IE- 400 Project

2019-2020 Fall

- In the project description, we were asked to find the shortest path using Manhattan distance between data points given. However, there were blocks whose coordinates are given. Our path would not pass through these blocks.

Mathematical Model of Problem

We thought the given points as vertices of a directed graph. Therefore, if there is an edge between two points, it means that drilling equipment moves between these two vertices.

Parameters

Y_{ij} = { 1 if there exists a path between point i and j

0 if no path exists between point i and j }

C_{ij} = {Manhattan distance between point i and j}

n = {Number of points}

Decision Variables

X_{ij} = {1 if there exists an edge from i to j

0 if there is no edge from i to j }

U_{ij}: This variable is used to make sure that there is no subroutine in the path.

Objective Function

$\min \sum C_{ij} X_{ij}$ for all $1 \le i,j \le n$;

subject to:

$X_{ij} \le Y_{ij}$	for all $1 \le i,j \le n$;	(1)
$\sum X_{ij} = 1$	for all $1 \le i \le n$;	(2)
$\sum X_{ij} = 1$	for all $1 \le i \le n$;	(3)
$U_i - U_j + nX_{ij} \le n - 1$	for all $i, j = 2..n$;	(4)
$1 \le U_i \le n - 1$	for all $i = 2..n$;	(5)
$X_{ij} = \{0, 1\}$;	for all $1 \le i,j \le n$;	(6)

Constraint 1: We need to check whether a path exists between two points before we construct an edge. This constraint is added to satisfy this condition.

Constraint 2, 3: We need to make sure that there exists at least one edge coming towards vertex and going outside of a vertex to satisfy connectivity. However, we should also make that there is a continuous flow. Therefore, each vertex should have only one edge that is going outside from the vertex and one edge coming towards the vertex.

Constraint 4, 5: These constraints are added to make sure that the graph is has no subroutine. For these constraints, we used the formulation Miller-Tucker-Zemlin suggested for Traveling Salesman Problem [1].

Constraint 6: X_{ij} can only be 0 or 1.

Conclusion

If we think each point as a vertex named with their order in the excel file and if we trace our solution X_{ij}, our path will look like:

Path is:
1 -> 18 -> 26 -> 50 -> 20 -> 47 -> 5 -> 48 -> 28 -> 42 -> 30 -> 43 -> 7 -> 41 ->
16 -> 37 -> 49 -> 27 -> 46 -> 45 -> 32 -> 34 -> 39 -> 33 -> 3 -> 44 -> 10 -> 6 ->
29 -> 15 -> 31 -> 21 -> 13 -> 40 -> 24 -> 36 -> 25 -> 17 -> 23 -> 2 -> 9 -> 8 ->
38 -> 4 -> 19 -> 12 -> 11 -> 22 -> 35 -> 1

Optimal solution is:382.

Note: We used “Unique paths in a Grid with Obstacles” algorithm to find out whether there exists a path between two holes or not [2].

References

[1] Generating subtour elimination constraints for the Traveling Salesman Problem (n.d.). Retrieved from https://www.iosrjen.org/Papers/vol8_issue7/Version-1/D0807011721.pdf.

[2] Unique paths in a Grid with Obstacles. (2017, August 28). Retrieved from https://www.geeksforgeeks.org/unique-paths-in-a-grid-with-obstacles/.