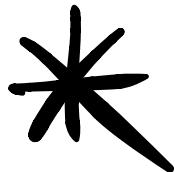


Objective Metatheory
of
Dependent Type Theories

J. Sterling © HOTTEST



References can be found in our draft:

"Gluing Models of Type Theory Along Flat Functors"

[S. Angiuli]

[⟨www.jonsterling.com⟩](http://www.jonsterling.com)

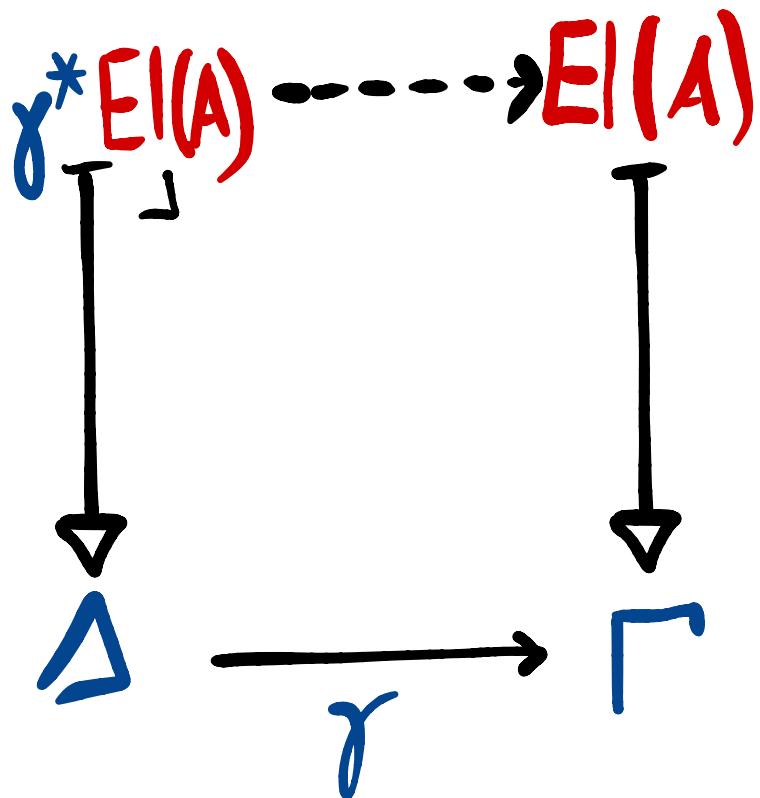
What is *Type*

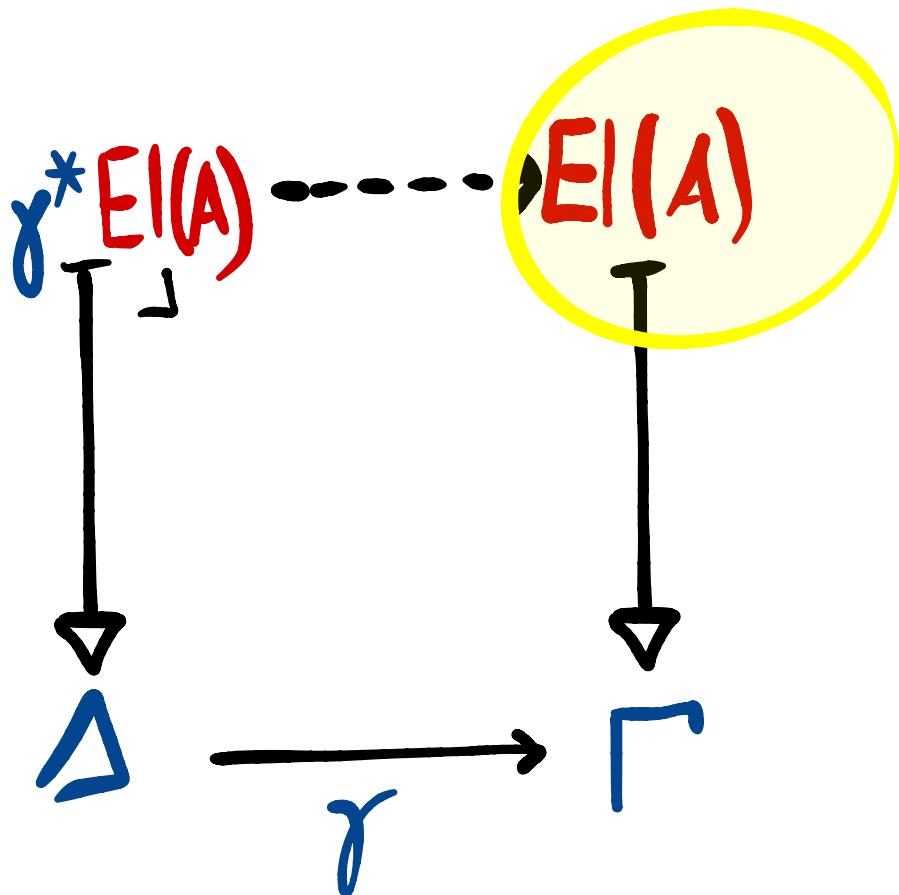
Theory ?

EI(A)



Γ





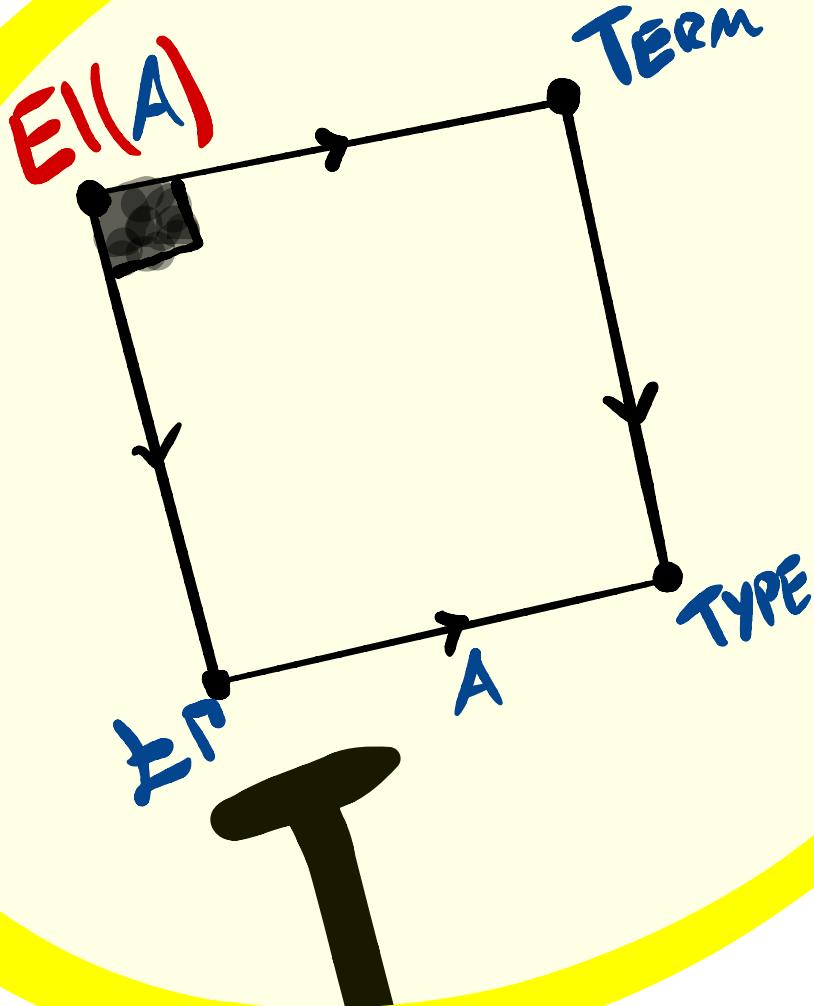
E(I) TYPE

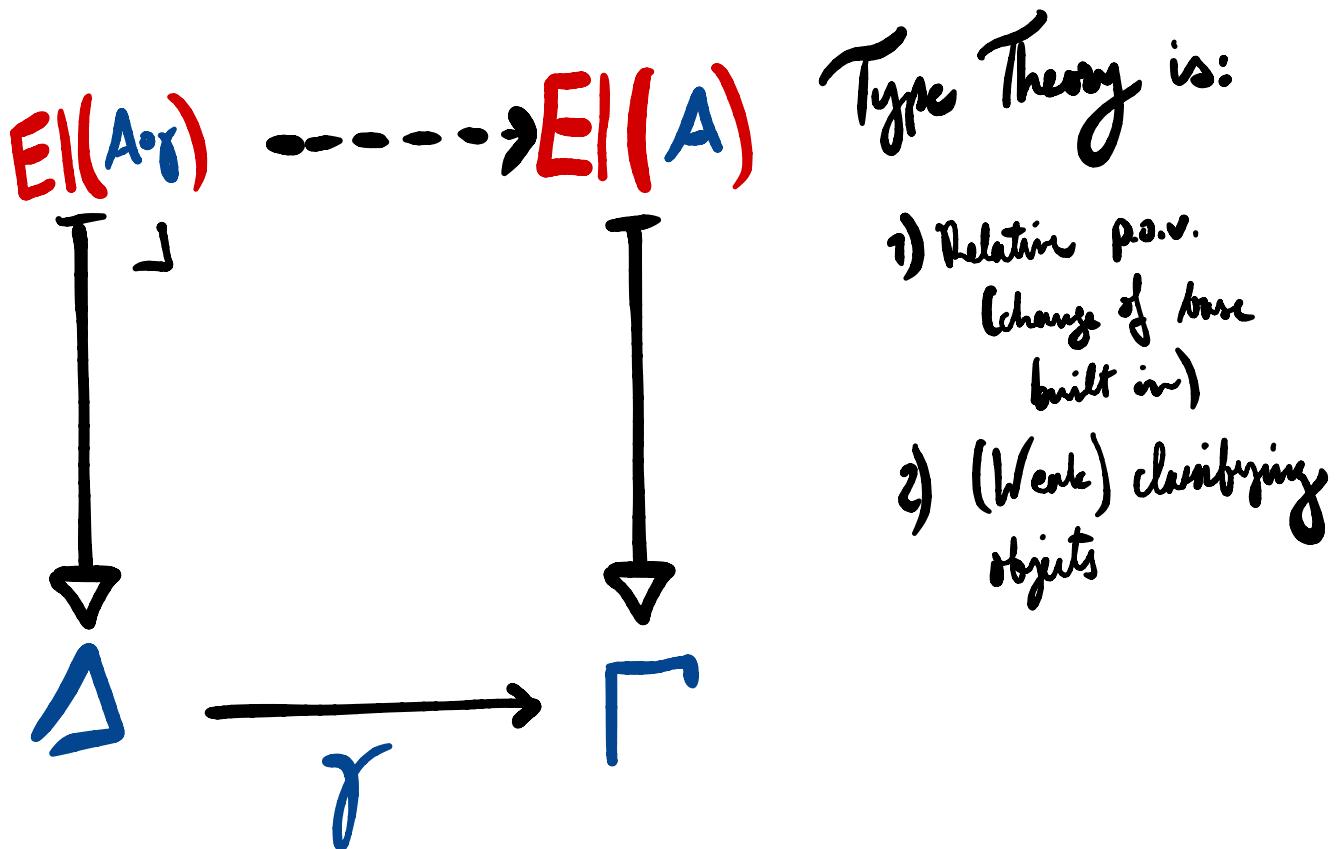
T

EII

T

XF
A
TYPE





Who is a Type Theorist?

Who is a Type Theorist?

* Study initial model

Who is a Type Theorist?

* Study non-type-theoretic
aspects of the initial model

Who is a Type Theorist?

* Study non-type-theoretic
aspects of the initial model

Admissibility, Metatheory

Initial Model

Preserves
 $\langle \cdot \vdash J \rangle$

$J ::=$

- | $(\Gamma \vdash A \text{ type})$
- | $(\Gamma \vdash M : A)$
- | $(\Gamma \vdash A = B \text{ type})$
- | $(\Gamma \vdash M = N : A)$

"type theoretic"

NOT PRESERVING:

$\langle J \vdash K \rangle$

Any Model

Example: $(\Gamma \vdash (A \rightarrow B) = (A' \rightarrow B' \text{ type})) \vdash (\Gamma \vdash A = A' \text{ type})$
holds in initial model, not elsewhere!

Emergent Structure of the Initial Model

injectivity of Π/Σ

Normalization



Proof Assistants!

Emergent Structure of the Initial Model



METATHEOREM

- * consistency
- * normalization
- * decidability
- :



SYNTAX

METATHEOREM



SYNTAX

SYNTAX

METATHEOREM



SYNTAX

UNIVERSAL SECTION

SYNTAX

METATHEOREM

100

MOTIVE

INITIALITY

SYNTAX

UNIVERSAL SECTION

SYNTAX ~~~~~

METATHEOREM

卷二

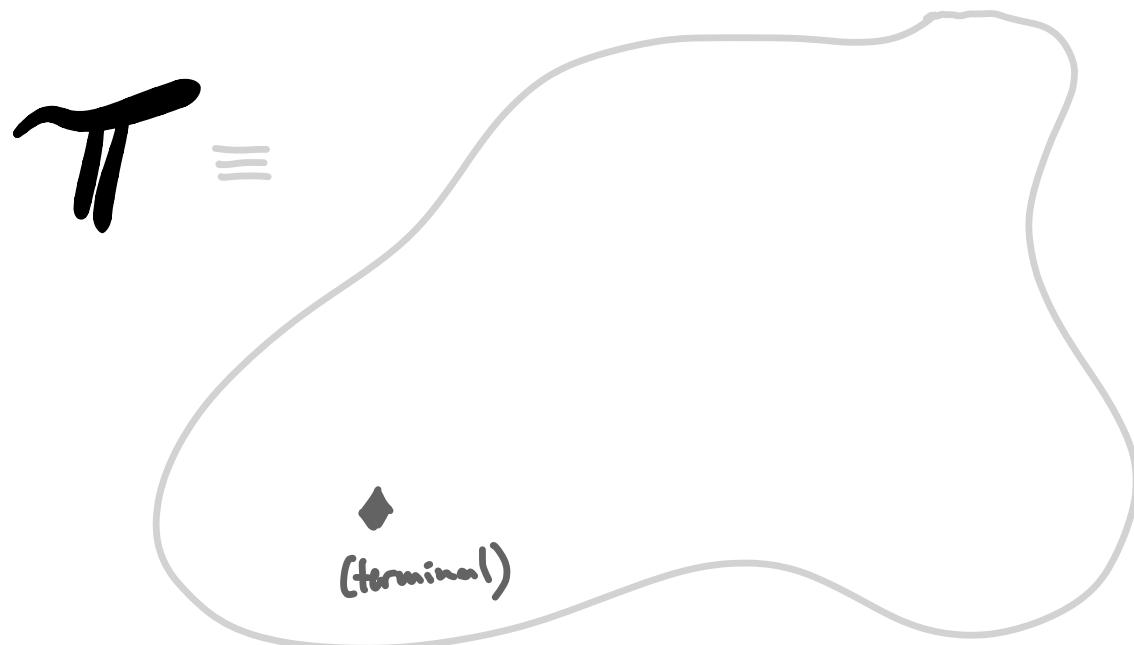
MOTIVE

INITIALITY

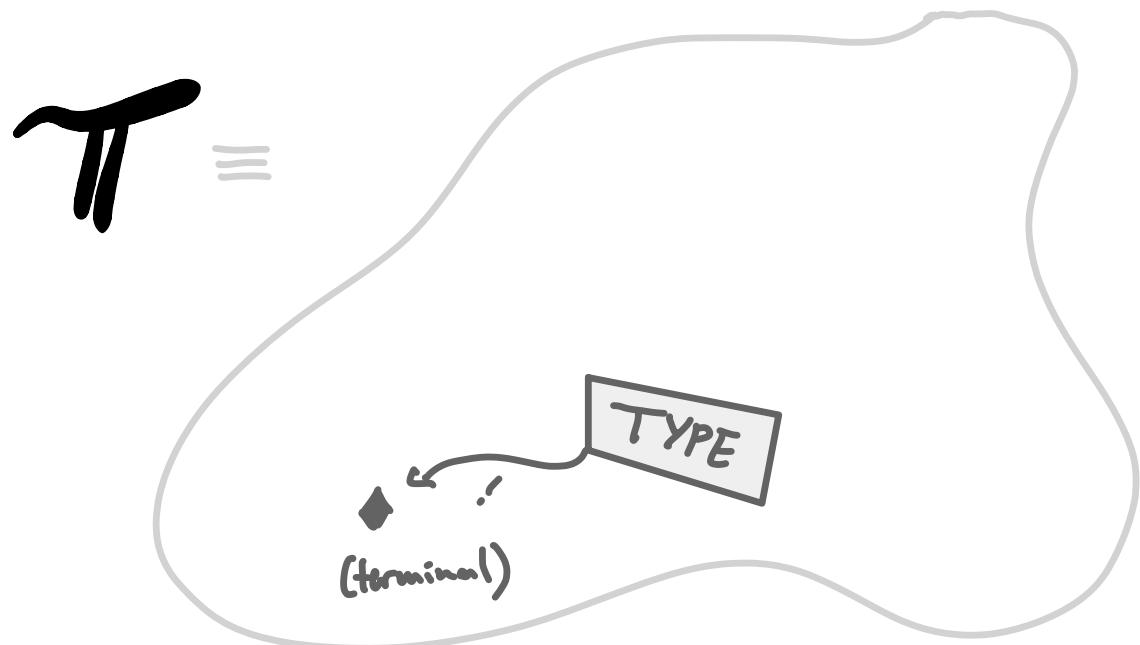
OBJECTIVES THAT'S KETTLEBELL!

Fully structural presentation independent

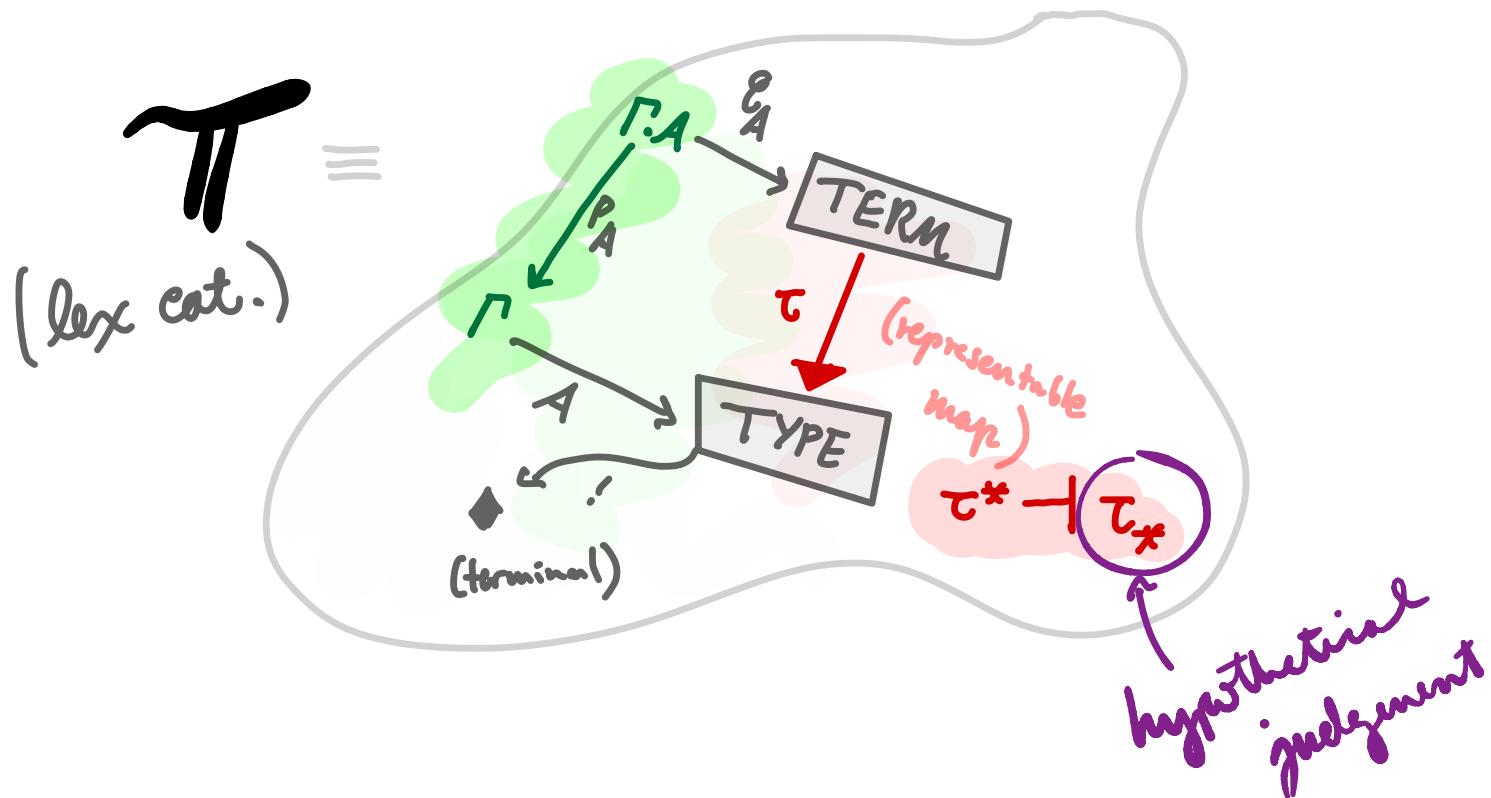
Kemura's General Framework



Kemura's General Framework

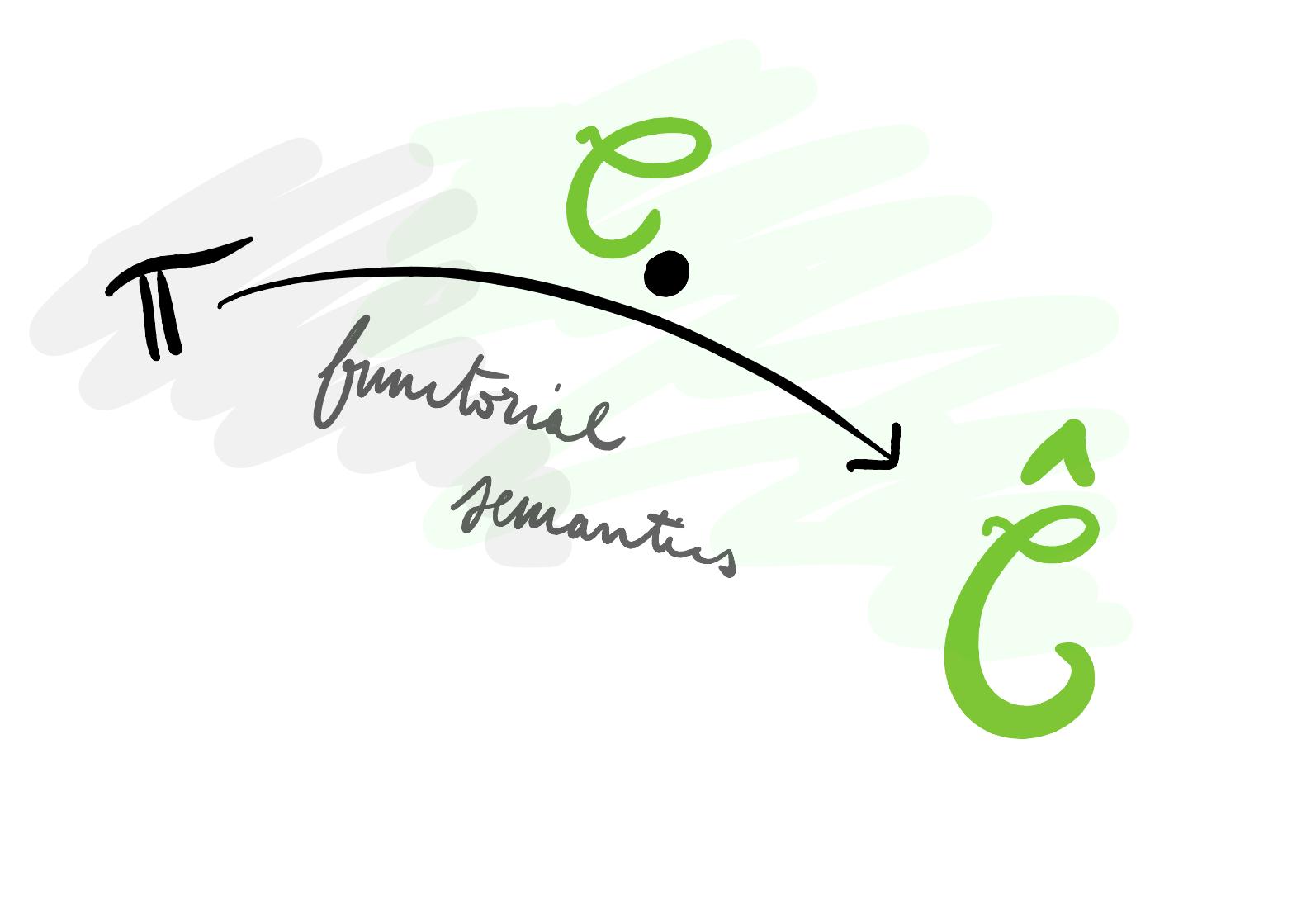


Kemura's General Framework

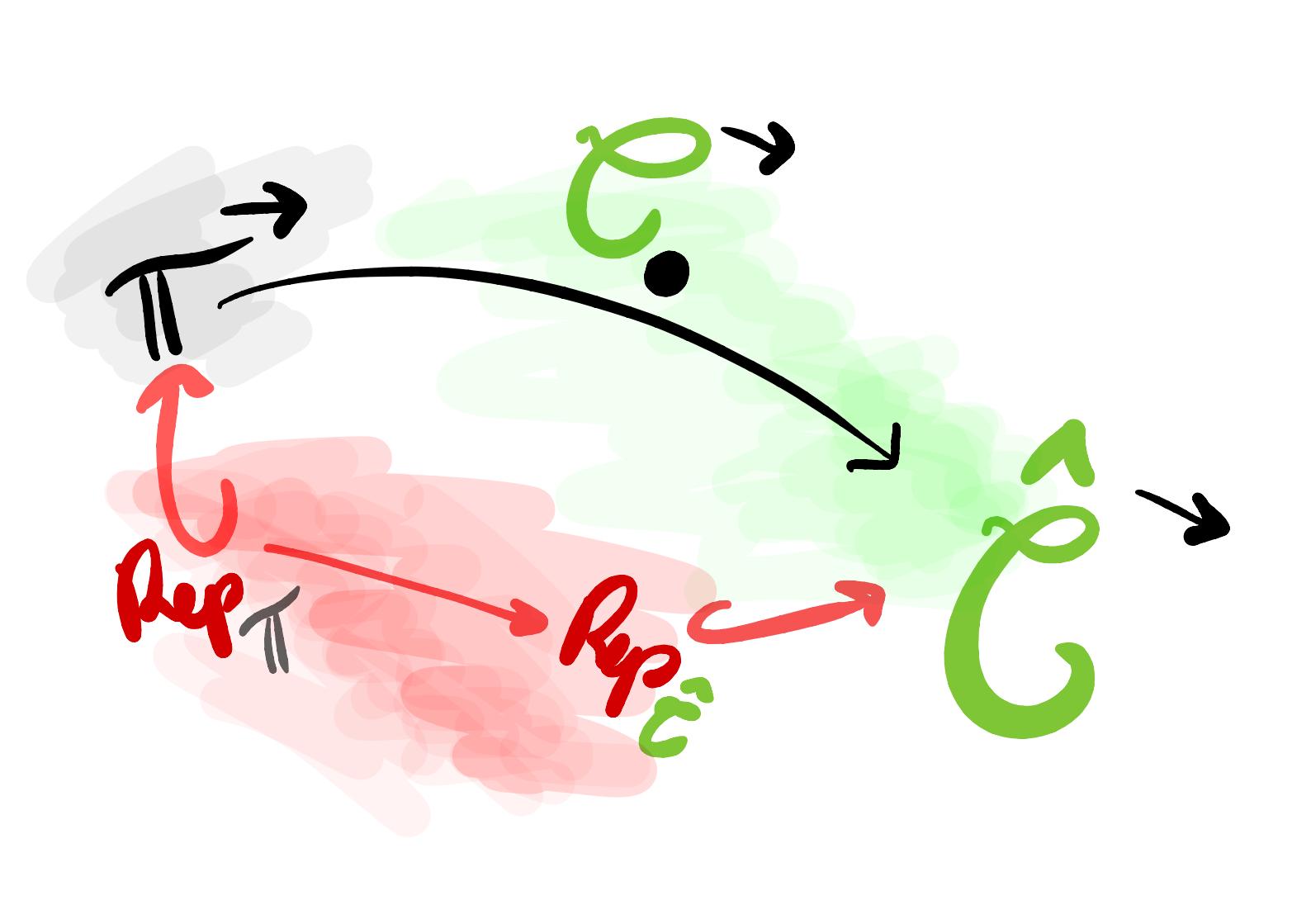


π

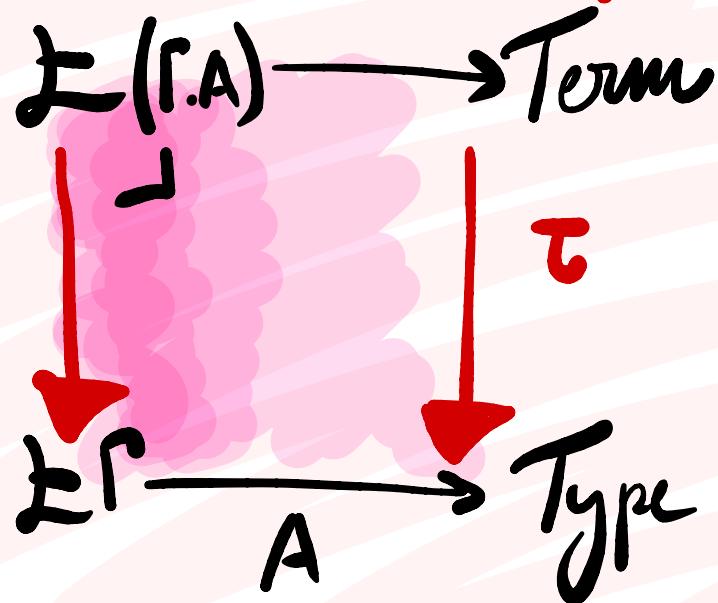
functional
semantics



\hat{C}

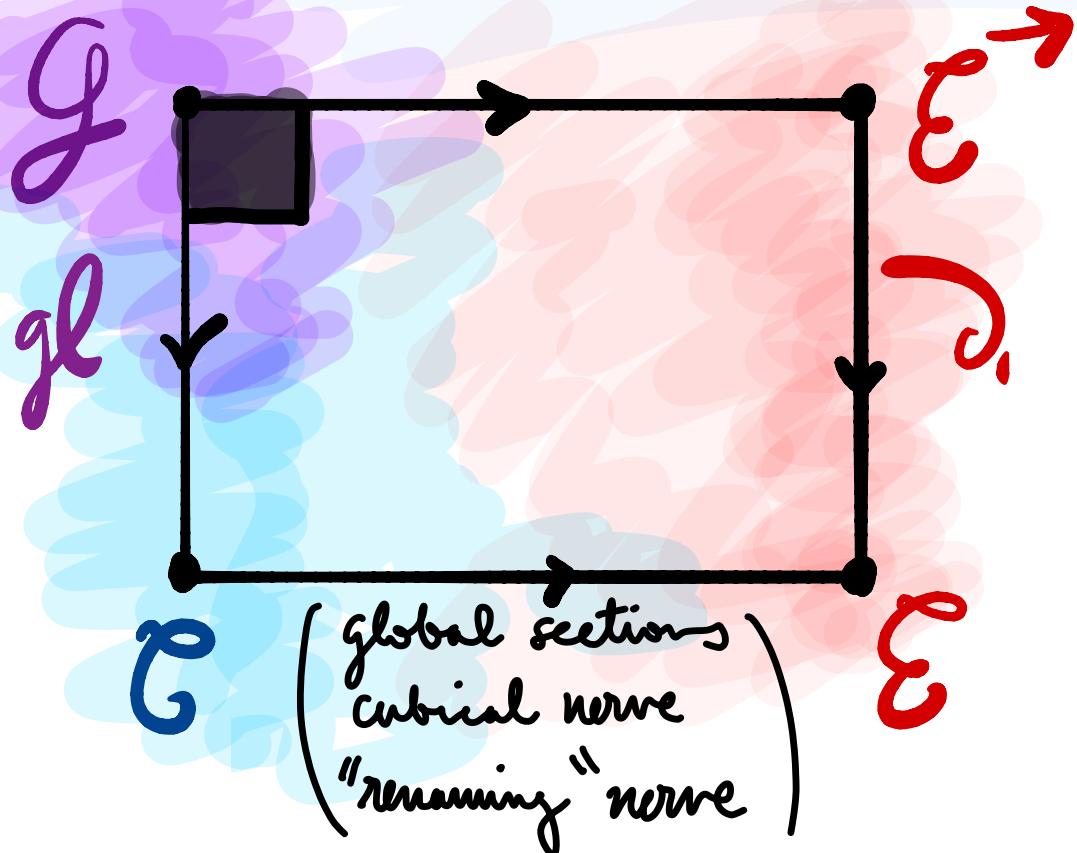


Rep. Maps à la Grothendieck:



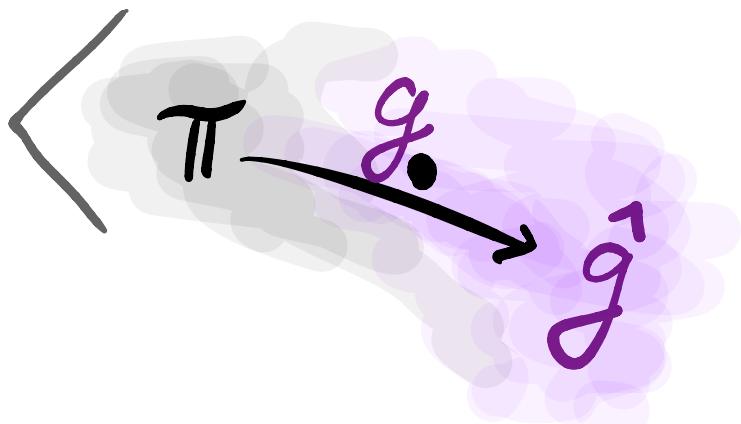
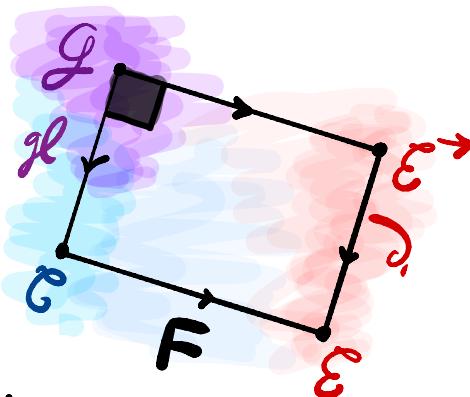
Anodyne's Natural Models

Let C be a model of Π .

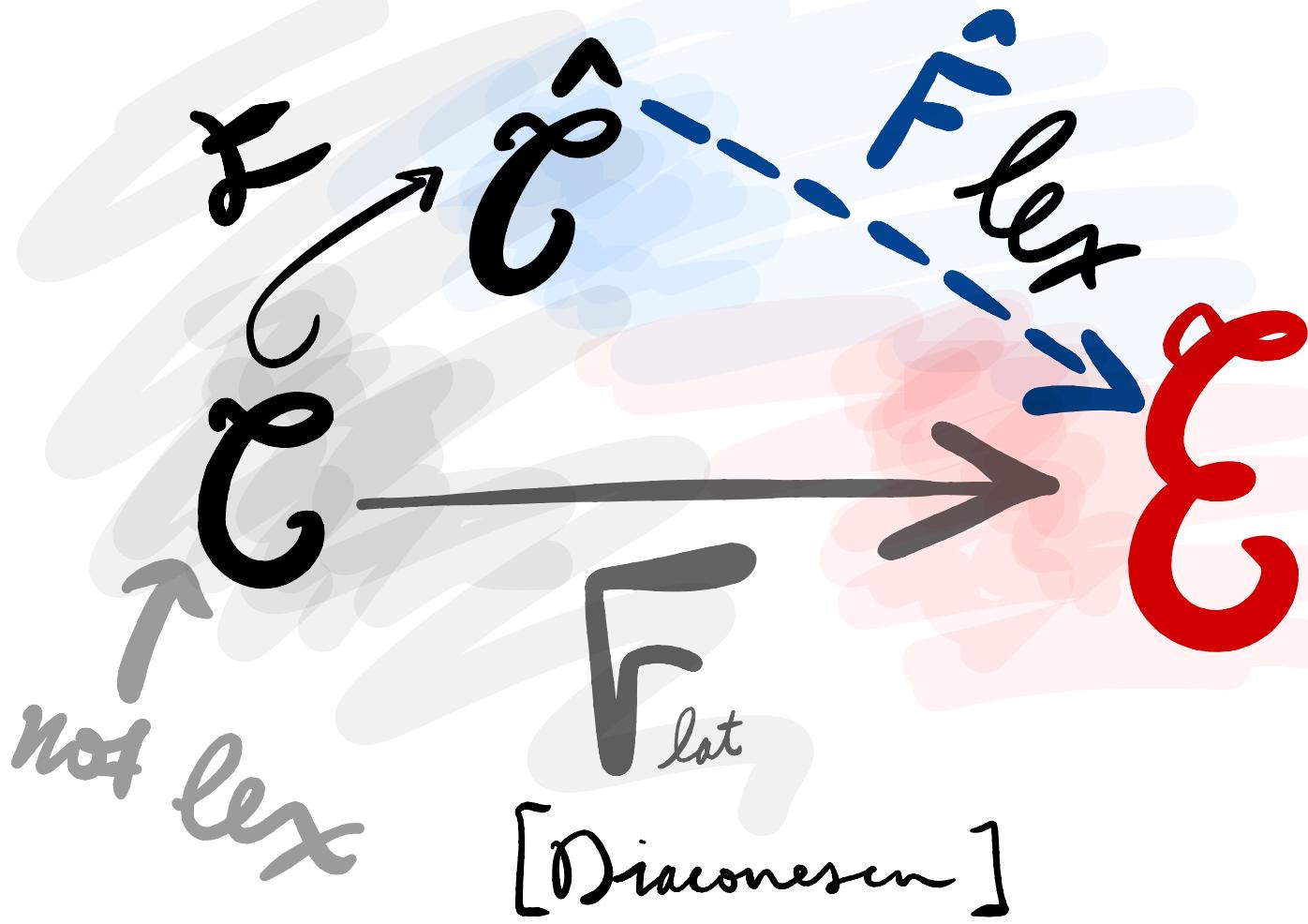


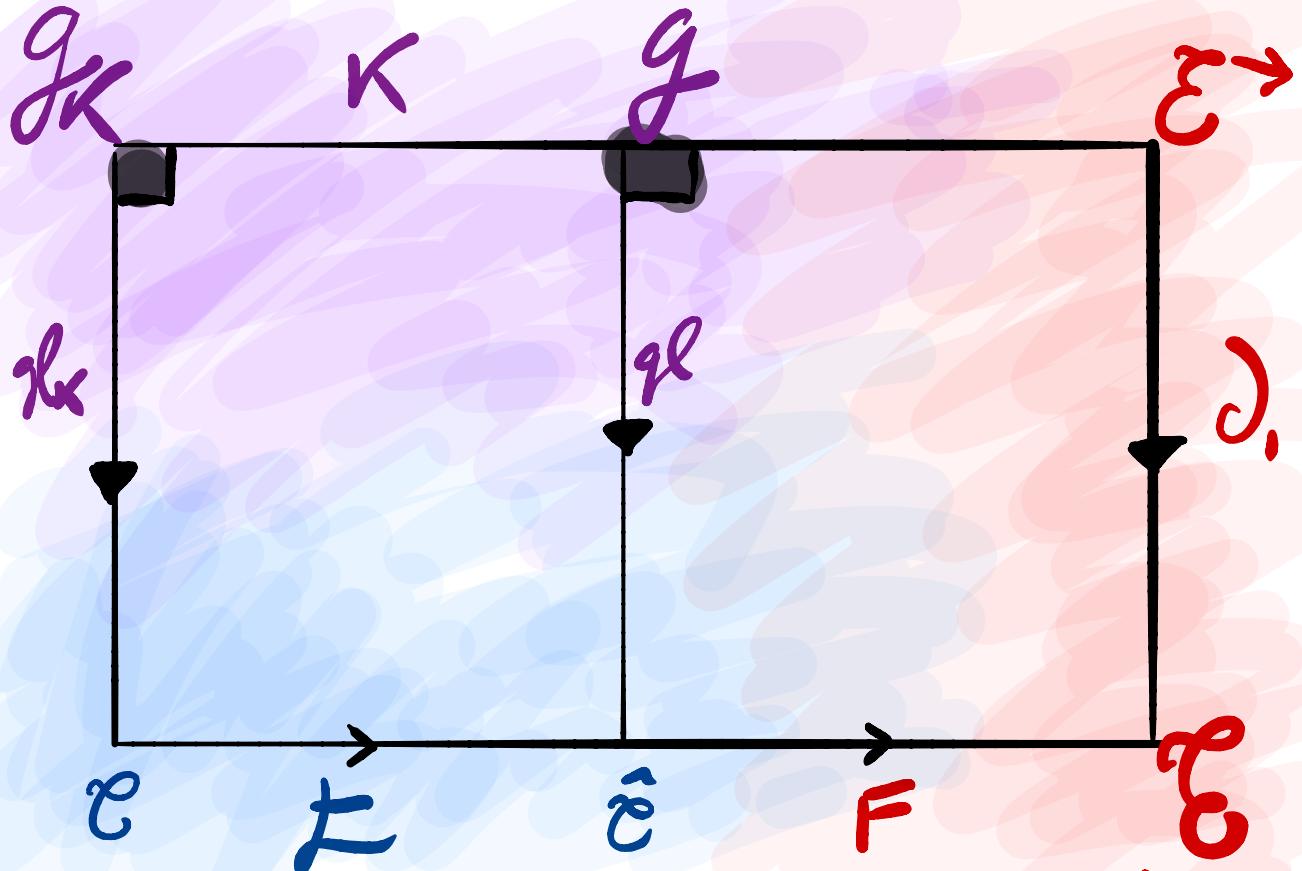
Theorem:

If F a flat functor
and \mathcal{E} a Grothendieck
topos, then \mathcal{G} carries
the structure of a model of
 Π , and gl preserves it.
(Angolini, S.)









$(\text{alg. morphism of topoi})$

\mathcal{G}_K

K

\mathcal{G}_K

C

L



$\mathcal{E} \rightarrow$

\mathcal{D}_L



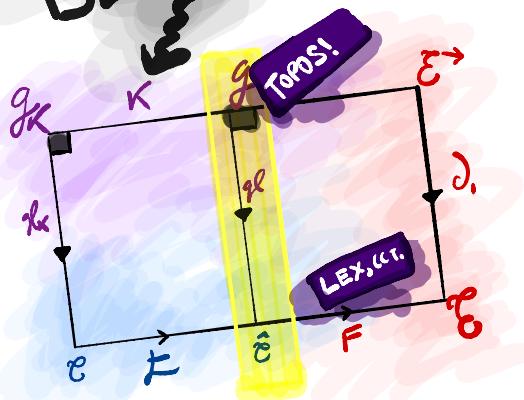
F

\mathcal{E}

(alg. morphism of
topoi)

DENSE!

[A., S.]

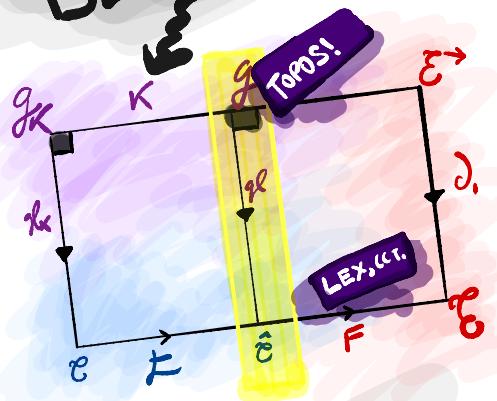


Therefore, we have
fully faithful nerve

$$g_N \rightarrow \hat{g}_K$$

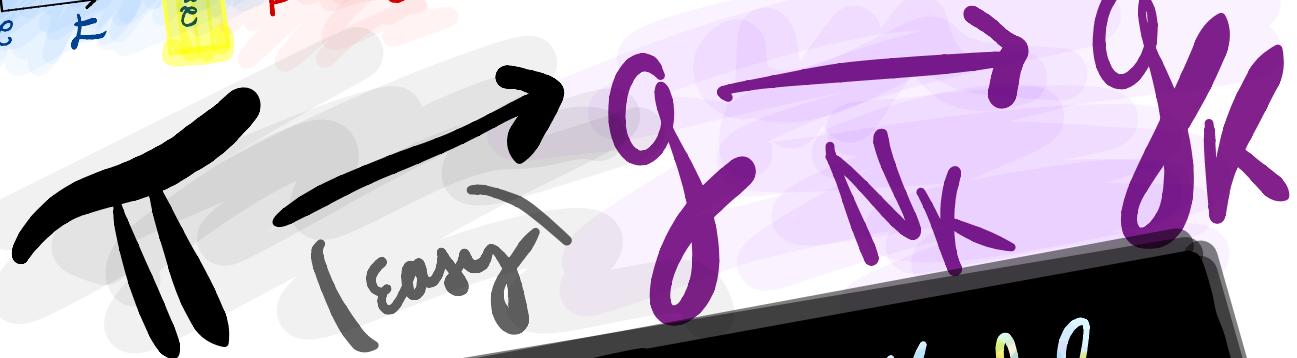
DENSE!

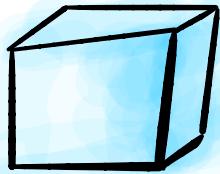
[A., S.]



THEREFORE:

fully faithful nerve



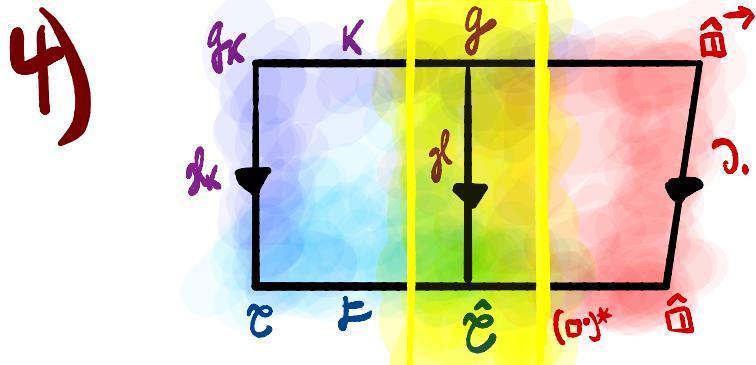


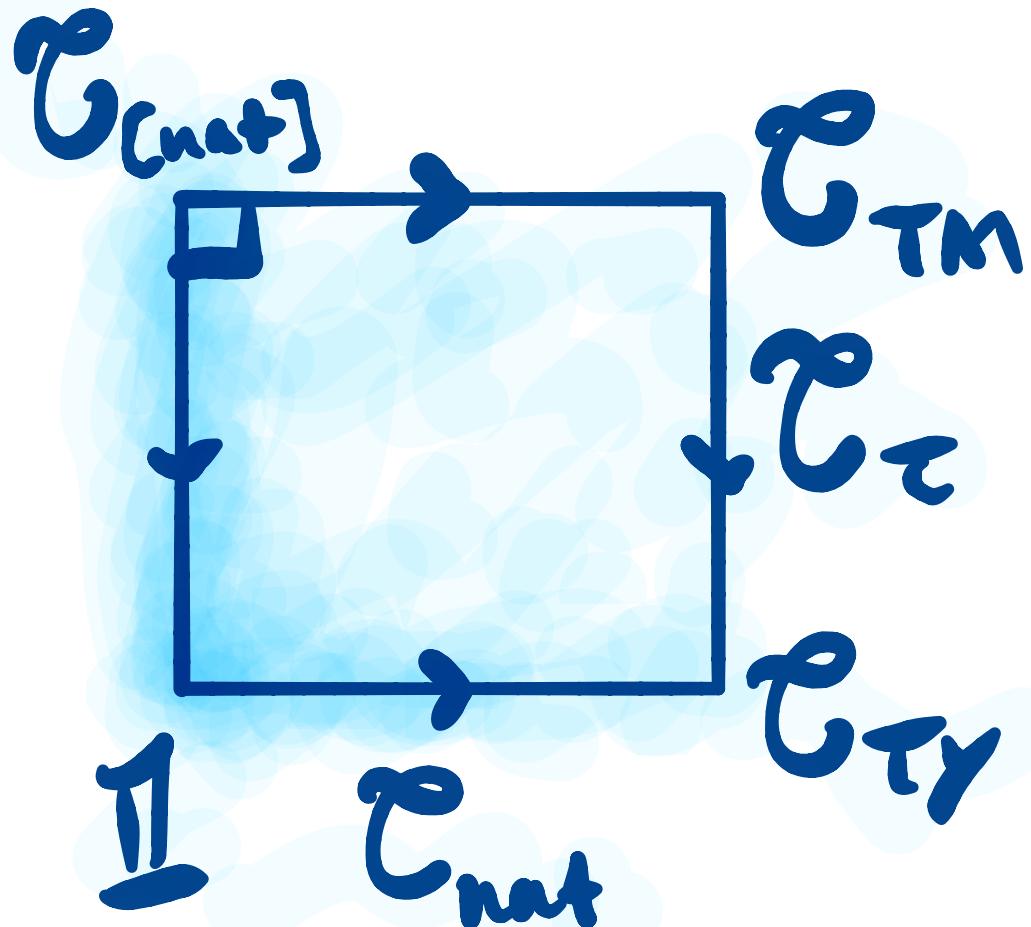
-ic!l Categori

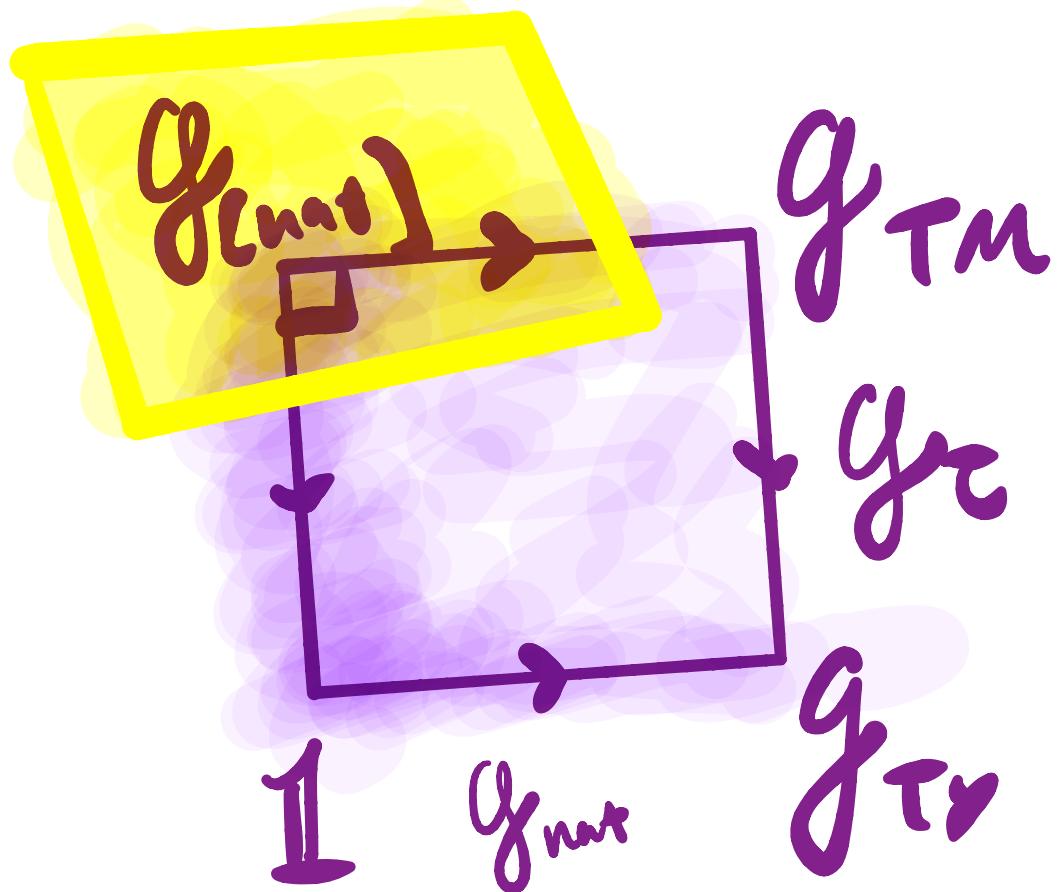
1) $I : \mathcal{C}$

2) $\square \xrightarrow{\text{f.p.}} \mathcal{C}$

3) $\mathcal{C} \xrightarrow{N_0} \square^1$
[Awodey + Fiore]







Gymnasium



$g[nat]$



$C[nat]$

(is)

universal
comparison
map

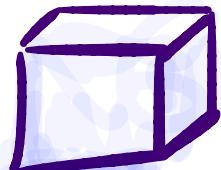
Π

$g[nat]$



$(\square)^*(C[nat])$

Corollary:



-ical Type Theory

has Canonicity



Next: cubical **normalization** (β/γ)

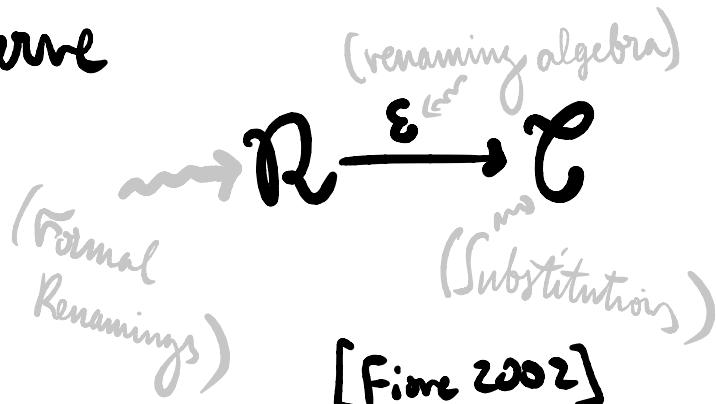
WHY: Essential for usability

(automatically discharge boundary conditions, cf.
Cubical Agda [e.g. [redd.it](#)])

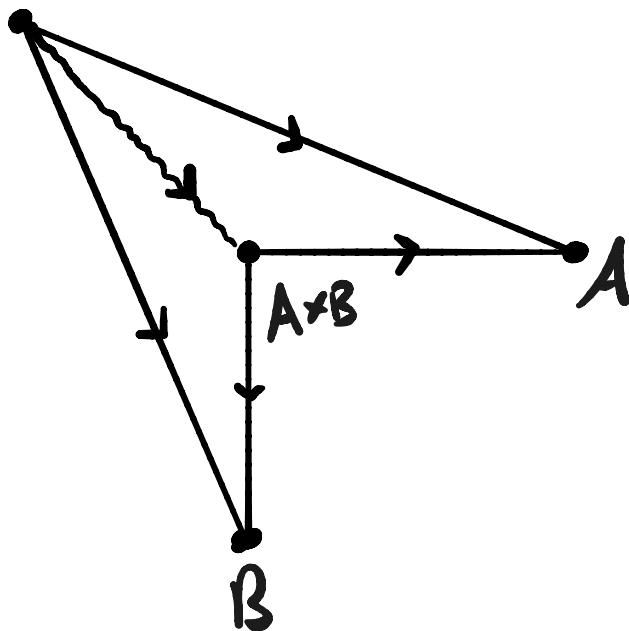
NOW: Glue along nerve

induced by

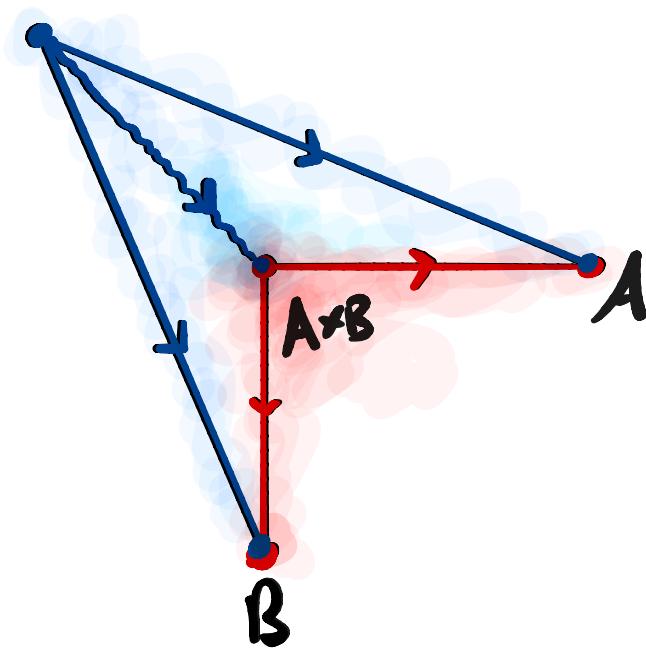
"formal renaming
algebra":



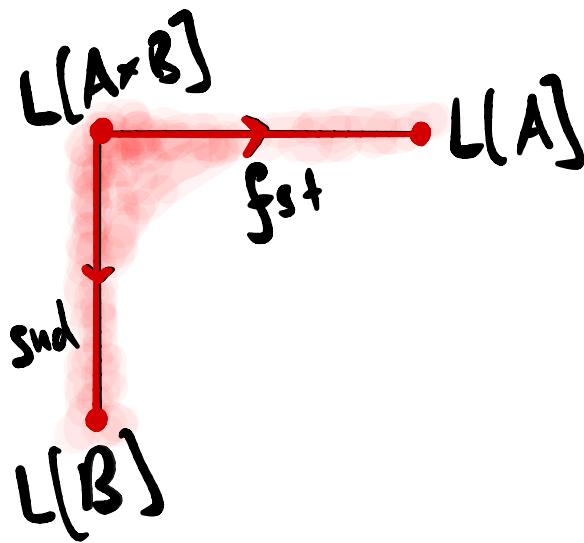
{ Gym }]



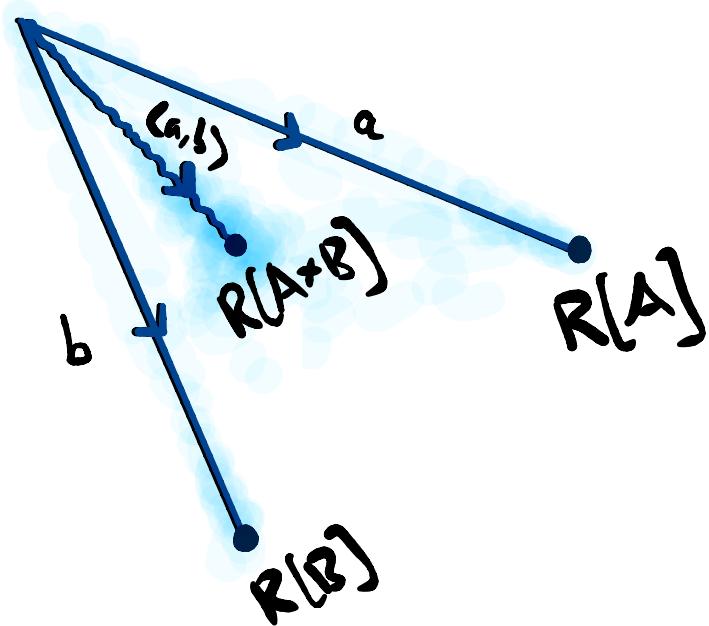
{ Gymfim }



{ Gymnasm }



"left normal forms"



"right normal forms"

