EEES012 MULTIDIMENSIONAL DIGITAL IMAGE PROCESSING TECHNIQUES HOMEWORK 1-MASK

Muhammet Nurullah Aydın
24830601410

1)

a) $\begin{bmatrix} 9 & 8 & 7 & 6 \\ 8 & 7 & 13 & 5 \\ 7 & 6 & 5 & 4 \\ 6 & 1 & 4 & 3 \end{bmatrix}$ input $\longleftrightarrow$ $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ filter $\Rightarrow$

$\begin{bmatrix} 9 & 9 & 8 & 7 & 6 & 6 \\ 9 & 9 & 8 & 7 & 6 & 6 \\ 8 & 8 & 7 & 13 & 5 & 5 \\ 7 & 7 & 6 & 5 & 4 & 4 \\ 6 & 6 & 1 & 4 & 3 & 3 \\ 6 & 6 & 1 & 4 & 3 & 3 \end{bmatrix}$ replicate-padded input

Filter applying steps:

(1,1) $\Rightarrow [9,9,8,8,9] \Rightarrow [8,8,9,9,9] \Rightarrow 9$ (order, find median)

(1,2) $\Rightarrow [9,8,7,7,8] \Rightarrow [7,7,8,8,9] \Rightarrow 8$

(1,3) $\Rightarrow [8,7,6,13,7] \Rightarrow [6,7,7,8,13] \Rightarrow 7$

(1,4) $\Rightarrow [7,6,6,5,6] \Rightarrow [5,6,6,6,7] \Rightarrow 6$

(2,1) $\Rightarrow [8,9,7,7,8] \Rightarrow [7,7,8,8,9] \Rightarrow 8$

(2,2) $\Rightarrow [8,8,13,6,7] \Rightarrow [6,7,8,8,13] \Rightarrow 8$

(2,3) $\Rightarrow [7,7,5,5,13] \Rightarrow [5,5,7,7,13] \Rightarrow 7$

(2,4) $\Rightarrow [13,6,5,4,5] \Rightarrow [4,5,5,6,13] \Rightarrow 5$

(3,1) $\Rightarrow [7,8,6,6,7] \Rightarrow [6,6,7,7,8] \Rightarrow 7$

(3,2) $\Rightarrow [7,7,5,1,6] \Rightarrow [1,5,6,7,7] \Rightarrow 6$

(3,3) $\Rightarrow [6,13,4,4,5] \Rightarrow [4,4,5,6,13] \Rightarrow 5$

(3,4) $\Rightarrow [5,5,4,3,4] \Rightarrow [3,4,4,5,5] \Rightarrow 4$

(4,1) $\Rightarrow [6,7,1,6,6] \Rightarrow [1,6,6,6,7] \Rightarrow 6$

(4,2) $\Rightarrow [6,6,4,1,1] \Rightarrow [1,1,4,6,6] \Rightarrow 4$

(4,3) $\Rightarrow [1,5,3,4,4] \Rightarrow [1,3,4,4,5] \Rightarrow 4$

(4,4) $\Rightarrow [4,4,3,3,3] \Rightarrow [3,3,3,4,4] \Rightarrow 3$

Output $\Rightarrow$ $\begin{bmatrix} 9 & 8 & 7 & 6 \\ 8 & 8 & 7 & 5 \\ 7 & 6 & 5 & 4 \\ 6 & 4 & 4 & 3 \end{bmatrix}$

b) We know the output of filter applied replicated input matrix from section (a). Thus we can use them to select min value:

$$\text{Output} \Rightarrow \begin{bmatrix} 8 & 7 & 6 & 5 \\ 7 & 6 & 5 & 4 \\ 6 & 1 & 4 & 3 \\ 1 & 1 & 1 & 3 \end{bmatrix}$$

c) We know the output of filter applied replicated-padded input matrix from section (a). Thus, we can use them to select max value.

$$\text{Output} \Rightarrow \begin{bmatrix} 9 & 9 & 13 & 7 \\ 9 & 13 & 13 & 13 \\ 8 & 7 & 13 & 5 \\ 7 & 6 & 5 & 4 \end{bmatrix}$$

d) We know the output of filter applied replicated-padded input matrix from section (a). Thus, we can obtain output by using them and weighting vector of the given order statistics filter. ($w_i = \{0, 1/3, 1/3, 1/3, 0\}$)

$(1,1) \Rightarrow \frac{1}{3}(9+8+8)$ $(1,2) \Rightarrow \frac{1}{3}(8+7+7)$ $(1,3) \Rightarrow \frac{1}{3}(7+6+13)$ $(1,4) \Rightarrow \frac{1}{3}(6+6+5)$

$(2,1) \Rightarrow \frac{1}{3}(9+7+7)$ $(2,2) \Rightarrow \frac{1}{3}(8+13+6)$ $(2,3) \Rightarrow \frac{1}{3}(7+5+5)$ $(2,4) \Rightarrow \frac{1}{3}(6+5+4)$

$(3,1) \Rightarrow \frac{1}{3}(8+6+6)$ $(3,2) \Rightarrow \frac{1}{3}(7+5+1)$ $(3,3) \Rightarrow \frac{1}{3}(13+4+4)$ $(3,4) \Rightarrow \frac{1}{3}(5+4+3)$

$(4,1) \Rightarrow \frac{1}{3}(7+1+6)$ $(4,2) \Rightarrow \frac{1}{3}(6+4+1)$ $(4,3) \Rightarrow \frac{1}{3}(5+3+4)$ $(4,4) \Rightarrow \frac{1}{3}(4+3+3)$

$$\text{Output} \Rightarrow \begin{bmatrix} 8 & 7 & 9 & 5 \\ 8 & 9 & 6 & 5 \\ 7 & 4 & 7 & 4 \\ 5 & 4 & 4 & 3 \end{bmatrix}$$

2)

$$\begin{bmatrix} 3 & 3 & 1 & 3 & 3 & 3 & 4 \\ 0 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 2 & 3 & 3 & 12 \\ 12 & 3 & 3 & 3 & 3 & 12 & 12 \\ 10 & 12 & 2 & 3 & 3 & 12 & 12 \\ 12 & 14 & 12 & 12 & 12 & 12 & 11 \\ 11 & 12 & 12 & 12 & 10 & 12 & 12 \end{bmatrix}$$

input

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

G_x G_y → Prewitt Gradient Operators

$|g(m,n)| = |g_x(m,n)| + |g_y(m,n)|$

Threshold = 20

⇒ Apply edge detector matrices to input matrix

⇒ (m=2, n=2)

$$\begin{bmatrix} 3 & 3 & 1 \\ 0 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3+0-3+0+0+0+1+3+3| = |1| = 1 \\ |G_y| &= |3+3+1+0+0+0-3-3-3| = |-2| = 2 \end{aligned}$$

$g = 2+1 = 3 < 20 \Rightarrow 0$

⇒ (m=2, n=3)

$$\begin{bmatrix} 3 & 1 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 2 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-3+0+0+0+3+3+2| = |-1| = 1 \\ |G_y| &= |3+1+3+0+0+0-3-3-2| = |-1| = 1 \end{aligned}$$

$g = 1+1 = 2 < 20 \Rightarrow 0$

⇒ (m=2, n=4)

$$\begin{bmatrix} 1 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 2 & 3 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-1-3-3+0+0+0+3+3+3| = |2| = 2 \\ |G_y| &= |1+3+3+0+0+0-3-2-3| = |-1| = 1 \end{aligned}$$

$g = 2+1 = 3 < 20 \Rightarrow 0$

⇒ (m=2, n=5)

$$\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 2 & 3 & 3 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-2+0+0+0+3+3+3| = |1| = 1 \\ |G_y| &= |3+3+3+0+0+0-2-3-3| = |1| = 1 \end{aligned}$$

$g = 1+1 = 2 < 20 \Rightarrow 0$

⇒ (m=2, n=6)

$$\begin{bmatrix} 3 & 3 & 4 \\ 3 & 3 & 3 \\ 3 & 3 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-3+0+0+0+4+3+12| = |10| = 10 \\ |G_y| &= |3+3+4+0+0+0-3-3-12| = |-8| = 8 \end{aligned}$$

$g = 10+8 = 18 < 20 \Rightarrow 0$

⇒ (m=3, n=2)

$$\begin{bmatrix} 0 & 3 & 3 \\ 3 & 3 & 3 \\ 12 & 3 & 3 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |0-3-12+0+0+0+3+3+3| = |-6| = 6 \\ |G_y| &= |0+3+3+0+0+0-12-3-3| = |-12| = 12 \end{aligned}$$

$g = 6+12 = 18 < 20 \Rightarrow 0$

⇒ (m=3, n=3)

$$\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 3 & 3 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-3+0+0+0+3+2+3| = |-1| = 1 \\ |G_y| &= |3+3+3+0+0+0-3-3-3| = |0| = 0 \end{aligned}$$

$g = 1+0 = 1 < 20 \Rightarrow 0$

$\Rightarrow (m=3, n=4)$

$$\begin{bmatrix} 3 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-3+0+0+0+3+3+3| = 0 \\ |G_y| &= |3+3+3+0+0+0-3-3-3| = 0 \\ g &= 0 < 20 \Rightarrow 0 \end{aligned}$$

$\Rightarrow (m=3, n=5)$

$$\begin{bmatrix} 3 & 3 & 3 \\ 2 & 3 & 3 \\ 3 & 3 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-2-3+0+0+0+3+3+12| = 10 \\ |G_y| &= |3+3+3+0+0+0-3-3-12| = 9 \\ g &= 19 < 20 \Rightarrow 0 \end{aligned}$$

$\Rightarrow (m=3, n=6)$

$$\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 12 \\ 3 & 12 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-3+0+0+0+3+12+12| = 18 \\ |G_y| &= |3+3+3+0+0+0-3-12-12| = 18 \\ g &= 36 \geq 20 \Rightarrow 1 \end{aligned}$$

$\Rightarrow (m=4, n=2)$

$$\begin{bmatrix} 3 & 3 & 3 \\ 12 & 3 & 3 \\ 10 & 12 & 2 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-12-10+0+0+0+3+3+2| = 17 \\ |G_y| &= |3+3+3+0+0+0-10-12-2| = 15 \\ g &= 32 \geq 20 \Rightarrow 1 \end{aligned}$$

$\Rightarrow (m=4, n=3)$

$$\begin{bmatrix} 3 & 3 & 2 \\ 3 & 3 & 3 \\ 12 & 2 & 3 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-12+0+0+0+2+3+3| = 10 \\ |G_y| &= |3+3+2+0+0+0-12-2-3| = 9 \\ g &= 19 < 20 \Rightarrow 0 \end{aligned}$$

$\Rightarrow (m=4, n=4)$

$$\begin{bmatrix} 3 & 2 & 3 \\ 3 & 3 & 3 \\ 2 & 3 & 3 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-2+0+0+0+3+3+3| = 1 \\ |G_y| &= |3+2+3+0+0+0-2-3-3| = 0 \\ g &= 1 < 20 \Rightarrow 0 \end{aligned}$$

$\Rightarrow (m=4, n=5)$

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 3 & 12 \\ 3 & 3 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-2-3-3+0+0+0+3+12+12| = 19 \\ |G_y| &= |2+3+3+0+0+0-3-3-12| = 10 \\ g &= 29 \geq 20 \Rightarrow 1 \end{aligned}$$

$\Rightarrow (m=4, n=6)$

$$\begin{bmatrix} 3 & 3 & 12 \\ 3 & 12 & 12 \\ 3 & 12 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-3+0+0+0+12+12+12| = 27 \\ |G_y| &= |3+3+12+0+0+0-3-12-12| = 9 \\ g &= 36 \geq 20 \Rightarrow 1 \end{aligned}$$

$\Rightarrow (m=5, n=2)$

$$\begin{bmatrix} 12 & 3 & 3 \\ 10 & 12 & 2 \\ 12 & 14 & 12 \end{bmatrix} \begin{aligned} |G_x| &= |-12-10-12+0+0+0+3+2+12| = 17 \\ |G_y| &= |12+3+3+0+0+0-12-14-12| = 20 \\ g &= 37 \geq 20 \Rightarrow 1 \end{aligned}$$

$\Rightarrow (m=5, n=3)$

$$\begin{bmatrix} 3 & 3 & 3 \\ 12 & 2 & 3 \\ 14 & 12 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-12-14+0+0+0+3+3+12| = 11 \\ |G_y| &= |3+3+3+0+0+0-14-12-12| = 29 \end{aligned}$$

$g = 40 \geq 20 \Rightarrow 1$

$\Rightarrow (m=5, n=4)$

$$\begin{bmatrix} 3 & 3 & 3 \\ 2 & 3 & 3 \\ 12 & 12 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-2-12+0+0+0+3+3+12| = 1 \\ |G_y| &= |3+3+3+0+0+0-12-12-12| = 27 \end{aligned}$$

$g = 28 \geq 20 \Rightarrow 1$

$\Rightarrow (m=5, n=5)$

$$\begin{bmatrix} 3 & 3 & 12 \\ 3 & 3 & 12 \\ 12 & 12 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-12+0+0+0+12+12+12| = 18 \\ |G_y| &= |3+3+12+0+0+0-12-12-12| = 18 \end{aligned}$$

$g = 36 \geq 20 \Rightarrow 1$

$\Rightarrow (m=5, n=6)$

$$\begin{bmatrix} 3 & 12 & 12 \\ 3 & 12 & 12 \\ 12 & 12 & 11 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-3-12+0+0+0+12+12+11| = 17 \\ |G_y| &= |3+12+12+0+0+0-12-12-11| = 8 \end{aligned}$$

$g = 25 \geq 20 \Rightarrow 1$

$\Rightarrow (m=6, n=2)$

$$\begin{bmatrix} 10 & 12 & 2 \\ 12 & 14 & 12 \\ 11 & 12 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-10-12-11+0+0+0+2+12+12| = 7 \\ |G_y| &= |10+12+2+0+0+0-11-12-12| = 11 \end{aligned}$$

$g = 18 < 20 \Rightarrow 0$

$\Rightarrow (m=6, n=3)$

$$\begin{bmatrix} 12 & 2 & 3 \\ 14 & 12 & 12 \\ 12 & 12 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-12-14-12+0+0+0+3+12+12| = 11 \\ |G_y| &= |12+2+3+0+0+0-12-12-12| = 19 \end{aligned}$$

$g = 30 \geq 20 \Rightarrow 1$

$\Rightarrow (m=6, n=4)$

$$\begin{bmatrix} 2 & 3 & 3 \\ 12 & 12 & 12 \\ 12 & 12 & 10 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-2-12-12+0+0+0+3+12+10| = 1 \\ |G_y| &= |2+3+3+0+0+0-12-12-10| = 26 \end{aligned}$$

$g = 27 \geq 20 \Rightarrow 1$

$\Rightarrow (m=6, n=5)$

$$\begin{bmatrix} 3 & 3 & 12 \\ 12 & 12 & 12 \\ 12 & 10 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-12-12+0+0+0+12+12+12| = 9 \\ |G_y| &= |3+3+12+0+0+0-12-10-12| = 16 \end{aligned}$$

$g = 25 \geq 20 \Rightarrow 1$

$\Rightarrow (m=6, n=6)$

$$\begin{bmatrix} 3 & 12 & 12 \\ 12 & 12 & 11 \\ 10 & 12 & 12 \end{bmatrix} \Rightarrow \begin{aligned} |G_x| &= |-3-12-10+0+0+0+12+11+12| = 10 \\ |G_y| &= |3+12+12+0+0+0-10-12-12| = 7 \end{aligned}$$

$g = 17 < 20 \Rightarrow 0$

Output Binary Image (5×5) ⇒

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$