Homework 0: Random Variables/Processes
due October 3, 23:59

1. Let X be a continuous random variable with pdf

$$f_X(x) = \begin{cases} kx, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Determine the value of k and sketch $f_X(x)$.

(b) Find and sketch the corresponding cdf $F_X(x)$.

(c) Find $P(1/4 < X \le 2)$.

2. Find the mean and variance of the random variable X of **Q1**.

3. The joint pdf of a bivariate random variable (X, Y) is given by

$$f_{XY}(x, y) = \begin{cases} k, & 0 < y \le x < 1 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Determine the value of k.

(b) Find the marginal pdf's of X and Y.

(c) Find $P(0 < X < 1/2, 0 < Y < 1/2)$.

(d) Find the conditional pdf's $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.

(e) Compute the conditional means $E(Y|x)$ and $E(X|y)$.

4. Let X and Y be defined by

$$X = \cos\Theta, \quad Y = \sin\Theta$$

where Θ is a random variable uniformly distributed over $(0, 2\pi)$.

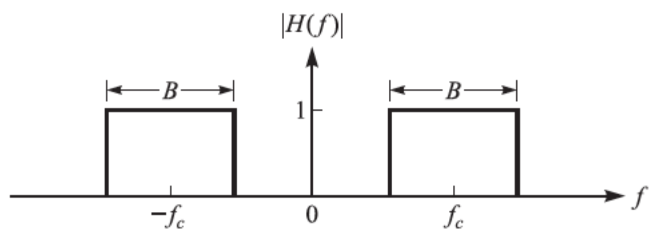
(a) Show that X and Y are uncorrelated.

(b) Show that X and Y are not independent.

5. The autocorrelation function of a stochastic process $X(t)$ is

$$R_X(\tau) = \frac{1}{2}N_0\delta(\tau)$$

Such a process is called white noise. Suppose $x(t)$ is the input to an ideal bandpass filter having the frequency response characteristic shown in the following figure. Determine the total noise power at the output of the filter.



6. A lowpass Gaussian stochastic process $X(t)$ has a power spectral density,

$$S(f) = \begin{cases} N_0, & |f| < B \\ 0, & \text{otherwise} \end{cases}$$

Determine the power spectral density and the autocorrelation function of $Y(t) = X^2(t)$.

7. Consider a band-limited zero-mean stationary stochastic process $X(t)$ with power density spectrum

$$S_X(f) = \begin{cases} 1, & |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

$X(t)$ is sampled at a rate $f_s = 1/T$ to yield a discrete-time process $X(n) \equiv X(nT)$.

(a) Determine the expression for the autocorrelation sequence of $X(n)$.

(b) Determine the minimum value of T that results in a white (spectrally flat) sequence.

8. Suppose that $N(t)$ is a zero-mean stationary narrowband process. The autocorrelation function of the equivalent lowpass process $Z(t) = X(t) + jY(t)$ is defined as

$$R_Z(\tau) = E[Z^*(t)Z(t+\tau)]$$

(a) Show that

$$E[Z(t)Z(t+\tau)] = 0$$

(b) Suppose $R_Z(\tau) = N_0\delta(\tau)$, and let

$$V = \int_0^T Z(t)dt$$

Determine $E[V^2]$ and $E[|V|^2]$.

Report and Submission:

You can prepare your reports either handwritten or on computer. Handwritten reports have to be scanned for upload. Make sure your scans are readable.

ALL SUBMISSIONS ARE ON MOODLE. Late submission is penalized with 10% per day.