Instrumental Variables

Annie Chen

February 26, 2020


Yeah, I'm using lagged variables as instruments
It's fine, I've tested the exclusion restriction
The IV correlates significantly with the treatment but not with the outcome

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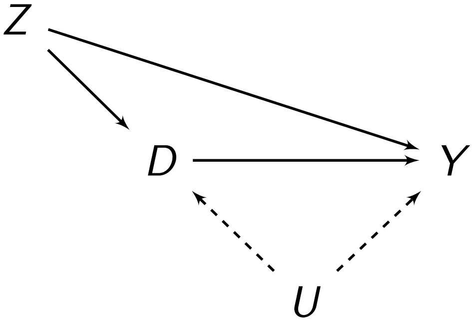
- Monotonicity
  - $D_i(1) \geq D_i(0)$ for all i

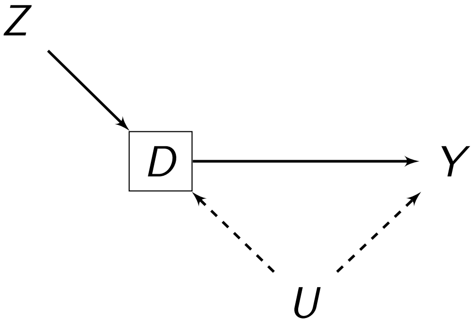
Valid Instruments

- Let's play a game: is Z a valid instrument that identifies a causal effect of D on Y?



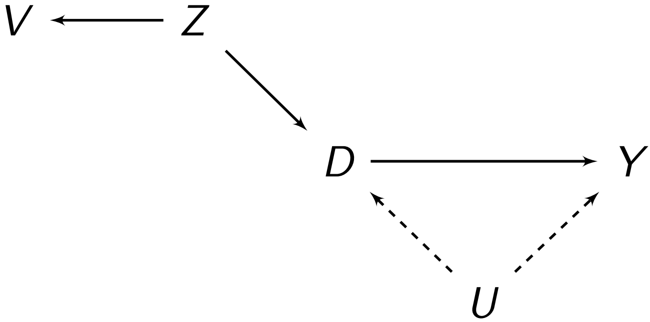
Valid Instruments

- Can we test the exclusion restriction by checking the association between Z and Y after conditioning on D?



Valid Instruments

- and here? Suppose the instrument Z is unobserved, but we have a proxy V.



Valid Instruments

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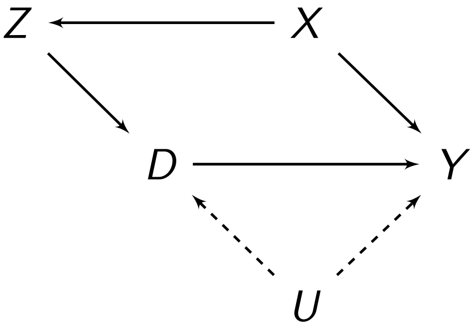

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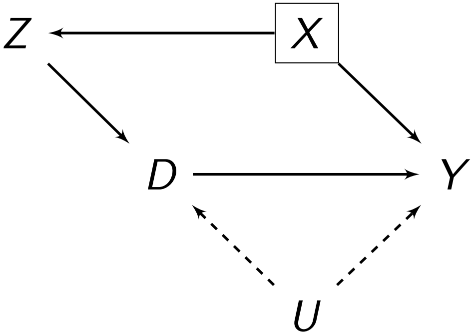
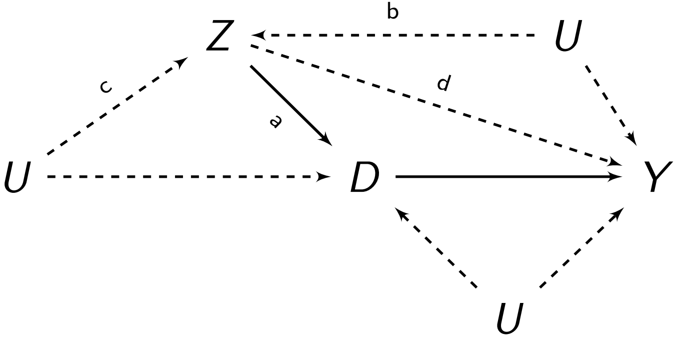


[1] Note that this is now a conditional instrument and we adjust our assumptions accordingly. I.e., $\{Y(z,d), D(1), D(0)\} \perp Z|X$

Violating IV Assumptions



Review LATE Framework

- For unit i, if $Z_i \in \{0, 1\}$ is the instrument (encouragement) and $D_i \in \{0, 1\}$ is treatment, then we have 4 principal strata (latent class variable C).

C_i	$D_i(z)$	
	$Z_i = 1$	$Z_i = 0$
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Always-takers	1	1
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- Can we determine which C unit i falls under from our observed data?
- Say $Z_i = 1$ and $D_i = 0$. What type(s) could unit i be?

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- Local Average Treatment Effect (LATE):[2] $= \frac{ITT}{ITT_d}$
- Can be estimated with 2SLS assuming monotonicity and excludability.

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- If instrument(s) are so weak that there is no first stage, the 2SLS distribution is centered on the plim of OLS.
- Run a joint-significance test (f-test) under the null that your instrument(s) is/are weak.
- The *rule of thumb* given by Staiger and Stock (1997) is a first-stage F-statistic > 10 (in which case, we may reject the null and conclude the instrument is relevant).[3]

[3] Stock and Yogo (2005) tabulate the critical values needed to reject null for some tolerable level of relative bias (between OLS and 2SLS estimator).

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  - Where $m < k$ = underidenfication, $m > k$ = overid, $m = k$ is just-identified.

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- Assumes homoskedasticity[4]
- Is this really a test for exogeneity?

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AER::IVREG()

```r
ivmod <- ivreg(y ~ x1 + w1 | w1 + z1 + z2, data)
summary(ivmod, diagnostics = TRUE)$diagnostics
```

```
##                    df1 df2  statistic      p-value
## Weak instruments     2  60 17.4114496 1.089283e-06
## Wu-Hausman           1  60 34.3521510 2.093826e-07
## Sargan               1  NA  0.1457748 7.026062e-01
```

ADDITIONAL NOTES

Limited Information Maximum Likelihood (LIML)

- Alternative to 2SLS is LIML estimator, which has better small sample properties than 2SLS