Binary Genetic Algorithm

11/03/2023

$$\min f(x) = \sum_{i=1}^{n} x_i \qquad x_i \in \{0,1\} \quad i = 1,2,3,\ldots,n$$

} Min One Problem

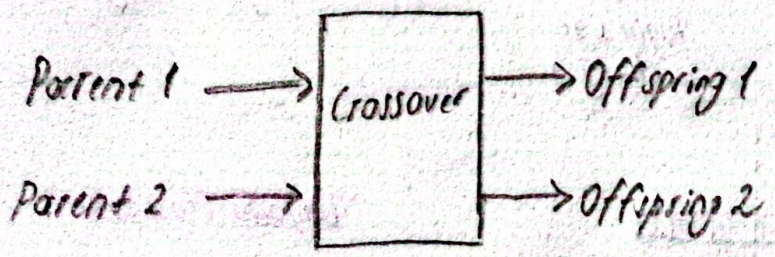
Solution to this problem $x_i^* = 0$

Genetic Algorithm

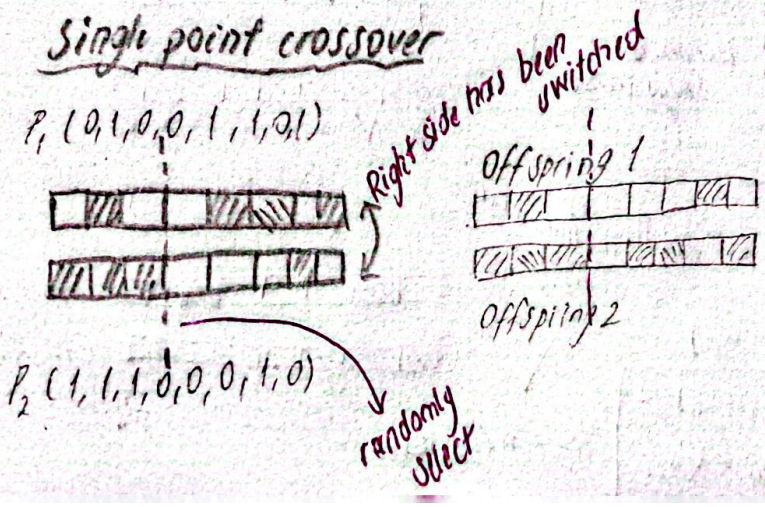
1) Initialization
2) Select parents & Crossover
3) Mutate Offsprings
4) Merge main population and offsprings
5) Evaluate, Sort & Select
6) Go to step 2, if it is needed

Crossover

At each crossover operation, # of offsprings should be an even number.



Single point crossover



Mutation

If you want you can increase the mutation rate!

Choose ↓ According to the problem

$x = (x_1, x_2, \dots x_n)$

$x_i \in \{0,1\}$ $\quad j \in \{1,2,\dots n\}$

Random index

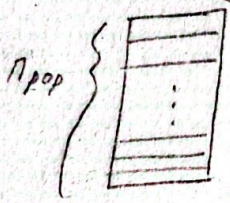
$x' = (x_1', x_2', \dots x_n')$

$$x_i' = \begin{cases} x_i & i \neq j \\ 1 - x_i & i = j \end{cases}$$

Parent Selection

1) Random Selection

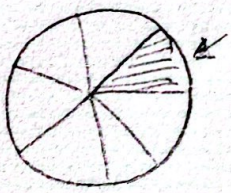
$k \in \{1,2,3,\dots n_{pop}\}$

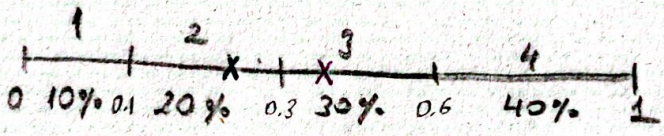


same probability for every member

2) Tournament Selection

3) Roulette Wheel Selection





The better the individual ↑ Selection chance ↑

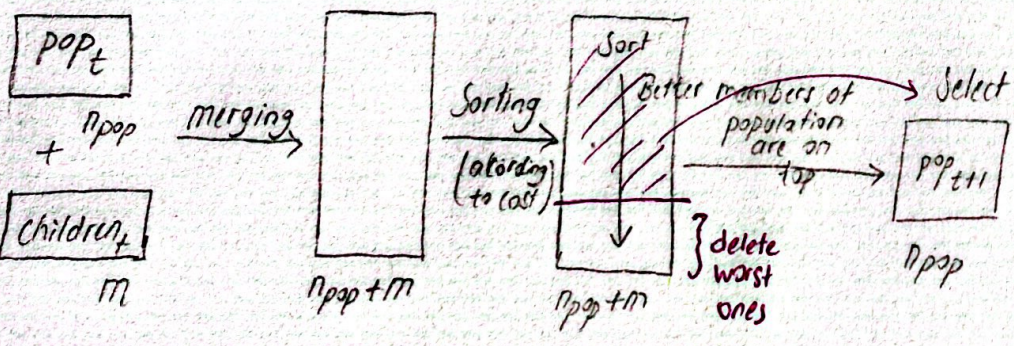
$P_1 = 0.1$ $\quad C_1 = 0.1$ x

$P_2 = 0.2$ $\quad C_2 = 0.3$ x $\quad$ 0.32 rand

$P_3 = 0.3$ $\quad C_3 = 0.6$ ✓

$P_4 = 0.4$ $\quad C_4 = 0.1$

rand → 0.36 $\quad$ rand → 0.22



If cost values of each member is,

Selection probabilities

$$\sum_i \pi_i = 1 \qquad \pi_i \propto e^{-\beta \frac{c_i}{\frac{1}{n}\sum c_i}}$$

$C_1 \longrightarrow \pi_1$

$C_2 \longrightarrow \pi_2$

$C_3 \longrightarrow \pi_3$

$\vdots$

$C_n \longrightarrow \pi_n$

$c_i \leq c_j \iff \pi_i \geq \pi_j$

$\pi_i \propto e^{-\beta c_i}$

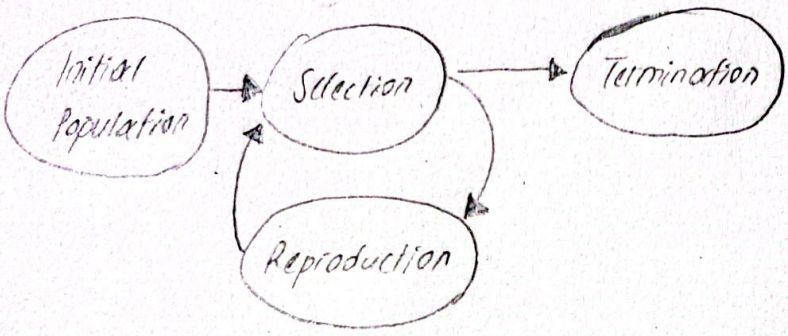
if $\beta = 0$ then $\pi_i = \frac{1}{n}$ / if $\beta \to \infty$ $\pi_i = \begin{cases} 1 & i = \text{best} \\ 0 & \text{otherwise} \end{cases}$

Genetic Algorithms (Evolutionary Algorithms)

* Solve optimization problems

$\min f(x) \rightarrow x^*$

$x = (x_1, x_2, \dots x_n) \in \mathbb{R}^n$



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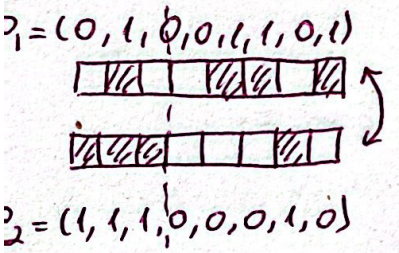
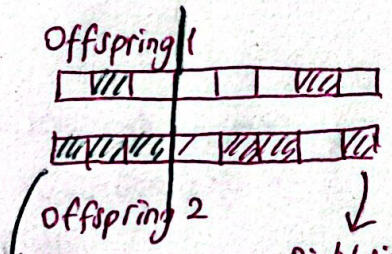
Crossover

Parent 1 → Crossover → Offspring 1
Parent 2 → Crossover → Offspring 2





Left side stays same

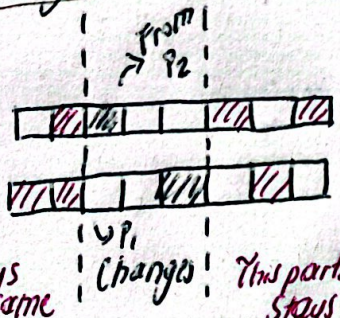
Right side is changed

Single Point Crossover

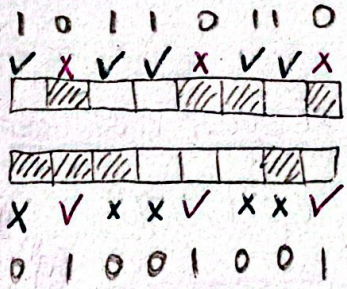
(Single Cutting Point)

Apply Double Point Crossover

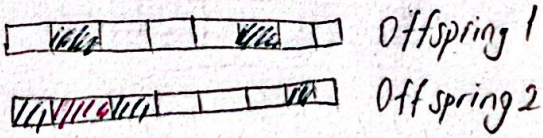
two cutting points selected randomly



Uniform Crossover



Write as mathematical eqns.



$X_1 = (x_{11}, x_{12}, x_{13}, \dots x_{1n})$
$X_2 = (x_{21}, x_{22}, x_{23}, \dots x_{2n})$ } parents

$y_1 = (y_{11}, y_{12}, \dots y_{1n})$
$y_2 = (y_{21}, y_{22}, \dots y_{2n})$

$\alpha = (\alpha_1, \dots \alpha_n)$

$y_{1i} = \alpha_i x_{1i} + (1-\alpha_i) x_{2i}$
$y_{2i} = (1-\alpha_i) x_{1i} + \alpha_i x_{2i}$

Uniform Crossover