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2015 - 2016 Fall Term
Deadline: 26.10.2015

FEEDBACK CONTROL SYSTEMS (KON 313E)

HOMEWORK ASSIGNMENT - 1

Question 1: Solve the problems given below.

a. $\ddot{y} + 4\dot{y} - 8 = 0 \quad \Longrightarrow \quad y(t) = ? \quad (\dot{y}(0) = y(0) = 0)$

b. $f(t) = \begin{cases} 0, & t < 0 \\ 3 & 0 \leq t < 1 \\ 4 - t, & 1 \leq t < 4 \\ 0, & t \geq 4 \end{cases} \quad \Longrightarrow \quad \mathcal{L}\{f(t)\} = ?$

c. $\mathcal{L}^{-1}\left\{\frac{3}{s^2+2s+10}\right\} = ?$

d. $\begin{aligned} \dot{\mathbf{x}}(t) &= \boldsymbol{A}\,\mathbf{x}(t) + \boldsymbol{B}\,\mathrm{u}(t) \\ \mathrm{y}(t) &= \boldsymbol{C}\,\mathbf{x}(t) + \boldsymbol{D}\,\mathrm{u}(t) \end{aligned} \quad \Longrightarrow \quad \frac{Y(s)}{U(s)} = ? \quad (\boldsymbol{x}(0) = 0)$

Question 2: A mechanical system is given in Figure 1. If the step input f is applied to the mass of M, the position (y) of the mass of M varies as in Figure 2.

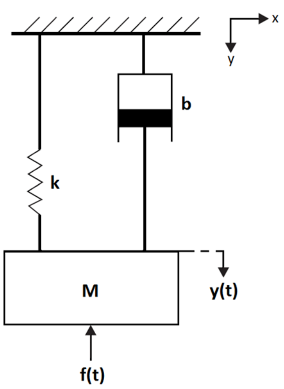

Figure 1. A mechanical system

a. Calculate the overshoot, peak time and gain of the system.

b. Obtain the open-loop transfer function G(s) and calculate the viscous friction coefficient (b), the spring constant (k) and the value of mass of M.

c. Calculate the damping ratio (ζ) and the natural frequency (ω_n).

d. It is expected to halve the peak time without changing the overshoot of system. Which parameter should be changed and what should its new value be?

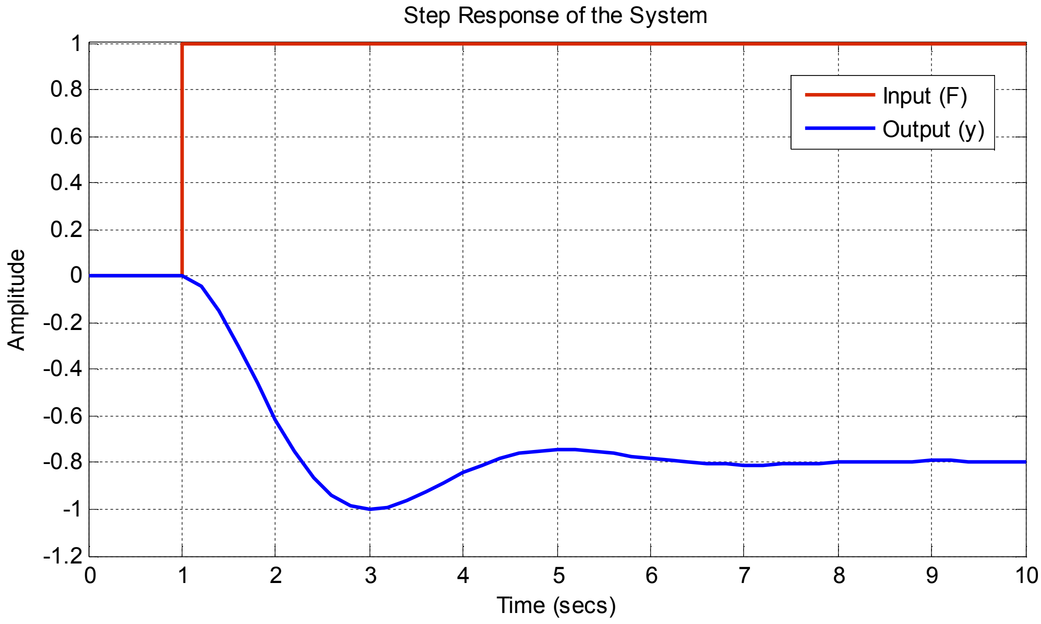

Figure 2. Step response of the system

<u>Question 3:</u> For a closed-loop system whose block diagram is given below,

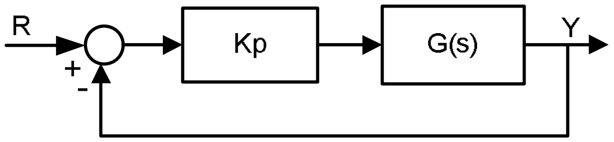

Figure 3. Closed-loop control system with unit feedback

It is known that G(s) is a second order system and has one zero. In addition, for $K_p = 1$ the closed-loop system poles are located at the points of $s_{1,2} = -\frac{3}{2} \pm j\frac{\sqrt{3}}{2}$ and for $K_p = 2$ the closed-loop system has double poles at the point of $s = -2$.

a. Find the G(s) open-loop transfer function.

b. What is the order of the system G(s)? Show the poles and zeros of the system in s-domain. Is the open-loop system stable? Explain.

c. Is there any K_p value which brings two of the system poles to the point of $s = -3$ in the closed-loop? Calculate if exists, otherwise indicate the reason.

d. Draw the open-loop step response of the system G(s) with the help of Matlab or Mathematica. After that plot the step response again but this time neglect the open-loop zero. According to this, discuss the effect of zero on the transient response.

Question 4: For a system whose input-output relation is given with the differential equation

$$y^{(4)}(t) + 10\,y'''(t) + 33\,y''(t) + 40\,y'(t) + 16\,y(t) = K\,u(t)$$

$(y'''(0) = y''(0) = y'(0) = y(0) = 0)$, explain if the expressions given below are "true" or "false" with admissible reasons.

a. The order of system is 4.

b. Roots of system are located on the imaginary axis.

c. System is stable.

d. System is converted to the closed-loop system with a unit feedback for $\mathrm{K} = 16$. Overshoot of the closed-loop system will be about 15% (Note that dominant poles have the greatest impact on system behaviour for higher order systems).

e. For the above case, settling time of the closed-loop system will be about 2 seconds.

f. $u(t) = x'(t) - x(t)$ is given $(x(0) = 0)$. The resulting new system **(**input x(t) - output y(t)**)** is an unstable system.

g. For the above case, the order of the system will be increased by 1.

h. Again for the above case, if the input signal $x(t) = 2$ is applied to the system, output of the system will be settled to a negative value $(K = 1)$.