How should we tax capital? Interaction between capital taxes and saving motives*

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Abstract

This paper examines the implications of capital income taxes and wealth taxes by distinguishing between two asset classes: one yielding greater financial returns and the other providing flow utility. Theoretical analysis reveals that both tax policies discourage saving and distort individuals' portfolio compositions. Optimal tax policies vary based on heterogeneity in time preference and non-monetary valuations of assets among households. Using numerical analysis calibrated to the U.S. economy, the paper identifies the optimal tax mix, suggesting a wealth subsidy aimed at benefiting low-income households more due to preference heterogeneity, coupled with an increase in capital income taxes to finance it.

JEL classification:

Keywords: Capital income tax, wealth tax, saving motives, optimal taxation

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1 Introduction

Capital taxation has been a subject of debate among economists and policymakers for years. Despite constituting a smaller share of overall tax revenue compared to income taxes, the impact of capital taxation remains significant (Bastani and Waldenström, 2020). It serves as a potential tool for raising additional revenue and redistributing between households.

Capital income taxes and wealth taxes represent two distinct approaches to taxing capital. While both aim to generate revenue from capital stock, they diverge in their application. Capital income taxes target the returns generated from investments and other capital assets, whereas wealth taxes typically focus on the net worth of individuals or households.

In this study, I explore the similarities and differences between capital income taxation and wealth taxation; incorporating an element of asset heterogeneity. Specifically, I consider assets that yield greater financial returns alongside those that offer non-monetary flow utility to households.

In a hypothetical scenario where returns on capital are uniform across households, capital income taxes and wealth taxes would essentially yield similar outcomes. Theoretically, a capital taxation policy relying solely on capital income taxes could be mirrored by another policy based solely on wealth taxation. However, empirical evidence suggests that there is persistent heterogeneity in capital returns (Fagereng et al., 2020; Bach et al., 2020). Therefore, it becomes crucial to distinguish the similarities and differences between these two forms of capital taxation and understand their implications from a policy-making standpoint.

A crucial factor contributing to heterogeneous returns is the variation in portfolio composition. For instance, Kuhn et al. (2020) emphasize the significance of household portfolio composition for wealth dynamics and inequality. They find that middle-income households' portfolios in the U.S. are primarily dominated by housing, whereas financial assets are more prevalent at the top income distribution. Hence, any heterogeneity in asset price changes or returns on these assets has a first-order effect on wealth distribution. Moreover, Bach et al. (2020) find that the heterogeneity in returns is the main driver of the recent increase in the top wealth shares in Sweden. This underscores the importance of understanding how differences in portfolio composition contribute to disparities in wealth accumulation and distribution.[1]

[1]The heterogeneity in returns on capital may also stem from scale-dependence, as suggested by Fagereng et al. (2020). This phenomenon could arise from fixed investment costs and learning-by-doing effects. Alternatively, certain innate characteristics may enable individuals to achieve higher returns on the same capital as in Guvenen et al. (2023). Schulz (2021) compares these two rationales and characterizes the optimal capital income tax rate based on sufficient statistics.

One possible explanation of these systemic differences in portfolio compositions could be the heterogeneity in how households value different types of assets. Heterogeneity in the relative valuation of different assets would lead to differences in households' portfolio compositions. Otherwise identical households would choose to invest their capital in different assets due to this heterogeneity, consequently earning different returns on the same amount of capital.

Several asset classes offer persistently lower returns even after adjusting for the higher risk associated with higher-return assets, yet are heavily invested in by households. Housing provides an example of such an asset class. For instance, Flavin and Yamashita (2002) show that a minimum housing constraint must be added to support large shares of housing in households' portfolios. Yamashita (2003) adds to this hypothesis by stating that "overinvestment in housing affects the financial portfolio of homeowners." Additionally, Pelizzon and Weber (2008) conclude that portfolios with large shares of housing are not efficient.[2]

Large portfolio shares of lower-return assets raise the question of why people would invest in these assets that yield lower returns, even when accounting for the higher risk that comes with assets with higher returns. One plausible explanation for this phenomenon could be the intrinsic flow utility associated with assets offering lower returns. In essence, individuals may choose to save not solely to preserve or increase financial value, but also to derive satisfaction from ownership itself. This is akin to the utility-in-the-wealth approach, albeit focused on specific components of the overall portfolio. In this scenario, it becomes rational for individuals to invest in assets with lower returns, as they derive utility from owning these assets during their possession.

To address the question of how capital should be taxed, I begin by presenting a simple and tractable model that combines households that have heterogeneous preferences across time and asset classes and a government that uses both capital income and wealth taxation to address capital stock, alongside labor income taxation. In this model, I explain the differences between the two taxation methods in terms of their effect on household saving behavior and portfolio allocation. Specifically, a positive wealth wedge discourages saving without affecting the portfolio decision, while a positive capital income wedge not only distorts the saving decision but also guides households towards investing more in assets with lower returns by altering the relative price of the two assets.

Then, I turn to the government's problem and solve for the welfare-maximizing allocations using appropriate wedges that distort household behavior. Without pref-

[2]Artwork is another example of an asset class with lower returns. Despite exhibiting lower risk-adjusted returns compared to other financial assets, there remains substantial demand for artwork (Baumol, 1986; Pesando, 1993; Mei and Moses, 2002; Korteweg et al., 2016).

erence heterogeneity, the Atkinson-Stiglitz theorem applies and capital accumulation decisions of households should not be distorted (Atkinson and Stiglitz, 1976). In the case where households do not share the same preferences, I characterize the optimal wealth and capital income distortions depending on the nature and extent of preference heterogeneity. Specifically, I demonstrate that if higher-income households display greater patience, the government should discourage wealth accumulation to leverage this information. Conversely, if low-income households place a higher relative value on the utility asset, implementing a positive capital income wedge to distort portfolio allocation is warranted. To counterbalance the impact of the capital income wedge on saving decisions, a negative wealth wedge is necessary.

Subsequently, I calibrate a life cycle model to the U.S. economy, focusing on preference heterogeneity among households. The analysis reveals that households with higher labor productivity tend to exhibit greater patience in making inter-temporal decisions, therefore saving relatively more in the asset that yields greater financial returns. Leveraging this calibrated heterogeneity, I simulate the outcomes of revenue-neutral tax reforms that vary the capital taxation mix. The findings indicate that relying solely on capital income taxation is inefficient; the government can enhance social welfare by using capital income taxes and wealth taxes at the same time. The optimal reform strategy identified involves implementing a wealth subsidy alongside an increase in the capital income tax rate. The negative wealth tax rate serves as a redistributive mechanism, reallocating resources from higher-income households, who predominantly save in higher-return assets, towards lower-income households, who hold relatively more utility-yielding assets in their portfolio. Yet, under fiscal pressure, indicated by the necessity to increase tax revenue, the optimal reform takes a different direction. The government should implement a wealth tax while decreasing the capital income tax rate, ensuring tax revenue remains constant. This adjusted reform results in diminished redistribution from higher-income households to lower-income ones.

Related literature. This study contributes to four strands of the literature. The first pertains to optimal capital taxation. Earlier studies conclude that capital should not be taxed for efficiency reasons (Atkinson and Stiglitz, 1976; Judd, 1985; Chamley, 1986). However, subsequent research indicates that taxing capital may be desirable due to factors such as uncertain or evolving labor productivity (Golosov et al., 2006), incomplete insurance markets (Conesa et al., 2009), multi-dimensional heterogeneity (Piketty and Saez, 2013), and preference heterogeneity (Diamond and Spinnewijn, 2011; Saez and Stantcheva, 2018; Ferey et al., 2023).[3] This study contributes by focusing on two

[3]See Bastani and Waldenström (2020) for a survey on optimal capital taxation.

different assets: one yielding greater financial return and the other providing larger flow utility.

Second, there is an emerging literature comparing capital income taxation with wealth taxation; however, it yields inconclusive results. For instance, Guvenen et al. (2023) suggests that replacing the current capital income tax in the U.S. with a wealth tax could enhance efficiency and welfare due to the "use-it-or-lose-it" effect, reallocating capital to individuals capable of generating higher returns. However, Boar and Midrigan (2023) argues that such a shift could result in equity losses outweighing efficiency benefits, as capital reallocation to successful entrepreneurs may exacerbate inequality and reduce social welfare. To contribute to this debate, I focus on preference heterogeneity as the underlying reason behind the heterogeneous returns on capital.

The third strand focuses on the effects of taxes on household portfolio allocation. Taxation, particularly capital taxation, significantly influences how households make portfolio allocation choices (Poterba and Samwick, 2003; Bergstresser and Poterba, 2004; Alan et al., 2010; Advani and Tarrant, 2021; Zoutman, 2018). This paper adds to this literature by studying the effects of both capital income and wealth taxation on household portfolio allocation.

The last strand pertains to wealth-in-the-utility. Fisher (1930) argues that people accumulate wealth for reasons beyond future consumption, such as increased social status, political influence, and satisfaction derived from the process of wealth accumulation itself, indicating a flow utility from wealth. This wealth-in-the-utility approach has been applied in various studies to explore asset pricing (Bakshi and Chen, 1996), unemployment in a business cycle model (Michaillat and Saez, 2022), optimal capital income taxation (Saez and Stantcheva, 2018), and the effects of wealth taxation (Jakobsen et al., 2020). This paper contributes by utilizing the wealth-in-the-utility approach to compare capital income and wealth taxes.

The remainder of this chapter is structured as follows. Section 2 introduces a two-period model and derives optimal allocations using a mechanism-design approach. In Section 3, a life cycle model is calibrated to the U.S. economy for determining the optimal mix of capital taxes through simulation analysis. Lastly, Section 4 presents concluding remarks.

2 Optimal Allocations in a Two-Period Model

In this section, I present a simple two-period model to explain the interaction between capital taxes and saving motives. The economy consists of two types of households:

those with high labor productivity (denoted by θ_h) and those with low labor productivity (denoted by θ_l). The households maximize their lifetime utility given their type and prices in the economy, whereas the government maximizes social welfare.

2.1 Households' Problem

Households live for two periods. In the first period, they work and earn labor income. They consume a portion of their earnings in the first period and save the remainder. Two types of assets are available for households to save in. The first type is termed the return asset (denoted by a_R), which yields financial returns that can be consumed in the second period. The second type is referred to as the utility asset (denoted by a_U). While this asset does not generate any financial returns[4], it offers non-monetary utility benefits to households. There are no mortality, earnings, or return risks factored in the model. The lifetime utility of households is given by

$$U_i(y, c_1, c_2, a_U) = u(c_1) + \beta_i u(c_2) + \phi_i(a_U) - v\left(\frac{y}{\theta_i}\right) \qquad i = l, h,$$

where $u(c_1)$ and $u(c_2)$ represent the utility from consumption in periods one and two, respectively. The consumption utility function $u(\cdot)$ is increasing and concave. β_i denotes the time discount rate. The subscript i highlights that households may have different discount rates. a_U denotes the level of savings in the utility asset. $\phi_i(a_U)$ captures the utility derived from the utility asset. Different households possibly have different valuations for the same level of utility asset. Similar to the consumption utility, the utility from wealth $\phi_i(\cdot)$ is also increasing and concave. $v\left(y/\theta_i\right)$ denotes the effort cost of earning y for a household with labor productivity θ_i. This cost is increasing and concave in labor effort. Moreover, the way the function is set up means that to earn the same labor income, a household with a higher labor productivity needs to exert less effort.

In the first period, the households consume a portion of their labor income and save the remainder. They can save both in the return and utility assets. In the second period, the households consume all their savings together with any financial returns.

[4]The fact that the utility asset does not yield financial returns serves as a normalization. Introducing different return rates for assets would yield similar insights. To align the model with reality, one might consider the return rate of the return asset in the model as the difference between the return rates of two assets in reality. Section 3 uses that approach.

The optimization problem of households is given by

$$\begin{aligned} \max_{y, c_1, c_2, a_U, a_R} \quad & U_i(y, c_1, c_2, a_U) \\ \text{s.t.} \quad & c_1 = y - a_R - a_U \\ & c_2 = a_R(1+r) + a_U \end{aligned} \tag{1}$$

where a_R denotes the savings in the return asset and r denotes the inter-period interest rate.

The existence of two assets with differing return rates introduces the possibility of return heterogeneity among households, measured relative to their total assets. A household that allocates a higher proportion of their savings to the return asset will receive greater financial returns in proportion to their overall assets. Return heterogeneity is also observed in other studies such as Guvenen et al. (2023), where it arises from individuals having different entrepreneurial productivities, and Schulz (2021), where it originates from size or type dependency.[5] In contrast, return heterogeneity arises from preference heterogeneity in this research.

Households make three decisions to solve the problem given in Equation (1): First, they decide on how much to earn and how much to enjoy leisure. Second, they decide on how much to consume in the first period and how much to save for the second period. Third, they decide on how to allocate their total savings in the return asset and utility asset. The first-order conditions with respect to labor income, return assets, and utility assets are given by

$$u'(c_1) - v'\left(\frac{y}{\theta_i}\right)\frac{1}{\theta_i} = 0 \tag{2}$$

$$-u'(c_1) + \beta_i u'(c_2)(1+r) = 0 \tag{3}$$

$$-u'(c_1) + \beta_i u'(c_2) + \phi_i'(a_U) = 0 \tag{4}$$

respectively.

The first condition regarding labor income states that households need to balance the marginal benefit of earning one more unit and its associated marginal cost. In other terms, the increase in consumption utility resulting from higher earnings should be offset by the decrease in utility caused by increased effort. The second condition regarding the return asset states the importance of optimizing saving decisions to maximize utility inter-temporally. Specifically, individuals should equalize the marginal

[5] Another paper that is closely related in this aspect is by Gahvari and Micheletto (2016). In their paper, the authors assume that individuals have exogenously different return rates and solve for the optimal capital income tax rate.

benefit of saving one more unit in the return asset, including its interest, against the marginal cost arising from reduced consumption in the first period. The third condition regarding the utility asset combines inter-temporal maximization with the utility derived from savings, emphasizing the fact that the utility asset does not provide any interest but increases the lifetime utility.

Equations (3) and (4) can be combined the emphasize the portfolio decision choice. Substituting $u'(c_1)$ yields

$$-\beta u'(c_2)r + \phi_i'(a_U) = 0 \tag{5}$$

The equation above captures the cost and benefit of allocating one more unit of savings into the utility asset rather than the return asset. The household would incur a loss in terms of second-period consumption due to forgone interest but would gain non-monetary benefits from the utility asset.

By utilizing the first-order conditions and the portfolio decision condition given in Equation (5), three wedges can be defined. These wedges distort the three decisions households make.

$$t_y = 1 - \frac{v'\left(\frac{y}{\theta_i}\right)\frac{1}{\theta_i}}{u'(c_1)} \tag{6}$$

$$t_w = 1 - \frac{u'(c_1) - \phi_i'(a_U)}{\beta_i u'(c_2)} \tag{7}$$

$$t_k = 1 - \frac{1}{r}\frac{\phi_i'(a_U)}{u'(c_1) - \phi_i'(a_U)} \tag{8}$$

The first wedge (labor income wedge) governs the household's decision on labor supply. When $t_y = 0$, Equation (6) simplifies to the first-order condition with respect to labor supply, indicating that the labor supply decision remains undistorted. The second wedge (wealth wedge) concerns the consumption decision between periods. When $t_w = 0$, Equation (7) is analogous to the first-order condition of the utility asset. In this scenario, households freely allocate their savings to the utility asset. The last wedge (capital income wedge) is about the decision to earn more interest income. The larger t_k is, the more distorted the household's decision to earn interest income becomes.

One crucial distinction between the wealth wedge and the capital income wedge is in the decisions they distort. The wealth wedge concerns the inter-temporal decision-making process between the first and second periods. It does not affect the relative prices of different assets; therefore, it does not impact the portfolio allocation decision. Conversely, the capital income wedge alters these relative prices and does affect the portfolio decision. Moreover, in addition to this distortion, the capital income wedge also influences the inter-temporal decision by making second-period consumption relatively more expensive due to the lower after-tax returns of the return asset.

These wedges can be implemented using a tax function that depends on labor income, capital income, and wealth. Suppose that the tax payment is calculated as a function with three arguments. It is given as $T(y_L, y_K, w)$ where y_L is labor income, y_K is capital income, and w is wealth. Then, the wedges can be interpreted as marginal tax rates of this tax payment function. That is, $t_j = \frac{\partial T(y_L, y_K, w)}{\partial j}$.

2.2 Government's Problem

The government maximizes social welfare, defined as the weighted sum of households' utilities. It assigns welfare weights $\tilde{f}^i$ to each household which may be different than their population weight. The social welfare is given by

$$\mathcal{W} = \sum_{i=l,h} \tilde{f}^i U_i(y^i, c_1^i, c_2^i, a_U^i) \tag{9}$$

To achieve the welfare-maximizing allocation, the government employs a direct mechanism. In essence, it assigns a set $(y^i, c_1^i, c_2^i, a_U^i)$ to household i. However, due to information asymmetry, the government cannot directly observe the household's type (labor productivity). It can only observe outcome variables which are labor income, consumption, and savings. Therefore, the government offers the entire menu of allocations to every household, ensuring that each household is incentivized to truthfully report their type.

The optimization problem of the government reads as

$$\begin{aligned} \max_{y^i, c_1^i, c_2^i, a_U^i} \quad & \mathcal{W} = \sum_{i=l,h} \tilde{f}^i U_i(y^i, c_1^i, c_2^i, a_U^i) \\ \text{s. t.} \quad & \sum_{i=l,h} f^i y^i \geq \sum_{i=l,h} f^i \left(c_1^i + \frac{c_2^i}{1+r} + \frac{r a_U^i}{1+r} \right) \\ & U_h(y^h, c_1^h, c_2^h, a_U^h) \geq U_h(y^l, c_1^l, c_2^l, a_U^l) \end{aligned} \tag{10}$$

The first constraint of this problem is the resource constraint. The aggregate income in the economy (represented by the LHS) must be greater than or equal to the net present value of aggregate consumption (represented by the RHS). The return asset allocated to households appears in the resource constraint even though it is not part of consumption. This is because the term $\frac{r a_U^i}{1+r}$ accounts for the opportunity cost of the utility asset. It reflects the cost the government incurs by allocating utility assets instead of return assets. In other words, it captures the forgone interest resulting from not saving in return assets.

The second constraint is called the incentive compatibility constraint. The LHS of the constraint captures the utility of a truthful reporting household with high labor productivity. The RHS is their utility when they mimic a household with low labor productivity.[6] The incentive compatibility constraint states that the utility of truthful reporting for high-productivity households must be greater than or equal to mimicking low-productivity households.

There could be another incentive compatibility constraint for low-productivity households as well; however, one can show that only one constraint can be binding in the optimum. If the ratio $\frac{\tilde{f}^l}{f^l}$ is sufficiently high (the government wants to redistribute towards low-productivity households), then only the constraint for high-productivity households is binding.

2.3 Optimal Distortions

After defining the problems of households and the government, one can characterize the optimal allocation that maximizes social welfare. The concept of the marginal social welfare weight proves useful in outlining and interpreting the optimal wedges that distort households' decisions. Specifically, the marginal social welfare weight for low-productivity households is defined as follows.

$$g^l \equiv \frac{\frac{\partial U_l}{\partial c_1}}{\lambda} \frac{\tilde{f}^l}{f^l}$$

where λ is the Lagrange coefficient for the resource constraint in Equation (10).[7] The marginal social welfare weight for low-productivity households measures the increase in welfare, depending on the relative welfare weight when the household is given one more unit of consumption in the first period. This measure is in terms of public funds. The higher g^l is, the more the government wants to redistribute towards low-productivity households.

Proposition 1. *The decision-making process of households with high productivity remains undistorted in the optimal allocation.*

$$t_y^h = 0, \qquad t_w^h = 0, \qquad t_k^h = 0$$

[6]This can be seen by observing the mismatch between the subscript of the utility function and its arguments.

[7]One interpretation of this coefficient is its representation of the marginal value of public funds. It quantifies the extent to which an additional unit of public funds can improve welfare. See Hendren and Sprung-Keyser (2020) for an overview.

Proof. See Appendix A. □

Proposition 1 summarizes the optimal allocation for high-productivity households. The solution to the government's optimization problem results in an allocation allowing them to make decisions without government intervention.

This finding is not unique, as the mechanism-design approach to redistributive taxation frequently yields the no-distortion-at-the-top result (for example, Mirrlees, 1971). Additionally, this result is local, applying only to the household with the highest productivity in cases involving more than two types of households.

Proposition 2. *All three decisions of households with low productivity need to be distorted in the optimal allocations. The optimal wedges are given by*

$$t_y^l = (g_l - 1)\frac{v'\left(\frac{y^l}{\theta_l}\right)\frac{1}{\theta_l} - v'\left(\frac{y^l}{\theta_h}\right)\frac{1}{\theta_h}}{u'(c_1^l)} \tag{11}$$

$$t_w^l = (g_l - 1)\left(\frac{\beta_h - \beta_l}{\beta_l} + \frac{\phi_h'(a_U^l) - \phi_l'(a_U^l)}{\beta_l u'(c_2^l)}\right) \tag{12}$$

$$t_k^l = -(g_l - 1)\frac{1+r}{r}\frac{\phi_h'(a_U^l) - \phi_l'(a_U^l)}{u'(c_1^l) - \phi_l'(a_U^l)} \tag{13}$$

Proof. See Appendix A. □

Proposition 2 outlines the optimal allocation for low-productivity households. To maximize social welfare, the government intervenes by distorting the decisions made by low-productivity households.

First, consider the labor income wedge. It is positive, assuming households do not share the same productivity level, implying that low-productivity households are incentivized to work less compared to a scenario without government intervention. This distortion is aimed at discouraging high-productivity households from mimicking low-productivity households. The larger the disparity between the marginal benefit of earning labor income and its associated marginal cost, the more costly it becomes for high-productivity households to mimic low-productivity ones.

Second, the wealth wedge is non-zero as long as there are differences between households in terms of discount rate or the valuation of the utility asset.[8]—unless two sources of heterogeneity exactly offset each other. The argument behind this

[8]Note that β_h is not necessarily larger than β_l. The subscripts merely denote their association with households of high and low productivity, respectively. For instance, if low-productivity households are more patient, then $\beta_l > \beta_h$. The same principle applies to $\phi_i'(a_U)$.

is similar to the one concerning the labor income wedge. With heterogeneities in discount rates or valuations of the utility asset, high-productivity households would ideally prefer a different wealth level when mimicking low-productivity households. Therefore, distorting the wealth decision discourages high-productivity households from mimicking.

Third, the capital income wedge is non-zero if households differ in terms of the valuation of the utility asset. In this scenario, the government once again distorts the decision of low-productivity households to deter high-productivity households from mimicking them. By imposing a wedge in capital income, the government aims to create a disincentive for high-productivity households This distortion serves to maintain a more efficient allocation of resources across households.

From the optimal allocation, several insights emerge. First of all, in cases where households differ solely in terms of labor productivity and have identical preferences in other aspects, the only non-zero wedge is the labor income wedge. This implies that the government does not distort either the saving decision or the portfolio choice. This result aligns with the Atkinson-Stiglitz theorem (Atkinson and Stiglitz, 1976), which states that with utility functions exhibiting separable labor effort, welfare can be maximized solely through direct labor income taxes, without any need for indirect taxes.

Another crucial insight is that the nature of preference heterogeneity among households, if present, determines which decisions are distorted in the optimal allocation. This is because the presence of wealth and capital income wedges proves useful for the government insofar as they can relax the incentive compatibility constraint. Specifically, these wedges discourage the high-productivity households from mimicking, therefore extending the redistribution possibilities for the government.

Corollary 1. *Suppose that households differ in terms of labor productivity and discount rate but value the utility asset the same. Then, the wealth wedge is non-zero for the low-productivity household, whereas the capital income wedge is zero. If households with higher income are also more patient ($\beta_h > \beta_l$), then the wealth wedge is positive.*

Proof. Setting $\phi_h'(a_U) = \phi_l'(a_U)$ in Equations (12) and (13) yields the result. □

Corollary 1 addresses a scenario where preference heterogeneity between households is inter-temporal, indicating that one type of household places relatively more importance on one period. However, both types of households share the same relative valuation between the return asset and the utility asset. Therefore, the government should distort the inter-temporal decision, for which the wealth wedge is useful in this context. For instance, if high-productivity households are more patient ($\beta_h > \beta_l$), they would prefer to save more than low-productivity households when mimicking them. To discourage this behavior, the government distorts the inter-temporal decision of low-productivity households by introducing a positive wealth wedge. However, the government should not distort the portfolio choice in this scenario, thus the capital income wedge remains zero.

Corollary 2. *Suppose that households differ in terms of labor productivity and the valuation of the utility asset but share the same discount rate. Then, both the wealth wedge and the capital income wedge are non-zero for low-productivity households but with different signs. Moreover, their impact on the households' inter-temporal decision counteracts each other, leaving it undistorted. If households with lower income prefer the utility asset more ($\phi_l'(a_U) > \phi_h'(a_U)$), then the capital income wedge is positive and the wealth wedge is negative.*

Proof. Setting $\beta_h = \beta_l$ and plugging in the optimal wedges from Equations (12) and (13) into their definitions in Equations (7) and (8) yield the result. □

In another scenario where preference heterogeneity is in terms of relative asset valuation, Corollary 2 summarizes the optimal allocation. Here, one type of household values the utility asset relatively more (or less), while all households share the same valuation between different periods. Therefore, the government should alter the portfolio decision while keeping the inter-temporal decision undistorted. The capital wedge distorts the relative prices of different assets to achieve this goal; however, it also distorts the inter-temporal decision. The wealth wedge, appropriately sized, reverses this distortion on the inter-temporal decision, restoring it to its undistorted state.

Lastly, if households have different discount rates and valuations of the utility asset, both the inter-temporal decision and the portfolio decision should be distorted. The direction and magnitude of these distortions depend on the extent and direction in which households have different preferences.

3 Simulation Analysis

Governments often opt for simpler and more practical tax instruments over more complex and non-linear ones. The tax instrument used for the optimality analysis in Section 2 is an example of a highly complex approach. It requires the use of three non-linear taxes, which may depend on each other and all choices made in the current period as well as previous periods. In this section, I will present a numerical analysis where governments rely on simpler linear taxes that are not interdependent. The tax rates depend only on the current choice of households that is specific for that tax rate.

3.1 Model

In the model, households have a finite lifespan, which can be interpreted as a specific number of years. The households enter into the model without any assets. During the initial years of their life, they work and generate labor income. They have the option to save a portion of their earnings in either the return asset or the utility asset. Both types of assets accrue interest annually, with the return of the return asset assumed to be higher than that of the utility asset, that is, $r_R > r_U$. This assumption's critical implication is that households save in both types of assets in equilibrium as long as the marginal utility from the utility asset is decreasing and the largest for zero input.[9]

In the later stages of their life, households transition into retirement and cease earning labor income. Nonetheless, they continue to earn capital income through their savings each year. During retirement, households consume their accumulated savings and eventually die in a predetermined period without any mortality risk in earlier periods. Given that they have no incentive to retain positive assets at the end of their lives[10], they deplete their savings entirely in the final period.

Households maximize their lifetime utility subject to yearly budget constraints given tax rates, interest rates, and their preferences. The optimization problem is given by

$$\begin{aligned} \max_{y,c,a} \; & U^{LT} = \sum_{i=t}^{T_w} \beta_i^t \left[u(c_{w,t}) + \phi_i(a_{U,t}) - v\left(\frac{y_t}{\theta_i}\right)\right] + \sum_{t=T_w+1}^{T_w+T_r} \beta_i^t \left[(u(c_{r,t}) + \phi(a_{U,t}) \right] \\ \text{s. t.} \; & a_{R,0} = a_{U,0} = 0 \\ & y_t(1-\tau_y) + a_{R,t-1}\overline{R}_R + a_{U,t-1}\overline{R}_U = c_{w,t} + a_{R,t} + a_{U,t}, \quad t \in [1, 2, \dots, T_W] \\ & a_{R,t-1}\overline{R}_R + a_{U,t-1}\overline{R}_U = c_{r,t} + a_{R,t} + a_{U,t}, \quad t \in [T_W+1, T_W+2, \dots, T_W+T_R] \end{aligned} \tag{14}$$

where β_i is discount rate, θ_i is labor productivity, ϕ_i is utility from wealth function, y_t represents annual labor income, $c_{w,t}$ and $c_{r,t}$ represent annual consumption during working life and retirement, and $a_{R,t}$ and $a_{U,t}$ represent return and utility assets at the end of a year. Similar to Section 2, households may have preference heterogeneity in terms of labor productivity, discount rate, and valuation of the utility asset. τ_y is the constant marginal tax rate on labor income. $\overline{R}_i = (1-\tau_w)(1 + r_i(1-\tau_k))$ represents the gross after-tax return of asset i. It depends on the before-tax return rate of the asset as well as both the capital income and wealth tax rates.

The first summation in the lifetime utility function captures the working life of

[9]Put formally, $\phi_i''(a_U) < 0$ and $\lim_{a_U \to 0} \phi_i'(a_U) = +\infty$.

[10]Other than mortality risk, another reason why households may retain positive assets at the time of death is the bequest motive. This motive can help explain observed wealth inequality, as affluent households may be more inclined to accumulate and maintain larger asset holdings to pass on to future generations. (De Nardi, 2004)

households. It accounts for the fact that earning labor income incurs costs for households, with the level of costliness varying based on their labor productivity. In each year of their working life, households derive utility from their consumption as well as from the stock of their utility assets at the year's end. The second summation pertains to the retirement of households. Similar to their working life, during retirement, households continue to derive utility from their annual consumption and the stock of their utility assets at the end of each year.

The problem outlined above presents significant complexity, making it challenging to address both analytically and numerically. However, it is possible to simplify the problem by making several assumptions, thereby reducing its dimensions and allowing a more focused examination of the central question: What is the significance of households' saving motives in determining the approach governments should adopt in taxing capital?

One such assumption is to restrict households to choose only one labor income level throughout their working life.[11] This simplification helps abstract away from the complexities of life cycle considerations on labor supply, allowing for a simpler analysis of the interaction between saving motives and optimal capital taxation policies.

Another assumption is to set the net saving of households in both assets to be constant. This implies that households save a fixed portion of their earnings in return and utility assets each year, maintaining the same proportion consistently. Consequently, their annual consumption remains constant throughout their working life. Similarly, during retirement, households withdraw from their assets in a fixed amount and proportion to maintain constant annual consumption. This simplification streamlines the analysis by ensuring a stable consumption pattern within working life and retirement, yet allowing households to adjust their consumption between the two periods.

With the two assumptions explained above, the households' lifetime utility maximization problem can be simplified as follows:

$$
\begin{aligned}
\max_{y, c_w, c_r, a_U, a_R} \quad & \beta_i^w \left[u(c_w) - v\left(\frac{y}{\theta_i}\right) \right] + \beta_i^r \left[u(c_r) \right] + \beta_i^\phi \phi_i(a_U) + \phi_i^C \\
\text{s. t.} \quad & c_w = y(1-\tau_y) - \delta_R a_R - \delta_U a_U \\
& c_r = \sigma_R a_R + \sigma_U a_U
\end{aligned}
\tag{15}
$$

where y, c_w, and c_r represent annual labor income, consumption during working life, and consumption during retirement, respectively. β_i^w and β_i^r are the composite discount rates of working life and retirement. These parameters not only capture time preference

[11]This assumption can be slightly relaxed by considering an evolving profile of earnings instead of assuming a constant income level throughout households' working lives.

but also reflect the varying durations of working life and retirement. Findeisen and Sachs (2017) adopt a similar composite discount rate approach to accommodate the differing durations of education and working life. The composite discount rates are given by

$$\beta_i^w = \sum_{t=1}^{T_w} (\beta_i)^t \,, \qquad \beta_i^r = \sum_{t=T_w+1}^{T_w+T_r} (\beta_i)^t$$

β^ϕ is the lifetime discount rate of the utility from wealth. Similarly, ϕ_i^C is a constant term that depends not on the level but the life cycle pattern of the utility asset holdings. Therefore, ϕ_i^C also depends on tax and interest rates which determine this pattern.[12]

In the optimization problem, a_R and a_U denote the amount of savings households have at the time of retirement. The simplifying assumption of constant labor income and consumption allows for the characterization of the whole life cycle pattern of asset holdings. Hence, it is not necessary to keep track of the amount of savings at the end of each period. δ_i measures the necessary decrease in annual working life consumption to increase the households' holdings of asset i by unit at the time of retirement, or vice versa. Similarly, σ_i measures the resulting increase in annual retirement consumption when households hold one more unit of asset i at the time of retirement.

The ratio of $\frac{\sigma_i}{\delta_i}$ can be interpreted as the inter-period gross after-tax interest rate of asset i between retirement and working life. For instance, when a household decreases their annual working-life consumption by one unit to save more in the return asset, they can increase their annual retirement consumption by $\frac{\sigma_R}{\delta_R}$ units.

3.2 Calibration

To align the model with real-world data, I use the *Survey of Consumer Finances* (SCF) provided by the Federal Reserve, specifically the wave from 2019. This survey, conducted every three years, is a cross-sectional household survey that is representative of the entire population. In particular, the SCF data includes information on labor income and households' balance sheets. The latter is crucial in characterizing the type and extent of preference heterogeneity observed among households.

I partition the households' balance sheets into two distinct categories: the utility asset, which provides non-monetary benefits, and the return asset, which offers greater financial returns. The utility asset consists of primary residences along with any associated mortgages and home equity loans. The return asset includes all other

[12] An explanation in detail regarding the life cycle pattern of utility asset holdings and the resulting discount rate can be found in Appendix B.

items on the balance sheet, including financial assets (stocks, bonds, mutual funds, etc.), remaining non-financial assets (additional properties, business assets, etc.), and any remaining debts not associated with the primary residence.

In addition to the data from the SCF, several other parameters must be calibrated to fully characterize the model numerically. Table 1 provides the summary of these parameters.

Parameter		Value	Source
Labor income tax rate	τ_y	24.7%	OECD (2023)
Capital income tax rate	τ_k	20%	*Long-term capital gains tax rate*
Wealth tax rate	τ_w	0%	*No wealth taxation*
Interest of return asset	r_R	9.31%	Jordà et al. (2019)
Interest of utility asset	r_U	5.86%	Jordà et al. (2019)
Working life (yrs.)	T_w	42	Federal Reserve (2019)
Retirement (yrs.)	T_r	18	World Bank (2022)

Table 1: Calibration Parameters

Notes: The table presents the parameters used for the initial calibration of the observed data. The first panel details the tax and interest rates, while the second panel is about the life cycle parameters of households.

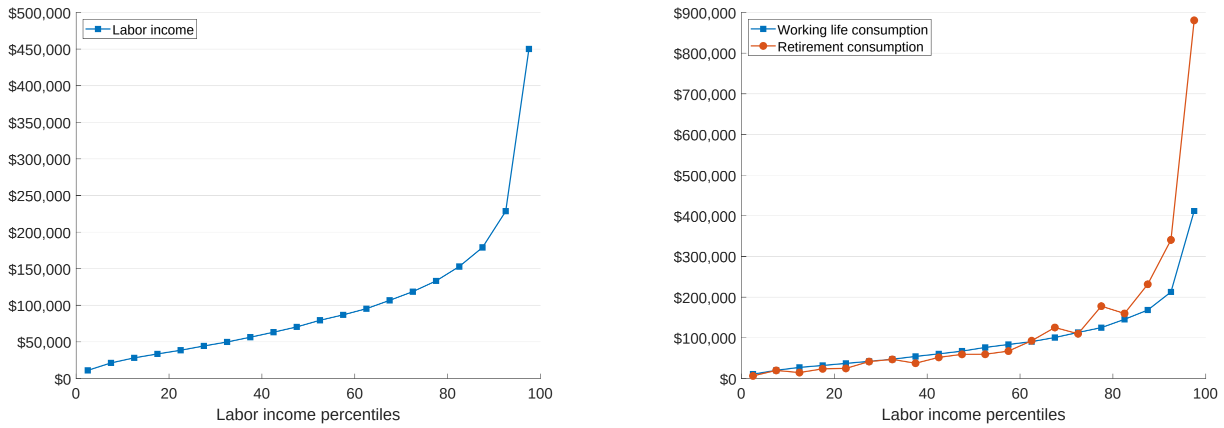
Among the most important parameters are the linear tax rates. For the linear labor income tax rate, I use data from the OECD to identify the average wage tax in the U.S. The linear capital income tax rate is given by the long-term capital income tax rate in the U.S. tax code. As there is currently no wealth taxation in the U.S., the wealth tax rate is assumed to be zero.

For the interest rates of different assets, I rely on the findings of Jordà et al. (2019), who calculate the post-1980 return rate of equity to be 9.31% p.a., which I adopt as the interest rate of the return asset. Additionally, they estimate the return rate of housing to be 5.86% p.a. I use this value as the interest rate of the utility asset in my model.

I assume that households enter the model (and therefore the labor market) at the age of 20. With a median retirement age of 61 (Federal Reserve, 2019), the resulting length of the working life is 42 years. Considering an average life expectancy of 79 years (World Bank, 2022), the average length of retirement for U.S. households is 18 years.

The cross-sectional nature of the SCF data presents an additional challenge when using it to calibrate a life cycle model. To address this challenge, I employ the following method. Initially, I focus on the households whose heads are between 20 and 61 years old, as these individuals are considered to be in the labor market based on the model's calibrations. Additionally, households without any reported labor income are excluded from the analysis. By examining a household's labor income and savings alongside their age, I can extrapolate their savings and portfolio composition throughout their entire working life.[13] This extrapolation is made possible by assuming constant labor income and consumption levels throughout the working years. Subsequently, retirement consumption is estimated by assuming that households deplete their savings at the end of their life.

Another challenge posed by the SCF data is that different households are observed at different points in their labor market experience. To address this, the calibration strategy proceeds as follows: First, the relevant sample is identified and divided into four age groups. Then, households within each age group are ranked according to their position in the labor income distribution. For each point along this within-age-group labor income distribution, the labor income, assets at retirement, and implied consumption during working life and retirement are calibrated. Finally, the weighted average of these calibrations across different age groups yields the final results of the calibration.[14]



(a) Labor Income (b) Consumption

Figure 1: Calibrated Labor Income and Consumption

Notes: The left figure illustrates the calibrated annual labor income of households with respect to labor income percentiles. The right figure illustrates the calibrated annual consumption of households in working life and retirement with respect to labor income percentiles.

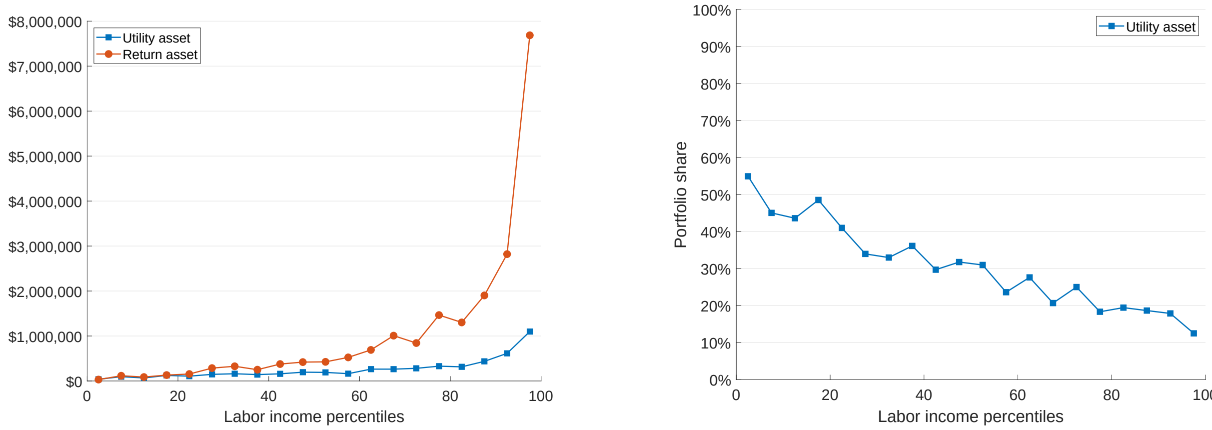
The left panel in Figure 1 illustrates the calibration results regarding labor income and consumption. The calibrated annual labor income of U.S. households spans from $11,000 for the bottom 5% to $490,000 for the top 5% of the labor income distribution, with a median household earning approximately $79,000 annually.

[13]In particular, this method may predict a negative level of working life consumption for households with significant savings and relatively lower labor income. In my sample, households with predicted negative consumption constituted around 1% of the entire dataset. These households are excluded from the analysis.

[14]See Figures 12 and 13 in Appendix D.1 for a comparison between this calibration method and a simpler one that uses a pooled sample.

Unsurprisingly, the right panel in Figure 1 depicts a positive correlation between annual consumption levels and household rank in the labor income distribution. Higher labor-income households are more likely to have higher consumption levels both during their working life and retirement.

Notably, the relationship between consumption levels during working life and retirement varies across different households. While households towards the lower end of the labor income distribution tend to consume more during their working life, those at the top consume significantly more during retirement relative to their working life. For instance, households with the highest labor income consume over twice as much in retirement compared to their working life—$880,000 versus $410,000, respectively.



(a) Assets at Retirement (b) Portfolio Share

Figure 2: Calibrated Assets

Notes: The left figure illustrates the calibrated values of household savings in the utility asset and the return assets with respect to labor income percentiles. The saving values are adjusted to represent the values at the time of retirement (i.e. the peak value of savings). The right figure illustrates the household portfolio composition, which is defined as the ratio of utility assets to the net worth, at the time of retirement with respect to labor income percentiles.

Figure 2 presents the calibration outcomes regarding household assets. In the left panel, the accumulated savings of households at retirement are depicted. The panel distinguishes between return and utility assets, giving an idea of households' portfolio allocation. As expected, a strong positive gradient in savings can be observed based on labor income. Lower-income households retire with minimal to no savings, while those at the upper end of the labor income distribution save significant amounts in both types of assets. For instance, households with the highest labor income retire with assets of more than $8,700,000 in total.

The right panel of Figure 2 provides further insight into the variations in households' portfolio allocation. It depicts the portfolio share of the utility asset, calculated as the value of utility assets at retirement relative to net worth. A notable result of the calibration is as follows: households in the lowest 20% of the distribution allocate approximately half of their savings to utility assets, whereas this proportion decreases to around 20% for those in the highest 20%. This decline in the utility asset share persists across the entire distribution and exhibits an almost monotonic pattern.

The heterogeneity observed in portfolio allocation among households could arise from various factors. For instance, households with lower labor productivity may have a stronger preference for the utility asset. Alternatively, households with higher labor productivity may display greater patience, leading them to favor the return asset with higher financial returns in the future over the utility asset, which provides non-monetary utility earlier in life.

To shed light on the underlying reasons for the observed portfolio allocation heterogeneity, I utilize the results from Figures 1 and 2 and calibrate the simplified model outlined previously. To achieve this, I introduce further functional form assumptions to the lifetime utility function presented in Equation (15). Table 2 provides an overview of the utility parameter values obtained from the literature, which will be used in the calibration process.

Parameter		Value	Source
Frisch elasticity	ε	0.5	Chetty et al. (2011)
Curvature of consumption utility	γ	2	Calvet et al. (2021)
Curvature of wealth utility	μ	2	–

Table 2: Utility Parameters Used for Calibration

Notes: The table shows the utility parameters that are taken from the literature and the assumptions to calibrate labor productivity, discount rate, and relative utility from wealth.

Firstly, I assume that the utility of annual consumption follows a constant-relative-risk-aversion (CRRA) function with a curvature parameter (γ) set to 2, consistent with Calvet et al. (2021). The utility of annual consumption is given by

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$$

Additionally, the work effort function is assumed to be iso-elastic with a Frisch elasticity (ε) of 0.5, following Chetty et al. (2011). The work effort function is represented as

$$v\left(\frac{y}{\theta_i}\right) = \frac{\left(\frac{y}{\theta_i}\right)^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$$

As for the utility of wealth, there is no widely accepted functional form in the literature. Therefore, I assume that it also exhibits the CRRA property, similar to the utility of annual consumption, with the same curvature parameter. However, I introduce a coefficient (ξ_i) that multiplies the utility of wealth. This coefficient allows for different weights for utility from consumption and wealth. By allowing this parameter to vary across households, the model accommodates the possibility of heterogeneous preferences toward the utility asset. The utility of wealth is given by

$$\phi_i(a_U) = \xi_i \frac{a_U^{1-\mu} - 1}{1-\mu}$$

Three utility parameters still require calibration using data: labor productivity (θ_i), discount rate (β_i), and relative utility of wealth (ξ_i). While these parameters may vary among households, the three first-order conditions of households' optimization problem given in Equation (15) precisely determine the values of these utility parameters for each household.

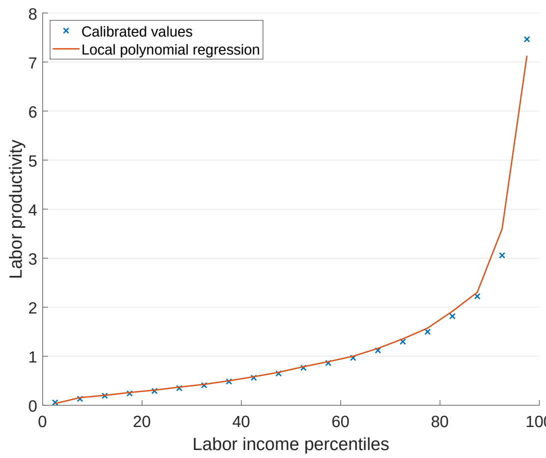
Figure 3 illustrates the calibrated utility parameters across the labor income distribution. In the top left panel, the labor productivity is depicted, with calibrated values increasing monotonically with labor income. Additionally, the observed increase demonstrates a convex pattern, indicating a fatter right tail in the labor income distribution.

The top right panel illustrates the discount rate. The calibration reveals a clear positive correlation between labor income and the discount rate, indicating that higher labor productivity corresponds to a higher discount rate.[15] This implies that households with higher labor productivity tend to be more patient, assigning relatively greater importance to future periods.

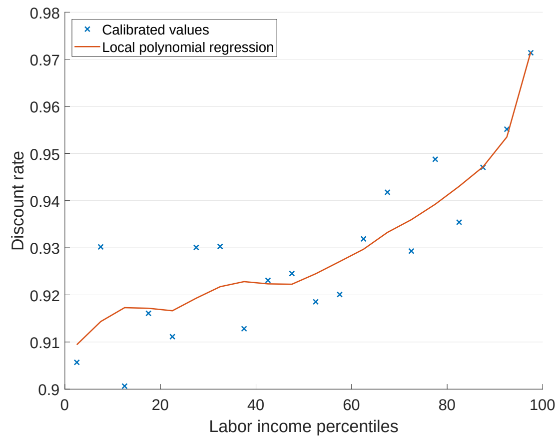
In the bottom panel, the relative utility of wealth is depicted. Unlike labor productivity and discount rate, the relationship between the relative utility of wealth and labor income is not as strong. However, there is a weak U-shaped relationship between the relative utility of wealth and labor income.

The findings from Figure 3 suggest that the heterogeneity in portfolio allocation between households with different levels of labor income can be attributed to the heterogeneity in their levels of patience. Specifically, higher-income households, characterized by greater labor productivity, exhibit higher levels of patience. These households with higher discount rates place greater importance on their future consumption. As a result, they are inclined to allocate a larger proportion of their savings to the return asset rather than the utility asset, which brings higher financial returns in later periods

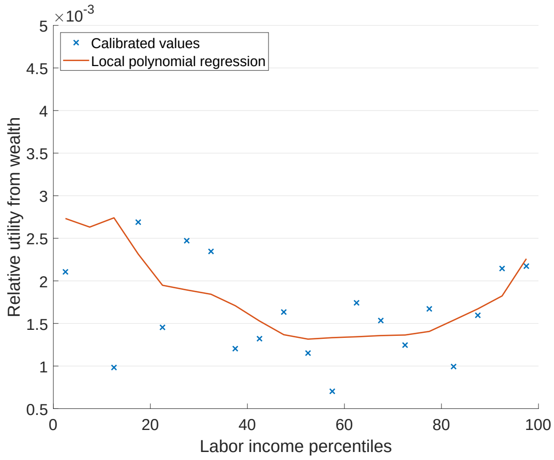
[15] Indeed, the Pearson correlation coefficient between these two utility parameters is notably high at 81.4%,



(a) Labor Productivity (θ)



(b) Discount Rate (β)



(c) Relative Utility from Wealth (ξ)

Figure 3: Calibrated Utility Parameters

Notes: The blue markers in the top left figure illustrate the calibrated labor productivity values for each household, the ones in the top right figure illustrate the calibrated discount rate values for each household, and the ones in the bottom figure illustrate the calibrated relative utility from wealth values for each household. All figures are with respect to labor income percentiles. The red lines in all figures illustrate the estimated values using a first-order locally-weighted regression.

of life.

3.3 Simulation

With the calibrated utility parameters[16] in hand, I now focus on the policy side. To determine the optimal capital taxation policy given households' preferences, I aim to compute the combination of capital income and wealth tax rates, that maximizes social welfare while keeping tax revenue constant. This approach abstracts from questions

[16] I utilize smoothed utility parameters estimated through first-order locally-weighted regressions to simulate outcomes.

regarding tax revenue allocation and instead assumes that the government exogenously requires a predetermined amount of tax revenue. This amount is given by the total tax revenue the government raises in the baseline economy. This exercise can be interpreted as the search for the most efficient means of raising tax revenue using different capital taxation instruments.

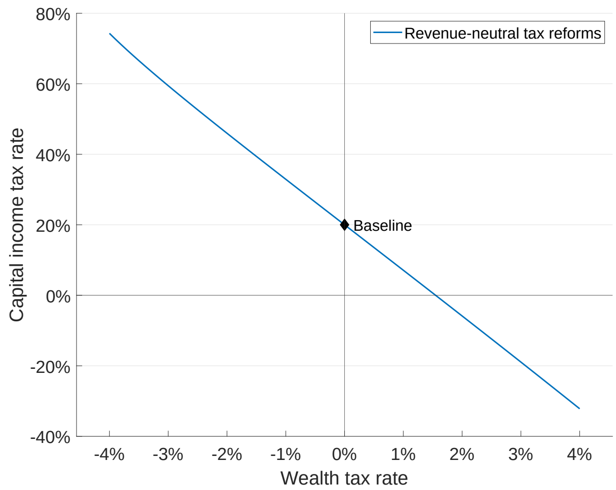


Figure 4: Revenue-Neutral Capital Taxation Reforms

Notes: The figure illustrates the family of wealth-capital income tax rate pairs where the total tax revenue remains constant. Any tax reform along the blue line, which involves an increase in the capital income tax rate coupled with a decrease in the wealth tax rate (or vice versa), does not change the total tax revenue. The black diamond denotes the current state, where the wealth tax rate is 0% and the capital income tax rate is 20%.

Figure 4 illustrates the pairs of capital income and wealth tax rates that maintain constant tax revenue. The labor income tax rate is assumed to remain constant. For a given wealth tax rate, the capital income tax rate that keeps the tax revenue constant is computed, considering households' re-optimization based on the new pair of capital tax rates. The numerical optimization method for households' utility maximization problem is detailed in Appendix C.

When implementing a tax reform that raises the wealth tax rate, it requires a corresponding reduction in the capital income tax rate to hold the level of tax revenue constant, and vice versa. The relationship between changes in these tax rates is predominantly linear. However, non-linearities become apparent near extreme values, such as when the capital income tax rate approaches 100%.

3.3.1 Optimal Capital Taxation Policy

To measure the impact of a particular revenue-neutral tax reform on social welfare, I employ a macro-level measure denoted as $\overline{CE}$. This measure represents a fixed proportional consumption transfer provided to every household in every year. The value of this transfer is calculated such that when it is distributed in the baseline economy, the resulting social welfare matches that of the economy following the tax reform.

$$\sum_i U_i(y_i', c_i', a_i') = \sum_i U_i(y_i, c_i \cdot (1 + \overline{CE}), a_i) \tag{16}$$

where (y_i, c_i, a_i) represents households' optimal choice in the baseline economy in terms of their labor income, consumption patterns, and portfolio decisions. (y_i', c_i', a_i') represents the re-optimized choices of the same households after a tax reform.

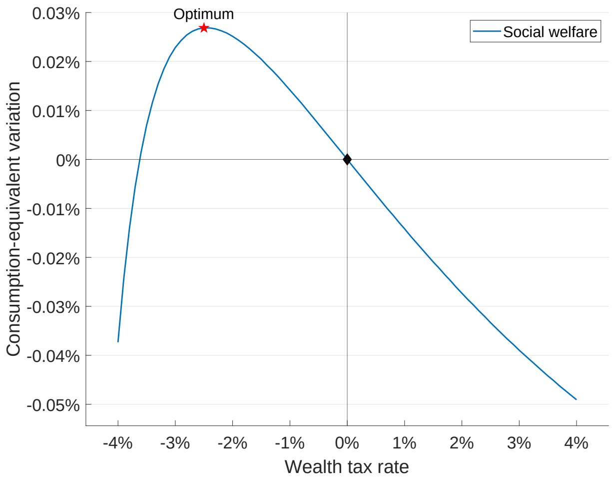


Figure 5: Social Welfare After Tax Reforms

Notes: The figure illustrates the social welfare, which is defined as the unweighted sum of all households' utilities, as a function of the wealth tax rate. For each wealth tax rate, the capital income tax rate is adjusted to maintain revenue neutrality. The y-axis represents the fixed proportional consumption transfer to all households such that the average lifetime utility is equal to that of the reform economy. The red star represents the optimal reform where the new wealth tax rate is -2.5%. The corresponding capital income tax rate is 52.6%.

Figure 5 illustrates the effect of revenue-neutral tax reforms on social welfare. First of all, it's evident that the baseline economy, without wealth taxation, is not the most efficient approach for raising tax revenue. By marginally subsidizing wealth while increasing the capital income tax rate to maintain constant tax revenue, social welfare can be increased. However, excessively subsidizing wealth and relying heavily on increased capital income tax revenue is not advantageous either, as it may lead to a decrease in social welfare, even below the baseline level. The optimal policy is given by an annual wealth subsidy rate of 2.5%, combined with a rise in the capital income tax rate to 52.6% to generate the additional tax revenue required. This optimal reform yields a welfare increase equivalent to a fixed proportional consumption transfer to all households by 0.027%.

At first glance, this finding may seem to contradict the predictions outlined in Section 2. The numerical calibration analysis reveals that there is a strong correlation between labor productivity and time preference of households. If households with higher income also exhibit greater patience, Corollary 1 states the optimal wealth wedge is positive while the optimal capital income wedge is zero. Nonetheless, it is crucial to acknowledge two significant distinctions between the approaches employed in the previous and current sections. Firstly, the theoretical analysis hinges on optimal distortions, which can solely be interpreted within the framework of non-linear tax rates, dependent on all observable variables simultaneously. Moreover, non-zero wedges prove useful insofar as they dissuade the high-productivity households from mimicking low-productivity households, unlike the linear tax rates studied in this section, which can facilitate redistribution. Secondly, Section 2 centers on the fully optimum solution, whereas this section focuses only on the optimal capital mix given the labor income tax rate and the exogenous revenue requirement.

The main channel through which capital income and wealth tax rates affect social welfare is through the variation in after-tax returns of different assets. In the baseline economy, the annual after-tax return of the return asset is 7.45%, while that of the utility is only 4.69%.[17] Following the optimal reform involving a wealth subsidy and a higher capital income tax rate, the after-tax return of the return asset decreases more than 40 basis points to 7.02%. Conversely, the after-tax return of the utility asset increases by almost 70 basis points to 5.35%.[18] The reason why the after-tax return of the return asset decreases after the optimal reform, while that of the utility asset increases, is that the return asset yields greater before-tax returns. Therefore, the change in its after-tax returns due to the increase in the capital income tax rate overcomes the effect of the increase in the wealth subsidy rate.

3.3.2 Redistributive Effects

To investigate the redistributive impacts of the optimal tax reform, I slightly adjust the macro-level measure used previously to assess the effect of a tax reform on social welfare. Instead, I employ a household-level measure, denoted by CE_i. This measure evaluates the effect of a reform on one household, representing a proportional consumption transfer to that household in every year. Similar to $\overline{CE}$, this measure is calculated such that when it is paid to a household, the resulting lifetime utility of that household is equivalent to that of the economy following the tax reform.

$$U_i(y_i', c_i', a_i') = U_i(y_i, c_i \cdot (1 + CE_i), a_i) \qquad \forall i \tag{17}$$

[17] Note that the annual before-tax returns of the return and utility assets are calibrated at 9.31% and 5.86%, respectively.

[18] See Figure 14 in Appendix D.2 for an illustration of how a given revenue-neutral capital taxation reform alters the after-tax return rates.

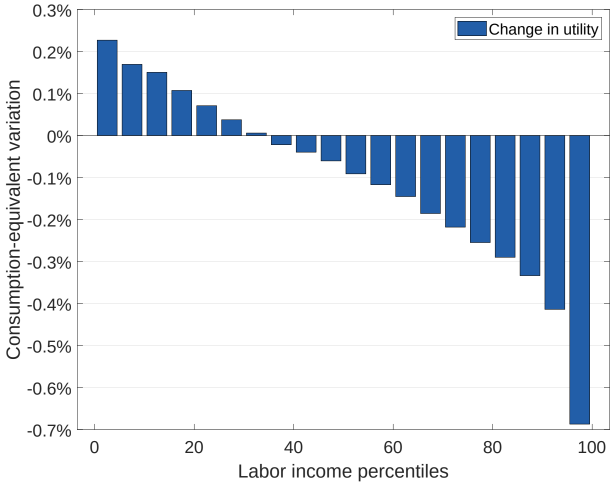


Figure 6: Change in Household Utility

Notes: The figure illustrates the change in household lifetime utility after the optimal reform with respect to the baseline. The y-axis represents the proportional consumption transfer to each household such that their lifetime utility is equal to that in the economy after the optimal reform.

Figure 6 illustrates how the optimal reform impacts households across various points of the labor income distribution. Households with lower labor incomes benefit from the reform, while those at the top of the labor income distribution experience a decrease in utility. Specifically, households in the bottom 5% of the labor income distribution require a consumption transfer of more than 0.2% of their annual consumption, indicating an improvement in their utility. Conversely, households at the very top incur a loss equivalent to about 0.7% of their annual consumption.

The initial observation of Figure 6 might suggest a decrease in social welfare after the optimal reform, given that more households experience a loss compared to those that gain, and the absolute value of the equivalent variation for those who lose is larger. However, it's important to consider that an additional unit of consumption holds greater importance when the initial consumption level is low, as is the case for households at the bottom of the distribution. Therefore, the increase in welfare resulting from a consumption transfer to low-income households outweighs the decrease in welfare due to reduced consumption among high-income households. To make a fair comparison of CE_i values across different households, they should be weighted by the corresponding households' marginal utility of consumption. Indeed, the macro-level measure $\overline{CE}$ indicates that the positive impact on low-income households outweighs the negative impact on high-income households after the optimal reform.

Figure 7 provides a breakdown of the utility change illustrated in Figure 6, differentiating between various decisions of households. This breakdown helps explain how the optimal reform can enhance social welfare while maintaining constant tax revenue.

All households enjoy an increase in lifetime utility thanks to an increase in their

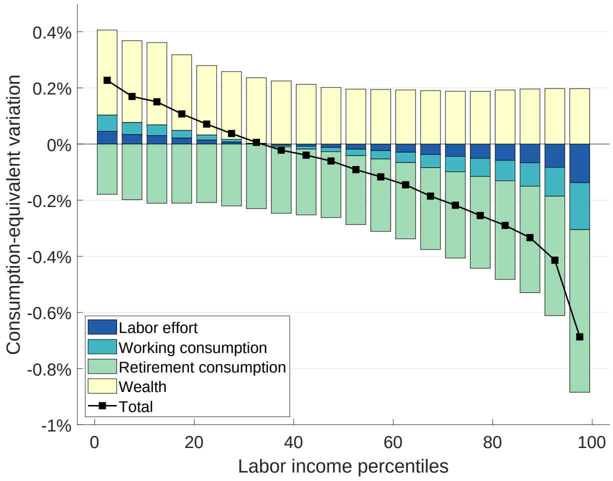


Figure 7: Changes in Sub-utilities

Notes: The figure illustrates the decomposed change in household lifetime utility after the optimal reform with respect to the baseline. The decomposition captures the changes in each part (consumption, wealth, and labor effort) of the lifetime utility. The y-axis represents the proportional consumption transfer to each household such that their lifetime utility is equal to that in the economy after the optimal reform. The black line represents the net change in households' lifetime utility.

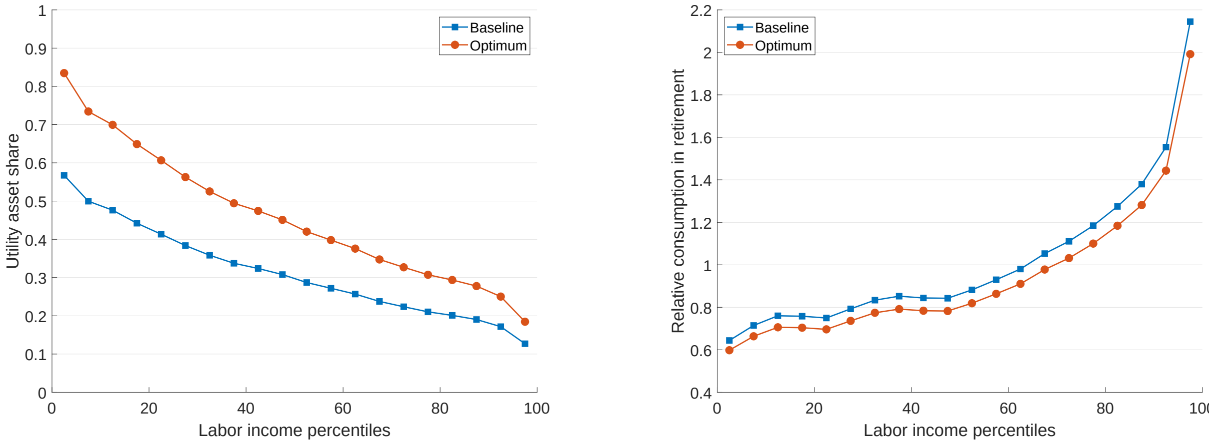
utility asset. Following the reform, the higher after-tax return of the utility asset makes it more attractive for households. Another common adjustment across households is the reduction in retirement consumption. This reduced consumption arises from the transition from the return asset, whose after-tax return decreases after the reform, to the utility asset. However, since the after-tax return of the return asset is still greater than that of the utility asset, households forgo the difference as the opportunity cost, resulting in lower levels of consumption in retirement.

Additionally, households with higher incomes find it optimal to increase their labor supply while reducing their working-life consumption. Conversely, lower-income households work less and yet consume more during their working life. These behavioral changes can be attributed to income effects. Households at the top of the income distribution hold a substantial amount of return assets, resulting in higher tax burdens post-reform due to the raised capital income tax rate. With a decrease in their lifetime resources, they choose to work more and also save more for their retirement. On the other hand, households at the bottom of the income distribution, who hold relatively more utility assets, see an increase in their after-tax return. With this boost in lifetime resources, they choose to work less and save less.[19]

In essence, the optimal reform, which involves raising the capital income tax rate while providing a wealth subsidy, facilitates a redistribution from households with a

[19] The effects that stem from the changes in household behavior are smaller in magnitude compared to the direct mechanical effects resulting from the changes in the after-tax return rates. See Figure 15 in Appendix D.3 for a decomposition.

relatively larger share of return assets in their portfolio, to those with a larger share of utility assets. Given that households with higher labor productivity tend to hold more return assets in their portfolio, the optimal reform effectively transfers resources from these higher-income households to those with lower labor income.



(a) Change in Utility Asset Share

(b) Change in Relative Consumption

Figure 8: Changes in Portfolio Composition and Saving Behavior

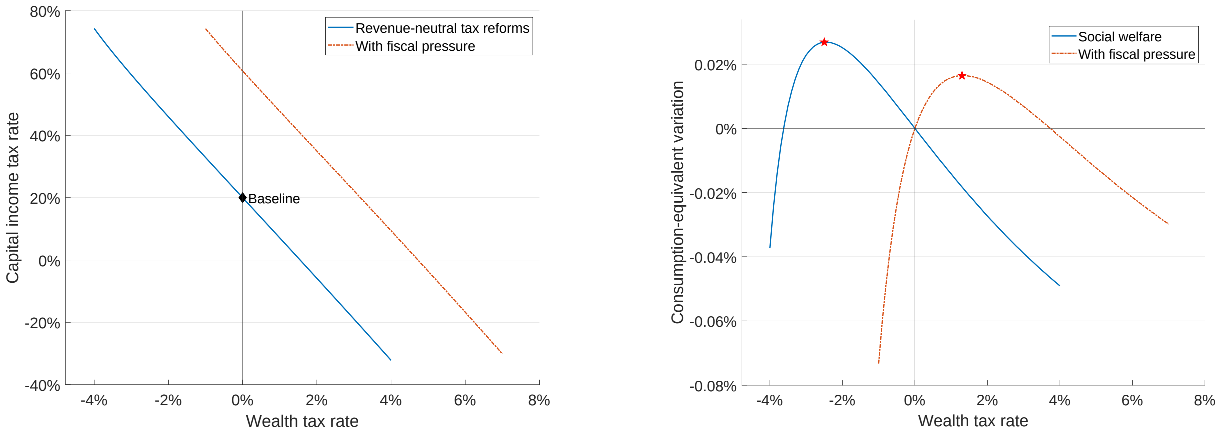
Notes: The left figure illustrates the change in the household portfolio composition, which is defined as the ratio of utility assets to the net worth, after the optimal reform with respect to the baseline. The right figure illustrates the change in household consumption composition after the optimal reform with respect to the baseline. The y-axes represent the ratio of average annual consumption during retirement to the average annual consumption during working life.

Figure 8 provides further insights into how households adjust their behavior following the optimal reform. In the left panel, the portfolio share of the utility asset is depicted for households across the labor income distribution in both the baseline economy and the economy resulting from the reform. All households increase their allocation to the utility asset as it becomes more desirable post-reform. Notably, lower-income households exhibit a larger increase in the utility asset's portfolio share compared to higher-income households. For instance, the utility asset constitutes over 80% of the portfolio for households at the bottom of the income distribution, while it represents less than 20% for the highest-income households.

In the right panel of Figure 8, the change in relative consumption during retirement is depicted, defined as the ratio of annual retirement consumption to annual working-life consumption. With the relative price of the return asset increasing after the reform, all households reduce their relative consumption in retirement. Households with higher labor income continue to consume significantly more annually during retirement—— almost twice as much for households with the highest labor income. This is because they value consumption in later periods more due to high discount rates.

3.3.3 Effects of Fiscal Pressure

To investigate the impact of fiscal pressure on the government, defined as the necessity to raise more tax revenue, I conduct a follow-up numerical analysis. Using the same baseline calibration and utility parameters as before, I examine a scenario where the government is required to raise an additional 20% tax revenue to cover an additional exogenous spending requirement.



(a) Revenue-Neutral Tax Reforms

(b) Social Welfare After Tax Reforms

Figure 9: Impact of Fiscal Pressure

Notes: The figure illustrates the impact of fiscal pressure. The left panel depicts the families of revenue-neutral wealth-capital income tax rate pairs in both the baseline scenario and a scenario with a 20% higher revenue requirement. The right panel displays social welfare in both scenarios. The reference case for the baseline scenario is the current state with a 0% wealth tax rate and a 20% capital income tax rate. In contrast, the reference case for the fiscal pressure scenario is where the capital income tax rate is increased to 60.1% to generate a 20% increase in total tax revenue.

Figure 9 illustrates the impact of a 20% additional revenue requirement on revenue-neutral capital taxation reforms and social welfare. In the left panel, it is observed that in the absence of wealth taxation, the capital income tax rate needs to rise to 60.1% to fulfill the additional revenue requirement.[20] Consequently, the graph of the family of revenue-neutral reforms shifts to the upper right due to fiscal pressure. In the fiscal pressure scenario, the capital income tax rate that balances the budget, given a wealth tax rate, is higher compared to the baseline scenario.

A significant finding emerges from the right panel. As previously discussed, in the baseline scenario, the optimal tax reform involves implementing a negative wealth tax rate (a wealth subsidy) alongside an increased capital income tax rate to maintain the same tax revenue. However, the same does not hold under fiscal pressure. If the government is required to generate 20% additional tax revenue, the optimal capital

[20]This considerable increase in the capital income tax rate stems from the smaller share of capital tax revenue relative to labor income tax revenue.

reform takes the opposite trajectory. In this scenario, the government should introduce a positive wealth tax while reducing the capital income tax rate.

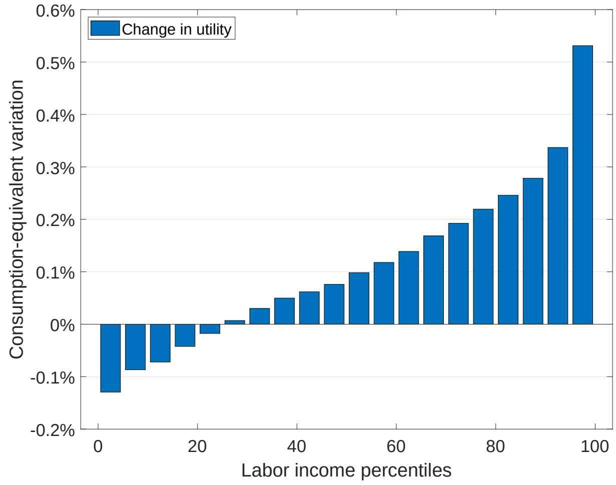


Figure 10: Change in Household Utility in Case of Fiscal Pressure

Notes: The figure illustrates the change in household lifetime utility after the optimal reform with respect to a scenario where the capital income tax increased to 60.1% to generate a 20% increase in total tax revenue. The y-axis represents the proportional consumption transfer to each household such that their lifetime utility is equal to that in the economy after the optimal reform.

In the baseline scenario, the optimal reform enhances welfare by serving as a redistributive mechanism. This reallocation effectively shifts resources from higher-income households to lower-income households. On the other hand, Ayaz et al. (2023) argue that raising additional tax revenue and redistributing between households present two conflicting objectives in tax-transfer system design. Examining Figure 10, it becomes evident that the same dichotomy applies when considering how to tax capital. After implementing the optimal reform, the utility of higher-income households improves, whereas those with lower incomes experience losses. The need to raise additional tax revenue limits the government's redistributive capacity. Consequently, the optimal reform decreases redistribution under fiscal pressure.

4 Conclusion

In this study, I investigate the optimal taxation policy within an economy where households make decisions about labor supply, consumption, and portfolio allocation. A key premise of the analysis is to distinguish between two different types of assets. One type yields greater financial returns, while the other provides flow utility to households as long as it remains in their portfolio. The analysis reveals several important insights into how capital taxation affects households and what the optimal capital taxation policy should be given households' preference heterogeneity.

First, I found that wealth taxation distorts households' inter-temporal decisions while leaving their portfolio allocation decisions unaffected. On the other hand, capital income taxation impacts both inter-temporal decision-making and portfolio allocation. Therefore, the selection of the appropriate capital taxation strategy depends on the underlying preference heterogeneity among households. If temporal preferences vary but relative asset valuations remain consistent across households, wealth taxation proves effective, distorting the inter-temporal margin, while preserving the portfolio allocation margin. Conversely, when households exhibit heterogeneous valuations towards different assets but share the same inter-temporal preferences, capital income taxation becomes useful to address this heterogeneity. Then, its inter-temporal distortion should be corrected by using wealth taxation appropriately.

Second, the calibration of a life cycle model with linear taxes to the U.S. economy reveals that, aside from labor productivity, households mainly differ in their discount rates. Those with higher labor productivity exhibit higher valuations for later periods of their life. As a result, lower-income households tend to hold more of the asset that brings them flow utility over the other generating greater financial returns. In this case, the government has the potential to enhance welfare, while maintaining the same tax revenue, by implementing a dual approach: subsidizing wealth alongside an increase in capital income taxes. This proposed reform acts as a transfer mechanism, redirecting resources from households primarily invested in assets yielding higher returns to those inclined towards assets providing flow utility. In contrast, when faced with an additional revenue requirement, the government's redistributive capacity is constrained, leading to a reversal in the optimal reform direction. In the scenario, where the government aims to increase tax revenue by 20%, the optimal approach involves introducing a positive wealth tax alongside a reduction in the capital income tax rate, effectively reducing the redistribution.

The differences between the outcomes of the theoretical and numerical approaches stem from their focus on different tax instruments. In the theoretical analysis, the emphasis lies on the optimal allocation, where non-linear labor income taxes are particularly effective in achieving desired redistribution outcomes. Both capital income taxes and wealth taxes prove useful insofar as they discourage households with higher labor productivity from mimicking those with lower labor productivity. However, in the simulation analysis, a linear labor income taxation is utilized, which is not as effective. In this context, the optimal mix of capital taxation can significantly enhance welfare by redistributing resources more efficiently.

The findings of this study have important implications for policymakers. By understanding how different capital taxation policies affect household behavior, policymakers can design more efficient and/or equitable tax systems. Moreover, the simulation results highlight a possible efficiency improvement in the U.S. economy. It is possible to enhance social welfare while achieving the same level of tax revenue.

While this study provides valuable insights, it is not without limitations. The theoretical analysis focuses on complex and interdependent tax instruments that might be hard to implement. On the other hand, the simulation analysis focuses on separable linear tax functions, which, may oversimplify real-world tax systems. Striking a balance between tax systems that are too complex to implement and too simple to achieve better outcomes is crucial for designing effective tax policies.

In conclusion, this study contributes to the literature on optimal capital taxation by providing an analysis of different capital taxation methods. By considering a distinction between asset types and preference heterogeneity among households, it offers valuable insights into how capital taxes should be designed taking preference heterogeneity into account.

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A Solution to the Government's Problem

Recall the government's welfare maximization problem in the simple model with two households using the mechanism-design approach. The problem is spelled out in (10). The Lagrange function for this optimization problem is given by

$$\begin{aligned}\mathcal{L}(y^i, c_1^i, c_2^i, a_U^i; \lambda, \mu) = &\sum_{i=l,h} \tilde{f}^i U_i(y^i, c_1^i, c_2^i, a_U^i) \\ &+ \lambda \left[\sum_{i=l,h} f^i \left(y^i - c_1^i - \frac{c_2^i}{1+r} - \frac{r a_U^i}{1+r} \right) \right] \\ &+ \mu \left[U_h(y^h, c_1^h, c_2^h, a_U^h) - U_h(y^l, c_1^l, c_2^l, a_U^l) \right] \end{aligned} \tag{18}$$

Eight first-order conditions need to hold at the optimum. They are with respect to labor income, first-period consumption, second-period consumption, and the utility asset for both households with high and low labor productivity. The conditions are given by

$$y^h: \quad -\tilde{f}^h v'\left(\frac{y^h}{\theta_h}\right)\frac{1}{\theta_h} + \lambda f^h - \mu v'\left(\frac{y^h}{\theta_h}\right)\frac{1}{\theta_h} = 0 \tag{19a}$$

$$c_1^h: \quad \tilde{f}^h u'(c_1^h) - \lambda f^h + \mu u'(c_1^h) = 0 \tag{19b}$$

$$c_2^h: \quad \tilde{f}^h \beta_h u'(c_2^h) - \lambda f^h \frac{1}{1+r} + \mu \beta^h u'(c_2^h) = 0 \tag{19c}$$

$$a_U^h: \quad \tilde{f}^h \phi_h'(a_U^h) - \lambda f^h \frac{r}{1+r} + \mu \phi_h'(a_U^h) = 0 \tag{19d}$$

$$y^l: \quad -\tilde{f}^l v'\left(\frac{y^l}{\theta_l}\right)\frac{1}{\theta_l} + \lambda f^l + \mu v'\left(\frac{y^h}{\theta_l}\right)\frac{1}{\theta_l} = 0 \tag{19e}$$

$$c_1^l: \quad \tilde{f}^l u'(c_1^l) - \lambda f^l - \mu u'(c_1^l) = 0 \tag{19f}$$

$$c_2^l: \quad \tilde{f}^l \beta_l u'(c_2^l) - \lambda f^l \frac{1}{1+r} - \mu \beta_h u'(c_2^l) = 0 \tag{19g}$$

$$a_U^l: \quad \tilde{f}^l \phi_l'(a_U^l) - \lambda f^l \frac{r}{1+r} - \mu \phi_h'(a_U^l) = 0 \tag{19h}$$

Tackle the allocation for the household with high productivity. First, divide the first-order conditions for y^h by the one for c_1^h to solve for the income wedge.

$$
\begin{aligned}
\frac{(\tilde{f}^h + \mu)v'\left(\frac{y^h}{\theta_h}\right)\frac{1}{\theta_h}}{(\tilde{f}^h + \mu)u'(c_1^h)} &= \frac{\lambda f^h}{\lambda f^h} \\
\frac{v'\left(\frac{y^h}{\theta_h}\right)\frac{1}{\theta_h}}{u'(c_1^h)} &= 1 \\
\implies t_y^h &= 0
\end{aligned} \tag{20}
$$

Second, subtract the first-order condition for a_U^h from the one for c_1^h, then divide by the one for c_2^h to solve for the wealth wedge.

$$
\begin{aligned}
\frac{(\tilde{f}^h + \mu)(u'(c_1^h) - \phi_h'(a_U^h))}{(\tilde{f}^h + \mu)\beta_h u'(c_2^h)} &= \frac{\lambda f^h \left(1 - \frac{r}{1+r}\right)}{\lambda f^h \frac{1}{1+r}} \\
\frac{u'(c_1^h) - \phi_h'(a_U^h)}{\beta_h u'(c_2^h)} &= 1 \\
\implies t_w^h &= 0
\end{aligned} \tag{21}
$$

Third, divide the first order condition for c_1^h by the one for a_U^h to solve for the capital income wedge.

$$
\begin{aligned}
\frac{(\tilde{f}^h + \mu)u'(c_1^h)}{(\tilde{f}^h + \mu)\phi_h'(a_U^h)} &= \frac{\lambda f^h}{\lambda f^h \frac{r}{1+r}} \\
\frac{u'(c_1^h)}{\phi_h'(a_U^h)} &= \frac{1+r}{r} \\
\frac{1}{r}\frac{\phi_h'(a_U^h)}{u'(c_1^h) - \phi_h'(a_U^h)} &= 1 \\
\implies t_k^h &= 0
\end{aligned} \tag{22}
$$

Now, tackle the allocation for the household with low productivity. To do so, take the first order condition for c_1^l and rearrange using the definition of the marginal social welfare weight g^l.

$$
\begin{aligned}
\tilde{f}^l {u'_1}^l - \lambda f^l - \mu {u'_1}^l &= 0 \\
\frac{{u'_1}^l}{\lambda}\frac{\tilde{f}^l}{f^l} - 1 - \frac{\mu {u'_1}^l}{\lambda f^l} &= 0 \\
\implies g^l &= \frac{\tilde{f}^l}{\tilde{f}^l - \mu} \qquad (23)
\end{aligned}
$$

First, divide the first-order conditions for y^l by the one for c_1^l to solve for the income wedge.

$$
\begin{aligned}
\frac{(\tilde{f}^l - \mu) v'\left(\frac{y^l}{\theta_l}\right)\frac{1}{\theta_l} + \mu\left(v'\left(\frac{y^l}{\theta_l}\right)\frac{1}{\theta_l} - v'\left(\frac{y^l}{\theta_h}\right)\frac{1}{\theta_h}\right)}{(\tilde{f}^l - \mu) u'(c_1^l)} &= \frac{\lambda f^l}{\lambda f^l} \\
\frac{\mu}{\tilde{f}^l - \mu}\frac{\left(v'\left(\frac{y^l}{\theta_l}\right)\frac{1}{\theta_l} - v'\left(\frac{y^l}{\theta_h}\right)\frac{1}{\theta_h}\right)}{u'(c_1^l)} &= 1 - \frac{v'\left(\frac{y^l}{\theta_l}\right)\frac{1}{\theta_l}}{u'(c_1^l)} \\
\implies t_y^l &= (g^l - 1)\frac{\left(v'\left(\frac{y^l}{\theta_l}\right)\frac{1}{\theta_l} - v'\left(\frac{y^l}{\theta_h}\right)\frac{1}{\theta_h}\right)}{u'(c_1^l)} \qquad (24)
\end{aligned}
$$

Second, subtract the first-order condition for a_U^l from the one for c_1^l, then divide by the one for c_2^l to solve for the wealth wedge.

$$
\begin{aligned}
\frac{(\tilde{f}^l - \mu)(u'(c_1^l) - \phi'_l(a_U^l)) + \mu(\phi'_h(a_U^l) - \phi'_l(a_U^l))}{(\tilde{f}^l - \mu)\beta_l u'(c_2^l) - \mu(\beta_h - \beta_l)u'(c_2^l)} &= \frac{\lambda f^l\left(1 - \frac{r}{1+r}\right)}{\lambda f^l \frac{1}{1+r}} \\
(u'(c_1^l) - \phi'_l(a_U^l)) &= \beta_l u'(c_2^l) - \frac{\mu}{\tilde{f}^l - \mu}\left[(\beta_h - \beta_l)u'(c_2^l) + (\phi'_h(a_U^l) - \phi'_l(a_U^l))\right] \\
\frac{(u'(c_1^l) - \phi'_l(a_U^l))}{\beta_l u'(c_2^l)} &= 1 - \frac{\mu}{\tilde{f}^l - \mu}\left(\frac{\beta_h - \beta_l}{\beta_l} + \frac{\phi'_h(a_U^l) - \phi'_l(a_U^l)}{\beta_l u'(c_2^l)}\right) \\
\frac{\mu}{\tilde{f}^l - \mu}\left(\frac{\beta_h - \beta_l}{\beta_l} + \frac{\phi'_h(a_U^l) - \phi'_l(a_U^l)}{\beta_l u'(c_2^l)}\right) &= 1 - \frac{(u'(c_1^l) - \phi'_l(a_U^l))}{\beta_l u'(c_2^l)} \\
\implies t_w^l &= (g^l - 1)\left(\frac{\beta_h - \beta_l}{\beta_l} + \frac{\phi'_h(a_U^l) - \phi'_l(a_U^l)}{\beta_l u'(c_2^l)}\right) \qquad (25)
\end{aligned}
$$

Third, divide the first-order condition for c_1^l by the one for a_U^l to solve for the capital income wedge.

$$
\begin{aligned}
\frac{(\tilde{f}^l - \mu)\phi_l'(a_U^l) - \mu(\phi_h'(a_U^l) - \phi_l'(a_U^l))}{(\tilde{f}^l - \mu)u'(c_1^l)} &= \frac{\lambda f^l \frac{r}{1+r}}{\lambda f^l} \\
(1+r)\left[(\tilde{f}^l - \mu)\phi_l'(a_U^l) - \mu(\phi_h'(a_U^l) - \phi_l'(a_U^l))\right] &= r(\tilde{f}^l - \mu)u'(c_1^l) \\
(\tilde{f}^l - \mu)\phi_l'(a_U^l) - (1+r)\mu(\phi_h'(a_U^l) - \phi_l'(a_U^l)) &= r(\tilde{f}^l - \mu)(u'(c_1^l) - \phi_l'(a_U^l)) \\
\frac{1}{r}\frac{(\tilde{f}^l - \mu)\phi_l'(a_U^l)}{(\tilde{f}^l - \mu)(u'(c_1^l) - \phi_l'(a_U^l))} &= 1 + \frac{1+r}{r}\frac{\mu(\phi_h'(a_U^l) - \phi_l'(a_U^l))}{(\tilde{f}^l - \mu)(u'(c_1^l) - \phi_l'(a_U^l))} \\
-\frac{\mu}{\tilde{f}^l - \mu}\frac{1+r}{r}\frac{\phi_h'(a_U^l) - \phi_l'(a_U^l)}{u'(c_1^l) - \phi_l'(a_U^l)} &= 1 - \frac{1}{r}\frac{\phi_l'(a_U^l)}{u'(c_1^l) - \phi_l'(a_U^l)} \\
\implies t_k^l &= -(g^l - 1)\frac{1+r}{r}\frac{\phi_h'(a_U^l) - \phi_l'(a_U^l)}{u'(c_1^l) - \phi_l'(a_U^l)} \qquad (26)
\end{aligned}
$$

B Lifecycle Pattern of Assets

The simplified version of the household utility maximization problem, as presented in Section 3, assumes that households receive a fixed labor income annually throughout their working life. Moreover, each year, they allocate the same amount and proportion of their income towards the return asset and the utility asset. Considering that their savings in asset i yield a gross after-tax return of $\overline{R}_i$, the amount of asset i owned by a household at the end of year t during their working life is given by

$$
a_{i,t} = \overline{R}_i a_{i,t-1} + \delta_i a_{i,T_w}, \qquad t \in [1, T_w] \qquad (27)
$$

where a_{i,T_w} is the amount of savings in asset i at the time of retirement. δ_i measures how much the consumption during working life needs to decrease to increase a_{i,T_w} by one unit.

Using backward induction for Equation (27) for each year from year 1 up to year t, and leveraging the initial condition $a_{i,0} = 0$, we derive

$$
a_{i,t} = \delta_i a_{i,T_w} \cdot \sum_{0=1}^{t-1} (\overline{R}_i)^k \qquad (28)
$$

Setting $t = T_w$ allows us to calculate δ_i.

$$
-\frac{\partial c_1}{\partial a_{i,T_w}} = \delta_i = \frac{1}{\sum_{t=0}^{T_w-1} (\overline{R}_i)^t} \qquad (29)
$$

Once the value of δ_i is determined given on $\overline{R}_i$, we can calculate the amount of asset i a household will own at the time of their retirement using Equation (27). During working life, the ratio of asset i holdings at period t to the retirement amount is given by

$$\frac{a_{i,t}}{a_{i,T_w}} = \frac{\sum_{k=0}^{t-1}(\overline{R}_i)^k}{\sum_{t=0}^{T_w-1}(\overline{R}_i)^t} \tag{30}$$

After their retirement, households start to withdraw from their savings to fund their retirement consumption. However, their remaining savings at the end of each year continue to yield a financial return. The amount of asset i owned by a household at the end of year t during their working life is given by

$$a_{i,t} = \overline{R}_i a_{i,t-1} - \sigma_i a_{i,T_w}, \qquad t \in [T_w + 1, T_w + T_r] \tag{31}$$

σ_i measures how much the increase in consumption during retirement when a_{i,T_w} increases by one unit.

Once again, using backward induction, this time for Equation (31) for each year from year T_w for t years up to year $t + T_w$ yields

$$a_{i,t+T_w} = a_{i,T_w}(\overline{R}_i)^t - \sigma_i a_{i,T_w} \sum_{k=1}^{t-1}(\overline{R}_i)^k \tag{32}$$

Setting $t = T_r$ and leveraging the asset depletion condition $a_{i,T_w+T_r} = 0$ allows us to calculate σ_i.

$$\frac{\partial c_2}{\partial a_{i,T_w}} = \sigma_i = \frac{(\overline{R}_i)^{T_r}}{\sum_{k=0}^{T_r-1}(\overline{R}_i)^k} \tag{33}$$

During retirement, the ratio of asset i holdings at year $t + T_w$ to the retirement amount is given by

$$\frac{a_{i,t+T_w}}{a_{i,T_w}} = \frac{\sum_{k=t}^{T_r-1}(\overline{R}_i)^k}{\sum_{k=0}^{T_r-1}(\overline{R}_i)^k} \tag{34}$$

Figure 11 summarizes the life cycle pattern of both the return asset and the utility asset, employing Equations (28) and (32).

Additionally, we can also calculate the implied inter-period between working life and retirement.

$$-\frac{\partial c_2}{\partial c_1} = \frac{\sigma_i}{\delta_i} = (\overline{R}_i)^{T_r} \cdot \frac{\sum_{t=0}^{T_w-1}(\overline{R}_i)^t}{\sum_{t=0}^{T_r-1}(\overline{R}_i)^t} \tag{35}$$

The ratio $\frac{\sigma_i}{\delta_i}$ measures the increase in retirement consumption when households reduce their working life consumption by one unit and allocate those savings to asset i. The

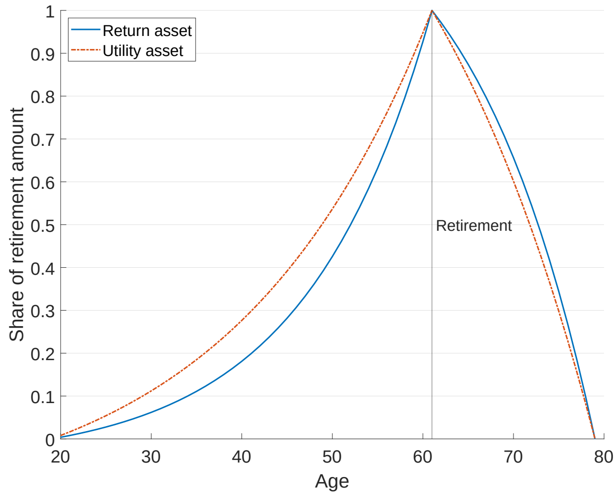


Figure 11: Lifecycle Pattern of Assets

Notes: The figure illustrates the life cycle pattern of the return and utility assets as a ratio of their value at households' retirement. The blue line shows the evolution of the return asset, and the red line shows the evolution of the utility asset. For this figure, the calibrated values of tax and interest rates are utilized. The annual interest of the return asset is 9.31% and the annual interest of the utility asset 5.35%. The capital income tax rate is given by 20%, whereas the wealth tax rate is 0%.

inter-period gross interest rate for the return asset is calibrated to be 26.78, while that for the utility asset is 10.42. It's important to note that these gross return rates reflect financial yields over an extended period and also consider the fact that retirement is shorter than working life. Therefore, even without any annual returns, a household can increase their retirement consumption by $\frac{T_w}{T_r} > 1$ units by reducing their working life consumption by one unit.

B.1 Lifetime Discount Rate of the Utility Asset

The lifetime utility of wealth can be expressed as the discounted sum of the annual utility of wealth at the end of each period.

$$\phi_i^{LT} = \sum_{t=1}^{T_w} \beta_i^t \phi_i \left(a_{U,t}\right) + \sum_{t=T_w+1}^{T_w+T_r-1} \beta_i^t \phi \left(a_{U,t}\right) \tag{36}$$

The utility of wealth is assumed to exhibit CRRA property, given by

$$\phi_i(a_{U,t}) = \xi_i \frac{a_{U,t}^{1-\mu} - 1}{1-\mu}$$

Using this functional form assumption, we can calculate the utility of wealth for any fraction k of the utility asset at retirement.

$$
\begin{aligned}
\phi_i(k \cdot a_{U,T_w}) &= \xi_i \frac{(k \cdot a_{U,T_w})^{1-\mu} - 1}{1-\mu} \\
&= k^{1-\mu} \cdot \xi_i \frac{a_{U,T_w}^{1-\mu} - 1}{1-\mu} + \frac{k^{1-\mu} - 1}{1-\mu} \\
&= k^{1-\mu} \cdot \phi_i(a_{U,T_w}) + \phi(k)
\end{aligned} \tag{37}
$$

Setting $k = \frac{a_{U,t}}{a_{U,T_w}}$ allows us to calculate the utility of wealth at any given period in terms of the utility of wealth at retirement.

$$
\phi(a_{U,t}) = \left(\frac{a_{U,t}}{a_{U,T_w}}\right)^{1-\mu} \phi(a_U) + \phi\left(\frac{a_{U,t}}{a_{U,T_w}}\right)
$$

Then, the lifetime utility of wealth reads as

$$
\begin{aligned}
\phi_i^{LT} = &\underbrace{\left(\sum_{t=1}^{T_w} \beta_i^t \left(\frac{a_{U,t}}{a_{U,T_w}}\right)^{1-\mu} + \sum_{t=T_w+1}^{T_w+T_r-1} \beta_i^t \left(\frac{a_{U,t}}{a_{U,T_w}}\right)^{1-\mu}\right)}_{\equiv \beta_i^{\phi}} \cdot \phi_i\left(a_{U,T_w}\right) + \\
&\underbrace{\left(\sum_{t=1}^{T_w} \beta_i^t \phi_i \left(\frac{a_{U,t}}{a_{U,T_w}}\right)^{1-\mu} + \sum_{t=T_w+1}^{T_w+T_r-1} \beta_i^t \phi_i \left(\frac{a_{U,t}}{a_{U,T_w}}\right)^{1-\mu}\right)}_{\equiv \phi_i^C}
\end{aligned} \tag{38}
$$

Equation (30) provides information about the ratio $\frac{a_{U,t}}{a_{U,T_w}}$ for working life, while Equation (34) does the same for retirement.

C Individual Optimization Algorithm

Consider the following constrained utility maximization problem

$$
\begin{aligned}
\max_{c_1, c_2, y, a_U, a_R} \quad & U(c_1, c_2, y, a_U, a_R) \\
\text{s.t.} \quad & c_1 = y - a_U - a_R \\
& c_2 = a_U + a_R R
\end{aligned} \tag{39}
$$

where c_1 and c_2 represent the first and the second period consumption, y represents the labor income, a_U represents the utility asset, and a_R represents the return asset.

This problem is constructed without taxes to keep the expressions simpler; however, the following arguments can be also made for a maximization problem including income and capital taxes as long as there is no cross-dependency between any two taxes.

The Lagrange function of this optimization problem is given by

$$\mathcal{L} = U(c_1, c_2, y, a_U, a_R) - \lambda_1(y - c_1 - a_U - a_R) - \lambda_2(a_U + a_R - c_2). \tag{40}$$

First-order conditions for five choice variables and two Lagrange coefficients are given by

$$\begin{aligned}
c_1 :& \quad U'_{c_1} + \lambda_1 = 0 && \text{(41a)}\\
c_2 :& \quad U'_{c_2} + \lambda_2 = 0 && \text{(41b)}\\
y :& \quad U'_y - \lambda_1 = 0 && \text{(41c)}\\
a_U :& \quad U'_{a_U} + \lambda_1 - \lambda_2 = 0 && \text{(41d)}\\
a_R :& \quad U'_{a_R} + \lambda_1 - \lambda_2 R = 0 && \text{(41e)}\\
\lambda_1 :& \quad y = c_1 + a_U + a_R && \text{(41f)}\\
\lambda_2 :& \quad a_U + a_R R = c_2. && \text{(41g)}
\end{aligned}$$

where $U'_k = \frac{\partial U}{\partial k}$ is the partial derivative of lifetime utility with respect to k.

A set of choice variables and Lagrange coefficients that satisfy the seven first-order conditions above yields the utility-maximizing and feasible individual choice.

Combining Equations (41a) and (41c) gives us the optimality condition that governs the choice between the first-period consumption and the labor income.

$$U'_y = -U'_{c_1} \tag{42}$$

On the other hand, combining Equations (41a), (41b), and (41e), and making use of the fact that a_R does not affect individual utility ($U'_{a_R} = 0$), we can deduce the optimality condition that governs the choice between the first and the second-period consumption.

$$U'_{c_2} = U'_{c_1}\frac{1}{R} \tag{43}$$

Similarly, combining Equations (41a), (41d), and (41e) yields the following condition.

$$U'_{a_U} = U'_{c_1}\frac{R-1}{R} \tag{44}$$

If we assume the utility function is weakly separable, that is, the partial derivative of a choice variable does not depend on other choice variables, then we can rewrite Equations (42), (43), and (44) to obtain a closed-form solution of the labor income, the second-period consumption, and the utility asset in terms of the first-period consumption.

$$y = {U'_y}^{-1}(-U'_{c_1}) \equiv \tilde{y}(c_1) \tag{45}$$

$$c_2 = {U'_{c_2}}^{-1}\left(U'_{c_1}\frac{1}{R}\right) \equiv \tilde{c_2}(c_1) \tag{46}$$

$$a_U = {U'_{a_U}}^{-1}\left(U'_{c_1}\frac{R-1}{R}\right) \equiv \tilde{a_U}(c_1) \tag{47}$$

The pairs on each of these closed-form solutions are consistent with the respective optimality conditions. However, they are not necessarily feasible. For example, any pair (c_1, c_2) that satisfies $c_2 = \tilde{c_2}(c_1)$ is consistent with the optimality condition between c_1 and c_2. But, no claim can be made on the *feasibility* of those pairs.

To ensure that the individual choices are feasible, we can obtain another expression for c_2 using the budget constraints. First, consider Equation (41f) and use the closed form solutions we found in Equations (45) and (47).

$$\begin{aligned} &\tilde{y}(c_1) = c_1 + \tilde{a_U}(c_1) + a_R \\ \implies\ & a_R = \tilde{y}(c_1) - c_1 - \tilde{a_U}(c_1) \equiv \hat{a_R}(c_1) \end{aligned} \tag{48}$$

Then, plug in Equations (47) and (48) into Equation (41g).

$$\tilde{a_U}(c_1) + \hat{a_R}(c_1) = c_2 \equiv \hat{c_2}(c_1) \tag{49}$$

Equation (49) gives us a set of pairs (c_1, c_2) that are consistent with the optimality conditions between c_1 and y; as well as c_1 and a_U. The fact that we used the budget constraints to obtain $\hat{c_2}(c_1)$ ensures that the choice of c_2 is feasible given the optimal choices of y and a_U.

On the one hand, we have a closed-form expression of c_2 that is consistent with *the optimality condition* between c_1 and c_2 from Equation (46). On the other hand, we have another expression that is consistent with the optimality conditions of y and a_U, and also *feasible* according to the budget constraints from Equation (49). If there exists a c_1^*

such that

$$\tilde{c_2}(c_1^*) = \hat{c_2}(c_1^*), \tag{50}$$

then the choice set $(c_1^*, \tilde{c_2}(c_1^*), \tilde{y}(c_1^*), \tilde{a_U}(c_1^*), \tilde{a_R}(c_1^*))$ solves the constrained utility maximization problem given in Equation (39).

D Additional Figures

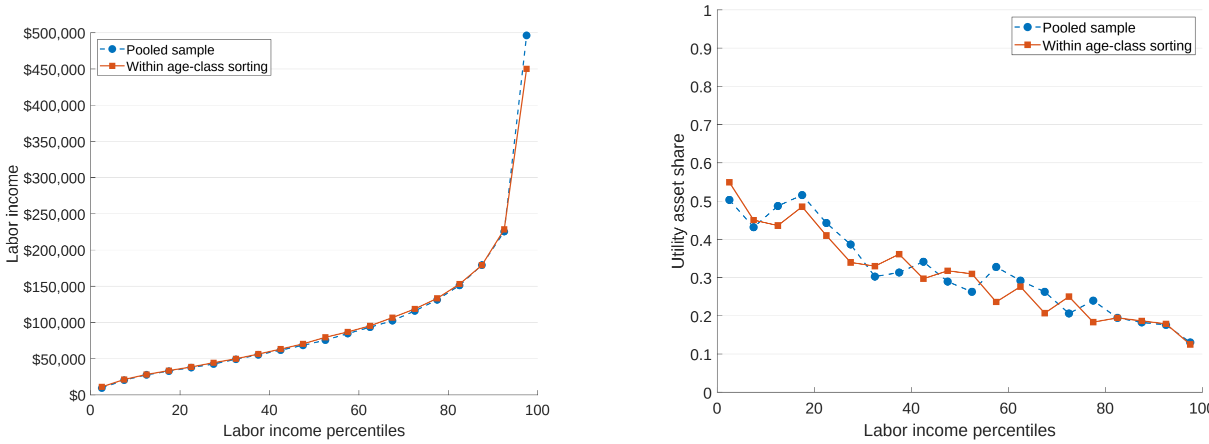
D.1 Within Age-Class Calibration Method

(a) Survey Weights

(b) Labor Income Within Classes

Figure 12: Age-Class Grouping

Notes: The left figure depicts the survey weights of each age class as a ratio of the total survey weights of the whole sample. The right figure illustrates the income distribution with each age class. The blue dashed line shows the 25th percentile, the red line shows the median, and the yellow dashed line shows the 75th percentile of the within-age-class labor income distribution.

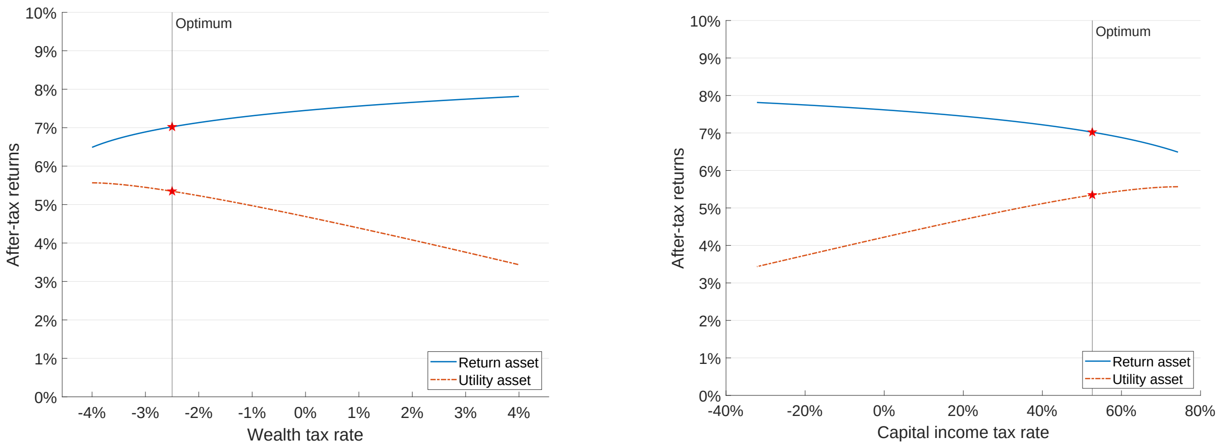


(a) Labor Income (b) Portfolio Share

Figure 13: Comparison of Two Methods

Notes: The figure compares the calibration results using the within-age-class sorting method and the pooled sample. The left panel depicts the calibrated labor income and the right panel depicts the calibrated utility asset share along the labor income distribution. In both panels, the blue dashed line shows the result when using the pooled sample and the red line shows the result when using the within-age-class sorting method

D.2 After-Tax Return of Assets



(a) Respecting Wealth Tax Rate (b) Respecting Capital Income Tax Rate

Figure 14: Change in After-Tax Returns

Notes: Both figures illustrate the change in after-tax returns of both the return asset and the utility asset. The left panel pertains to variations in the wealth tax rate, while the right panel pertains to variations in the capital income tax rate. In both figures, the other capital tax rate is adjusted to maintain revenue neutrality. The before-tax return of the return asset is calibrated at 9.31%, while that of the utility asset is at 5.86%. The red stars denote the after-tax returns following the optimal reform. Specifically, the optimal after-tax return of the return asset is 7.02%, and that of the utility asset is 5.35%.

D.3 Decomposition of Utility Change into Mechanical and Behavioral Effects

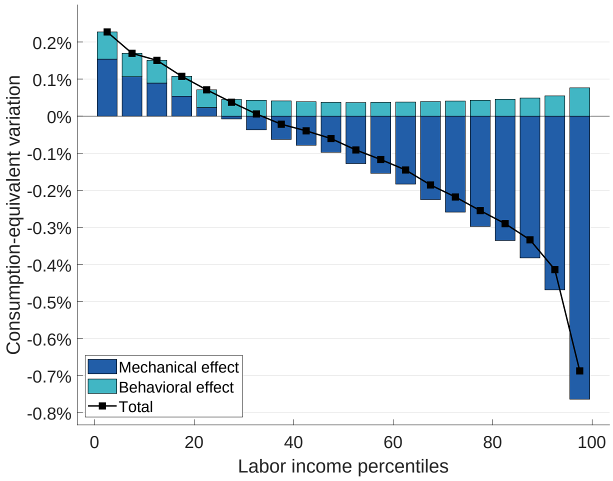


Figure 15: Mechanical and Behavioral Changes

Notes: The figure illustrates the decomposed change in household lifetime utility after the optimal reform with respect to the baseline. The decomposition captures the mechanical effect, which is defined by the change in utility keeping households' behavior fixed, and the behavioral effect, which is defined by the change in utility due to behavioral responses. The y-axis represents the proportional consumption transfer to each household such that their lifetime utility is equal to that in the economy after the optimal reform. The black line represents the net change in households' lifetime utility.