EEE5012 MULTIDIMENSIONAL DIGITAL IMAGE PROCESSING TECHNIQUES

HOMEWORK II - TRANSFORMATIONS

Muhammet Nurullah Aydın
24830601410

1)

$$\begin{bmatrix} 0 & 10 & 7 & 5 \\ 0 & 2 & 9 & 12 \\ 4 & 2 & 2 & 6 \\ 10 & 3 & 9 & 15 \end{bmatrix}$$ original image

$$\begin{bmatrix} 2 & 9 & 10 & 5 \\ 0 & 1 & 6 & 1 \\ 3 & 6 & 2 & 6 \\ 11 & 3 & 14 & 14 \end{bmatrix}$$ distorted image

$\Rightarrow$ Mean Absolute Error (MAE) $= \frac{1}{N}\sum_{i=1}^{N} |O_i - D_i|$

N = pixel number

$$MAE = \frac{1}{16}\left[|10-2| + |10-9| + |7-10| + \dots + |15-14|\right]$$

$$MAE = \frac{33}{16} = 2.0625$$

$\Rightarrow$ Mean Square Error (MSE) $= \frac{1}{N}\sum_{i=1}^{N} (O_i - O_d)^2$

$$MSE = \frac{1}{16}\left[(0-2)^2 + (10-9)^2 + \dots (15-14)^2\right] = \frac{189}{16} = 11.8125$$

$\Rightarrow$ Peak Signal-to-Noise Ratio (PSNR) $= 10.\log_{10}\left(\frac{MAX_I^2}{MSE}\right)$

where $MAX_I = 2^B - 1 \Rightarrow 2^4 - 1 = 15$

$$PSNR = 10\log_{10}\left(\frac{15^2}{11.8125}\right) = 10\log_{10}\left(\underbrace{\frac{225}{11.8125}}_{19.04}\right) \approx 10 \cdot 1.28 = 12.8\,dB$$

2)

$$\begin{bmatrix} 7 & 3 & 4 & 1 \\ 1 & 2 & 0 & 3 \\ 4 & 2 & 2 & 1 \\ 0 & 3 & 5 & 1 \end{bmatrix} \xrightarrow{T_1(r) = \text{round}(5\sqrt{r})} \begin{bmatrix} 13 & 9 & 10 & 5 \\ 5 & 7 & 0 & 9 \\ 10 & 7 & 7 & 5 \\ 0 & 9 & 11 & 5 \end{bmatrix}$$

$$\xrightarrow{T_2(r) = 15 - 2r} \begin{bmatrix} 1 & 9 & 7 & 13 \\ 13 & 11 & 15 & 9 \\ 7 & 11 & 11 & 13 \\ 15 & 9 & 5 & 13 \end{bmatrix}$$

3)

a)

$$\begin{bmatrix} 2 & 9 & 10 & 0 \\ 7 & 1 & 6 & 1 \\ 10 & 15 & 2 & 6 \\ 11 & 3 & 8 & 10 \end{bmatrix}$$ original image

$\xrightarrow{\text{zero padding}}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 9 & 10 & 0 & 0 \\ 0 & 7 & 1 & 6 & 1 & 0 \\ 0 & 10 & 15 & 2 & 6 & 0 \\ 0 & 11 & 3 & 8 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \frac{1}{4}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ Filter 1

⇒Applying Filter 1 to zero padded original image

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 9 & 10 & 0 & 0 \\ 0 & 7 & 1 & 6 & 1 & 0 \\ 0 & 10 & 15 & 2 & 6 & 0 \\ 0 & 11 & 3 & 8 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow (0\times0)+(0\times1)+(0\times0)+(0\times1)+(2\times0)+(9\times1)+(0\times0)+(7\times1)+(1\times0) = 16$

$\Rightarrow \frac{1}{4}\times 16 = 4$

...

When the filter is applied to each raws and columns, it can be obtained the output like below;

$$\begin{bmatrix} 4 & 3.25 & 3.75 & 2.75 \\ 3.25 & 9.25 & 3.5 & 3 \\ 8.25 & 4 & 8.75 & 3.25 \\ 3.25 & 8.5 & 3.75 & 3.5 \end{bmatrix} \Rightarrow \text{Output image}$$

b)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 9 & 10 & 0 & 0 \\ 0 & 7 & 1 & 6 & 1 & 0 \\ 0 & 10 & 15 & 2 & 6 & 0 \\ 0 & 11 & 3 & 8 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \frac{1}{4}\begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

Filter 2

$\Rightarrow (0\times-1)+(0\times0)+(0\times-1)+(0\times0)+(2\times4)+(9\times0)+(0\times-1)+(7\times0)+(1\times-1) = 7 \Rightarrow \frac{7}{4} = 1.75$

...

When the filter is applied to each row and column, it can be obtained the output like below;

$$\begin{bmatrix} 1.75 & 5.25 & 9.5 & -1.5 \\ 1 & -5 & -1.5 & -2 \\ 9 & 7 & -1.75 & 2.5 \\ 7.25 & 0 & 2.75 & 9.5 \end{bmatrix} \Rightarrow \text{output image}$$

c)

$$\frac{1}{4}\begin{bmatrix} -1 & 1 & -1 \\ 1 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix} \Rightarrow \text{Filter 3}$$

, Since the filter 3 is sum of the Filter 1 and Filter2, we can obtain output image by adding filtered image 1 and filtered image 2.

$$\begin{bmatrix} 4 & 3.25 & 3.75 & 2.75 \\ 3.25 & 9.25 & 3.5 & 3 \\ 8.25 & 4 & 8.75 & 3.25 \\ 3.25 & 8.5 & 3.75 & 3.5 \end{bmatrix} + \begin{bmatrix} 1.75 & 5.75 & 9.5 & -1.5 \\ 1 & -5 & -1.5 & -2 \\ 9 & 7 & -1.75 & 2.5 \\ 7.25 & 0 & 2.75 & 9.5 \end{bmatrix} = \begin{bmatrix} 5.75 & 9 & 13.25 & 1.25 \\ 4.25 & 4.25 & 2 & 1 \\ 17.25 & 11 & 7 & 5.75 \\ 10.5 & 8.5 & 6.5 & 13 \end{bmatrix} \Rightarrow \text{output image}$$

/2