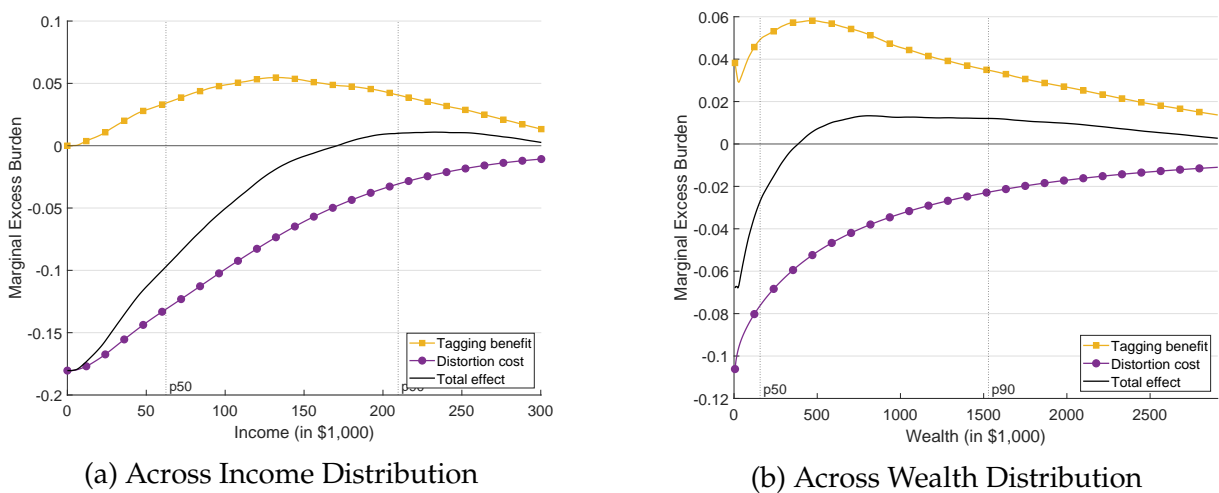


Figure 6: Decomposition of Tagging Benefit and Distortion Cost

*Notes:* The left panel illustrates the decomposition of the impact of extending univariate income tax reforms with a wealth threshold set at the 50th percentile of the wealth distribution. Conversely, the right panel illustrates the decomposition of the impact of extending univariate wealth tax reforms with an income threshold at the 50th percentile of the income distribution. In both panels, the yellow and purple lines represent the tagging benefit and distortion cost, respectively.

Figure 6 provides additional insight into the fiscal impact of joint reforms by decomposing their effects into tagging benefit and distortion cost. In the left panel, the effect of extending income tax reforms with a wealth threshold at the 50th percentile is illustrated, while the right panel focuses on extending wealth tax reforms with an income threshold at the 50th percentile. Essentially, the left panel decomposes the difference between the blue line and the black dashed line from Figure 5a, and the right panel decomposes the difference between the blue line and the black dashed line from



Figure 5b.

We observe that the distortion cost of joint reforms decreases monotonically with increasing income and wealth. In the case of income taxation, this suggests that the additional distortion on wealth accumulation decreases as the level at which the income tax reform is implemented increases. This occurs because the additional revenue scales down slower than the efficiency costs via wealth accumulation, given the positive correlation between income and wealth. A similar argument applies to wealth tax reforms.

In contrast, the tagging benefit exhibits an inverse U-shape in both cases. This suggests that the correlation between income and wealth is strongest in the middle of the income and wealth distributions, resulting in a peak tagging benefit for joint tax reforms.

Combining these two effects yields the total impact of joint tax reforms compared to univariate tax reforms. In both cases, the efficiency costs of joint reforms are higher if the reform targets the low-to-middle part of the respective distribution. However, joint reforms provide a fiscal benefit compared to univariate reforms if they target the upper part of the income or wealth distributions.

#### 4.4 Welfare Effects

We can also compute the welfare effects of univariate and joint reforms using our theoretical framework. This necessitates adopting a normative stance on society's preference for redistribution. Rather than assuming a specific social welfare function like utilitarianism or Rawlsianism, we posit that the baseline taxation, consisting of two distinct univariate tax functions, reflects society's preference for redistribution. We assume that these univariate tax functions are separately calibrated to maximize social welfare.

With this assumption of separable optimality, any elementary univariate tax reform should theoretically have a neutral effect on social welfare ( $\Delta W_Y(y) = 0$ ). Therefore, Equation (11) allows us to derive the normative welfare weights across the entire income distribution.<sup>12</sup>

$$\overline{g}_Y(y) = 1 + \frac{\Delta R^E(y)}{\Delta R^M(y)} \quad (20)$$

where  $\overline{g}_Y(y) = \frac{\int_y^\infty g_Y(y') f_Y(y') dy'}{1 - F_Y(y)}$  is the average welfare weight of those who have a higher income than  $y$ . Similarly, the corresponding condition for univariate wealth tax reforms

---

<sup>12</sup>This approach is referred to as the inverse-optimum approach and used in the literature to estimate the redistributive preferences of political parties (Jacobs et al., 2017) or to study the effects of fiscal pressure (Ayaz et al., 2023).

enables the derivation of welfare weights ( $\bar{g}_A(a)$ ) across the entire wealth distribution.<sup>13</sup>

Constructing joint welfare weights ( $\bar{g}(y, a)$ ) at a specific point along the joint distribution of income and wealth is not straightforward. Firstly, given the separate welfare weights determined using the separable optimality assumption, the set of joint weights is not unique. One approach to deriving these joint weights is to assume that the relative weights at distinct points along the income distribution are independent of wealth, and vice versa. Mathematically, this assumption is expressed as

$$\bar{g}(y, a) = \bar{g}_Y(y) \cdot \bar{g}_A(a) \quad (21)$$

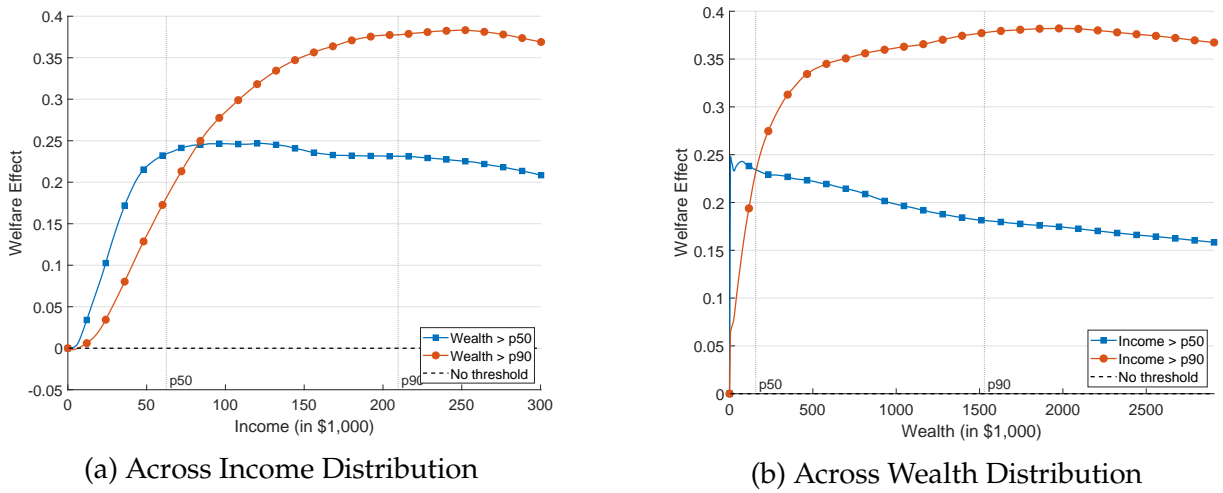


Figure 7: Marginal Excess Burden of Elementary Joint Tax Reforms

*Notes:* The left panel illustrates the welfare effect per dollar of revenue of tax reforms carried out at different income levels. The black dashed line represents univariate income tax reforms. Blue and red lines represent joint reforms with wealth thresholds set at the 50th and 90th percentiles. The right panel illustrates the welfare effect per dollar of revenue of tax reforms carried out at different wealth levels. A black dashed line represents univariate wealth tax reforms. Blue and red lines represent joint reforms with income thresholds set at the 50th and 90th percentiles.

Figure 7 visually summarizes our findings regarding the welfare effects of joint and univariate reforms in terms of public funds. Univariate income and wealth tax reforms without thresholds are depicted as having a neutral effect on social welfare. This outcome is inherent in the assumption that separate univariate tax functions are optimized to calculate distinct welfare weights, implying that any elementary univariate tax reform neither enhances nor diminishes social welfare.

Analyzing income tax reforms extended with a wealth threshold unveils their potential to enhance welfare, a benefit not achievable by univariate income tax reforms across the income distribution. Furthermore, setting the wealth threshold higher at the

<sup>13</sup>See Appendix D.2 for an illustration of the estimated welfare weights across the income and wealth distributions.

90th percentile instead of the 50th percentile amplifies this improvement. However, it's essential to note that the welfare effects are expressed per dollar of tax revenue, and the absolute effects of these reforms may differ. For instance, considering an income tax increase at the 50th percentile of the income distribution, limiting this reform above the 50th percentile of the wealth distribution reduces the additional revenue by 30.3%, while setting a wealth limit at the 90th percentile decreases the additional revenue by 81.4%.

Extending wealth tax reforms with an income threshold similarly demonstrates the potential to enhance welfare. Under the separate optimality assumption, joint tax reforms characterized by a wealth threshold at the 90th percentile of the wealth distribution can improve welfare by 8 to 24 cents per dollar of tax revenue depending on the chosen level for the income threshold.

## 5 Conclusion

We explore the implications of integrating income and wealth taxation, highlighting key reasons why governments may find it advantageous to design these systems in tandem.

Firstly, we demonstrate the efficiency benefits of linking wealth taxes to current income using a simplified two-period model. In scenarios where labor income is uncertain, applying wealth taxes based on varying income levels results in different levels of distortion on saving behavior. This approach provides an additional incentive for governments to adjust saving taxes, lowering them for individuals with lower incomes and raising them for those with higher incomes, beyond the typical insurance motive.

Secondly, we investigate the fiscal and social impacts of joint tax reforms employing a sufficient-statistic approach based on elasticities of taxable income and wealth. By deriving theoretical formulas for the marginal excess burden and welfare effects of joint reforms based on those of univariate income and wealth tax reforms, we argue that the classical "tagging benefit" logic applies when income and wealth are positively correlated. However, the joint reforms we analyze also introduce additional "distortion costs," the dominance of which depends on the joint distribution of income and wealth, particularly the strength of their correlation.

Subsequently, our empirical analysis reveals that employing joint tax reforms may be preferable to univariate reforms, particularly in regions where the correlation between income and wealth is stronger. By leveraging our theoretical formulations on real-world data, we observe that joint reforms incur lower efficiency costs in such re-

gions. Furthermore, we compute the welfare effects of joint reforms without adopting a normative stance regarding redistribution preferences. Instead, we assume that the observed separate systems for income and wealth taxation reflect society's preferences. Our computations indicate that even in the absence of univariate welfare-improving tax reforms, joint tax reforms have the potential to further enhance social welfare.

## References

- Advani, Arun and Hannah Tarrant**, “Behavioural Responses to a Wealth Tax,” *Fiscal Studies*, 2021, 42 (3-4), 509–537.
- Akerlof, George A.**, “The Economics of “Tagging” as Applied to the Optimal Income Tax, Welfare Programs, and Manpower Planning,” *American Economic Review*, 1978, 68 (1), 8–19.
- Albanesi, Stefania and Christopher Sleet**, “Dynamic Optimal Taxation with Private Information,” *Review of Economic Studies*, 2006, 73, 1–30.
- Alesina, Alberto, Andrea Ichino, and Loukas Karabarbounis**, “Gender-Based Taxation and the Division of Family Chores,” *American Economic Journal: Economic Policy*, 2011, 3 (2), 1–40.
- Ayaz, Mehmet, Lea Fricke, Clemens Fuest, and Dominik Sachs**, “Who Should Bear the Burden of COVID-19 Related Fiscal Pressure? An Optimal Income Taxation Perspective,” *European Economic Review*, 2023, 153, 104381.
- Bastani, Spencer, Sören Blomquist, and Luca Micheletto**, “The Welfare Gains of Age-related Optimal Income Taxation,” *International Economic Review*, 2013, 54 (4), 1219–1249.
- Boadway, Robin and Pierre Pestieau**, “Tagging and Redistributive Taxation,” *Annales d’Économie et de Statistique*, 2006, (83/84), 123–147.
- Bourguignon, François and Amedeo Spadaro**, “Tax–Benefit Revealed Social Preferences,” *The Journal of Economic Inequality*, 2012, 10 (1), 75–108.
- Brülhart, Marius, Jonathan Gruber, Matthias Krapf, and Kurt Schmidheiny**, “Behavioral Responses to Wealth Taxes: Evidence from Switzerland,” *American Economic Journal: Economic Policy*, 2022, 14 (4), 111–150.
- Chamley, Christophe**, “Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives,” *Econometrica*, 1986, 54 (3), 607–622.
- Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber**, “Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins,” *American Economic Review*, 2011, 101 (3), 471–475.
- Conesa, Juan Carlos, Sagiri Kitao, and Dirk Krueger**, “Taxing Capital? Not a Bad Idea after All!,” *American Economic Review*, 2009, 99 (1), 25–48.
- Cremer, Helmuth, Firouz Gahvari, and Jean-Marie Lozachmeur**, “Tagging and Income Taxation: Theory and an Application,” *American Economic Journal: Economic Policy*, 2010, 2 (1), 31–50.
- , **Pierre Pestieau, and Jean-Charles Rochet**, “Capital Income Taxation When Inherited Wealth Is Not Observable,” *Journal of Public Economics*, 2003, 87 (11), 2475–2490.

- Diamond, Peter A.**, "Optimal Income Taxation: An Example with a U-shaped Pattern of Optimal Marginal Tax Rates," *American Economic Review*, 1998, 88 (1), 83–95.
- Farhi, Emmanuel and Ivan Werning**, "Insurance and Taxation over the Life Cycle," *Review of Economic Studies*, 2013, 80 (2), 596–635.
- Feenberg, Daniel and Elisabeth Coutts**, "An Introduction to the TAXSIM Model," *Journal of Policy Analysis and Management*, 1993, 12 (1), 189.
- Ferey, Antoine, Benjamin B. Lockwood, and Dmitry Taubinsky**, "Sufficient Statistics for Nonlinear Tax Systems with General Across-Income Heterogeneity," 2023. Unpublished manuscript.
- Golosov, Mikhail, Aleh Tsyvinski, and Nicolas Werquin**, "A Variational Approach to the Analysis of Tax Systems," Working Paper w20780, National Bureau of Economic Research, Cambridge 2014.
- , **Narayana Kocherlakota, and Aleh Tsyvinski**, "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, 2003, 70 (3), 569–587.
- Hansen, Emanuel**, "Optimal Income Taxation with Labor Supply Responses at Two Margins: When Is an Earned Income Tax Credit Optimal?," *Journal of Public Economics*, 2021, 195, 104365.
- Heathcote, Jonathan and Hitoshi Tsujiyama**, "Optimal Income Taxation: Mirrlees Meets Ramsey," *Journal of Political Economy*, 2021, 129 (11), 3141–3184.
- Jacobs, Bas, Egbert L.W. Jongen, and Floris T. Zoutman**, "Revealed Social Preferences of Dutch Political Parties," *Journal of Public Economics*, 2017, 156, 81–100.
- Jacquet, Laurence and Etienne Lehmann**, "How to Tax Different Incomes?," Working Paper 9324, CESifo, Munich 2021.
- , —, and **Bruno Van der Linden**, "Optimal Redistributive Taxation with Both Extensive and Intensive Responses," *Journal of Economic Theory*, 2013, 148 (5), 1770–1805.
- Jakobsen, Katrine, Kristian Jakobsen, Henrik Kleven, and Gabriel Zucman**, "Wealth Taxation and Wealth Accumulation: Theory and Evidence from Denmark," *The Quarterly Journal of Economics*, 2020, 135 (1), 329–388.
- Jordà, Òscar, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick, and Alan M Taylor**, "The Rate of Return on Everything, 1870–2015," *The Quarterly Journal of Economics*, 2019, 134 (3), 1225–1298.
- Judd, Kenneth L.**, "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 1985, 28 (1), 59–83.
- Kleven, Henrik Jacobsen and Claus Thustrup Kreiner**, "The Marginal Cost of Public Funds: Hours of Work versus Labor Force Participation," *Journal of Public Economics*, 2006, 90 (10-11), 1955–1973.

- , —, and **Emmanuel Saez**, “The Optimal Income Taxation of Couples,” *Econometrica*, 2009, 77 (2), 537–560.
- Mankiw, N. Gregory and Matthew Weinzierl**, “The Optimal Taxation of Height: A Case Study of Utilitarian Income Redistribution,” *American Economic Journal: Economic Policy*, 2010, 2 (1), 155–176.
- Mirrlees, J. A.**, “An Exploration in the Theory of Optimum Income Taxation,” *Review of Economic Studies*, 1971, 38 (2), 175–208.
- Piketty, Thomas**, *Capital in the Twenty-First Century*, Cambridge: The Belknap Press of Harvard Univ. Press, 2014.
- and **Emmanuel Saez**, “A Theory of Optimal Inheritance Taxation,” *Econometrica*, 2013, 81 (5), 1851–1886.
- Rothschild, Casey and Florian Scheuer**, “Redistributive Taxation in the Roy Model,” *The Quarterly Journal of Economics*, 2013, 128 (2), 623–668.
- Sachs, Dominik, Aleh Tsyvinski, and Nicolas Werquin**, “Nonlinear Tax Incidence and Optimal Taxation in General Equilibrium,” *Econometrica*, 2020, 88 (2), 469–493.
- Saez, Emmanuel**, “Using Elasticities to Derive Optimal Income Tax Rates,” *Review of Economic Studies*, 2001, 68 (1), 205–229.
- and **Gabriel Zucman**, “Progressive Wealth Taxation,” *Brookings Papers on Economic Activity*, 2019, 2019 (Fall), 437–511.
- and **Stefanie Stantcheva**, “A Simpler Theory of Optimal Capital Taxation,” *Journal of Public Economics*, 2018, 162, 120–142.
- Scheuer, Florian and Joel Slemrod**, “Taxing Our Wealth,” *Journal of Economic Perspectives*, 2021, 35 (1), 207–230.
- Seim, David**, “Behavioral Responses to Wealth Taxes: Evidence from Sweden,” *American Economic Journal: Economic Policy*, 2017, 9 (4), 395–421.
- Spiritus, Kevin, Etienne Lehmann, Sander Renes, and Floris T. Zoutman**, “Optimal Taxation with Multiple Incomes and Types,” Working Paper 9534, CESifo, Munich 2022.
- Stiglitz, Joseph E.**, “Self-Selection and Pareto Efficient Taxation,” *Journal of Public Economics*, 1982, 17 (2), 213–240.
- Straub, Ludwig and Iván Werning**, “Positive Long-Run Capital Taxation: Chamley-Judd Revisited,” *American Economic Review*, 2020, 110 (1), 86–119.
- Zoutman, Floris T.**, “The Elasticity of Taxable Wealth: Evidence from the Netherlands,” 2018. Unpublished manuscript.

## A Optimal State-Dependent Saving Tax Rates

Recall the optimization problem given in Equation (7). The Lagrangian function of this problem is given by

$$\mathcal{L} = u(c_1) + \sum_{i=h,l} p_i u(c_{2i}) + \lambda ((p_l \tau_l + p_h \tau_h) a - G)$$

where  $\lambda$  measures the shadow price of relaxing the budget constraint by a unit exogenously. The first-order conditions are

$$\tau_l : -p_l u'_{2l} a + \lambda \left( (p_l \tau_l + p_h \tau_h) \frac{\partial a}{\partial \tau_l} + p_l a \right) = 0 \quad (22)$$

$$\tau_h : -p_h u'_{2h} a + \lambda \left( (p_l \tau_l + p_h \tau_h) \frac{\partial a}{\partial \tau_h} + p_h a \right) = 0 \quad (23)$$

Rearrange the first-order condition with respect to the low-state tax rate.

$$\begin{aligned} p_l \left( -\frac{u'_{2l}}{\lambda} + 1 \right) a - (p_l \tau_l + p_h \tau_h) \varepsilon_{a,1-\tau_l} \frac{a}{1-\tau_l} &= 0 \\ \frac{p_l \tau_l + p_h \tau_h}{p_l (1-\tau_l)} &= \frac{1 - \frac{u'_{2l}}{\lambda}}{\varepsilon_{a,1-\tau_l}} \\ \Rightarrow \frac{\tau_l}{1-\tau_l} &= \frac{1}{\varepsilon_{a,1-\tau_l}} \cdot \left( 1 - \frac{u'(c_{2l})}{\lambda} \right) \cdot \frac{p_l}{p_l + p_h \frac{\tau_h}{\tau_l}} \end{aligned} \quad (24)$$

Similarly, the first-order condition with respect to the high-state tax rate can be rearranged to obtain

$$\frac{\tau_h}{1-\tau_h} = \frac{1}{\varepsilon_{a,1-\tau_h}} \cdot \left( 1 - \frac{u'(c_{2h})}{\lambda} \right) \cdot \frac{p_h}{p_l \frac{\tau_l}{\tau_h} + p_h} \quad (25)$$

Combining the two optimality conditions shows how the government should set these two tax rates concerning one another.

$$\begin{aligned} \frac{\frac{p_l \tau_l + p_h \tau_h}{p_l (1-\tau_l)}}{\frac{p_l \tau_l + p_h \tau_h}{p_h (1-\tau_h)}} &= \frac{\varepsilon_{a,1-\tau_h}}{\varepsilon_{a,1-\tau_l}} \frac{1 - \frac{u'_{2l}}{\lambda}}{1 - \frac{u'_{2h}}{\lambda}} \\ \Rightarrow \frac{1 - \frac{u'_{2h}}{\lambda}}{1 - \frac{u'_{2l}}{\lambda}} &= \frac{\varepsilon_{a,1-\tau_h}}{\varepsilon_{a,1-\tau_l}} \frac{p_l (1-\tau_l)}{p_h (1-\tau_h)} \end{aligned} \quad (26)$$



## B Proof of Corollary 2

To demonstrate the corollary, begin with the first-order conditions provided in Equations (22) and (23). Scale the condition concerning the low-state tax rate by  $-1$  and the condition concerning the high-state tax rate by  $\frac{p_l}{p_h}$  and then combine the two conditions. This scaling ensures that the resulting tax reform does not alter the immediate tax revenue. Assuming that  $\tau_l = \tau_h = \tau$ , the total change in the Lagrangian becomes

$$\begin{aligned} -\frac{\partial \mathcal{L}}{\partial \tau_l} + \frac{\partial \mathcal{L}}{\partial \tau_h} \frac{p_l}{p_h} &= p_l a u'(c_{2l}) - p_h a u'(c_{2h}) \frac{p_l}{p_h} + \lambda \left( -\tau \frac{\partial a}{\partial \tau_l} + \tau \frac{\partial a}{\partial \tau_l} \frac{p_l}{p_h} - p_l a + p_h a \frac{p_l}{p_h} \right) \\ &= p_l a (u'(c_{2l}) - u'(c_{2h})) + \lambda \tau \left( -\frac{\partial a}{\partial \tau_l} + \frac{\partial a}{\partial \tau_l} \frac{p_l}{p_h} \right) \end{aligned}$$

Using the definition of the saving elasticity,

$$-\frac{\partial \mathcal{L}}{\partial \tau_l} + \frac{\partial \mathcal{L}}{\partial \tau_h} \frac{p_l}{p_h} = p_l a (u'(c_{2l}) - u'(c_{2h})) + \lambda \frac{\tau}{1 - \tau} \left( \varepsilon_{a, 1 - \tau_l} a - \varepsilon_{a, 1 - \tau_h} a \frac{p_l}{p_h} \right) \quad (27)$$

The first term in this addition is positive, if marginal utility is diminishing. Assuming the condition in Equation (6) holds, the second term is also positive. Consequently, the tax reform outlined in the corollary results in an improvement in the government's objective.

To illustrate the last statement of the corollary, consider the limit as  $\lambda \rightarrow \infty$ . In this scenario, the first term becomes insignificant, reflecting the government's disregard for the insurance motive. However, the positivity of the second term ensures that tax revenue increases following the tax reform.

## C Marginal Excess Burden of Elementary Joint Tax Reforms

The marginal excess burden of elementary joint tax reforms depends on three separate effects of the reform on tax revenue. Consider the additional tax revenue raised from individuals whose income and wealth are higher than the threshold values that characterize the joint tax reform. The additional revenue collected from these individuals is given by

$$\begin{aligned} \Delta R^M(y, a) &= \int_y^\infty \int_a^\infty \Delta T f(y', a') da' dy' \\ &= \Delta T \cdot \bar{F}(y, a) \end{aligned} \quad (28)$$

where  $\Delta T$  represents the increase in an individual's tax liability and  $\bar{F}(y, a)$  represents the exceedance function of the joint income and wealth distribution.

Two distortions on labor supply and wealth accumulation decrease tax revenue. They are given by

$$\Delta R_Y^E(y, a) = \int_a^\infty -\varepsilon_{y,1-T_Y'} \frac{y'}{1 - \tau_Y(y', a)} \tau_Y(y', a) \frac{\Delta T}{\Delta y} \Delta y f(y, a') da' \quad (29)$$

$$\Delta R_A^R(y, a) = \int_y^\infty -\varepsilon_{a,1-T_A'} \frac{a'}{1 - \tau_A(y, a')} \tau_A(y, a') \frac{\Delta T}{\Delta a} \Delta a f(y', a) dy' \quad (30)$$

If the initial tax schedule consists of two univariate tax functions, the marginal tax rate of income does not depend on wealth, and vice versa.

$$\tau_Y(y) = \tau_Y(y, a') \quad \forall a' \quad (31)$$

$$\tau_A(a) = \tau_A(y', a) \quad \forall y' \quad (32)$$

We can simplify the total effect of two distortions on tax revenue using the identities above

$$\Delta R_Y^E(y, a) = -\Delta T \cdot y \cdot \varepsilon_{y,1-T_Y'} \cdot \frac{\tau_Y(y)}{1 - \tau_Y(y)} \cdot f(y|a' > a) \quad (33)$$

$$\Delta R_A^R(y, a) = -\Delta T \cdot a \cdot \varepsilon_{a,1-T_A'} \cdot \frac{\tau_A(a)}{1 - \tau_A(a)} \cdot f(a|y' > y) \quad (34)$$

where  $f_Y(y|a' > a)$  and  $f_A(a|y' > y)$  conditional probability distribution functions of income and wealth. According to Equation (13), the marginal excess burden of elementary joint reforms is given by

$$\begin{aligned} MEB_{\text{joint}}(y, a) = & y \cdot \varepsilon_{y,1-T_Y'} \cdot \frac{\tau_Y(y)}{1 - \tau_Y(y)} \cdot \frac{f(y|a' > a)}{\bar{F}(y, a)} \\ & + a \cdot \varepsilon_{a,1-T_A'} \cdot \frac{\tau_A(a)}{1 - \tau_A(a)} \cdot \frac{f(a|y' > y)}{\bar{F}(y, a)} \end{aligned} \quad (35)$$

We can show that the exceedance function of the joint distribution is identical to the exceedance functions of conditional income and wealth distributions. Using the defi-

dition of marginal distributions yields the result.

$$\begin{aligned}
\bar{F}(y, a) &= \int_y^\infty \int_a^\infty f(y', a') da' dy' \\
&= \int_y^\infty f_Y(y' | a' > a) dy' \\
&= 1 - F_Y(y | a' > a)
\end{aligned} \tag{36}$$

Similarly,

$$\begin{aligned}
\bar{F}(y, a) &= \int_a^\infty \int_y^\infty f(y', a') dy' da' \\
&= \int_a^\infty f_A(a' | y' > y) da' \\
&= 1 - F_A(a | y' > y)
\end{aligned} \tag{37}$$

## D Additional Figures and Tables

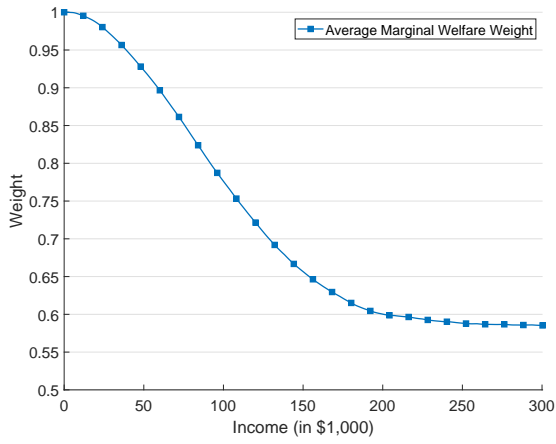
### D.1 Mean Income and Wealth Across the Joint Distribution

			Wealth groups		
			Bottom 50%	Mid 40%	Top 10%
Income groups	Bottom 50%	I	30,700	39,000	39,800
		W	41,400	400,000	3,563,300
	Mid 40%	I	99,500	112,500	129,700
		W	75,100	489,000	3,086,300
	Top 10%	I	280,400	295,900	592,300
		W	105,000	716,600	8,146,300

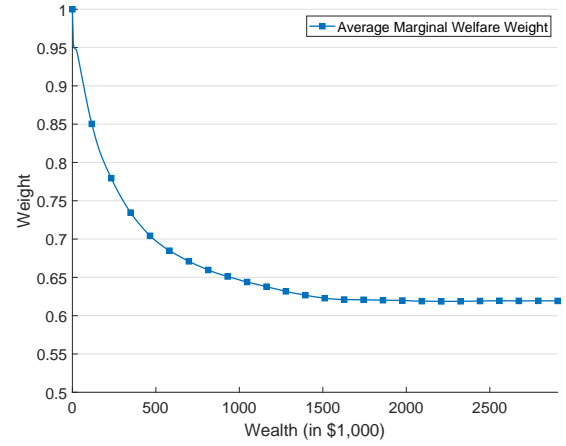
Table 3: Mean Income and Wealth Across the Joint Distribution

*Notes:* The table summarizes the mean income and wealth values along the joint distribution of income and wealth. The income distribution is divided into three bins. The bottom 50% represents households up to the 50th percentile, the mid 40% represents households between the 50th and 90th percentile. The top 10% represents households above the 90th percentile of the income distribution. The same division is also applied to the wealth distribution. The first number above in each cell, denoted by I, represents the average income in U.S. dollars in the respective joint bin. The second number below, denoted by W, represents the average wealth in U.S. dollars in the same bin.

## D.2 Calibrated Welfare Weights



(a) Across Income Distribution



(b) Across Wealth Distribution

Figure 8: Calibrated Average Marginal Social Welfare Weights

*Notes:* The figure illustrates the calibrated average marginal social welfare weights obtained through the inverse-optimum approach. The left panel corresponds to the income distribution, while the right panel corresponds to the wealth distribution. These calibrated weights indicate the welfare gain when individuals above a certain income or wealth threshold receive an additional unit of consumption.