EE475 Homework 3

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I. Mean vs Median

A. a

All pixels except those in the center of a run of 3 equal horizontal pixels will change. Counting, there are only 15 pixels that will not change, so 49 will change. On figure 1 one can see the 15 pixels that will not change. The max gray level increase is from 14 to

$$\frac{1}{3}\times 14+\frac{2}{3}\times 41=32 \tag{1}$$

so largest gray level increase is

$$32-14=18 \tag{2}$$

(at pixel (3,4) using matrix notation).

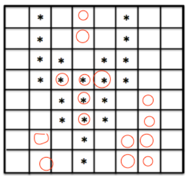


Fig. 1: pixels that will change their values

B. b

Counting the pixels which are equal to the median of the set of three horizontal pixels with itself as the pixel in the center of the three. There are 6 such in the first and 2nd rows, 7 in the 3rd, 7th and 8th rows, and 8 in the 4th-6th rows for a total of 57 that don't change. That leaves 7 that change which can be seen on figure 2. The largest gray level increase is 41- 14=27 (again at pixel (3,4)).

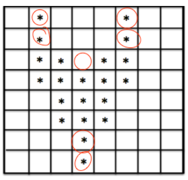


Fig. 2: pixels that will change their values.

II. Connectivity and Path Length

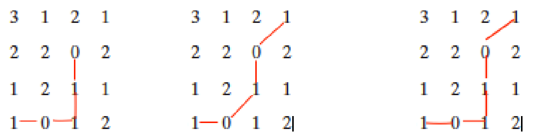
(a) When V = 0,1, 4-path does not exist between p and q because it is impossible to get from p to q by traveling along points that are both 4-adjacent and also have values from V. // Figure 3.a shows this condition; it is not possible to get to q. The shortest 8-path is shown on Figure 3.b. Its length is 4. The length of the shortest m- path is 5. It is shown on Figure 3.c

(b)One possibility for the shortest 4-path when V =1,2 is shown on Figure 4.a, its length is 6. One possibility for the shortest 8-path is shown in Figure 4.b; its length is 4. The length of a shortest m-path is 6 (Figure 4.c).

III. Estimation of Noise Parameters

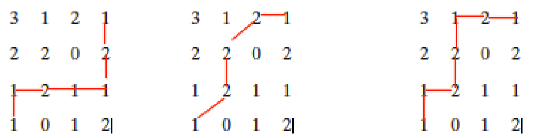
A. a

See Figure 5.



(a) 4- path (b) 8- path (c) m- path

Fig. 3: shortest 4-,8- abd m- paths for V = 0, 1



(a) 4- path (b) 8- path (c) m- path

Fig. 4: shortest 4-,8- abd m- paths for V = 1, 2

B. b

For Lena, a=0.0282 and b=448.3965. For BBC, a=19.0648 and b=675.0768. For Lena the calculated parameters check with the the true parameters. For BBC not. This is because of the higher pixel's gray scale values of BBC image. There are a lot of pixels on BBC image whose gray scale values bigger than 200. When one adds Rayleigh noise it causes overflowing. Because of overflowing there is a loss of noise in other words there is an amount of noise whose effect one cannot observe. This is what expected and one can confirm this looking at the pixel's gray scale values of contaminated BBC image. There are lot of pixels whose gray values are 255. On the other hand this is not the case looking at the pixel's gray scale values on contaminated Lena image.

IV. Gaussian Noise

Higher PSNR means more noise removed. However it is slightly biased towards over blurred results. Although a higher PSNR generally indicates that the reconstruction is of higher quality, in some cases it may not. It performs poorly compared to SSIM when it comes to estimating the quality of images as perceived by humans. SSIM has been developed to have a quality reconstruction metric that also takes into account the similarity of the edges between the denoised image and the ideal one. To have a good SSIM measure, an algorithm needs to remove the noise while also preserving the edges of the objects. On this context it's meaningful that with contraharmonic filter denoised image on figure 7.h has the lowest SSIM value. It has the least similarity with the original image. It's clearly more luminious and have the least similarity according to my visual judgement.

5x5 median filter results in a blurry image, and hence it has clearly lower SSIM value although it's PSNR value is slightly higher than other filters' outputs.

From the calculated values one may conclude that the best filter type to filter out gaussian noise is average filter or gaussian filter with smaller standard deviation or an alpha trimmed filter. My visual judgement would choose gaussian filter with the smaller sigma, because with average filter denoised image shown on figure 7.a is more blurry than 7.b and the one cannot

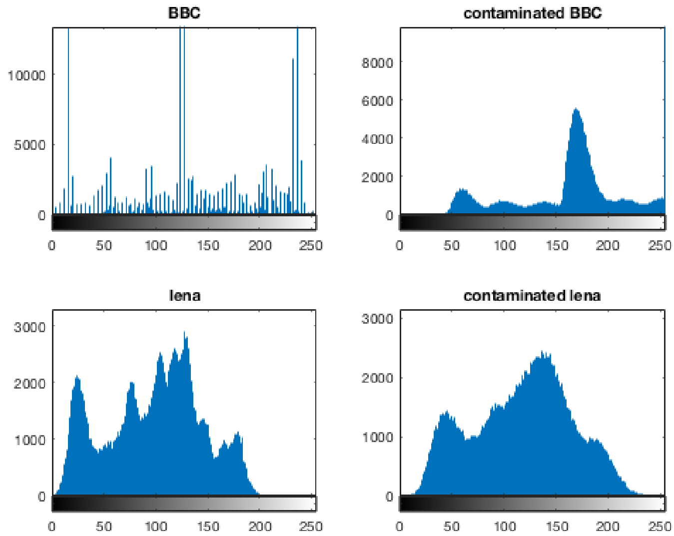


Fig. 5: Histograms for Lena and BBC, respectively.

distinguish well the parked cars and window details looking at the output of alpha trimmed filter. Gaussian filter with σ = 1 would be a perfect choice.

By the way although SSIM and PSNR values are calculated completely with different methods and it's more complicated to compute SSIM, an analytical link between PSNR and SSIM was identified in a research paper.

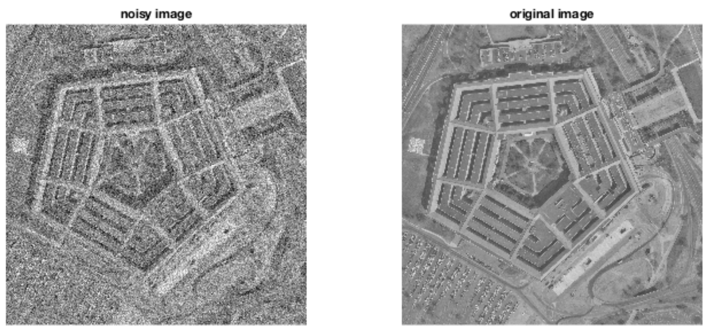


Fig. 6: Noisy(Gaussian N(0,1000)) and original images, respectively.

Filter	PSNR	SSIM
5x5 box	25.2422	0.5480
Gauss($\sigma = 1$)	25.3459	0.5488
Gauss($\sigma = 2.5$)	25.2502	0.5359
5x5 median	25.0468	0.5166
5x5 $\alpha trimmed(p = 3)$	25.2447	0.5473
5x5 geometric mean	23.5262	0.5336
5x5 harmonic mean	21.9110	0.5066
Contraharmonic with Q=2	21.3642	0.5183

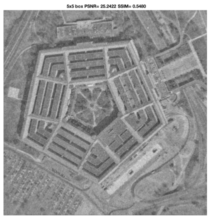
TABLE I: PSNR and SSIM values of corresponding filters.

V. SALT & PEPPER NOISE

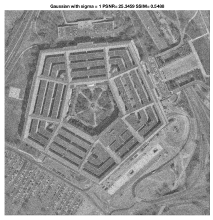
A. 2

B. 3

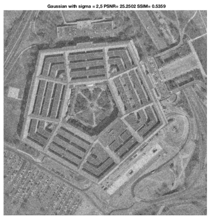
Best p values are calculated on MATLAB code empirical. I assumed that higher PSNR value results in better p value so I calculated the p value where the PSNR of the denoised image with certain filter size is the highest.



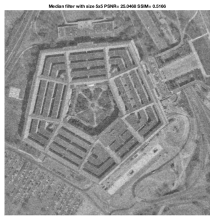
(a) 5x5 box filter



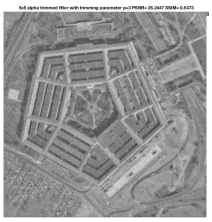
(b) Gaussian filter with σ = 1



(c) Gaussian filter with σ = 2,5

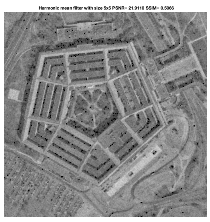


(d) median filter with size 5x5



(e) 5x5 alpha trimmed filter with p=3

(f) geometric mean filter 4x4



(g) harmonic mean 5x5

(h) contraharmonic with Q=2

Fig. 7: denoised images

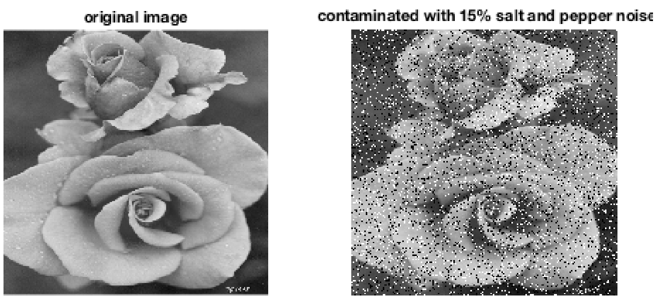


Fig. 8: original and noisy image

C. 4

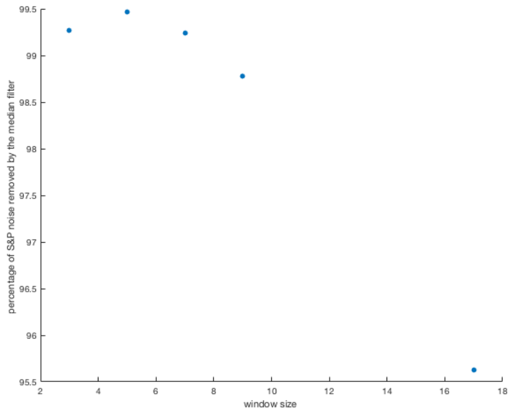


Fig. 9: plot of question 2

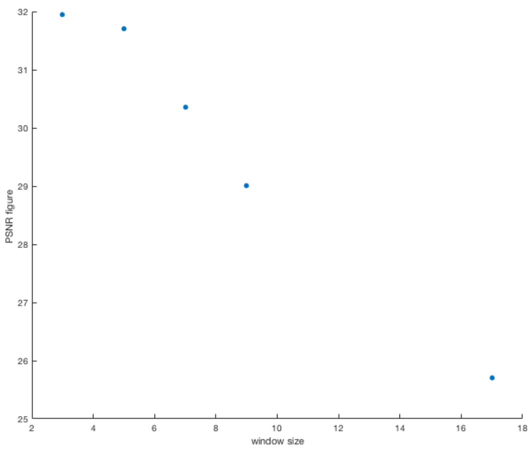


Fig. 10: plot of question 3

(a) denoised with 3x3 median filter

(b) 3x3 α trimmed filter with p = 4

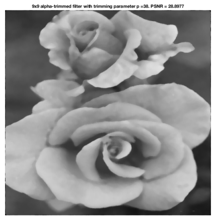
(c) denoised with 5x5 median filter

(d) 5x5 α trimmed filter with p = 11

(e) denoised with 7x7 median filter

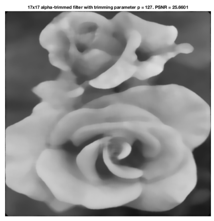
(f) 7x7 α trimmed filter with p = 23

(g) denoised with 9x9 median filter



(h) 9x9 α trimmed filter with p = 38

(i) denoised with 17x17 median filter



(j) 17x17 α trimmed filter with p = 127

Fig. 11: denoised images

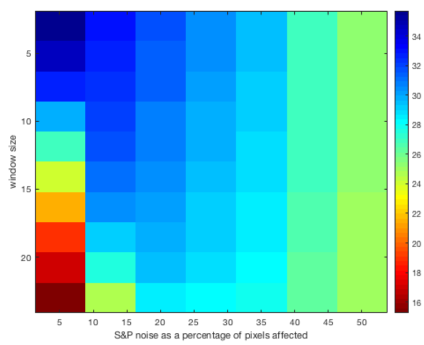


Fig. 12: Heatmap