

One hundred and fifty-two new families of Newtonian periodic planar three-body orbits

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The famous three-body problem can be traced back to Isaac Newton in 1680s. In the 300 years since this “three-body problem” was first recognized, just three families of solutions had been found, until 2013 when Šuvakov and Dmitrašinović [Phys. Rev. Lett. **110**, 114301 (2013)] made a breakthrough to find 13 new distinct periodic orbits, which belong to 11 new families of Newtonian planar three-body problem with equal mass and zero angular momentum. In this letter, we numerically obtained 164 families of Newtonian planar periodic three-body orbits with equal mass and zero angular momentum, including the well-known Figure-eight family found by Moore in 1993, the 11 families found by Šuvakov and Dmitrašinović in 2013, and 152 completely new families that have been never reported. With the definition of the average period $\bar{T} = T/k$, where k is the length of the so-called free group element, these 164 families of the periodic three-body orbits suggest that there should exist the quasi Kepler’s third law $\bar{R} \propto |E|^{-1} = 0.56 \bar{T}^{2/3}$, where \bar{R} is the mean of hyper-radius of the three-body system and E is its total kinetic and potential energy, respectively. The movies and pictures of the periodic three-body orbits in the real space and the corresponding close curves in the “shape sphere” can be found via the website: <http://numericaltank.sjtu.edu.cn/three-body/three-body.htm>

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The famous three-body problem [1] can be traced back to Isaac Newton in 1680s. According to Poincaré [2], a three-body system is not integrable in general. Besides, orbits of three-body problem are often chaotic [3], say, sensitive to initial conditions [2], although there exist periodic orbits in some special cases. In the 300 years since this “three-body problem” [1] was first recognized, just three families of periodic solutions had been found, until 2013 when Šuvakov and Dmitrašinović [4] made a breakthrough to find 13 new distinct periodic orbits belonging to 11 new families of Newtonian planar three-body problem with equal mass and zero angular momentum. Before their elegant work, only three families of periodic three-body orbits were found: (1) the Lagrange-Euler family, the simplest ones discovered by Lagrange and Euler in 18th century; (2) the Broucke-Hadjidemetriou-Hénon family [5–10]; (3) the Figure-eight family, first discovered numerically by Moore [11] in 1993 and rediscovered by Chenciner and Montgomery [12] in 2000, and then extended to the rotating case [13–16]. Up until now, only these 14 families of periodic orbits have been found in collisionless planar three-body problem.

Furthermore, Dmitrašinović and Šuvakov [17] found topological dependence of Kepler’s third law for the three-body problem with vanishing angular momentum. Janković and Dmitrašinović [18] studied topological dependence of Kepler’s third law in the Broucke-Hadjidemetriou-Hénon Family with nonzero angular momentum.

Recently, Šuvakov and Dmitrašinović [19] specifically illustrated their numerical strategies used in [4]. They suggested that more new periodic solutions are expected to be found when the length of period is longer than 100. In this letter, we used a different numerical approach to solve the same problem, i.e. Newtonian planar three-body problem with equal mass and zero angular momentum, but gained 164 families of periodic orbits, including the well-known Figure-eight found by Moore [11], the 11 families found by Šuvakov and Dmitrašinović [4], and 152 new families that have been never reported. Among these 152 new families, only two have periods greater than 100.

The motions of Newtonian planar three-body system are governed by the Newton’s second law and the gravitational law:

$$\ddot{\mathbf{r}}_i = \sum_{j=1, j \neq i}^3 \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_i - \mathbf{r}_j|^3}, \quad i = 1, 2, 3 \quad (1)$$

where \mathbf{r}_i and m_j are the position vector and mass of the i th body ($i = 1, 2, 3$), G is the Newtonian gravity coefficient, and the dot denotes the derivative with respect to the time t , respectively. Following Šuvakov and Dmitrašinović [4], let us consider a planar Newtonian three-body system with zero angular momentum in the case of $G = 1$ and $m_1 = m_2 = m_3 = 1$. With the scaled zero-angular-momentum initial conditions, we consider the initial positions of three bodies with collinear configurations and one body in the middle between the other two, say,

$$\mathbf{r}_1(0) = (x_1, x_2) = -\mathbf{r}_2(0), \quad \mathbf{r}_3(0) = (0, 0),$$

and the initial velocities

$$\dot{\mathbf{r}}_1(0) = \dot{\mathbf{r}}_2(0) = (v_1, v_2), \quad \dot{\mathbf{r}}_3(0) = -2\dot{\mathbf{r}}_1(0).$$

So, the initial conditions are specified by four parameters (x_1, x_2, v_1, v_2) . Write $\mathbf{y}(t) = (\mathbf{r}_1(t), \dot{\mathbf{r}}_1(t))$. A periodic solution with the period T is the root of the equation $\mathbf{y}(T) - \mathbf{y}(0) = 0$, where T is unknown. Note that $x_1 = -1$ and $x_2 = 0$ correspond to the normal case considered by Šuvakov and Dmitrašinović [4]. So, mathematically speaking, we search for the periodic orbits of the same three-body problem using a larger degree of freedom than Šuvakov and Dmitrašinović [4].

Firstly, like Šuvakov and Dmitrašinović [4], we use the grid search method to find candidates of the approximated initial conditions $\mathbf{y}(0) = (x_1, x_2, v_1, v_2)$ for periodic orbits. As is well known, the grid search method suffers from the curse of dimensionality. In order to reduce the dimension of the search space, we set the initial positions $x_1 = -1$ and $x_2 = 0$. Then, we search the initial conditions of periodic orbits in the two dimensional search plane: $v_1 \in [0, 1]$ and $v_2 \in [0, 1]$. We set 1000 points in each dimension and thus have 1000×1000 grid points in the square search plane. With these different 10^6 initial conditions, the motion equations are integrated up to the time $T_0 = 100$ by means of the ODE solver dop853 developed by Hairer et al. [20], which is based on an explicit Runge-Kutta method of order 8(5,3) in double precision with adaptive step size control. The corresponding initial conditions and the period T are chosen as the candidates when the return proximity function

$$|\mathbf{y}(t) - \mathbf{y}(0)| = \sqrt{\sum_{i=1}^4 (y_i(t) - y_i(0))^2} \quad (2)$$

is less than 10^{-1} .

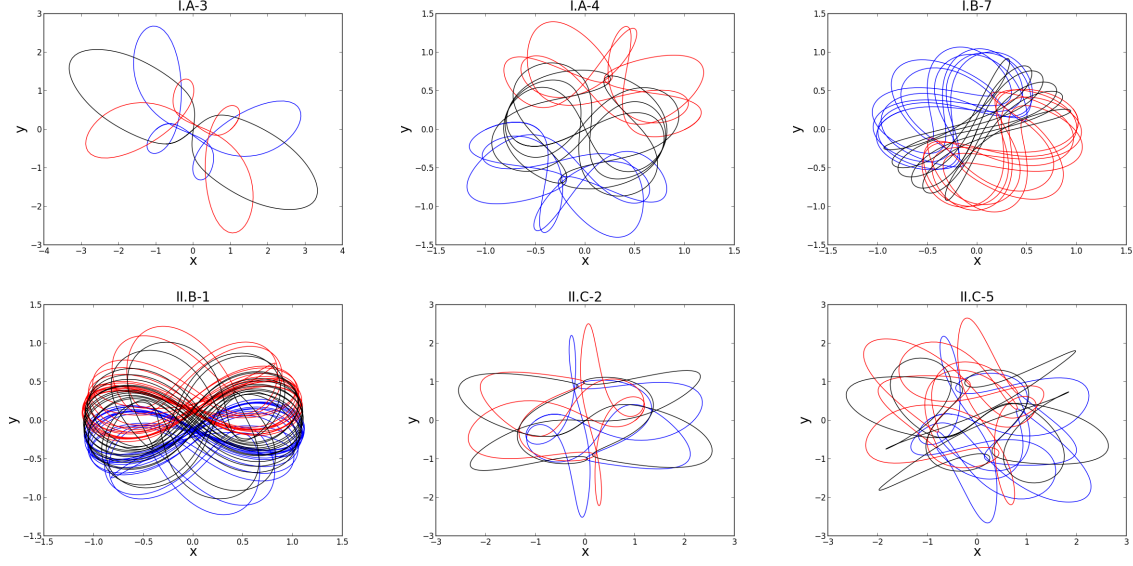


FIG. 1. (color online.) Brief overview of the six newly-found families of periodic three-body orbits.

TABLE I. The initial velocities, periods T and the free group elements of some newly-found periodic orbits of the three-body system with equal mass and zero angular momentum in the case of $\mathbf{r}_1(0) = (-1, 0) = -\mathbf{r}_2(0)$, $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$ and $\mathbf{r}_3(0) = (0, 0)$, $\dot{\mathbf{r}}_3(0) = (-2v_1, -2v_2)$ when $G = 1$ and $m_1 = m_2 = m_3 = 1$.

Class and number	v_1	v_2	T	free group element
I.A-3	0.6150407229	0.5226158545	37.3205235945	$(Ba)^2(bA)^2$
I.A-4	0.5379557207	0.3414578545	26.918669616	$abaBABabABababAB$
I.B-7	0.1862378160	0.5787138661	33.6414187604	$BAb^2(ABab)^2ABa^2(BAb)^2$
II.B-1	0.3962186234	0.5086826315	96.4358796119	$a(baBA)^7b(abAB)^7$
II.C-2	0.5647061130	0.5368389792	54.7501910522	$B(aBAB)^3A$
II.C-5	0.5512729728	0.5504821832	86.941701946	$B(aBAB)^2bABa(aBAB)^2A$

Secondly, we correct these candidates of the initial conditions for periodic orbits by means of the Newton-Raphson method [21–23]. At this stage, the motion equations are solved numerically by means of the same ODE solver dop853 developed by Hairer et al. [20]. A periodic orbit is found when the level of the return proximity function (2) is less than 10^{-6} . Note that, different from the numerical approach in [4], not only the initial velocity (v_1, v_2) but also the initial position (x_1, x_2) are also modified. In other words, our numerical approach allows (x_1, x_2) to deviate from its initial guess $(-1, 0)$. With such kind of larger degree of freedom, our approach gives 137 families of periodic orbits with the return proximity function less than 10^{-6} , including the well-known Figure-eight found by Moore [11], the 10 families found by Šuvakov and Dmitrašinović [4], and 126 completely new families that have been never reported.

Then, following Šuvakov and Dmitrašinović [4], we use the topological identification and classification method suggested by Montgomery [24] to identify these periodic orbits. The positions $\mathbf{r}_1, \mathbf{r}_2$ and \mathbf{r}_3 of the three-body corresponds to a unit vector \mathbf{n} in the so-called “shape sphere” with the Cartesian components

$$n_x = \frac{2\boldsymbol{\rho} \cdot \boldsymbol{\lambda}}{R^2}, \quad n_y = \frac{\lambda^2 - \rho^2}{R^2}, \quad n_z = \frac{2(\boldsymbol{\rho} \times \boldsymbol{\lambda}) \cdot \mathbf{e}_z}{R^2},$$

where $\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2)$, $\boldsymbol{\lambda} = \frac{1}{\sqrt{6}}(\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)$ and the hyper-radius $R = \sqrt{\rho^2 + \lambda^2}$. A periodic orbit of the three-body system gives a closed curve on the shape sphere, which can be characterized by its topology with three punctures (two-body collision points). With one of the punctures as the “north pole”, the sphere can be mapped onto a plane by a stereographic projection. And then a closed curve can be mapped onto a plane with two punctures and its topology can be described by the so-called “free group element” (word) with letters a (a clockwise around right-hand-side puncture) and b (a counter-clockwise around left-hand-side puncture), and their inverses $a^{-1} = A$ and $b^{-1} = B$. The free group word reading algorithm is described in the Appendix of Ref. [19].

Following Šuvakov and Dmitrašinović [4], we can also divide these periodic orbits into different classes according

to their geometric and algebraic symmetries. There are two types of geometric symmetries in the shape space: (I) the reflection symmetries of two orthogonal axes — the equator and the zeroth meridian passing through the “far” collision point; and (II) a central reflection symmetry about one point — the intersection of the equator and the aforementioned zeroth meridian. Besides, Šuvakov and Dmitrašinović [4] found three types of algebraic exchange symmetries for the free group elements: (A) the free group elements are symmetric with $a \leftrightarrow A$ and $b \leftrightarrow B$, (B) free group elements are symmetric with $a \leftrightarrow b$ and $A \leftrightarrow B$, and (C) free group elements are not symmetric under either (A) or (B).

Similar to the 11 families found by Šuvakov and Dmitrašinović [4], the 137 families of the periodic orbits can be divided into four classes: I.A, I.B, II.B and II.C, as shown in Table S II - IV in Supplementary material [25]. Note that we regard the periodic orbits and all its satellites as the same family, therefore the “moth I” orbit and its satellite “yarn” orbit in [4] belong to the same family.

Note that the initial positions $\mathbf{r}_1 = (x_1, x_2)$ in Table S II - IV depart from $(-1, 0)$ a little. However, it is well-known that, if $\mathbf{r}_i(t)$ ($i = 1, 2, 3$) denotes a periodic orbit with the period T of a three-body system, then $\mathbf{r}_i^*(t^*) = \alpha \mathbf{r}_i(t)$, $\mathbf{v}_i^* = \mathbf{v}_i/\sqrt{\alpha}$, where $t^* = \alpha^{3/2} t$, is also a periodic orbit with the period $T^* = \alpha^{3/2} T$ for arbitrary $\alpha > 0$. Thus, through coordinate transformation and then the scaling of the spatial and temporal coordinates, we can always enforce $(-1, 0)$, $(1, 0)$ and $(0, 0)$ as the initial positions of the body-1, 2 and 3, respectively, with the initial velocities $\dot{\mathbf{r}}_1(0) = \dot{\mathbf{r}}_2(0)$ and $\dot{\mathbf{r}}_3(0) = -2\dot{\mathbf{r}}_1(0)$, corresponding to zero angular momentum. The corresponding initial velocities of the periodic orbits are listed in Tables S V - VII in Supplementary material [25]. It should be emphasized that these 137 families contain the well-known Figure-eight found by Moore [11] and the 10 families found by Šuvakov and Dmitrašinović [4], and especially the 126 completely new families that have been never reported. All of these 137 families belong to the four classes, i.e. I.A, I.B, II.B and II.C, as shown in Tables S V - VII. Note that, due to the scaling of temporal coordinate, we gained two periodic orbits, namely I.A-41 and I.B-45, with periods a little greater than 100.

However, one family reported by Šuvakov and Dmitrašinović [4] was *not* found among these 137 periodic orbits. So, at least one periodic orbit was lost at this stage. It is well-known that three-body problem is not integrable in general [2] and besides might be rather sensitive to initial conditions, i.e. the butterfly-effect [3]. For example, Hoover et al. [26] compared numerical simulations of a chaotic Hamiltonian system given by five symplectic and two Runge-Kutta integrators in *double precision*, and found that “all numerical methods are susceptible”, “which severely limits the maximum time for which chaotic solutions can be accurate”, although “all of these integrators conserve energy almost *perfectly*” and “they also reverse back to the initial conditions even when their trajectories are *inaccurate*”. In fact, there exist many examples which clearly indicate that numerical noises might have great influence on nonlinear dynamic systems. Currently, some numerical approaches were developed to gain reliable numerical results of chaotic dynamic systems in a long (but finite) interval of time. One of them is the so-called “Clean Numerical Simulation” (CNS) [27, 28], which is based on the arbitrary order of Taylor expansion method [29–32] in arbitrary precision [33, 34], and more importantly, a check of solution verification in a given interval of time by comparing it with a new result gained with even smaller numerical noise.

At first, using the obtained initial conditions, we checked the 137 periodic orbits by means of the high-order Taylor series method in the 100-digit precision with truncation errors less than 10^{-70} , and guaranteed that they are indeed periodic orbits. In addition, we further found 27 new families of periodic orbits (with the periods less than 100) by means of the Newton-Raphson method [21–23] for the correction of candidates of the initial conditions, and using the high-order Taylor series method in 100-digit precision with truncation errors less than 10^{-70} for the evolution of motion equations, instead of the ODE solver dop853 [20] based on the Runge-Kutta method in double precision. Besides, we use the CNS with even smaller round-off error (in 120-digit precision) and truncation error (less than 10^{-90}) to guarantee the reliability of these 27 families. It is found that all of these 27 families belong to the four classes I.A, I.B, II.B and II.C. One of them belongs to the 11 families found by Šuvakov and Dmitrašinović [4], but the other 26 families have never been reported, to the best of our knowledge.

So, we totally found 164 periodic orbits, including the Figure-eight found by Moore [11], the 11 families found by Šuvakov and Dmitrašinović [4], and the 152 new families that have been never reported. Note that only two new families among them have periods a little great than 100. The free group elements of all 164 families are shown in Table S VIII-X in Supplementary material [25]. Due to the limited length, only six new families are listed in Table I, and their real space orbits are shown in FIG. 1. The movies and pictures of the 164 families of periodic orbits can be found via the website: <http://numericaltank.sjtu.edu.cn/three-body/three-body.htm>.

For a two-body system, there exists the so-called Kepler’s third law $a \propto T^{2/3}$, where T is the period and a is the semi-major axis of its orbit. For a three-body system, Šuvakov and Dmitrašinović [4] mentioned that there should exist the relation $T^{2/3}|E| = \text{constant}$, where E denotes the total kinetic and potential energy of the three-body system. But they pointed out that “the constant on the right-hand-side of this equation is not universal”, which may depend on “both of the family of the three-body orbit and its angular momentum”. However, with the definition of the average period $\bar{T} = T/k$, where k is the length of free group word of periodic orbit of a three-body system with equal mass and zero angular momentum, the 164 families of periodic orbits approximately satisfy such a generalized

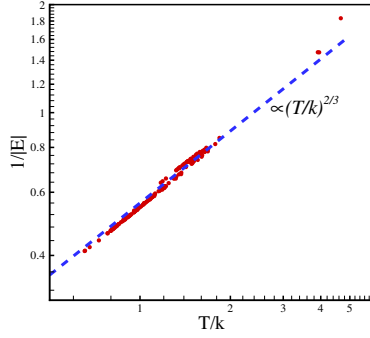


FIG. 2. (color online). The inverse of total energy $1/|E|$ and average period T/k approximately fall in with a power law with exponent $2/3$, where k is the length of free group word. Symbols: the 164 families of periodic orbits; dashed line: $1/|E| = 0.56 \cdot (T/k)^{2/3}$.

Kepler's third law $\bar{R} \propto |E|^{-1} = 0.56 \bar{T}^{2/3}$, as shown in FIG. 2, where \bar{R} is the mean of hyper-radius of the three-body system.

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- [1] Z. E. Musielak and B. Quarles, Rep. Prog. Phys. **77**, 065901 (30pp) (2014).
 - [2] J. H. Poincaré, Acta Math. **13**, 1 (1890).
 - [3] E. N. Lorenz, Journal of the Atmospheric Sciences **20**, 130 (1963).
 - [4] M. Šuvakov and V. Dmitrašinović, Phys. Rev. Lett. **110**, 114301 (2013).
 - [5] R. Broucke and D. Boggs, Celestial mechanics **11**, 13 (1975).
 - [6] R. Broucke, Celestial mechanics **12**, 439 (1975).
 - [7] J. D. Hadjidemetriou and T. Christides, Celestial mechanics **12**, 175 (1975).
 - [8] J. D. Hadjidemetriou, Celestial mechanics **12**, 255 (1975).
 - [9] M. Hénon, Celestial mechanics **13**, 267 (1976).
 - [10] M. Hénon, Celestial mechanics **15**, 243 (1977).
 - [11] C. Moore, Phys. Rev. Lett. **70**, 3675 (1993).
 - [12] A. Chenciner and R. Montgomery, Annals of Mathematics **152**, 881 (2000).
 - [13] M. Nauenberg, Physics Letters A **292**, 93 (2001).
 - [14] A. Chenciner, J. Fejoz, and R. Montgomery, Nonlinearity **18**, 1407 (2005).
 - [15] R. Broucke, A. Elpe, and A. Riaguas, Chaos, Solitons & Fractals **30**, 513 (2006).
 - [16] M. Nauenberg, Celestial Mechanics and Dynamical Astronomy **97**, 1 (2007).
 - [17] V. Dmitrašinović and M. Šuvakov, Physics Letters A **379**, 1939 (2015).
 - [18] M. R. Janković and V. Dmitrašinović, Phys. Rev. Lett. **116**, 064301 (2016).
 - [19] V. Dmitrašinović and M. Šuvakov, American Journal of Physics **82**, 609 (2014).
 - [20] E. Hairer, G. Wanner, and S. P. Norsett, **8** (1993).
 - [21] S. C. Farantos, Journal of Molecular Structure: THEOCHEM **341**, 91 (1995).
 - [22] M. Lara and J. Pelaez, Astronomy and Astrophysics **389**, 692 (2002).
 - [23] A. Abad, R. Barrio, and A. Dena, Phys. Rev. E **84**, 016701 (2011).
 - [24] R. Montgomery, Nonlinearity **11**, 363 (1998).
 - [25] see the Supplementary material, .
 - [26] W. Hoover and C. Hoover, Computational Methods in Science and Technology **21**, 109 (2015).
 - [27] S. Liao, Tellus A **61**, 550 (2009).
 - [28] S. Liao, Communications in Nonlinear Science and Numerical Simulation **19**, 601 (2014).
 - [29] D. Barton, I. Willem, and R. Zahar, Comput. J. **14**, 243 (1971).
 - [30] G. Corliss and Y. Chang, ACM Trans. Math. Software **8**, 114 (1982).
 - [31] Y. F. Chang and G. F. Corhss, Computers Math. Applic. **28**, 209 (1994).
 - [32] R. Barrio, F. Blesa, and M. Lara, Computers & Mathematics with Applications **50**, 93 (2005).
 - [33] P. Oyanarte, Computer Physics Communications **59**, 345 (1990).
 - [34] D. Viswanath, Physica D **190**, 115 (2004).

Supplementary information for “One hundred and fifty-two new families of Newtonian periodic planar three-body orbits”

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The initial conditions and the periods T of the newly-found 152 families of the periodic orbits of the three-body system with equal mass and zero angular momentum are given here in details, together with the trajectories, the free group elements and their length k of these periodic orbits. There exist 11 tables in the supplementary material. The movies and pictures of the periodic orbits in the real space and the corresponding close curves in the “shape sphere” can be found via the website: <http://numericaltank.sjtu.edu.cn/three-body/three-body.htm>

Table S I. The 12 previously-found orbits and their corresponding class and number defined in this letter.

Class and name	free group element	Class and number in this letter
I.A Figure-eight	abAB	I.A-1
I.A butterfly I and II	$(ab)^2(AB)^2$	I.A-2
I.A bumblebee	$b^2(ABab)^2A^2(baBA)^2baB^2(abAB)^2a^2(BAba)^2BA$	I.A-17
I.B moth I	baBABabABA	I.B-1
I.B butterfly III	$(ab)^2(ABA)(ba)^2(BAB)$	I.B-2
I.B moth II	$(abAB)^2A(baBA)^2B$	I.B-5
I.B moth III	ABabaBABabaBAbabABabab	I.B-6
I.B goggles	$(ab)^2ABBA(ba)^2BAAB$	I.B-3
I.B butterfly IV	$((ba)^2(AB)^2)^6A((ba)^2(BA)^2)^6B$	I.B-49
I.B dragonfly	$b^2(ABabAB)a^2(BAbaBA)$	I.B-4
II.C yin-yang I (a) and (b)	$(ab)^2ABAbA(BAB)$	II.C-1
II.C yin-yang II (a) and (b)	$(abaBAB)^3abaBAbab(ABAbab)^3(AB)^2$	II.C-27

Table S II. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of $\mathbf{r}_1(0) = (x_1, x_2) = -\mathbf{r}_2(0)$, $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$ and $\mathbf{r}_3(0) = (0, 0)$, $\dot{\mathbf{r}}_3(0) = (-2v_1, -2v_2)$ when $G = 1$ and $m_1 = m_2 = m_3 = 1$. The Figure-eight found by Moore [9] and the 11 families found by Šuvakov and Dmitrašinović [4] are marked in blue. The 26 new families found by the high-order Taylor series method in 100-digit precision are marked in red.

Class and number	x_1	x_2	v_1	v_2	T
I.A-1	-1.0024277970	0.0041695061	0.3489048974	0.5306305100	6.3490473929
I.A-2	-1.0005576155	-0.0029240248	0.3064392516	0.1263673939	6.2399303613
I.A-3	-0.9826997937	-0.0315203302	0.6030557226	0.5466747521	36.3842947805
I.A-4	-0.9964559619	0.0051918896	0.5406827989	0.3392497094	26.7762404680
I.A-5	-0.9720963681	-0.0704958088	0.3964121098	0.2935656454	19.9650778710
I.A-6	-0.9999327738	-0.0008223485	0.4422571974	0.4238918567	35.8298692301
I.A-7	-0.9977227094	0.0070021715	0.1222984917	0.1004607487	15.6909239194
I.A-8	-0.9994554344	-0.0011995337	0.4091701188	0.3634132078	33.8401262309
I.A-9	-0.9987682260	-0.0031341566	0.5251116596	0.2519279723	43.7891455476
I.A-10	-1.0135928065	0.0369002019	0.4191609820	0.2663106893	34.9824204443
I.A-11	-1.0019260810	0.0020911852	0.4369275314	0.4443694754	49.7389219446
I.A-12	-1.0005140398	-0.0001156871	0.1181029973	0.5846001931	48.3095979022
I.A-13	-1.0020551612	0.0043916449	0.4775162156	0.3764636910	53.6371875645
I.A-14	-1.3147318816	0.7132945256	0.3713149918	-0.0060886985	63.5000850224
I.A-15	-0.9929272851	-0.0134998193	0.3925874406	0.3595501581	45.6518283178
I.A-16	-0.9985908494	-0.0022240402	0.4333321596	0.4619833415	63.9809834569
I.A-17	-1.0074958476	0.0081648176	0.1883232887	0.5834831526	64.2532204831
I.A-18	-1.0035505512	0.0069356949	0.4600346659	0.4054759860	66.7080558750
I.A-19	-0.9975373781	-0.0067760199	0.4137670183	0.2945195624	47.7502255665
I.A-20	-1.0023416890	0.0034690771	0.0987694721	0.5606002909	53.8791265298
I.A-21	-0.9944059892	-0.0160342344	0.4053380361	0.2509213812	48.0899422464
I.A-22	-1.0051040678	0.0094190694	0.4322835083	0.4671381437	79.0046932990
I.A-23	-1.0005990303	0.0006513647	0.4139830510	0.3474219895	61.9003772551
I.A-24	-1.0022464885	-0.0005079367	0.4897501985	0.4042162458	85.6738583738
I.A-25	-0.9996665302	-0.0003500779	0.1981083820	0.5762280189	73.2401681272
I.A-26	-0.9946395214	-0.0199130330	0.3963758249	0.1931593194	48.2910929099
I.A-27	-1.0128122834	-0.0049944126	0.4110801302	0.2954958184	62.5062638452
I.A-28	-0.9983642281	-0.0029915248	0.0473206001	0.5908205345	78.9582358872
I.A-29	-1.0012940616	0.0040175714	0.3460993325	0.3914034723	67.5194380903
I.A-30	-1.0001268768	-0.0002768989	0.2201401036	0.5718474139	85.8099256919
I.A-31	-1.0012673904	0.0023779503	0.4447554832	0.3810128399	82.5188557424
I.A-32	-0.9877512802	-0.0430400097	0.3964569812	0.2378986017	61.3902686322
I.A-33	-1.0172548423	0.0480118437	0.4209823631	0.3230893830	77.0997278616
I.A-34	-1.0007553891	0.0016489085	0.4577398645	0.4065094686	95.9895491214
I.A-35	-1.0049087422	0.0155206574	0.3987835408	0.1626357009	63.1005696104
I.A-36	-1.0012474113	0.0030153067	0.4166996961	0.2957105307	74.9297037600
I.A-37	-1.0002982171	-0.0008001808	0.1034021115	0.5907158341	92.3300512832
I.A-38	-0.9977885909	-0.0055266460	0.4124824487	0.2735925337	75.7782102494
I.A-39	-1.0004497905	0.0025599575	0.3976005814	0.3697808266	85.9927903991
I.A-40	-0.9983014289	-0.0053078120	0.4064430497	0.2412089372	76.0294647048
I.A-41	-0.9679361060	-0.1104962526	0.0672594569	0.6030209734	98.8657694414
I.A-42	-0.9984824236	-0.0062824747	0.4005780292	0.2049425482	76.2294965022
I.A-43	-1.0016614175	0.0039687130	0.4166059221	0.2962897857	88.2694596541
I.A-44	-1.0001378487	0.0004564237	0.4189450671	0.2768485374	90.4399218097
I.A-45	-0.9999058470	0.0041754333	0.3967334937	0.1569524375	76.5828835512
I.A-46	-1.0056329618	0.0176753713	0.4127890336	0.2437542476	90.7691206861
I.A-47	-0.9999108695	0.0015640018	0.4050458197	0.2219737238	90.1694565806

Table S III. Initial conditions and periods T of the periodic three-body orbits for class I.B and II.B in the case of $\mathbf{r}_1(0) = (x_1, x_2) = -\mathbf{r}_2(0)$, $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$ and $\mathbf{r}_3(0) = (0, 0)$, $\dot{\mathbf{r}}_3(0) = (-2v_1, -2v_2)$ when $G = 1$ and $m_1 = m_2 = m_3 = 1$. The Figure-eight found by Moore [9] and the 11 families found by Šuvakov and Dmitrašinović [4] are marked in blue. The 26 new families found by the high-order Taylor series method in 100-digit precision are marked in red.

Class and number	x_1	x_2	v_1	v_2	T
I.B-1	-0.9989071137	-0.0001484864	0.4646402601	0.3963456869	14.8698954200
I.B-2	-1.1770534081	-0.5225957568	0.2446132140	0.3305126876	20.2667904949
I.B-3	-0.9996174046	0.0028671814	0.0836823874	0.1276739661	10.4589089289
I.B-4	-0.9859387540	-0.0288038725	0.0637911125	0.5950102821	20.8385742723
I.B-5	-0.9997857309	-0.0003533584	0.4390528231	0.4531713698	28.6600597106
I.B-6	-1.0043366457	0.0085104316	0.3857847594	0.3732858410	26.0089024096
I.B-7	-0.9995693352	-0.0012868539	0.1855324998	0.5790776181	33.6197306295
I.B-8	-1.0012380956	0.0018757155	0.4297132024	0.4742147154	42.9096450556
I.B-9	-0.9869994424	-0.0329384519	0.4080709664	0.2901166521	27.1495760745
I.B-10	-0.9999193334	-0.0002717176	0.3980126012	0.1762535971	27.8200493126
I.B-11	-1.0010436520	0.0029859847	0.2318334372	0.5686672120	46.1393431256
I.B-12	-0.9912922078	-0.0169276058	0.4238823678	0.3638554791	41.8211429240
I.B-13	-0.9955318366	-0.0057317839	0.3560089302	0.5008649399	49.3004616047
I.B-14	-1.0000328444	0.0003978205	0.4098850694	0.4390517046	54.0599247160
I.B-15	-0.9820178956	-0.0439094593	0.4045186605	0.3089615770	40.0817543149
I.B-16	-0.9793316018	-0.0343328490	0.2415686630	0.5788831000	57.4029776575
I.B-17	-0.9852740589	-0.0640855916	0.3872339266	0.1809512432	40.9983837590
I.B-18	-0.9954789776	-0.0090056972	0.4069832833	0.3543167976	53.9614235401
I.B-19	-1.0003135452	0.0005407130	0.2778712767	0.5598575722	72.3315394965
I.B-20	-1.0080911586	0.0212978481	0.4217594107	0.2832267055	55.5464441478
I.B-21	-0.9999536614	-0.0001750829	0.4109939999	0.4030802063	71.3800291841
I.B-22	-0.9686776407	-0.0608196023	0.3991611243	0.2827980458	52.9430984041
I.B-23	-1.0025047383	0.0016376144	0.1294943334	0.5833026191	75.8581653606
I.B-24	-1.0092396488	0.0396030643	0.4075995186	0.1878601935	56.3933245589
I.B-25	-0.9991779003	-0.0023293910	0.4524794248	0.4173526023	87.5501725839
I.B-26	-1.0120493442	0.0327654798	0.4224042986	0.2810350321	69.3540290982
I.B-27	-0.9958261057	-0.0180235617	0.4746698475	0.3848074753	92.0041366553
I.B-28	-1.0012573546	0.0022897677	0.4290649202	0.4627444811	98.8359617418
I.B-29	-1.0016467940	0.0007588511	0.1554933398	0.5786962880	87.0908236211
I.B-30	-0.9910057079	-0.0251737916	0.3331962796	0.2861128695	60.4345154079
I.B-31	-1.0091765176	0.0201754445	0.4157410078	0.2569669976	70.1042225630
I.B-32	-0.9998130400	-0.0053468765	0.3750465942	0.0261694007	54.9599616485
I.B-33	-0.9961710033	-0.0074269978	0.4107912477	0.3486701313	82.1507596077
I.B-34	-0.9962761072	-0.0172343437	0.4737986398	0.1688124971	79.5685727648
I.B-35	-0.9919037360	-0.0188701087	0.4100106195	0.3071172647	80.1888154296
I.B-36	-0.9990532732	-0.0043820662	0.3204762525	0.2061939336	62.0002762587
I.B-37	-1.0025451082	0.0013669961	0.4039796728	0.3652787606	94.1439172549
I.B-38	-1.0002032247	0.0006453603	0.3939791513	0.1344605101	69.7600199827
I.B-39	-0.9997462078	-0.0011028163	0.2901713884	0.2612727959	69.7400917634
I.B-40	-0.9902765570	-0.0307512535	0.4894639489	0.2365260776	97.2705134964
I.B-41	-1.0000822875	0.0003330901	0.3650078095	0.0603887946	67.6590068332
I.B-42	-1.0020843723	0.0171698170	0.4206012554	0.2909314421	95.0451372238
I.B-43	-0.9973221994	-0.0091941904	0.4017883631	0.2174805839	82.9624661836
I.B-44	-1.0018919789	0.0061735438	0.3010143797	0.2902774095	80.5992058621
I.B-45	-0.9894653911	-0.0608004298	0.4883359248	0.1669356426	99.0978825987
I.B-46	-1.0000325339	0.0003221382	0.3029750443	0.1969660658	73.2099935849
I.B-47	-0.9982822070	-0.0052079203	0.4095156641	0.2589467566	96.6187276779
I.B-48	-1.0003976445	0.0030603406	0.3938702763	0.1281065458	83.7700535678
I.B-49	-1.0017029160	0.0041736578	0.3501405955	0.0778141747	79.6790864881
I.B-50	-0.9996145603	-0.0017302155	0.4010821936	0.2015884635	97.1896385782
I.B-51	-1.0002894289	0.0009291890	0.2668487750	0.2777035009	89.3197319785
I.B-52	-0.9992941098	-0.0024372579	0.2931395758	0.1922320514	84.7295120544
I.B-53	-1.0012336598	0.0060424877	0.3977399716	0.1646648075	97.6151914585
I.B-54	-0.9986567557	-0.0089510385	0.3924139983	0.1289496286	97.5107476946
I.B-55	-0.9999453732	-0.0008999222	0.4030162257	0.0473267119	99.9599973732
II.B-1	-0.9795529757	-0.0288228080	0.3849595037	0.5254027227	93.5540138135
II.B-2	-1.0007518096	0.0076238027	0.2537011397	0.2647081365	99.9335612373

Table S IV. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of $\mathbf{r}_1(0) = (x_1, x_2) = -\mathbf{r}_2(0)$, $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$ and $\mathbf{r}_3(0) = (0, 0)$, $\dot{\mathbf{r}}_3(0) = (-2v_1, -2v_2)$ when $G = 1$ and $m_1 = m_2 = m_3 = 1$. The Figure-eight found by Moore [9] and the 11 families found by Šuvakov and Dmitrašinović [4] are marked in blue. The 26 new families found by the high-order Taylor series method in 100-digit precision are marked in red.

Class and number	x_1	x_2	v_1	v_2	T
II.C-1	-0.9826146484	-0.0411837391	0.2710001824	0.3415940623	10.6927139709
II.C-2	-1.0016879974	0.0026309175	0.5656359883	0.5349017518	54.8891607912
II.C-3	-1.0010192913	0.0028805303	0.5229031501	0.2251886482	26.3597995195
II.C-4	-0.9979319887	-0.0029629112	0.2006675819	0.4109167331	20.9554821689
II.C-5	-0.9961434009	-0.0050071036	0.5495563197	0.5543125804	86.4408761911
II.C-6	-1.0007357274	0.0020118982	0.5238826063	0.3409361213	44.9799754767
II.C-7	-1.0000747816	0.0003915745	0.4738937949	0.4310750400	46.8999197399
II.C-8	-1.0001704251	-0.0022610973	0.2427965934	0.2510445612	22.3391739136
II.C-9	-1.0006221621	0.0012247344	0.1740037507	0.1138359814	21.7738630579
II.C-10	-0.9975148730	0.0001297649	0.1450629144	0.5435551180	37.9212316240
II.C-11	-0.9943383314	-0.0057812831	0.1331341835	0.5429944488	37.2103034833
II.C-12	-1.0020231439	0.0037713135	0.5551632149	0.2698044462	53.1094005605
II.C-13	-1.0032605106	0.0045167438	0.4472991527	0.4896750344	62.8530057118
II.C-14	-0.9985578294	-0.0027678151	0.5362144152	0.3723306084	60.4301224826
II.C-15	-1.0029500764	0.0070627083	0.5213493496	0.3491337692	58.2492464733
II.C-16	-1.0031391560	0.0039466348	0.2468215905	0.5672081394	53.1896103937
II.C-17	-1.0000862446	0.0003055319	0.5164852735	0.1636660959	43.4802911792
II.C-18	-1.0008489261	0.0013290610	0.4057627077	0.4954415922	62.3794063369
II.C-19	-0.9998645606	0.0006210543	0.1869988183	0.2043525957	27.7197406792
II.C-20	-0.9986419306	0.0064065752	0.3051083709	0.3559741487	37.1024009040
II.C-21	-1.0007988437	0.0014317851	0.5048230051	0.3964978849	70.9198301370
II.C-22	-0.9998260895	-0.0001772606	0.2660412682	0.5521309238	62.9700624858
II.C-23	-1.0019501117	0.0020107888	0.2874388693	0.5596327476	67.7844044591
II.C-24	-1.0012529935	0.0028881095	0.0892701310	0.3610325208	35.8845749685
II.C-25	-1.0027353913	0.0005157812	0.5153175411	0.1440310535	52.4270825010
II.C-26	-1.0044088408	0.0008204369	0.3965893119	0.4982442012	74.8967977689
II.C-27	-0.9974591829	0.0014434640	0.4178305701	0.3301492680	55.5769080376
II.C-28	-0.9996200271	-0.0008020030	0.2661357628	0.3362393813	43.5093590242
II.C-29	-0.9999611432	0.0017175644	0.4299740470	0.3730083098	64.8696729632
II.C-30	-1.0002778408	-0.0007336310	0.2540745616	0.2155375716	40.6100821848
II.C-31	-1.0043264234	0.0085351855	0.5081224548	0.3842496864	86.9841516452
II.C-32	-1.0022103816	0.0102733510	0.4147469694	0.4067682790	73.1187153646
II.C-33	-0.9883083190	-0.0266620699	0.4097218737	0.3197223480	59.1311936461
II.C-34	-0.9982384101	-0.0026965767	0.4471973476	0.4612592418	96.5838877286
II.C-35	-0.9951304413	-0.0113745372	0.4152046229	0.3089526360	66.2706254945
II.C-36	-0.9997799146	-0.0000871689	0.2709972127	0.5711878526	93.0111567688
II.C-37	-1.0046985762	0.0162831677	0.3530834895	0.2445162178	57.2474723960
II.C-38	-1.0008608869	0.0011280380	0.3788005974	0.5094274206	98.7297713440
II.C-39	-1.0012100852	0.0028099578	0.4277768060	0.3389118542	80.1840191342
II.C-40	-1.0178257900	0.0160007544	0.1778768589	0.5724956206	96.7051205494
II.C-41	-0.9999159782	-0.0012779406	0.4758965652	0.2693288357	85.3853381329
II.C-42	-1.0005146883	0.0178470909	0.4338014225	0.1146294454	66.8767469903
II.C-43	-0.9969262199	-0.0055327948	0.4135974680	0.3039779928	78.9201147757
II.C-44	-0.9997689776	-0.0006983641	0.2708168388	0.3123404924	63.3716725823
II.C-45	-1.0058335497	0.0141374874	0.4263183336	0.2920871571	88.0387952421
II.C-46	-1.0004035587	0.0031565296	0.3287145259	0.1543359704	63.6946794825
II.C-47	-1.0001981436	-0.0005327229	0.2949051506	0.2388052958	65.2492675927
II.C-48	-1.0025031994	0.0052121421	0.4082798530	0.3518144302	95.1326222384
II.C-49	-1.0083313286	-0.0286778328	0.4077992979	0.3099009593	94.2024974068
II.C-50	-0.9991000611	-0.0029159692	0.2874417121	0.2789397413	74.2602225931
II.C-51	-1.0019070615	0.0078677169	0.2848590503	0.2308658034	70.7113437472
II.C-52	-0.9996332993	-0.0028910173	0.4471116841	0.1069668572	83.9891019124
II.C-53	-0.9947326912	-0.0128306041	0.4093239532	0.3063748779	97.6687075761
II.C-54	-0.9997446546	-0.0003123011	0.3351127698	0.3541842111	96.8684615916
II.C-55	-0.9993349134	-0.0032458410	0.2754489605	0.2297037522	76.0001131456
II.C-56	-1.0003357519	0.0051959956	0.4507218273	0.0976302349	92.2314398981
II.C-57	-1.0019558752	0.0070044294	0.2809143305	0.2689766307	80.0099426092
II.C-58	-1.0015893090	0.0071301440	0.2737543421	0.2225574422	82.0867499689
II.C-59	-0.9989466432	-0.0050002701	0.2716591775	0.2221618499	87.8794229689
II.C-60	-0.9998090862	-0.0001193521	0.2769776134	0.2163748880	94.4991196762

Table S V. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of $\mathbf{r}_1(0) = (-1, 0) = -\mathbf{r}_2(0)$, $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$ and $\mathbf{r}_3(0) = (0, 0)$, $\dot{\mathbf{r}}_3(0) = (-2v_1, -2v_2)$ when $G = 1$ and $m_1 = m_2 = m_3 = 1$. The Figure-eight found by Moore [9] and the 11 families found by Šuvakov and Dmitrašinović [4] are marked in blue. The 26 new families found by the high-order Taylor series method in 100-digit precision are marked in red.

Class and number	v_1	v_2	T
I.A-1	0.3471168881	0.5327249454	6.3259139829
I.A-2	0.3068934205	0.1255065670	6.2346748391
I.A-3	0.6150407229	0.5226158545	37.3205235945
I.A-4	0.5379557207	0.3414578545	26.9186696160
I.A-5	0.4112926910	0.2607551013	20.7490650080
I.A-6	0.4425908552	0.4235138348	35.8334644158
I.A-7	0.1214534165	0.1012023800	15.7440944954
I.A-8	0.4094945913	0.3628231655	33.8677507235
I.A-9	0.5255769251	0.2501253528	43.8698538026
I.A-10	0.4121028725	0.2833837497	34.2470499687
I.A-11	0.4364192674	0.4457095477	49.5954033190
I.A-12	0.1182009612	0.5847367662	48.2723717545
I.A-13	0.4763527642	0.3789434497	53.4714914343
I.A-14	0.4027121690	0.2100155085	34.7120153702
I.A-15	0.3960494651	0.3529413931	46.1340735475
I.A-16	0.4340543928	0.4606927255	64.1162217970
I.A-17	0.1842784887	0.5871881740	63.5343529785
I.A-18	0.4580378828	0.4093753024	66.3519736810
I.A-19	0.4152505707	0.2913461808	47.9254977196
I.A-20	0.0969422653	0.5615968396	53.6899444096
I.A-21	0.4082108156	0.2436851904	48.4868513719
I.A-22	0.4289870678	0.4723797648	78.3984979261
I.A-23	0.4138807521	0.3477955684	61.8447789156
I.A-24	0.4905050535	0.4044215155	85.3859524958
I.A-25	0.1982770999	0.5760625510	73.2768117405
I.A-26	0.3991287659	0.1847081193	48.6673769352
I.A-27	0.4151691260	0.2953409098	61.3228288140
I.A-28	0.0490506729	0.5901941115	79.1518362736
I.A-29	0.3447503346	0.3930446386	67.3877746400
I.A-30	0.2203123981	0.5718227262	85.7935924161
I.A-31	0.4441311511	0.3823106072	82.3618800834
I.A-32	0.4041322114	0.2191641945	62.4468134238
I.A-33	0.4089912496	0.3457125084	75.0210999574
I.A-34	0.4572423635	0.4074171858	95.8806923191
I.A-35	0.3972193787	0.1691985386	62.6275833821
I.A-36	0.4160674674	0.2971499303	74.7892110946
I.A-37	0.1038901209	0.5907210858	92.2887207815
I.A-38	0.4135366646	0.2710056059	76.0285224477
I.A-39	0.3967429300	0.3708809839	85.9343829011
I.A-40	0.4073762182	0.2388431491	76.2219729678
I.A-41	0.1334658448	0.5838277552	102.8155095976
I.A-42	0.4015585101	0.2022664462	76.4010838847
I.A-43	0.4157753114	0.2981866614	88.0489003051
I.A-44	0.4188475683	0.2770588072	90.4212103693
I.A-45	0.3960577146	0.1586009694	76.5926988984
I.A-46	0.4096220179	0.2516961122	89.9866883856
I.A-47	0.4046803380	0.2225972157	90.1813477230

Table S VI. Initial conditions and periods T of the periodic three-body orbits for class I.B and II.B in the case of $\mathbf{r}_1(0) = (-1, 0) = -\mathbf{r}_2(0)$, $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$ and $\mathbf{r}_3(0) = (0, 0)$, $\dot{\mathbf{r}}_3(0) = (-2v_1, -2v_2)$ when $G = 1$ and $m_1 = m_2 = m_3 = 1$. The Figure-eight found by Moore [9] and the 11 families found by Šuvakov and Dmitrašinović [4] are marked in blue. The 26 new families found by the high-order Taylor series method in 100-digit precision are marked in red.

Class and number	v_1	v_2	T
I.B-1	0.4644451728	0.3960600146	14.8943051743
I.B-2	0.4059155671	0.2301631260	13.8671234361
I.B-3	0.0833000718	0.1278892555	10.4648495256
I.B-4	0.0805842255	0.5888360898	21.2723373956
I.B-5	0.4391659182	0.4529676431	28.6692709402
I.B-6	0.3834435199	0.3773636946	25.8392363356
I.B-7	0.1862378160	0.5787138661	33.6414187604
I.B-8	0.4290898149	0.4753132903	42.8299661050
I.B-9	0.4149129608	0.2746187551	27.6646471048
I.B-10	0.3980444335	0.1761383334	27.8234143343
I.B-11	0.2302567240	0.5696545030	46.0668997027
I.B-12	0.4281877764	0.3550351874	42.3641397966
I.B-13	0.3580870041	0.4976954491	49.6315066172
I.B-14	0.4097171234	0.4392219554	54.0572550607
I.B-15	0.4143481483	0.2881031095	41.1260553556
I.B-16	0.2590629830	0.5643154612	59.1751946306
I.B-17	0.3956370780	0.1544499871	41.7884305464
I.B-18	0.4092519851	0.3498343142	54.3261092062
I.B-19	0.2776121408	0.5600955198	72.2975181708
I.B-20	0.4174078020	0.2932839393	54.8606819583
I.B-21	0.4110550483	0.4029989043	71.3849893061
I.B-22	0.4099321826	0.2534182814	55.3680130042
I.B-23	0.1287022901	0.5842440794	75.5738965721
I.B-24	0.4019178915	0.2047154472	55.5565256091
I.B-25	0.4532653563	0.4161260139	87.6578887763
I.B-26	0.4156793762	0.2964031093	68.0656402822
I.B-27	0.4805889690	0.3753996716	92.5604403814
I.B-28	0.4282751070	0.4640165419	98.6494595687
I.B-29	0.1551825144	0.5792904049	86.8760972509
I.B-30	0.3388749658	0.2763529104	61.2294979456
I.B-31	0.4124421756	0.2664662522	69.1294839452
I.B-32	0.3751487886	0.0241612662	54.9741990396
I.B-33	0.4125928452	0.3449403658	82.6214161579
I.B-34	0.4757948431	0.1603050376	79.9971544120
I.B-35	0.4141289440	0.2980760641	81.1505833629
I.B-36	0.3212269533	0.2046903079	62.0875306826
I.B-37	0.4039945439	0.3662946692	93.7855168318
I.B-38	0.3939323748	0.1347283909	69.7387381695
I.B-39	0.2904226486	0.2609195136	69.7665857553
I.B-40	0.4942684807	0.2201949355	98.6353340741
I.B-41	0.3650027028	0.0605128530	67.6506508279
I.B-42	0.4160188002	0.2984267206	94.7278887029
I.B-43	0.4032437210	0.2134855934	83.2915115056
I.B-44	0.2995058112	0.2924056736	80.3687190155
I.B-45	0.4954934838	0.1360770719	100.4005086563
I.B-46	0.3029165156	0.1970668629	73.2064153230
I.B-47	0.4105107221	0.2565879477	96.8662425654
I.B-48	0.3935556862	0.1293368487	83.7195247407
I.B-49	0.3501125857	0.0793401820	79.4749538995
I.B-50	0.4013534478	0.2008553668	97.2456382734
I.B-51	0.2666293299	0.2779915428	89.2809106554
I.B-52	0.2935043434	0.1914491983	84.8189274623
I.B-53	0.3969872473	0.1671666821	97.4321722312
I.B-54	0.3932974661	0.1253456024	97.7016625145
I.B-55	0.4030477277	0.0469627166	99.9681279479
II.B-1	0.3962186234	0.5086826315	96.4358796119
II.B-2	0.2517755120	0.2667371937	99.8166259190

Table S VII. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of $\mathbf{r}_1(0) = (-1, 0) = -\mathbf{r}_2(0)$, $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$ and $\mathbf{r}_3(0) = (0, 0)$, $\dot{\mathbf{r}}_3(0) = (-2v_1, -2v_2)$ when $G = 1$ and $m_1 = m_2 = m_3 = 1$. The Figure-eight found by Moore [9] and the 11 families found by Šuvakov and Dmitrašinović [4] are marked in blue. The 26 new families found by the high-order Taylor series method in 100-digit precision are marked in red.

Class and number	v_1	v_2	T
II.C-1	0.2827020949	0.3272089716	10.9633031497
II.C-2	0.5647061130	0.5368389792	54.7501910522
II.C-3	0.5225201635	0.2268083872	26.3193848826
II.C-4	0.2016783093	0.4098955437	21.0205158972
II.C-5	0.5512729728	0.5504821832	86.9417019465
II.C-6	0.5233890828	0.3421147815	44.9302453896
II.C-7	0.4737427040	0.4312766992	46.8946539631
II.C-8	0.2433845587	0.2505166924	22.3333787897
II.C-9	0.1739184309	0.1140843875	21.7535340992
II.C-10	0.1448119296	0.5428981413	38.0630301501
II.C-11	0.1359037483	0.5406786957	37.5276119206
II.C-12	0.5547060649	0.2721678509	52.9480727303
II.C-13	0.4458173723	0.4924872346	62.5459036895
II.C-14	0.5368578744	0.3705761039	60.5607353967
II.C-15	0.5196491506	0.3533207146	57.9902773954
II.C-16	0.2449726867	0.5690681065	52.9395192839
II.C-17	0.5164575299	0.1638309455	43.4746638318
II.C-18	0.4052765310	0.4961906801	62.2999748903
II.C-19	0.1868592137	0.2044548811	27.7253651277
II.C-20	0.3026158825	0.3576846960	37.1769633336
II.C-21	0.5044568709	0.3973785294	70.8348253915
II.C-22	0.2661160110	0.5520357440	62.9864913052
II.C-23	0.2865945055	0.5607550065	67.5864018030
II.C-24	0.0882838085	0.3615155443	35.8170122339
II.C-25	0.5159476361	0.1444933284	52.2126924444
II.C-26	0.3970546540	0.4996659112	74.4041641681
II.C-27	0.4168220336	0.3303332949	55.7893112814
II.C-28	0.2663548692	0.3359619580	43.5341483568
II.C-29	0.4293246983	0.3737393096	64.8733105441
II.C-30	0.2542679242	0.2153811120	40.5931469775
II.C-31	0.5059387393	0.3894005400	86.4180127242
II.C-32	0.4110200317	0.4114629190	72.8712103518
II.C-33	0.4158187604	0.3068035415	60.1507436615
II.C-34	0.4480473854	0.4596449839	96.8391326694
II.C-35	0.4177015960	0.3034552740	66.7511112061
II.C-36	0.2710171846	0.5711013679	93.0418702843
II.C-37	0.3499168589	0.2508093737	56.8351605485
II.C-38	0.3783890881	0.5100736088	98.6023217127
II.C-39	0.4270829576	0.3403174863	80.0382219001
II.C-40	0.1703649368	0.5803609520	94.1583429936
II.C-41	0.4762205782	0.2687092179	85.3959959932
II.C-42	0.4318334213	0.1223893080	66.8092062353
II.C-43	0.4146425708	0.3012162581	79.2835610170
II.C-44	0.2710036741	0.3121152228	63.3936161404
II.C-45	0.4234217026	0.2989326748	87.2610777514
II.C-46	0.3282929621	0.1554041101	63.6556668110
II.C-47	0.2950615495	0.2386718494	65.2298654293
II.C-48	0.4069563704	0.3543774452	94.7746114120
II.C-49	0.4182604701	0.2994822960	92.9809890454
II.C-50	0.2881254754	0.2779750581	74.3601049133
II.C-51	0.2833115180	0.2333212957	70.5062877151
II.C-52	0.4473380629	0.1056541774	84.0347942344
II.C-53	0.4121687384	0.3002886778	98.4332144983
II.C-54	0.3351806004	0.3540343105	96.9055687195
II.C-55	0.2761024483	0.2287323888	76.0753942955
II.C-56	0.4502872480	0.0999875045	92.1831437265
II.C-57	0.2793033126	0.2712019544	79.7728571001
II.C-58	0.2723827379	0.2246817404	81.8883337850
II.C-59	0.2726258099	0.2206843436	88.0168046313
II.C-60	0.2769769990	0.2163211708	94.5261869045

Table S VIII. The free group elements and their length k for the periodic three-body orbits.

Class, number	k	free group element
I.A-1	4	BaBA
I.A-2	8	BAbaBaBA
I.A-3	8	BaBabAbA
I.A-4	16	BabaBAbaBAbaBA
I.A-5	20	BAbabABABababABaBA
I.A-6	24	BabaBAbaBAbaBAbaBAbaBA
I.A-7	24	BABAabABBAbaBAABabBA
I.A-8	28	BabaBAbaBAbaBAbaBAbaBAbaBA
I.A-9	32	BababABABabABABabABaBAbaBA
I.A-10	32	BAbabABABabABABababABaBAbaBA
I.A-11	32	BabABABaBAbaBAbaBAbaBAbaBAbaBA
I.A-12	36	BabAAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-13	36	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-14	36	BAbabABABabABABabABABabABABabBA
I.A-15	40	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-16	40	BabABABaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-17	44	BabABaaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-18	44	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-19	44	BAbabABABabABABabABABabABABabABABabBA
I.A-20	44	BabAAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-21	48	BAbabABABabABABabABABabABABabABABabBA
I.A-22	48	BabABABaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-23	52	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-24	52	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-25	52	BabABaaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-26	52	BAbabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-27	56	BAbabABABabABABabABABabABABabABABabBA
I.A-28	60	BabAAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-29	60	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-30	60	BabABABaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-31	60	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-32	64	BAbabABABabABABabABABabABABabABABabBA
I.A-33	64	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-34	64	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-35	68	BAbabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-36	68	BAbabABABabABABabABABabABABabABABabBA
I.A-37	68	BabAAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-38	72	BAbabABABabABABabABABabABABabABABabBA
I.A-39	72	BabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-40	76	BAbabABABabABABabABABabABABabABABabBA
I.A-41	76	BabAAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-42	80	BAbabABABabABABabABABabABABabABABabBA
I.A-43	80	BAbabABABabABABabABABabABABabABABabBA
I.A-44	84	BAbabABABabABABabABABabABABabABABabBA
I.A-45	84	BAbabaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
I.A-46	88	BAbabABABabABABabABABabABABabABABabBA
I.A-47	92	BAbabABABabABABabABABabABABabABABabBA

Table S IX. Continuation of Table S VIII. The free group elements and their length k for the periodic orbits.

Class and number	k	free group element
I.B-1	10	BabaBababa
I.B-2	14	BAbabABABabaBA
I.B-3	16	BAbaabABABabbaBA
I.B-4	16	BabAAbaBAbabBBabA
I.B-5	18	BabABaBaBaBABabA
I.B-6	22	BabaBABabaBAbabABababA
I.B-7	24	BabABaaBaBaBaBAbbABabA
I.B-8	26	BabABabaBAbaBAbabABabA
I.B-9	26	BAbabABAbabABABabaBABabaBA
I.B-10	30	BAbabaBABabABABababABababaBA
I.B-11	32	BabABabAAbaBAbaBAbaBBabABabA
I.B-12	34	BabaBABabaBABabaBAbabABabABababA
I.B-13	34	BabABabABabaBAbaBAbaBABabABabA
I.B-14	38	BabaBAbaBABabABabaBAbabABabABababaBA
I.B-15	38	BAbabABabABAbaBAbaBAbaBABabaBA
I.B-16	40	BabABabABaaBAbaBAbaBAbaBabbABabABabA
I.B-17	46	BAbabaBABababABABababABABabaBABababaBA
I.B-18	46	BabaBABabaBABabaBABabaBAbabABababABababA
I.B-19	48	BabABabABabAAbaBAbaBAbaBAbaBBabABabABabA
I.B-20	50	BAbabABabABAbaBAbabABABabaBABabaBABabaBA
I.B-21	54	BabaBAbabABAbaBABabaBAbabABabaBAbaBAbaBAbaBA
I.B-22	54	BAbabABababaBABababABabABAbaBAbaBAbaBABabaBA
I.B-23	56	BabAAbaBAbaBabbABabABabAAbaBAbaBBabABabABabA
I.B-24	58	BAbabABABababABABababABABabaBABabaBABababABABababA
I.B-25	58	BabaBAbaBABabABABaBABabaBAbabABABabaBABabABabA
I.B-26	62	BAbabABabABAbaBABabABABabaBABabaBABabaBA
I.B-27	62	BabaBAbabABAbaBABabABAbaBABabABabABabABababA
I.B-28	62	BabABAbaBAbaBABabABabABAbaBABabABabABabaBAbaBABabA
I.B-29	64	BabABaaBAbaBAbaBBabABabABaaBAbaBAbaBAbaBabbABabA
I.B-30	66	BAbabABabABAbaBABabABAbaBABabABABabaBABabaBA
I.B-31	66	BAbabABababaBABabaBABabABABabaBABababABababaBABabA
I.B-32	66	BAbabaBABabaBABABababABABababABABabABABababABABababA
I.B-33	70	BabaBABABabaBABabaBAbaBABabaBAbabABAbaBABabABAbaBABabA
I.B-34	74	BAbabABABababABABabaBAbaBAbaBABabABABababABABabaBA
I.B-35	74	BAbabABabABAbaBABabABABabABABabaBABabaBABabaBABabaBA
I.B-36	74	BAbabaBABabABABababaBABabABABabABAbaBABabABabABababA
I.B-37	78	BabaBABABabaBABabaBABabaBABabABAbaBABabaBABabABAbaBABabA
I.B-38	78	BAbabaBABabaBABabABABababABABabABAbaBABabaBABabaBABabaBA
I.B-39	82	BAbabABABabaBABabABAbaBABabaBABababABABabaBABababABABabaBA
I.B-40	82	BAbabABababaBABababABABabABAbaBABabaBABabABAbaBABabaBA

Table S XI. Continuation of Table S X. The free group elements and their length k for the periodic orbits.

Class and number k : free group element	
III.C-21	42 BabaBAbaBABabABaBaBAbaBABabABabABaBaBABabA
III.C-22	44 BabABabABabABaaBAbaBAbaBAbaBAbaBABabABabABabA
III.C-23	44 BabABabABabAAbaBAbaBAbaBAbaBAbaBABabABabABabA
III.C-24	44 BAbaBAbaBABabABABabABaBAbaBAbaBAbaBABabABabA
III.C-25	44 BAbaBAbaBABabABABabABaBAbaBAbaBAbaBABabABabA
III.C-26	46 BabABabABabaBAbaBAbaBAbaBAbaBAbaBABabABabABabA
III.C-27	48 BabaBABabaBAbaBAbaBAbaBAbaBAbaBABabABabABabABA
III.C-28	48 BAbaBAbaBABaBAbaBAbaBAbaBAbaBABabABabABabABA
III.C-29	50 BabaBABabABABabABABabABaBAbaBAbaBAbaBABabABabA
III.C-30	52 BAbaBAbaBABabABABabABABabABaBAbaBAbaBABabABabA
III.C-31	52 BabaBAbaBABabABABabABabABaBAbaBAbaBABabABabA
III.C-32	54 BabaBABabABABabABABabABabABaBAbaBAbaBABabABabA
III.C-33	54 BAbaBAbaBABabABABabABABabABaBAbaBAbaBABabABabA
III.C-34	58 BabABABaBAbaBABabABabABABabABabABaBAbaBABabABabA
III.C-35	60 BAbaBAbaBABabABABabABABabABaBAbaBAbaBABabABabA
III.C-36	60 BabABabABaaBAbaBAbaBAbaBABabABabABabABabABabA
III.C-37	62 BAbaBAbaBABabABABabABABabABaBAbaBAbaBABabABabA
III.C-38	62 BabABabABABaBAbaBAbaBAbaBAbaBAbaBABabABabABabA
III.C-39	66 BabaBABabaBAbaBAbaBAbaBAbaBAbaBABabABabABabABabA
III.C-40	68 BabABaaBAbaBAbaBABabABabABaBAbaBAbaBABabABabABabA
III.C-41	70 BababABABabABABabaBAbaBAbaBABabABabABabABabABabA
III.C-42	70 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-43	72 BAbaBAbaBABabABABabABABabABaBAbaBAbaBABabABabA
III.C-44	72 BAbaBAbaBABabABABabABABabABaBAbaBAbaBABabABabA
III.C-45	78 BAbaBAbaBABabABABabABABabABaBAbaBAbaBABabABabA
III.C-46	78 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-47	78 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-48	80 BabaBAbaBAbaBABabABabABabABabABabABabABabABabA
III.C-49	84 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-50	86 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-51	86 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-52	86 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-53	90 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-54	94 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-55	94 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-56	94 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-57	94 BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-58	102BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-59	110BAbaBAbaBABabABABabABABabABabABabABabABabABabA
III.C-60	118BAbaBAbaBABabABABabABABabABabABabABabABabABabA