2.

$$\boldsymbol{X(z) = \frac{10}{(z-1)(z-2)}}$$

If we want to inverse Z-transform, we use inverse transform integal.

$$x(kT) = \frac{1}{2\pi i}\oint_C X(z)\, z^{k-1} dz$$

So $x(kT) = Sum\ of\ Residues\ of\ X(z)$ and,

$$x(kT) = \lim_{z\to 1}\left[(z-1)\frac{20z^{k-1}}{(z-1)(z-2)}\right] + \lim_{z\to 2}\left[(z-2)\frac{20z^{k-1}}{(z-1)(z-2)}\right]$$

$$x(kT) = -20 + 10.2^k$$

3.

Analitically;

$$x(k+2) - x(k+1) + 0.25x(k) = u(k+2)$$

If we take the z-transform of the difference equation;

$$x(k+2) = z^2X(z) - z^2X(0) - zX(1)$$

$$x(k+1) = zX(z) - zX(0)$$

$$x(k) = X(z)$$

$$u(k+2) = z^2U(z) - z^2U(0) - zU(1)$$

If we rewrite the difference equation;

$$[z^2X(z) - z^2X(0) - zX(1)] - [zX(z) - zX(0)] + 0.25X(z) = z^2U(z) - z^2U(0) - zU(1)$$

We put the starting points $x(0) = 1$, $x(1) = 2$ and $u(k) = 1$ in the equation;

$$[z^2X(z) - z^2 - 2z] - [zX(z) - z] + 0.25X(z) = z^2U(z) - z^2 - z$$

$$X(z)(z^2 - z + 0.25) - z^2 - z = z^2U(z) - z^2 - z$$

And we get; $X(z) = \frac{z^2}{z^2 - z + 0.25} U(z)$

Since $U(z) = Z\{U(k)\} = \frac{z}{z-1}$;

$$X(z) = \frac{z^3}{(z-0.5)^2(z-1)}$$

$$x(kT) = \frac{1}{2\pi i} \oint_C X(z)\, z^{k-1} dz$$

$$\oint X(z)\, dz = 2\pi i \sum_{k=1}^{n} K(f, a_k)$$

Residue for the pole $K_1 = z_1 = 1$;

$$K_1 = \lim_{z=1} (z-1) \frac{z^3 \times z^{k-1}}{(z-0.5)^2(z-1)} = \frac{1}{(0.5)^2} = 4$$

Residue for the pole $K_2 = z_2 = ½$;

$$K_2 = \lim_{z=0.5} \frac{d}{dz} \left\{ (z-0.5)^2 \frac{z^3 \times z^{k-1}}{(z-0.5)^2(z-1)} \right\} = \lim_{z=0.5} \frac{d}{dz} \left\{ \frac{z^{k+2}}{(z-1)} \right\}$$

$$Res_{2,3} == \lim_{z=0.5} \frac{(k+1)z^{k+2} - (k+2)z^{k+1}}{(z-1)^2} = 4 \left[(k+1) \left(\frac{1}{2}\right)^{k+2} - (k+2) \left(\frac{1}{2}\right)^{k+1} \right]$$

$$x(kT) = \frac{1}{2\pi i} \oint_C X(z)\, z^{k-1} dz$$

$$x(kT) = \frac{1}{2\pi i} 2\pi i \sum_{k=1}^{n} K(f, a_k)$$

$$x(kT) = K_1 + K_2$$

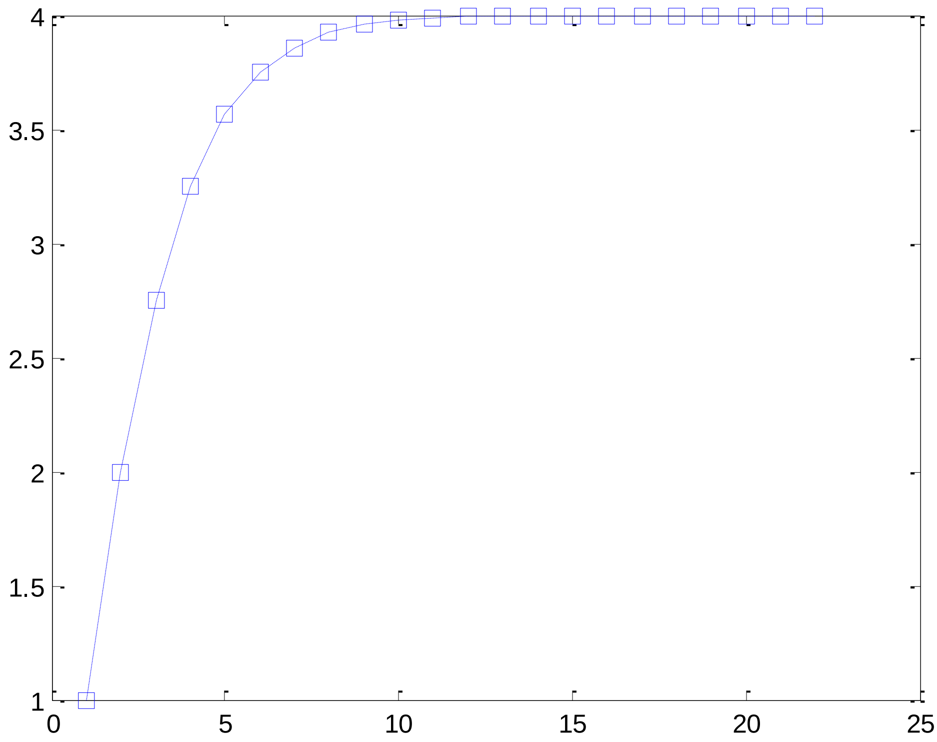
$$x(kT) = 4 \left[1 + (k+1) \left(\frac{1}{2}\right)^{k+2} - (k+2) \left(\frac{1}{2}\right)^{k+1} \right]$$

MATLAB:

>> % Computer Controlled Systems lecture Project 1, Question 3

>> % Due to: 29.02.2016   Group 4

```
>>
>> x(1)=1;
>> x(2)=2;
>> for i=1:1:20
x(i+2)=1+x(i+1)-0.25*x(i);
end
>> plot(x,'--s')
```



The system starts and settles at 4.

4)

$$X(s) = \frac{3(s+3)}{(s+1)(s+2)}$$

First we need to factorize the transfer function.

$$X(s) = \frac{A}{(s+1)} + \frac{B}{(s+2)} = \frac{6}{(s+1)} - \frac{3}{(s+2)}$$

After factorization, we can find the discrete time transfer function as below;

$$X(z) = \frac{2z}{(z+e^{-T})} + \frac{-z}{(z+e^{-2T})}$$

Also it can be found by residue theorem,

Respectively, we find the residues for poles;

$$Res_1 = \lim_{s=-1}(s+1)\frac{(s+3)}{(s+1)(s+2)}\frac{z}{z-e^{Ts}} = \frac{2z}{z-e^{T}}$$

$$Res_2 = \lim_{s=-2}(s+2)\frac{(s+3)}{(s+1)(s+2)}\frac{z}{z-e^{Ts}} = \frac{-z}{z-e^{2T}}$$

$$X(z) = Res_1 + Res_2$$

$$X(z) = \frac{2z}{z-e^{-T}} + \frac{-z}{z-e^{-2T}}$$