

EEE5012 MULTIDIMENSIONAL DIGITAL IMAGE PROCESSING  
TECHNIQUES HOMEWORK III - HISTOGRAM

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1)

$$\begin{bmatrix} 0 & 10 & 7 & 5 \\ 0 & 2 & 9 & 12 \\ 4 & 2 & 2 & 6 \\ 10 & 3 & 9 & 15 \end{bmatrix} \xRightarrow{\text{Convert binary}} \begin{bmatrix} 0000 & 1010 & 0111 & 0101 \\ 0000 & 0010 & 1001 & 1100 \\ 0100 & 0010 & 0010 & 0110 \\ 1010 & 0011 & 1001 & 1111 \end{bmatrix}$$

original image

Bitplane 1 (MSB):

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Bitplane 2:

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bitplane 3:

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Bitplane 4 (LSB):

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

2)

$$\begin{bmatrix} 6 & 13 & 12 & 13 \\ 12 & 6 & 7 & 12 \\ 13 & 7 & 7 & 12 \\ 14 & 11 & 11 & 14 \end{bmatrix} \Rightarrow \text{Original Image}$$

a) Full-scale contrast stretch

Min value of all pixels = 6  
Max. " " " " = 14

$$s = \frac{(r - r_{\min})}{(r_{\max} - r_{\min})} \times (s_{\max} - s_{\min}) + s_{\min}, \text{ where } r = \text{input pixel}$$

$$s = \frac{(r - 6)}{(14 - 6)} (15 - 0) + 0 = \frac{15(r - 6)}{8}$$

$$\begin{aligned} 4 \text{ bit} & \leftarrow s_{\max} = 2^4 - 1 = 15 \\ s_{\min} & = 0 \\ r_{\min} & = 6 \\ r_{\max} & = 14 \end{aligned}$$

$$\text{Original Image} \Rightarrow s(r) \Rightarrow \begin{bmatrix} 0 & 13 & 11 & 13 \\ 11 & 0 & 2 & 11 \\ 13 & 2 & 2 & 11 \\ 15 & 9 & 9 & 15 \end{bmatrix}$$

b) We need to find Probability Density Functions (PDF) first;

$$6 \rightarrow 2/16 = 0.125 \quad 13 \rightarrow 3/16 = 0.1875$$

$$7 \rightarrow 3/16 = 0.1875 \quad 14 \rightarrow 2/16 = 0.125$$

$$11 \rightarrow 2/16 = 0.125$$

$$12 \rightarrow 4/16 = 0.25$$

⇒ Now, compute the Cumulative Distribution Function (CDF)

$$CDF(6) = 0.125$$

$$CDF(7) = 0.125 + 0.1875 = 0.3125$$

$$CDF(11) = 0.3125 + 0.125 = 0.4375$$

$$CDF(12) = 0.4375 + 0.25 = 0.6875$$

$$CDF(13) = 0.6875 + 0.1875 = 0.875$$

$$CDF(14) = 0.875 + 0.125 = 1$$

$$CDF(6) = 0.125 \times 15 = 1.875 \approx 2$$

$$CDF(7) = 0.3125 \times 15 = 4.6875 \approx 5$$

$$CDF(11) = 0.4375 \times 15 = 6.5625 \approx 7$$

$$CDF(12) = 0.6875 \times 15 = 10.3125 \approx 10$$

$$CDF(13) = 0.875 \times 15 = 13.125 \approx 13$$

$$CDF(14) = 1.0 \times 15 = 15$$

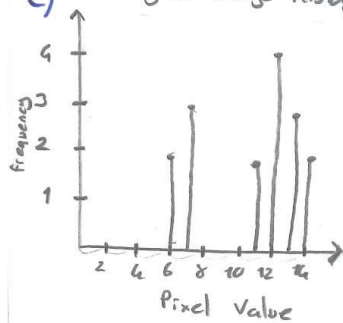
Normalize  
⇒

Original Image ⇒

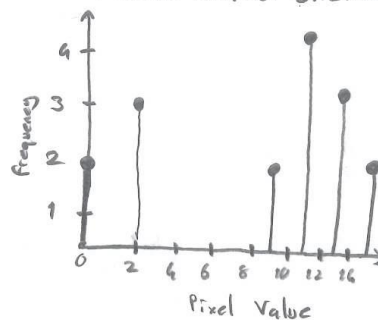
Histogram  
Equalization

2	13	10	13
10	2	5	10
13	5	5	10
15	7	7	15

c) Original Image Histogram



Full-scale contrast stretched image histogram



Histogram Equalized Image Histogram.

