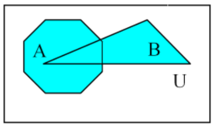
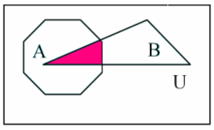
2-ma’ruza.To'plamlar ustida amallar

Eyler-Venn diagrammalari. To'plamlarni taqqoslash. To'plamlarning tengligi. To'plam quvvati. Teng quvvatli to'plamlar. To'plamlarning xossalari. To'plamlarning birlashmasi, kesishmasi, ayirmasi. Simmetrik ayirma. Sanoqli va kontinium quvvatli to'plamlar. Asosiy ayniyatlar.To'plamlarga doir asosiy ayniyatlarni taqqqoslashga doir misollar

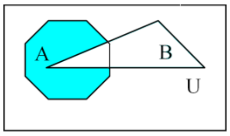
U univyersal to‘plamda quyidagi amallarni kiritamiz.



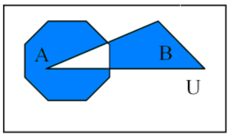
Ta’rif 1. A va B to‘plamlarning birlashmasi deb, bu to‘plamlarning hech bo‘lmaganda bittasiga tegishli bo‘lgan elementlardan iborat to‘plamga aytiladi va u $A \cup B$ kabi yoziladi, ya’ni Agar A,B$\in$**U** bolsa, u holda $A \cup B = \{\exists x:\ x \in A \text{ yoki } x \in B\}$. Ayrim hollarda A va B ning birlashmasi **yigindi** deb ham yuritiladi va A+B kabi belgilanadi.



Ta’rif 2. A va B to‘plamlarning kesishmasi (ko‘paytmasi) deb, ham A ga ham B ga tegishli elementlardan iborat to‘plamga aytiladi va $A \cap B$ ($A \cdot B$) kabi belgilanadi, ya’ni agar A,B$\in$ **U** bo‘lsa, u holda $A \cap B = \{\exists x:\_x \in A, x \in B\}$

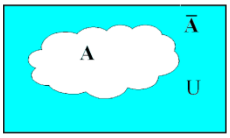


Ta’rif 3. A to‘plamdan B to‘plamning ayirmasi deb, A ning B ga tegishli bo‘lmagan elementlaridan iborat to‘plamga aytiladi va A\B kabi belgilanadi, ya’ni agar A,B$\in$ **U** bo‘lsa, u holda A\B=A-B= $\{\exists x:\_x \in A, x \notin B\}$



Ta’rif 4. A va B to‘plamlarning **simmetrik ayirmasi** (**halqali yig‘indisi**) deb, A to‘plamning B to‘plamga, B to‘plamning A to‘plamga tegishli bo‘lmagan elementlaridan iborat to‘plamga aytiladi va $A\Delta B$ kabi belgilanadi:

$$A\Delta B = A \oplus B = (A\backslash B) \cup (B\backslash A)$$

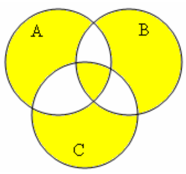


Ta’rif 5. U-universal to‘plamning A to‘plamga tegishli bo‘lmagan elementlaridan tuzilgan $\bar{\mathbf{A}}$ to‘plamga A to‘plamning **to‘ldiruvchisi** (**qarama-qarshisi**) deyiladi va quyidagicha aniqlanadi:

$$\bar{\mathbf{A}} = \mathbf{U\backslash A} = \{\exists x:\_x \in U, x \notin A\}$$

To‘plamlar ustida amalarda keltirilgan diagrammalarga **Eyler-Veynn diagrammalari** deyiladi. Ushbu kiritilgan amallar yordamida ayrim to‘plamlarni boshqalari orqali ifodalash mumkin, bunda birinchi bo‘lib to‘ldiruvchi amali, keyin kesishma va undan keyin yig‘indi va ayirma amallari bajariladi. Bu tartibni ozgartirish uchun qavslardan foydalaniladi. Shunday qilib to‘plamni boshqa to‘plamlar orqali amallar, qavslardan foydalanilgan holda ifodflash mumkin, bunday ifoda **to‘plamning analitik ifodasi** deyiladi.

Misol. Quyidagicha shtrixlangan to‘plamning analitik ifodasini A, B, C to‘plamlar orqali yozing.

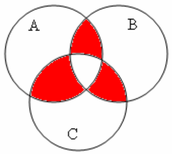


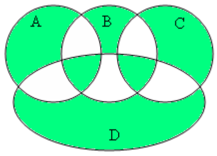
1-usul.

$(A\cap B\cap C)\cup(A\setminus(B\cup C))\cup(B\setminus(A\cup C))\cup(C\setminus A\setminus B)$

2-usul $A\Delta B\Delta C$

Misol. Quyidagicha shtrixlangan to‘plamlarning analitik ifodalarini A, B, C, D to‘plamlar orqali ifodalash talabaga taklif etiladi.





Eslatma. A va B to‘plamlar bitta **U-univyersum**ga tegishli bo‘lgandagina ular ustida amallar bajarilishi mumkin, agar ular turli xil univyersumlarga tegishli bo‘lsa, ya’ni $A\in U_1$ va $B\in U_2$ bo‘lsa, u holda ular ustida amallar bajarishdan oldin bitta universum ularning dekart ko‘paytmasi $U_1\times U_2$ ga o‘tiladi, keyin to‘plamlar ustida amallar bajarish mumkin bo‘ladi.

Misol. $A=\{1\}\subset U_1=\{1,2,3\}$ va $B=\{a,b\}\subset U_2=\{a,b,c\}$, $A\cap B-?$

Buning uchun U_1 va U_2 univyersumlar dekart ko‘paytmasini topib, undagi A va B to‘plamlar ko‘rinishini aniqlab olamiz:

$U_1\times U_2=\{<1,a>,<1,b>,<1,c>,<2,a>,<2,b>,<2,c>,<3,a>,<3,b>,<3,c>\}$, u holda

$A=\{<1,a>,<1,b>,<1,c>\}$, $B=\{<1,a>,<2,a>,<3,a>,<1,b>,<2,b>,<3,b>\}$

Endi A va B to‘plamlar ko‘paytmasini topishimiz mumkin: $A\cap B=\{<1,a>,<1,c>\}$

U-univyersal to‘plamning A, B, C to‘plam ostilari uchun quyidagi xossalar o‘rinli.

1.	$A\cup B=B\cup A$	Kommutativlik	11.	$A\cap A=A$	0 va 1 qonun lari
2.	$A\cap B=B\cap A$		12.	$A\cup\overline{A}=U$	
3.	$(A\cup B)\cup C=A\cup(B\cup C)$	Assotsiativlik	13.	$A\cap\overline{A}=\varnothing$	
4.	$(A\cap B)\cap C=A\cap(B\cap C)$		14.	$A\cup\varnothing=U$	
5.	$(A\cup B)\cap C=(A\cap C)\cup(B\cap C)$	distributivlik	15.	$A\cap U=A$	
6.	$(A\cap B)\cup C=(A\cup C)\cap(B\cup C)$		16.	$A\cup U=U$	
7.	$A\cap(A\cup B)=A$	Yutilish qonunlari	17.	$A\cap\varnothing=\varnothing$	
8.	$A\cup(A\cap B)=A$		18.	$\overline{U}=\varnothing$	
9.	$\overline{A\cap B}=\overline{A}\cup\overline{B}$	De Morgan qonunlari	19.	$\overline{\varnothing}=U$	
10.	$\overline{A\cup B}=\overline{A}\cap\overline{B}$		20.	$A\setminus B=A\cap\overline{B}$	Ayirish dan qutilish
	21.	$\overline{\overline{A}}=A$	Ikkilangan rad etish qonuni		

To‘plamlar ustida amallarning asosiy xossalariga ko‘ra algebraik ifodalarni soddalashtirish mumkin.

Misol. Ifodani soddalashtiring.

$\overline{A\cup(A\setminus\overline{B})\cup(\overline{A}\setminus\overline{B})}=\overline{A\cup(A\cap\overline{\overline{B}})\cup(\overline{A}\cap\overline{\overline{B}})}=\overline{A\cup(A\cap B)\cup(\overline{A}\cap B)}=\overline{A}\cap\overline{A\cap B}\cap\overline{\overline{A}\cap B}=$

$$= \overline{A} \cap (\overline{A} \cup \overline{B}) \cap (\overline{\overline{A}} \cup \overline{B}) = \overline{A} \cap (A \cup \overline{B}) = \overline{A} \cap A \cup \overline{A} \cap \overline{B} = \overline{A} \cap \overline{B}.$$

Yuqorida kiritilgan amallar va ularning xossalari yordamida ayrim to‘plamlardagi elementlar sonini bila turib, bu to‘plamlar ustida bajarilgan qandaydir amallardan iborat boshqa to‘plamlarning elementlari sonini hisoblash mumkin.