Homework 3: Phase and Timing Estimation
due November 14

1 Specifications

- The symbol constellation is QPSK with unit energy: $\phi_i \in \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$, for $i = 1, 2, 3, 4$.
- The transmitted signal is
$$s(t) = \sum_{n=1}^{N} g(t - nT)\cos(2\pi f_c t + \phi_n) \tag{1}$$
where $g(t)$ is a raised cosine pulse shaping filter with a roll-off factor of $\alpha = 0.5$, $T = 1\ \mu s$ is the symboling period, N is the number of transmitted symbols, $f_c = 1\ GHz$ is the carrier frequency and ϕ_n is the phase of the n^{th} information symbol.
- The received signal under perfect synchronization is
$$r(t) = \sum_{n=1}^{N} g(t - nT)\cos(2\pi f_c t + \phi_n) + n(t) \tag{2}$$
where $n(t)$ is the additive white Gaussian noise.
- For all cases, simply transmit N pilot symbols, i.e. the receiver knows the transmitted information before the communication starts. Choose the symbol $\phi_n = \pi/4$ for all n without loss of generality.

Now consider the following three cases.

2 Case 1

There is no timing delay but there is a carrier phase shift of $\phi = \pi/4$. The receiver is not aware of ϕ, therefore it has to be estimated. Design a receiver, which incorporates the *decision directed carrier phase estimate*, that is covered in class. The input to the estimator should be the received signal with phase error:

$$r(t) = \sum_{n=1}^{N} g(t - nT)\cos(2\pi f_c t + \phi + \phi_n) + n(t) \tag{3}$$

Now, simulate the communication system with the given parameters in "Specifications" for the signal-to-noise (SNR) ratios of $\{0, 2, 4, ..., 20\}$ dB and for $N = 1, 5, 10, 20$. For each SNR and N value, obtain the phase estimation $\hat{\phi}$ and the phase error at the receiver, $\phi_e = |\hat{\phi} - \phi|$. Then plot ϕ_e vs SNR for different N values on the same graph. You should end up with 4 curves on one single figure.

3 Case 2

There is no phase shift but there is a symbol timing error of $\tau = T/4$. Design a receiver, which incorporates the *decision directed maximum likelihood timing estimation*, that is covered in class. The input to the estimator should be the received signal with timing error:

$$r(t) = \sum_{n=1}^{N} g(t - \tau - nT) \cos(2\pi f_c t + \phi_n) + n(t) \tag{4}$$

Now, simulate the communication system with the given parameters in “Specifications” for the *SNR* values of $\{0, 2, 4, ..., 20\}$ dB and for $N = 1, 5, 10, 20$. For each *SNR* and N value, obtain the timing estimation $\hat{\tau}$ and the timing error at the receiver, $\tau_e = |\hat{\tau} - \tau|$. Then plot τ_e vs *SNR* for different N values on the same graph. You should end up with 4 curves on one single figure.

4 Case 3

Now, assume there is a timing error of $\tau = T/30$ and a delay induced phase shift. Design a receiver, which incorporates the *decision directed joint maximum likelihood timing and phase estimation*, that is covered in class. The input to the estimator should be the received signal with timing and phase error:

$$r(t) = \sum_{n=1}^{N} g(t - \tau - nT) \cos(2\pi f_c t + \phi + \phi_n) + n(t) \tag{5}$$

with $\phi = -2\pi f_c \tau$. Now, simulate the communication system with the given parameters in “Specifications” for *SNR* values of $\{0, 2, 4, ..., 20\}$ dB and for $N = 1, 5, 10, 20$. For each *SNR* and N value, obtain the timing error, τ_e, and the phase error at the receiver, ϕ_e, then plot τ_e vs *SNR* and ϕ_e vs SNR for different N values on two different graphs. You should end up with two figures, each with 4 curves.

5 Report and Submission

Prepare three different MATLAB scripts (one for each case). Make sure your code is understandable.

Gather all your plots in the same report and explain the results. Do not forget to label the axes and caption the plots. Your report does not need to exceed two pages.

Zip the three m-files with your report pdf and upload it on the moodle. Late submission is penalized with 10% per day.