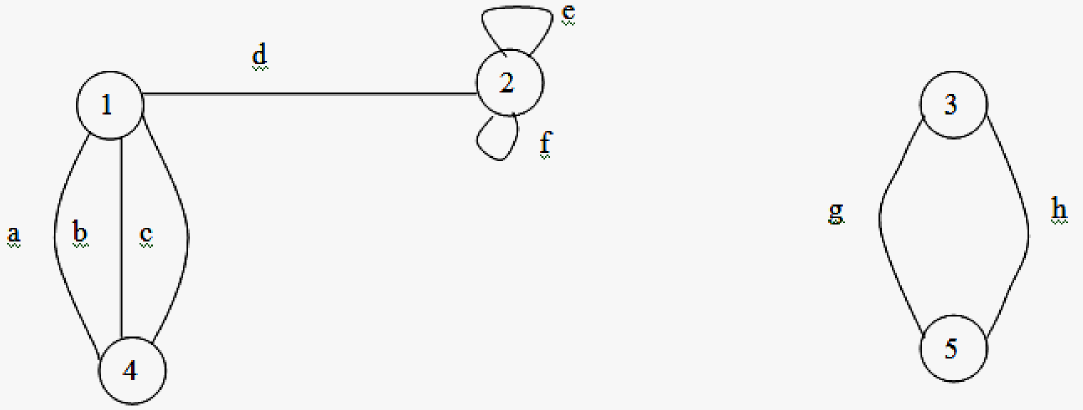
15-ma’ruza.Grafning berilish usullari. Qo'shnilik va insidentlik matrisalari. Graflarning izomorfligi

Endi umumiy xolda chekli, orientirlanmagan graflarni kiritamiz.

Ta’rif. Graf deb $G=(X,U,\varphi)$ tartiblangan uchlikka aytiladi, bu erda $X\neq\emptyset$ uchlar to‘plami U- qirralar to‘plami (chekli), $\varphi: U\Rightarrow X^2$ akslantirish, xar bir $u\Leftarrow U$ qirra uchun uning $x,y\in X$ uchlariga tartiblanmagan $\varphi(u)=xy$ juftlikni mos qo‘yadi. Agar $\varphi(u)=xx$ bo‘lsa, u xolda u qirra x uchdagi sirtmoq $\varphi(u)=x,y\wedge x\neq y$ bo‘lsa, u zveno deyiladi. Agar x va u uchlarning ikkalasi kamida bitta umumiy insident qirraga ega bo‘lsa, ular qo‘shni deyiladi. Agar u va v qirralar uchun u $\neq$ v va $\varphi(u)=\varphi(v)$ bo‘lsa, u xolda ular parallel deyiladi. Agar grafning uchlari X=(1,2,…, n) kabi tartiblangan bo‘lsa, u xolda uni $A(G)=(\alpha_{i_j})$ qo‘shnilik matritsasi yordamida berish mumkin, bu erda α_{i_j} shu I va J uchlarni tutashtiruvchi qirralar soni. Insidentlik matritsasi $B(G)=(\beta_{i_j})_m^n$ bo‘yicha grafni yagona ravishda tiklash mumkin:

$$\beta_{i_j}=\begin{cases}1, \text{ agar I uch va j qirra insident}\\ 0, \text{ aks xolda.}\end{cases}$$

Bunda i=1,2,…m va qirralar xam tartiblangan hisoblanadi, $U=(u_1,u_2,\ldots u_m)$



9-shakl.

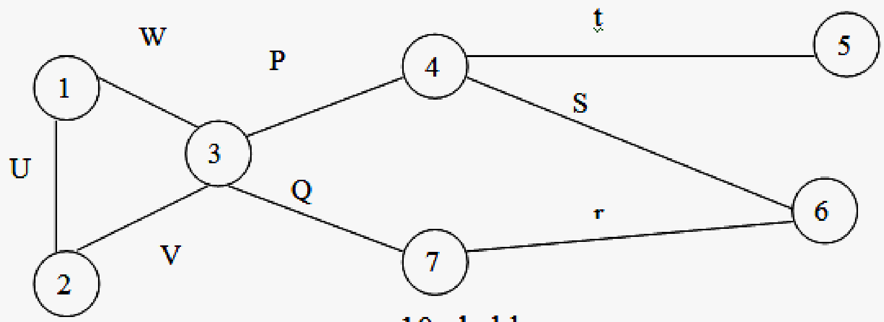
9 shaklda uchlari X=(1,2,3,4,5), qirralari U=(a,b,c,d,e,f,g,h) bo‘lgan $G=(X,U\,\psi)$ graf berilgan. Akslantirish esa ψ quyidagicha aniqlangan $\psi(a)=\psi(b)=\psi(c)=14,\psi(d)=12,\psi(e)=\psi(f)=22,\psi(g)=\psi(h)=35$. Bu graf multigraf deyiladi.

Bu graflar uchun

$$A(G)=\begin{pmatrix} 0 & 1 & 0 & 3 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2, \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{pmatrix},\quad B(G)=\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Marshrutlar, zanjirlar,ssikllar. Bog‘liqliliq.

Ta’rif. Oddiy G grafdagi $x_0u_1\, x_1u_2\ldots x_{l-1}u_lx_{ln}$ ketma ketlik uzunligi l ga teng bo‘lgan x_0 va x_l uchlarni tutashtiruvchi marshrut deyiladi, bunda $x_0,x_1,\ldots,x_l\in X, u_1,u_2,\ldots,u_l\in U$.
Agar $x_0 = x_l$ va $l\geq 1$bo‘lsa, marshrutssiklik xar xil bo‘lishi talab qilinmaydi.
Ta’rif. Qirralari xar xil bo‘lgan marshrut zanjir deb ataladi. siklik zanjir sikl deyiladi. Agar zanjirda x_0 va x_l lardan tashqari barcha uchlar xar xil bo‘lsa, u oddiy zanjir (sikl) deb ataladi.



10-shakl.

10 shakldagi grafda 3 v 2 u 1w3p4t5t4t5 va 3w 1u2v3p4t5t4t5 marshrutlar bir xil elementlardan tuzilgan bo‘lsada, lekin xar xildir, ularssiklik emas, va zanjir xam emasdir. 3w1u2v3p4 marshrut zanjir, lekin sodda emas,ssiklni tashkil etmaydi. 3w1u2v3p4s6r7q3 sodda bo‘lmaganssikldir.
3q 7r 6s 4p3 sodda sikldir.

Ta’rif. Agar G grafning x va u uchlari orasida xech bo‘lmaganda bitta zanjir mavjud bo‘lsa, u xolda ular tutashtirilgan deyiladi.

Grafning uchlari to‘plamida berilgan “tutashtirilganlik” munosabati refleksivlik, simmetriklik va tranzitivlik xossalariga ega, demak bu munosabat ekvivalentlikdir va grafning X uchlar to‘plamini $X_1,X_2,\ldots,X_k$ sinflarga ajratadi. Xar bir sinfga tegishli uchlar o‘zaro tutashtirilgandir. Turli sinflarga tegishli uchlar orasida zanjirlar yo‘q.

$G=(X,U)$ grafning $G_i=(X_i,V_i)$, $(i=1,2,\ldots,k)$ qism grafi uning bog‘liqli komponentasi deyiladi. Bog‘liqli G grafning uchlari to‘plami X ga masofa tushunchasini kiritish mumkin. i va j uchlar orasidagi masofa deb

$$d(i,j)=\min l_{[i,j]}$$

ga aytiladi, bu erda $l_{[i,j]}$, shu [i,j] zanjirning uzunligi va minimum barcha [i,j] zanjirlar bo‘yicha olinadi.

Kiritilgan d(i,j) uchun masofaning barcha xossalari (aksiomalari) bajariladi.

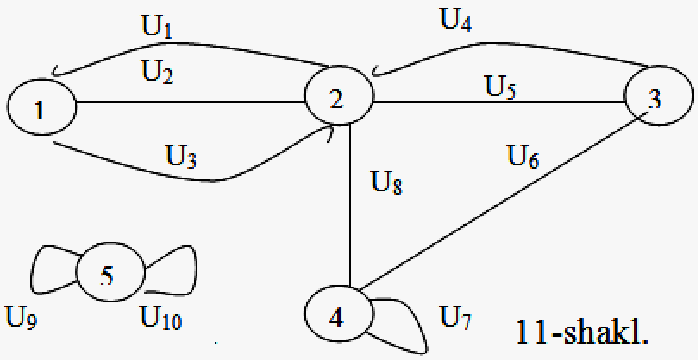
1) $d(i,i)=0$, $d(i,j)>0$, $i\neq j$

2) $d(i,j)= d(j,i)$

3) $d(i,j)+ d(j,k)\geq d(i,k)$

Demak X uchlar to‘plami metrik fazoni tashkil etadi.

$G=(X,U,\psi)$ multigraf berilgan bo‘lsin, bunda $X=(x_1,x_2,\ldots,x_n)$, $U=(u_1,u_2,\ldots,u_m)$, $A(G)=(\alpha_{i,j})$ qo‘shnilik matritsasi. Grafning X_i va X_j uchlarini tutashtiruvchi uzunligi l bo‘lgan turli xil marshrutlar sonini qaraymiz. Bu son $[A(G)]^l=(\alpha_{ij}^l)$ matritsaning $\alpha_{ij}^{(l)}$ elementiga teng.

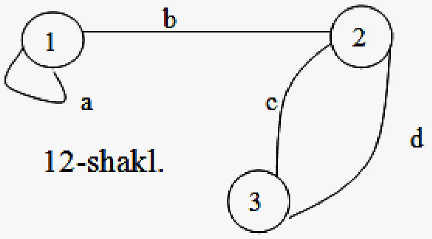


11-shakl.

$$A=\begin{pmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$A^2=\begin{pmatrix} 90 & 0 & 6 & 3 & 0 \\ 0 & 14 & 1 & 3 & 0 \\ 6 & 1 & 5 & 3 & 0 \\ 3 & 3 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}, \quad A^3=\begin{pmatrix} 0 & 42 & 3 & 9 & 0 \\ 42 & 15 & 31 & 18 & 0 \\ 3 & 31 & 5 & 9 & 0 \\ 9 & 18 & 9 & 9 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{pmatrix}$$

Masalan, x_1 uch bilan x_4 uchni tutashtiruvchi uzunliklari 2 ga teng bo‘lgan 3 ta marshrut bor va bu uchlarni tutashtiruvchi uzunliklari 3 ga teng 9 ta marshrut bor.
Marshrutlarni o‘zlarini aniqlash usulini sodda misolda ko‘rib o‘tamiz. 12- shakl. Bu grafning takomillashtirilgan qo‘shnilik matritsasini tuzamiz.



12-shakl.

$$A(u) = (a_{ij}(u)) = \begin{pmatrix} a & b & o \\ b & o & c+d \\ o & c+d & o \end{pmatrix}$$

Bu erda $a_{ij}(u)$ shu i va j uchlarni tutashtiruvchi qirralarning shartli yig‘indisi. Qirralar belgilarini nokommutativ yarim xalqaning yasovchilari deb qabul qilamiz.
A(u) matritsaning ketma-ket darajalarini topamiz.

$$\left[\mathrm{A(u)}\right]^2 = \begin{pmatrix} a^2+b^2 & ab & bc+bd \\ \mathrm{va} & b^2+c^2+cd+dc & 0 \\ cb+db & 0 & c^2+d^2+cd+dc \end{pmatrix}$$

$$\left[\mathrm{A(u)}\right]^2 = \begin{pmatrix} a^3+b^2a+ab^2 & a^2b+b^3+bc^2+ bcd+bdc+bd^2 & abc+abd \\ ba^2+b^3+c^2b+ d^2b+cdb+dcb & bab & b^2c+b^3+d^2c+cdc+ +dc^2+b^2d+c^2d+ d^3+cd^2+dcd \\ cba+dba & cd^2+db^2+c^3+d^2c+ cdc+dc^2+c^2d+ d^3+cd^2+dcd & 0 \end{pmatrix}$$

Masalan $[A(U)]^3$ matritsaning $\alpha_{21}^{(3)} = ba^2+b^3+d^2b+c^2b+cdb+dcb$ elementi x_2 bilan x_1 ni tutashtiruvchi uzunligi 3 ga teng bo‘lgan 6 ta marshrutni aniqlaydi.

$x_2bx_1ax_1ax_1$ $\quad$ $x_2bx_1bx_2bx_1$ $\quad$ $x_2cx_3cx_2bx_1$
$x_2dx_3dx_2bx_1$ $\quad$ $x_2cx_3dx_2bx_1$ $\quad$ $x_2dx_3cx_2bx_1$

GRAFLARNING IZOMORFLIGI

Ta’rif. Agar G va G' graflar uchlari to‘plamlari X va X' lar orasida o‘zaro bir qiymatli va uchlarning qo‘shnilik munosabatini saqlaydigan moslikni $(\leftrightarrow)$ o‘rnatish mumkin bo‘lsa, yaъni $\forall x, y \in x$ va ularga mos $x', y' \in x'(x \leftrightarrow x', y \leftrightarrow y')$ uchun $xy \in U \leftrightarrow x'y' \in U'$ bo‘lsa, u xolda bu graflar izomorf deyiladi.

Quyidagi graflar berilgan bo‘lsin.

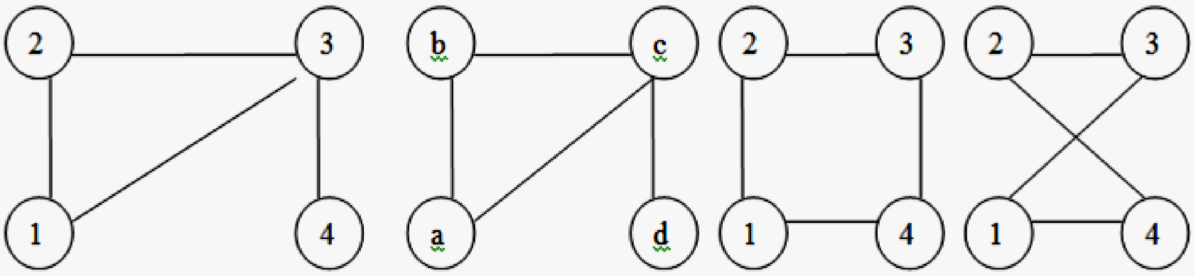
$G_{ig}=(I_n ui_t)$; i= 1, 2, 3, 4.

X_1=(1, 2, 3, 4) U_1=(ab,ac,bc,cd)

X_2=(a,b,c,d) U_2=(12,13,23,34)

X_3=(1, 2, 3, 4) U_3=(12,23,34,14)

X_4=(1, 2, 3, 4) U_4=(13,23,14,24)



Grafning analitik usulda berilishi usullari. Grafning matrisalar ko'rinishida berilishi. Qo'shnilik va insidentlik matrisalari. Qo'shnilik va insidentlik matrisalariga ko'ra grafni yasash. Izomorfizm tushunchasi. Graflarning izomorfligi