group is an ordered pair whose components are the set and the operation in question.

Definition: An ordered pair $(G, \circ)$, where G is a nonempty set and $\circ$ is a binary operation on G, is called a *group* provided the following hold.

(i) $\circ$ is a (well defined) binary operation on G. Thus, for any $a, b \in G, a \circ b$ is a uniquely determined element of G.

(ii) For all $a, b, c \in G$, we have $(a \circ b) \circ c = a \circ (b \circ c)$.

(iii) There is an element e in G such that

$$a \circ e = a \text{ for all } a \in G$$

and which is furthermore such that

(iv) for all $a \in G$, there is an x with

$$a \circ x = e.$$

When $(G, \circ)$ is a group, we also say that G is (or builds, or forms) a group with respect to $\circ$ (or under $\circ$). Since a group is an ordered pair, two groups $(G, \circ)$ and $(H, *)$ are equal if and only if G = H and the binary operation $\circ$ on G is equal to the binary operation $*$ on G (i.e., $\circ$ and $*$ are identical mappings from $G \times G$ into G). On one and the same set G, there may be distinct binary operations $\circ$ and $*$ under which G is a group. In this case, the groups $(G, \circ)$ and $(G, *)$ are distinct.

The four conditions (i)-(iv) of Definition 7.2 are known as the *group axioms.* The first axiom (i) is called the *closure axiom.* When (i) is true, we say G is *closed under* $\circ$.

A binary operation $\circ$ on a nonempty set G is said to be *associative* when (ii) holds. The associativity of $\circ$ enables us to write $a \circ b \circ c$ without ambiguity. Indeed, $a \circ b \circ c$ has first no meaning at all. We must write either $(a \circ b) \circ c$ or $a \circ (b \circ c)$ to denote a meaningful element in G. By associativity, we may and do make the convention that $a \circ b \circ c$ will mean $(a \circ b) \circ c = a \circ (b \circ c)$, for whether we read it as $(a \circ b) \circ c$ or $a \circ (b \circ c)$ does not make any difference. This would be wrong if $\circ$ were not associative. For instance, : (division) is not an associative operation on $\mathbb{Q} \setminus \{0\}$ and $(a : b) : c \neq a : (b : c)$ unless $c = 1$ (here $(a, b, c \in \mathbb{Q} \setminus \{0\})$). Thus $a : b : c$ is ambiguous.

An element e of a set G, on which there is a binary operation $\circ$, is called a *right identity element* or simply a *right identity* if $a \circ e = a$ for all a in