NUMERICAL OPTIMIZATION

KON428E Introduction to Optimal Control

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Outline

- **Numerical Optimization**
  - Hooke & Jeeves Algorithm
  - Genetic Algorithm
  - Big Bang-Big Crunch Optimization

- **Discussion About The Final Project**

Numerical Optimization

- **Optimization**

  - The selection of a best element (with regard to some criteria) from some set of available alternatives.

- **Numerical Optimization**

  - Fitness Function

  - Constraint Function

  - Iterative Solution

    $x(k) = x(k-1) + \text{correction function}$

Numerical Optimization

- **Local Optimization Methods**
  - Hooke & Jeeves Algorithm

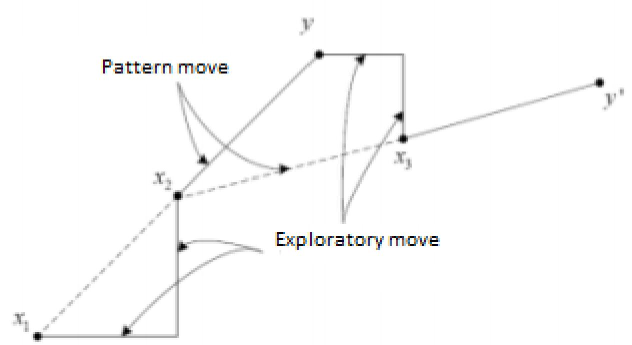
- **Global Optimization Methods**
  - Genetic Algorithm
  - Big Bang – Big Crunch Optimization Algorithm

Hooke & Jeeves Algorithm

- Aim is to find the minimum value of function f(x)
- **2 type of search:**

  - Exploratory search

  - Pattern search



- **Procedure**

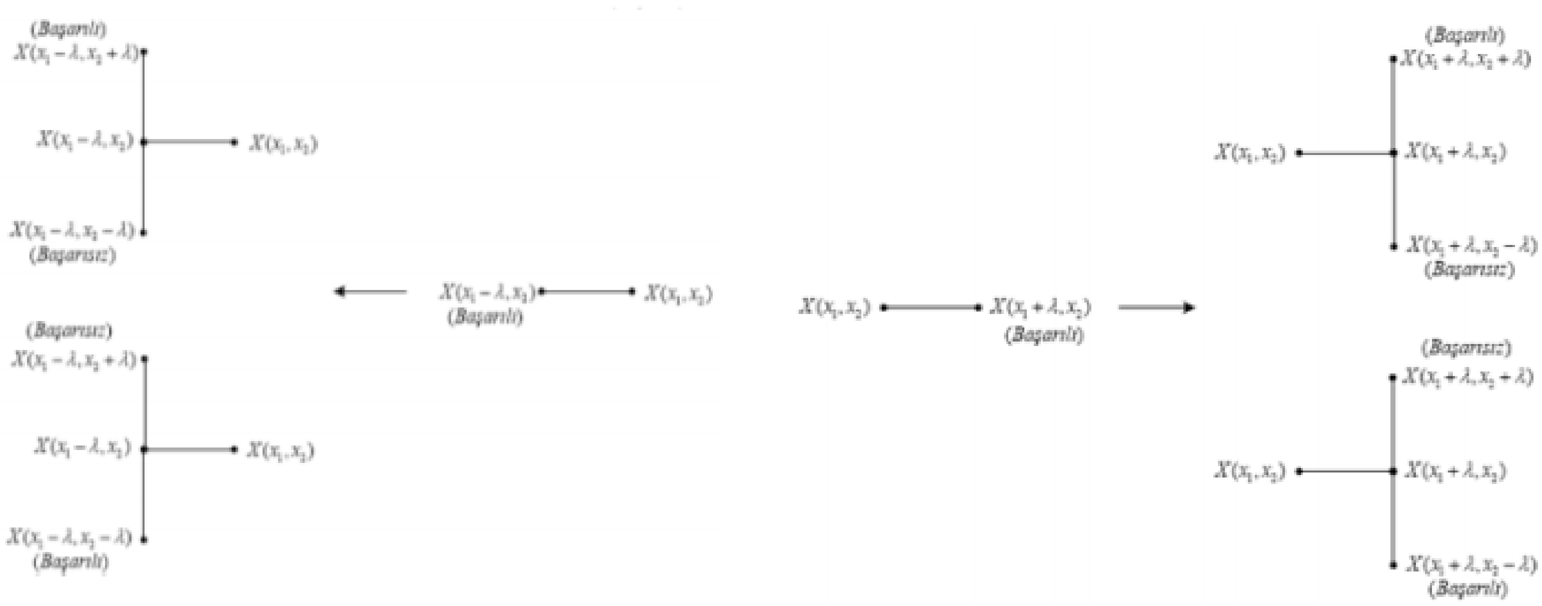
  - Initial point and step length is determined.

  - Exploratory and pattern move is applied.

  - If the value of function f(x) decreases after the exploratory move, it is called as successful point else it is called as unsuccessful point.

Hooke & Jeeves Algorithm

- **Exploratory Search:**



Hooke & Jeeves Algorithm

- After the exploratory search;

  - If a successful point is found, pattern search is done.

  - If all possible points are found to be unsuccessful, step length is halved and exploratory search is repeated.

- Algorithm is terminated if the minimum step length is reached.

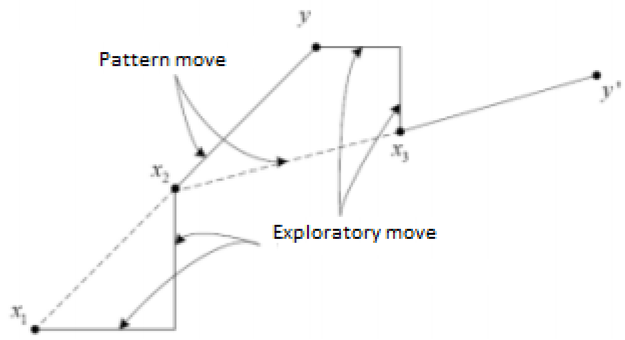
Hooke & Jeeves Algorithm

- **Pattern Search**

  - Let b_2 be a successful point after the exploratory search.

  - The new point, which is obtained by pattern search, is given as follows:

$$p_1 = b_2 + (b_2 - b_1) = 2\,b_2 - b_1$$



Hooke & Jeeves Algorithm

- Algorithm continues with the exploratory search near the point p_1 obtained in pattern search.
- If any of the points in the exploratory search is found to be successful, the new reference point is taken as this successful point.
- Otherwise, exploratory search is done near the previous point (b_2).

Example

Hooke & Jeeves metodu kullanarak aşağıda verilen fonksiyonun;

$$f(x) = 3x_1^2 - 2x_1x_2 + x_2^2 + 4x_1 + 3x_2$$

$b_1 = [0, 0]$ başlangıç değeri
$h_1 = h_2 = 1$ adım aralığı
$h_1 = h_2 < \frac{1}{4}$ durdurma şartı

için minimum noktasını bulunuz.

Example

Gerek koşul

$$\left.\begin{array}{l}\dfrac{\partial f}{\partial x_1} = 6x_1 - 2x_2 + 4 = 0 \\[2ex] \dfrac{\partial f}{\partial x_2} = 2x_2 - 2x_1 + 3 = 0\end{array}\right\} \underbrace{x_1 = -1.75 \;\; x_2 = -3.25}_{\textit{durağan nokta}}$$

Yeter koşul

$$\left.\begin{array}{ll}\dfrac{\partial f}{\partial x_1 \partial x_1} = 6 & \dfrac{\partial f}{\partial x_1 \partial x_2} = -2 \\[2ex] \dfrac{\partial f}{\partial x_2 \partial x_1} = -2 & \dfrac{\partial f}{\partial x_1 \partial x_2} = 2\end{array}\right\} H = \begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix} = 8 > 0$$

Example

$E(b_1)$:

$$\begin{aligned} &f[1,0] = 7 && (F) \\ &f[-1,0] = -1 && (S) \\ &f[-1,1] = 5 && (F) \\ &f[-1,-1] = -5 && (S) \end{aligned}$$

Yeni referans noktamız,

$b_2 = [-1,-1] \quad \rightarrow \quad f(b_2) = -5$

$p(b_2)$: $\quad p_1 = 2b_2 - b_1 = 2[-1,-1] - [0,0] = [-2,-2]$

Example

$E(p_1)$:

$$f[-1,-2] = -7 \quad (S)$$
$$f[-1,-1] = -5 \quad (F)$$
$$f[-1,-3] = -7 \quad (F)$$

Yeni referans noktamız,

$b_3 = [-1,-2] \quad \rightarrow \quad f(b_3) = -7$

$p(b_3)$: $\quad p_2 = 2b_3 - b_2 = 2[-1,-2] - [-1,-1] = [-1,-3]$

Example

$E(p_2)$:

$$f[0,-3] = 0 \quad (F)$$
$$f[-2,-3] = -8 \quad (S)$$
$$f[-2,-2] = -6 \quad (F)$$
$$f[-2,-4] = -8 \quad (F)$$

Yeni referans noktamız,

$$b_4 = [-2,-3] \quad \rightarrow \quad f(b_4) = -8$$

$p(b_4)$: $\quad p_3 = 2b_4 - b_3 = 2[-2,-3] - [-1,-2] = [-3,-4]$

Example

$E(p_3)$:

$$f[-2,-4] = -8 \quad (F)$$
$$f[-4,-4] = 4 \quad (F)$$
$$f[-3,-5] = -5 \quad (F)$$
$$f[-3,-3] = -3 \quad (F)$$

Referans noktamız değişmedi,

$$b_4 = [-2,-3] \quad \rightarrow \quad f(b_4) = -8$$

Example

$E(b_4)$:

$$f[-1,-3] = -7 \quad (F)$$
$$f[-3,-3] = -3 \quad (F)$$
$$f[-2,-2] = -6 \quad (F)$$
$$f[-2,-4] = -8 \quad (F)$$

$E(b_4)$ yani b_4 civardaki aramada da tüm noktalar başarısız olduğundan adım aralığı yarıya düşürülüp tekrar civar araması yapılmalı

$b_4 = [-2,-3]$ $\qquad h_1 = h_2 = \frac{1}{2}$

Example

$E(b_4)$:

$$f[-1.5,-3] = -8.25 \quad (F)$$
$$f[-1.5,-2.5] = -8 \quad (F)$$
$$f[-1.5,-3.5] = -8 \quad (F)$$

Yeni referans noktamız,

$b_5 = [-1.5,-3] \quad \rightarrow \quad f(b_5) = -8.25$

$p(b_5)$: $\quad p_4 = 2b_5 - b_4 = 2[-1.5,-3] - [-2,-3] = [-1,-3]$

Example

$E(b_5)$:

$$f[-1,-3] = -7 \qquad (F)$$
$$f[-2,-3] = -8 \qquad (F)$$
$$f[-1.5,-2.5] = -8 \quad (F)$$
$$f[-1.5,-3.5] = -8 \quad (F)$$

b_5 civarındaki aramada tüm noktalar başarısız olduğundan adım aralığı yarıya düşürülüp tekrar civar araması yapılmalı

$$b_5 = [-1.5,-3] \qquad h_1 = h_2 = \frac{1}{4}$$

Example

$E(b_5)$:

$$f[-1.25, -3] = -7.8125 \quad (F)$$
$$f[-1.75 \ -3] = -8.3125 \quad (S)$$
$$f[-1.75, -2.75] = -8.125 \quad (F)$$
$$f[-1.75, -3.25] = -8.375 \quad (S)$$

Yeni referans noktamız,

$$b_6 = [-1.75, -3.25] \quad \rightarrow \quad f(b_6) = -8.375$$

$p(b_6)$: $\quad p_5 = 2b_6 - b_5 = 2[-1.75, -3.25] - [-1.5, -3] = [-2, -3.5]$

Example

$E(p_5)$:

$$f[-2.25, -3.5] = -7.8125 \quad (F)$$
$$f[-1.75 \ - 3.5] = -8.3125 \quad (F)$$
$$f[-2, -3.75] = -8.1875 \quad (F)$$
$$f[-2, -3.25] = -8.1875 \quad (F)$$

Referans noktamız değişmedi,

$b_6 = [-1.75, -3.25] \quad \rightarrow \quad f(b_6) = -8.375$

Example

$E(b_6)$:

$$f[-1.5,-3.25] = -8.1875 \quad (F)$$
$$f[-2,-3.25] = -8.1875 \quad (F)$$
$$f[-1.75,-3] = -8.3125 \quad (F)$$
$$f[-1.75,-3.5] = -8.3125 \quad (F)$$

$E(b_6)$ tümüyle başarısız olduğundan adım aralığı yarıya düşürülür,

$$h_1 = h_2 = \frac{1}{8}$$

Durdurma kriteri sağlandığından algoritmadan çıkılır.

$$\boxed{b_6 = [-1.75,-3.25] \quad \rightarrow \quad f(b_6) = -8.375}$$

Big Bang-Big Crunch Optimization*

Step 1 (Big Bang):
- Initial population is generated.

Step 2:
- The fitness function values for all candidates are calculated.

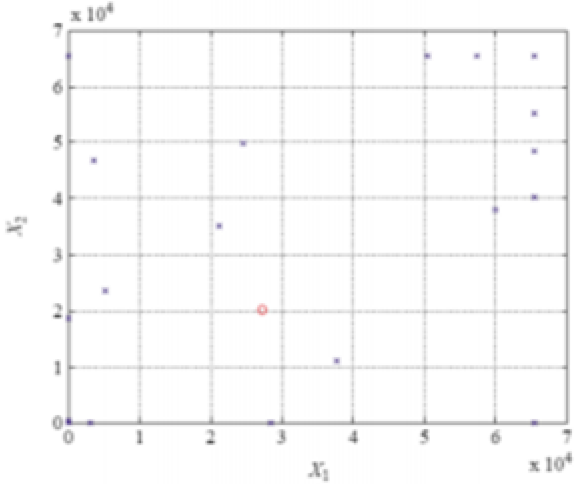
Step 3: (Big Crunch)
- Center of mass is calculated and is chosen as a new point (or the best candidate is chosen as a new point).

Step 4: (Big Bang)
- New candidates are randomly spread over near the point obtained in previous step.

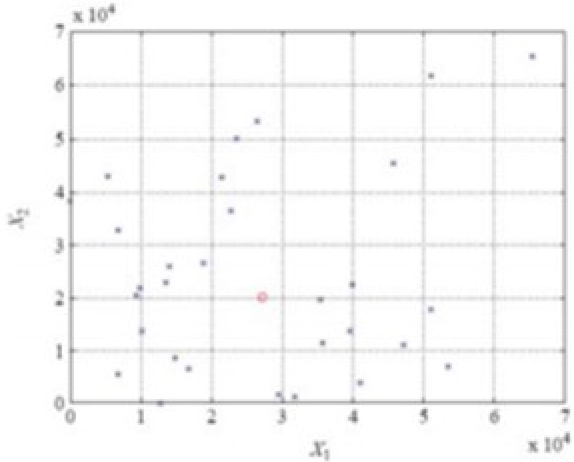
Step 5:
- Return back to step 2 and repeat until stopping criteria is provided.

*O.K.Erol and I.Eksin, "A new optimization method: Big Bang–Big Crunch" Advances in Engineering Software, 37, 106–111, 2006 http://www3.itu.edu.tr/~okerol/BBBC.html

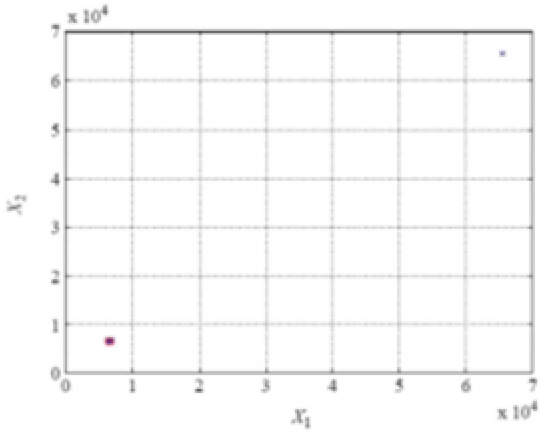
Big Bang-Big Crunch Optimization



The distribution of candidates over 2D search space



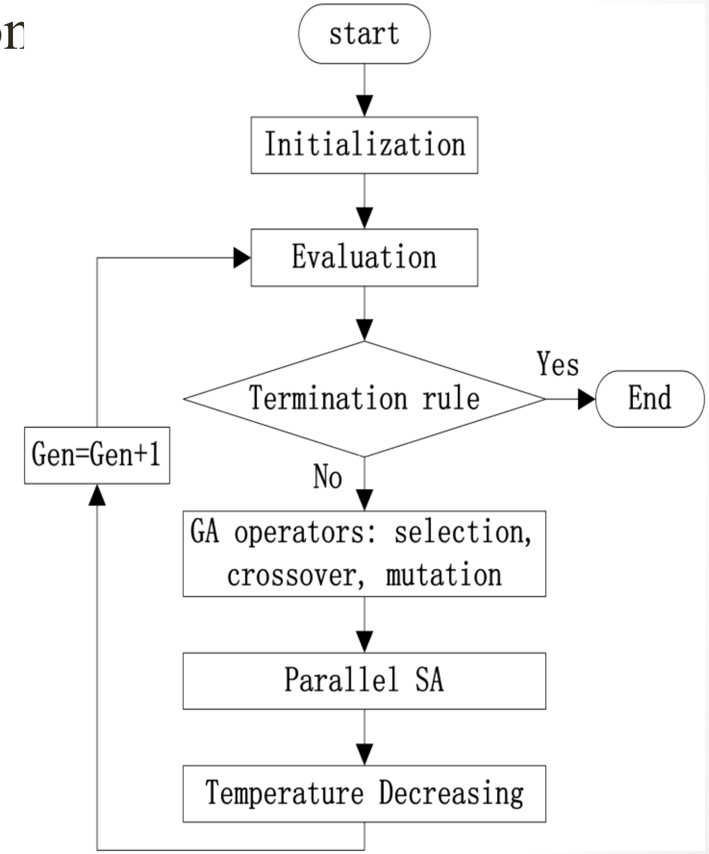
The new distribution after 4 iterations



The new distribution after 500 iterations

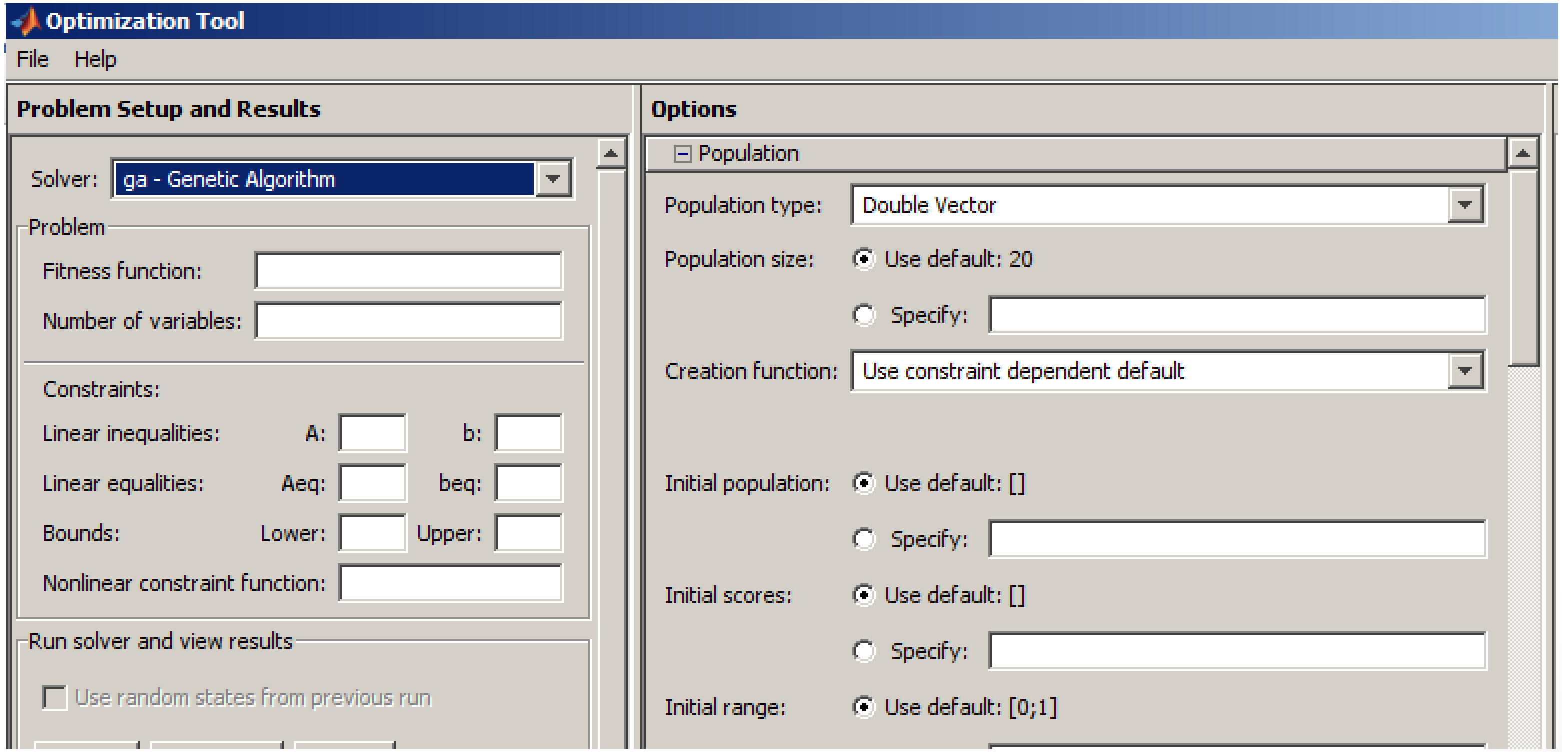
Genetic Algorithm

- Charles Darwin's Theory of Evolution
- Global search algorithm
- Population based
- Natural selection
  - Only the stronger individuals live.
- Genetic operators
  - Variations in population



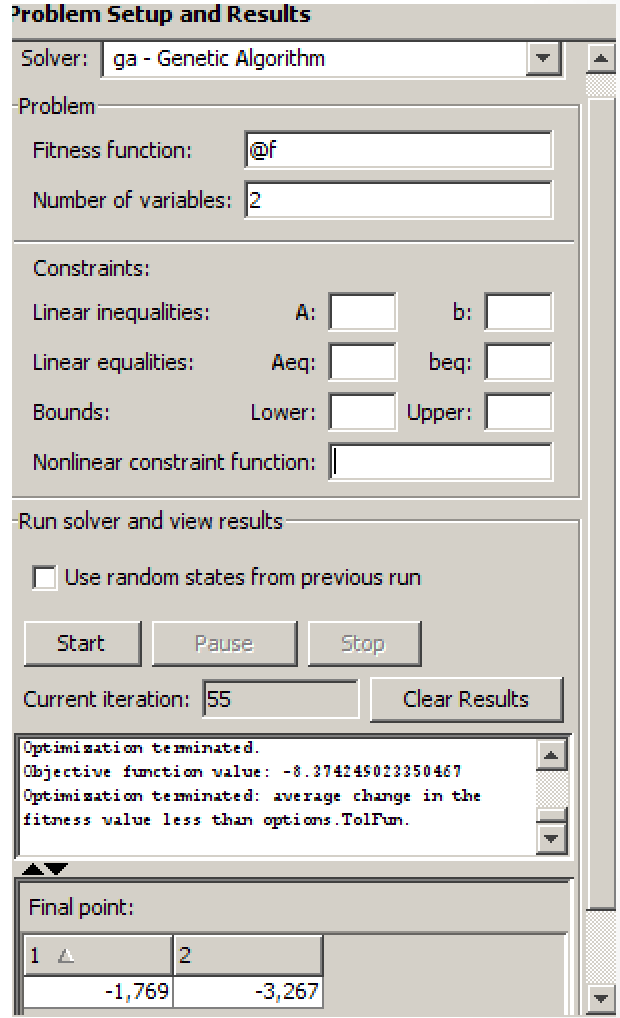
Genetic Algorithm

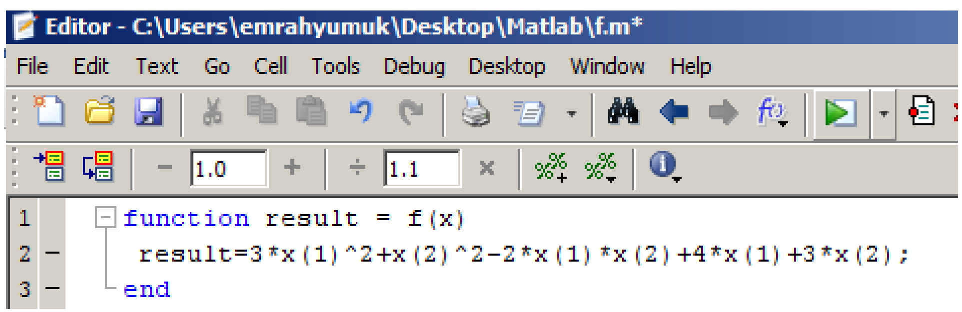
- Optimization Toolbox in MATLAB (gatool)



Genetic Algorithm

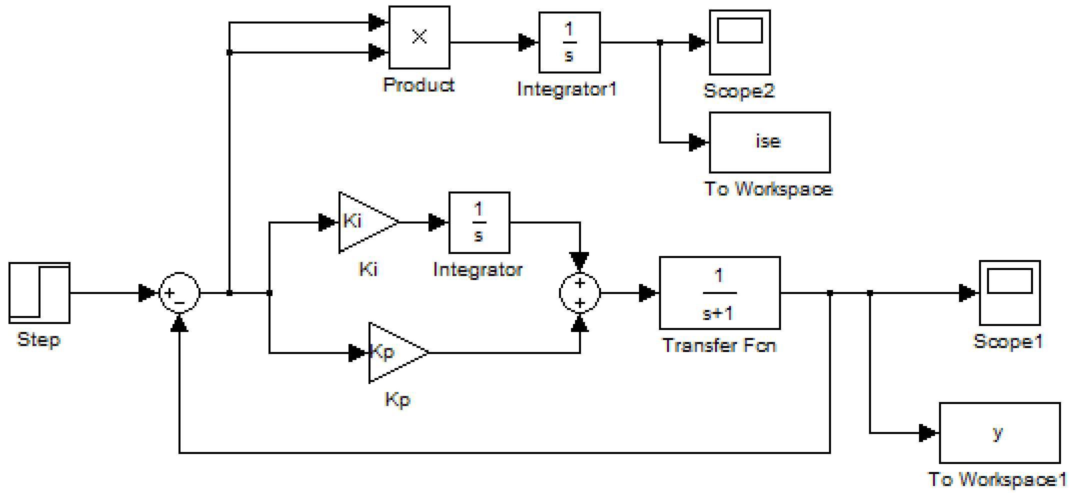
- Function name and the file name should be the same.





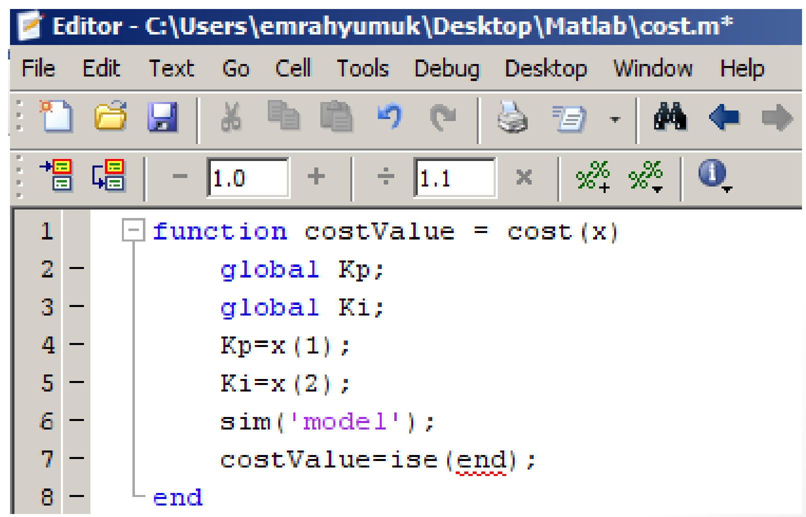
Genetic Algorithm

Fitness Function:
ISE
Variables:
Kp, Ki



Command window:

>> global Ki Kp




```
function costValue = cost(x)
    global Kp;
    global Ki;
    Kp=x(1);
    Ki=x(2);
    sim('model');
    costValue=ise(end);
end
```

About the Final Project

1. Hooke & Jeeves algorithm will be coded and will be tested on the example given in presentation.
2. - **Local search method**

   Hooke & Jeeves Algorithm

(Choosing of the initial points is critical!)

   - **Global optimization method**

   BBBC

   Genetic Algorithm

Each team will design a PI or PD controller using a local search method and one of the global algorithm given above.

About the Final Project

$$J_1 = \int e^2(t)dt$$
$$J_2 = \int te^2(t)dt$$
$J_3 =$ **Overshoot, peak time, settling time etc. (will be determined by yourself)**

- For 3 different fitness function, 3 different controllers will be designed.

3. Controller will be redesigned with the saturation element placed in system input.
4. For the J_1 fitness function, parameter optimization method will be used and will also be compared with numerical optimization results.