FUSION OF SUBJECTIVE OPINIONS THROUGH BEHAVIOR ESTIMATION

Gönül AYCI

October 2016

Problem Statement

- Important almost all decision-making process.
- A decision-maker collects information
- Information sources may provide
  - unreliable
  - malicious
  - noisy

Subjective Logic (SL) and Distribution

- Probabilistic logic
- Standard Logic: restriction
  - either **true** or **false**
- Expressed with degrees of uncertainty
- Directly compatible with binary logic
- Beta and Dirichlet distributions
- Defined on the interval [0, 1]

Subjective Opinions

- Binomial opinions
- Beliefs about propositions with degrees of uncertainty
- A binomial opinion about binary proposition is $\omega_x = (b_x, d_x, u_x, a_x, W)$.
- a is a base rate, which is set to $a_x = 0.5$
- W denotes non-informative prior weight, which is set to $W = 2$
- These components satify $b_x + d_x + u_x = 1$ and $b_x, d_x, u_x \epsilon [0, 1]$

Subjective Opinions

$$op(r,s) = (b,d,u) = \left(\frac{r}{r+s+2}, \frac{s}{r+s+2}, \frac{2}{r+s+2}\right) \tag{1}$$

$$r = \frac{2 * b}{u}, \tag{2a}$$

$$s = \frac{2 * d}{u} \tag{2b}$$

$$E_x = b_x + a_x u_x \tag{3}$$

Behaviors of Information Sources

- Provide their opinions in the form of Dirichlet parameters
- A set of information sources including two unreliable sources : **Bob** and **Carol**
  - $\alpha^{Bob:x} = \langle 25, 175 \rangle$, $[0.12, 0.87, 0.01, 0.5, 0.5, 2]$
  - $\alpha^{Carol:x} = \langle 38, 62 \rangle$, $[0.37, 0.61, 0.02, 0.5, 0.5, 2]$
- A reliable source : **John**
  - $\alpha^{John:x} = \langle 195, 5 \rangle$, $[0.98, 0.01, 0.01, 0.5, 0.5, 2]$

Modeling Information Sources

- Adoption of specific behaviors with a certain probability while sharing their opinions.
- Let us consider binomial opinion $\alpha^{s:x} = \langle \alpha_1, \alpha_2 \rangle$
- If the source
  - **Honest**: $\varphi_{h(\alpha^{s:x})} = \langle \alpha_1, \alpha_2 \rangle$
  - **Flip**: $\varphi_{f(\alpha^{s:x})} = \langle \alpha_2, \alpha_1 \rangle$
  - **Random**: $\varphi_{r(\alpha^{s:x})} = \langle 1, 1 \rangle$

Modeling Information Sources

- The behavior profile : **maximum likelihood method**
- Have opinions for **n** common propositions
- The likelihood function:

$$L\left(t^{s}|\alpha^{a:1},\alpha^{s:1},...,\alpha^{a:n},\alpha^{s:n}\right)=\prod_{x=1}^{n}\int_{p}\left(f\left(p|\alpha^{a:x}\right)\times\sum_{i=1}^{k}t_{i}^{s}\times f\left(p|m_{i}\left(\alpha^{s}\right.\right.\right. \tag{4}$$

- Calculate the value of the behavior profile of information sources: t^s
- Compute likelihood function: **log likelihood method**

Fusion of Opinions

- **Estimating Source Behavior**
  - To find an elementary vector z^s

$$L\left(\mathbf{z}^1, \ldots, \mathbf{z}^n | \alpha^{1:y}, \ldots, \alpha^{n:y}, \mathbf{t}^1, \ldots, \mathbf{t}^n\right) = \int_{\mathbf{p}} \prod_{s=1}^{n} \prod_{i=1}^{k} \left(t_i^s \times f\left(\mathbf{p} | m_i\left(\alpha^{s:y}\right)\right)\right)^{z_i^s} \tag{5}$$

    - NP-complete
    - Complexity $O\left(k^n\right)$: **k** is the number of behavior types and **n** is the number of information sources.

- **Estimating Ground Truth**
  - The likelihood function

$$L\left(\mathbf{p}|\alpha^{\mathbf{1}:\mathbf{y}},\ldots,\alpha^{\mathbf{n}:\mathbf{y}},\mathbf{z}^{\mathbf{1}},\ldots,\mathbf{z}^{\mathbf{n}}\right)=\prod_{s=1}^{n}\prod_{i=1}^{k} f\left(\mathbf{p}|m_i\left(\alpha^{s:y}\right)\right)^{z_i^s} \tag{6}$$

$$f\left(\mathbf{p}|\alpha^{+}\right) : \text{single Dirichlet distribution}$$

$$\alpha^{+} = Wa^{y} + \sum_{s=1}^{n}\sum_{i=1}^{k} z_i^s \times \left(m_i\left(\alpha^{s:y}\right) - Wa^{y}\right) \tag{7}$$

Evaluation

- Competent sources generate an opinion close to the ground truth
- The collected opinions are binomial

Simulated Behaviors

- Three types of behaviors; *Honest, Flipping,* and *Random Competent*
- Determine behavior probability vector
  - $p^s = (p^s_h, \, p^s_f, \, p^s_r)$
  - $p^s_h + p^s_f + p^s_r = 1.$
  - $p^s_i = 1 - 2\phi$ and $p^s_{ij} = \phi$ for any $j \epsilon \{h, \, f, \, r\} \setminus i$
    - $0 \leq \phi < 1/3.$
- Ratios of sources are fixed: $R_h = 0.2, R_f = 0.3, R_r = 0.5$

Evaluation

Benchmarking Fusion Methods

- Compare two fusion methods
  - Discounted Consensus (DC)
  - Behavioral Discounted Consensus (BDC)
- DC only trustworthy sources
- BDC other behaviors

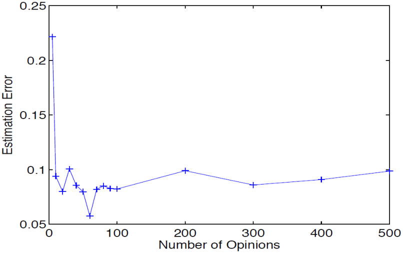
Behavior Estimation Results

- Compute the estimation error

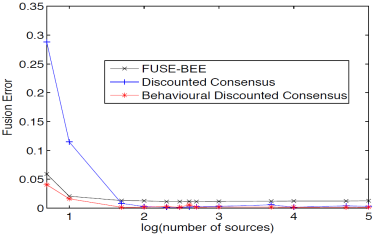
$$error\left(t^s|p^s\right) = \sqrt{\left(t_h^s - p_h^s\right)^2 + \left(t_f^s - p_f^s\right)^2 + \left(t_r^s - p_r^s\right)^2} \tag{8}$$

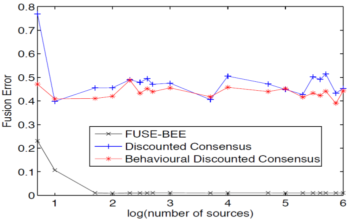
$$error\left(\omega_x|gt_x\right) = \sqrt{\left(b_x - gt_x\right)^2 + \left(d_x + gt_x - 1\right)^2 + \left(t_r^s - p_r^s\right)^2} \tag{9}$$

Simulation Results

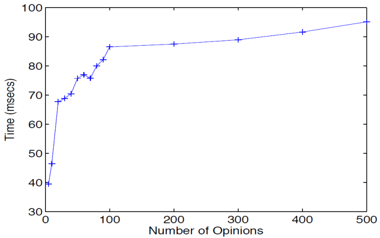


1. Average behavior estimation error.
2. Average fusion error when sources are consistent.
3. Average fusion error for $\phi = 0.15$.





Simulation Results



1. Average time used for behavior estimation.
2. Average time for clustering and fusion.
3. Average percentage error in estimating source behavior during fusion.