BITCOIN VS ETHEREUM AS DIVERSIFICATION TOOLS IN US EQUITY PORTFOLIOS

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List of Abbreviations

Abbreviation	Full Form
ADF	**Augmented Dickey-Fuller (test)**
BTC	**Bitcoin**
CI	**Confidence Interval**
CVaR	**Conditional Value at Risk**
DeFi	**Decentralized Finance**
ETH	**Ethereum**
ETF	**Exchange-Traded Fund**
H1	**Hypothesis 1**
H2	**Hypothesis 2**
MAR	**Minimum Acceptable Return**
MPT	**Modern Portfolio Theory**
MVP	**Minimum-Variance Portfolio**
NFT	**Non-Fungible Token**
PMPT	**Post-Modern Portfolio Theory**
TP	**Tangency Portfolio**
VaR	**Value at Risk**

ρ	**Spearman's Rank Correlation Coefficient (Rho)**

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Abstract

This paper investigates the dynamic role of Bitcoin (BTC) and Ethereum (ETH) to function as diversification assets within U.S. equity portfolios. It methodically resolves unaddressed methodology in previous literature, such as regime blindness, excessively focusing on the dominance of BTC, and post approval of cryptocurrency exchange-traded fund (ETF) products. This study particularly explores the possibility of cryptocurrencies to substantially changing how they are correlated with the S&P500 in different market regimes, as well as whether strategic allocation can improve risk-adjusted returns during bullish markets.

The analysis uses paired Spearman rank correlations to produce strong conclusions but then does a transformation with the Z score of Fisher to show relationships that are regime dependent. The paper goes on to use Modern Portfolio Theory (MPT) and Post-Modern Portfolio Theory (PMPT) to review the performance of portfolios in response to strategic cryptocurrency investments.

The daily observations of the BTC, ETH, and S&P 500 over 2019-2024 are considered empirical data, and the market regimes are defined using a 20 per cent threshold rule. The main results suggest that the correlation of BTC/ETH with S&P 500 is significantly lower in bull markets than in bear markets, which represents market regime dependent diversification. Despite the high positive correlation between BTC and ETH, their joint addition to the portfolio has always enhanced the risk-return of equity portfolio in bull regimes. Additionally, BTC allocations are better for tail risk as compared to ETH allocations of the same kind. Portfolios constructed dynamically as optimized portfolios performed better than fixed-weight constructs even with the rebalancing transaction costs considered. Such findings indicate that instead of being a safe investment option, BTC and ETH are strong

diversifiers, especially during market rises. The results provide practical recommendations to investors who may want to improve equity portfolios in the United States by incorporating emerging digital assets.

Chapter One: Introduction

This chapter offers background to the broad-based analysis of blockchain-based cryptocurrencies within the global financial system. It outlines the basic concept of the digital asset, specifically Bitcoin (BTC) and Ethereum (ETH), identifies the problem of research that arises out of literature, and outlines the research goals, questions and hypothesis behind the diversification of portfolio inclinations that include the two crypto currencies, BTC and ETH.

1.1 Background and Problem Statement

The cryptocurrencies that are built on a blockchain have changed as experimental digital assets to essential elements of the existing financial world (Zhang et al., 2023). BTC and ETH have capitalized on the market, securing significant institutional investors and regulatory attention in the past decade. Simultaneously, the capital inflow in the cryptocurrency market seems to have intensified by leaps and bounds (Jackon, 2024)**.** This blistering rise requires the focus of researchers who attempt to comprehend the place and usefulness of cryptocurrencies in the diversified investment portfolio (Bakry et al., 2021).

The empirical evidence on the portfolio advantages of cryptocurrencies is, however, inconclusive, characterized by conflicting evidence and a controversy on whether they can be useful diversification tools.

Some studies find indications of the existence of diversification gains associated with negative or low correlation in strongly bullish times (Platanakis & Urquhart, 2019 ; Mahmoudi, 2022 ; Koutrouli et al., 2025). Other investigations record how the correlation has risen during market slump (Wątorek et al., 2022 ; Wątorek et al., 2023). These conflicting results lead to a re-evaluation of their position to diversify into portfolio management systems. Furthermore, the majority of studies have concentrated on BTC but ETH taking over in the field of decentralized finance has been overlooked. Also, the comparative literature exploring the use of BTC and ETH to diversify a portfolio remains limited (Jiang et al., 2020 ; Ma et al., 2020). Finally, several of the studies were conducted prior to the existing trends of institutional adoption observed following the approval of BTC and ETH ETFs. This brings into consideration whether discovery is still a constant in a fast-changing marketplace today.

1.2 Research Objectives and Questions

This research aims to fill in few gaps in knowledge by taking a regime-aware analytical approach with a focus on bull market regimes. The first is to assess and compare the efficiency of BTC and ETH as diversification tool in U.S. equity portfolio by resolving two gaps in current literatures. The research questions behind the investigation are the following: Does BTC/ETH correlate with the S&P 500 in different market regimes, specifically do we expect weaker correlation when in bull markets than when in bear markets? What is the effect of the addition of BTC/ETH on the risk-adjusted performance (Sharpe, Sortino, CVaR) of U.S. equity portfolios specifically in bull markets? Addressing these questions, this study offers a more accurate, regime-specific view of the application of crypto in portfolio diversification and both theoretical background and practical advice to investors in the post-ETF world.

1.3 Research Hypotheses

Guided by the research questions and the theoretical frameworks of MPT and PMPT, the following hypotheses are proposed:

- Hypothesis 1 (H1): The relationship between both BTC and Eth with the S&P 500 is much lower during a bull market regime compared to a bear market regime.

  Hypothesis 2 (H2): 1-5 per cent of strategic allocation of BTC or/and ETH in a U.S.

Equity portfolio will statistically significantly improve risk-adjusted performance (the Sharpe, Sortino, CvaR ratios) when in bull market regimes.

1.4 Structure of the Thesis

This thesis is made up of 5 chapters. After this Chapter 1, introduction and Chapter 2 will address the literature and the theoretical framework related to this study. Chapter 3 will provide a data collection, regime types in the market and the choice of analytical models. The empirical results will be described in chapter 4. And, lastly, Chapter 5 will be a discussion of the findings, implications, limitations, future research and conclusions.

Chapter Two: Literature Review

This chapter begins with a critical synthesis concerning the diversification of cryptocurrency portfolios. It begins with the definition of the unique features of BTC and ETH and the theoretical background of this dissertation. This is followed by a detailed overview of the rising trend of institutional investment in digital assets, and the empirical evidence of their diversification impact, as well as various approaches used in earlier research. Finally, this chapter ends by defining gaps in the current research that this thesis has been intended to address.

2.1 The rise of Bitcoin and Ethereum with Institutional Integration

Bitcoins and Ethereum have undergone a slow transition to become not digital fads but the key components of global finance (Corbet et al., 2018). Popularly known as digital gold, BTC is a currency with enormous price fluctuations, and intricate market cycles, like bull and bear (Chan et al., 2023). Similar to all other cryptocurrency, BTC is deeply locked in a loop of speculation and stagnation, strongly controlled by its programmed quadrennial halving

On the other hand, the value of ETH lies in a more general application layer which is largely based in Decentralized Finance (DeFi) and Non-Fungible Tokens (NFTs) that are built upon programmable smart contracts. It is more susceptible to wider macroeconomic factors, including interest rate, and development of the tech sector than BTC (Krause, 2024). Not only does recent progress and the advent of Layer2 scaling solutions remove the sporadic network congestion of ETH, but in the long term, it also curbs soaring price fluctuations (Gudgeon et al., 2020). These underlying and technological differences indicate that BTC and ETH offer some diversification advantages and react to market changes in a diversified portfolio differently, and so they should be further explored (Ma et al., 2020).

Meanwhile, the rise in institutional involvement in the cryptocurrency market has implied that the asset category has ceased to be the object of retail speculation and has become more integrated into the mainstream financial market. As an example, the market capitalization of cryptocurrencies increases as a good precursor of more interconnections with the general financial system (Sahu et al., 2024). In the meantime, the newly authorized spot BTC exchange traded funds, enhancing the intersection of BTC and the U.S. equity market, especially S&P 500, during the initial months of 2024. This was also influenced by the increasing interest of Hedge Funds in the crypto asset market (Widarto et al., 2022 ; Mungo et al., 2024).

These Regulatory milestones mean that cryptocurrency is now accepted in general and motivates the majority of institutional investors to incorporate them into the investment procedure (Jackon, 2024 ; Sergio & Wedemeier, 2025). Moreover, this influx of institutional capital has required the full assessment of the risk and reward impact of the cryptocurrency, both in speculative interest and in the strategic placement in a diversified portfolio (DeVault & Wang, 2021).

2.2. Theoretical Framework

This study draws on four key theoretical perspectives to assess the diversification potential of BTC and ETH in U.S equity portfolio, particularly the S&P 500.

2.1.1 Modern Portfolio Theory (MPT)

The introduction of MPT by Markowitz in 1952 aimed to maximize expected return at a given level of risk focusing on the advantages of diversification (Şerban & Vrînceanu, 2025). The model assumes that a portfolio that has low or negative pairwise correlation can result in a portfolio with lower aggregate risk (Chen, 2023). In line with this, this paper uses MPT to build a portfolio that includes cryptocurrencies, based on the assumption that they have low co-movement with equities in bull markets. MPT can also be used to compute optimal allocation weights and risk-adjusted performance measures applied to conventional and crypto-diverse portfolios. It indicates that small investments in cryptocurrencies can increase the risk-adjusted returns of the entire portfolio based on strategies of MPT (Ma et al., 2020). As such, this paper applies both the Sharpe ratio, an MPT measure and MPT-optimal weights to analyze the measure hypothesis II.

2.1.2 Post- Modern Portfolio Theory (PMPT)

PMPT is an extension of MPT with an increased focus on downside risk as opposed to total risk. It addresses the limitations of MPT that assumes normally distributed returns with symmetric risk which are often violated by cryptocurrencies heightened kurtosis and asymmetry (Bouteska et al., 2023 ; Sahu et al., 2024). PMPT uses the Sortino Ratio and the Conditional Value at Risk (CVaR) to measure extreme losses, which apply to the highly volatile cryptocurrencies (Brayek et al., 2024 ; Kuang, 2025). An example is CVaR, which approximates losses in worst-case scenarios and provides risk-averse investors with an instrument to capped exposure in the event of extreme market stress (Brayek et al., 2024)**.** The explicit consideration of the asymmetry of the potential losses allows PMPT to attract risk-averse investors that focus on the downside risk, especially in bull markets (Deng, 2023).

This paper will therefore use PMPT measures to test Hypothesis II which states that cryptocurrencies will increase the performance of the portfolio in bull markets.

2.1.3 Behavioral Finance

Behavioral finance studies the ways in which the psychology and investor sentiment influence financial markets (Hadad & Kedar-Levy, 2022)**.** In contrast to conventional financial products, there is little regulation over cryptocurrencies as they are largely affected by speculation, hype cycles, and media coverage (Hackethal et al., 2021 ; Seabe et al., 2024).

They are overly speculative and thus cause a variety of price fluctuations that do not necessarily follow basic values, meaning the cryptocurrency economy is largely driven by investor sentiment (Bouteska et al., 2023 ; Seabe et al., 2024)

The analysis of regime-dependent and nonlinear correlations of the BTC, ETH, and S&P 500 equity portfolios is based on this framework. It helps to test the hypothesis that digital assets can respond differently to bull and bear markets depending on the expectations and behavior of investors (Almeida & Gonçalves, 2023)**.** Hypothesis I directly depends on such behavioral expectations, the regime-dependent role of cryptocurrencies.

2.3 Empirical Evidence on Cryptocurrency Diversification

Cryptocurrencies have presented a controversial issue in terms of whether they can offer diversification benefits to conventional investment portfolios (Bányai et al., 2024). During the initial years of BTC existence, the majority of the available empirical studies appeared to indicate a certain degree of diversification or hedge benefits in relation to BTC. Indicatively, BTC was identified as a global uncertainty index diversifier, but the effect was sensitive to state and may not always stand the test of crises (Bouri et al., 2016). Subsequently, BTC was classified as a potentially dangerous risk in the COVID-19 crash, as it was showing rising correlations with equities at the time when it should have been most diversified (Conlon & McGee, 2020). These contradictory results are a strong indication that the initial hype regarding the hedging abilities of BTC was probably prompted by the exact times that were being examined.

This conditionality has been pointed out in more developed research based on time-varying dependence models. As shown in the copula-ADCC-EGARCH model by Tiwari et al., (2019) BTC correlations with equities are low in normal times and explode in times of market stress. Bouri et al., (2018) also employed a copula-based design to demonstrate the asymmetric dependence where the dependence on co-movement is increasing in an uneven way during left-tail events. The wavelet analyses reveal that the magnitude and sign of co-movement vary over time, and short-run dynamics are not similar to long-run dynamics (Goodell & Goutte, 2021)a.

At the same time, the amount of evidence available on ETH is small in comparison with BTC. According to Mariana et al., (2020) ETH exhibited a safe-haven behavior in the short term in the initial stages of the COVID-19 pandemic. These results indicate that hedge/diversification characteristics of cryptocurrencies are regime and time dependent.

2.4 Regime-Dependency of Correlation and Portfolio Performance

A new collection of empirical data (Somoza & Didisheim, 2022 ; Patra & Singh, 2025). grows out of the pessimistic side of the argument that the diversification properties of cryptocurrencies are generally optimistic, as opined by early research during periods of market recessions or financial crisis; In fact, a portion of the current literature has stressed the fact that cryptocurrencies showed low or no correlation with assets like stock, bond, and commodities, particularly during stable market (Aslanidis et al., 2019 ; Harvey et al., 2022). On the other hand, when a bear market occurs in standard finance, the correlation factor can get quite high and the advantage of diversifying is lowered (Lahajnar & Rožanec, 2020 ; Ampountolas, 2023). This implies that cryptocurrencies exhibit diversification characteristics during the bull period and reduce or in fact disappear during such market declines (Koutrouli et al., 2025). The same assumption can also be seen in studies that underline the dynamic nature of cryptocurrencies throughout various market stages (Lahajnar & Rožanec, 2020). These results inspire this paper to examine correlation of crypto currencies with US equity namely S&P 500 index across the market regimes.

Moreover, these regime specific correlation results highlight the significance of strategic allocation of cryptocurrencies to enhance the overall portfolio performance and risk-adjusted return (Sepp, 2022). The mean variance efficiency of portfolios consisting of small BTC allocations was found to be increased by its historical high returns (Brauneis & Mestel, 2018 ; Platanakis & Urquhart, 2019).

Equally, there is some evidence that ETH may portend greater potential diversification advantage than BTC because of the different technological foundations and expanded application domains (Ma et al., 2020). Furthermore, other studies indicate that under certain market conditions ETH may be an excellent short-run investment (Jiang et al., 2020). This observation shows that the most optimal cryptocurrency is not predetermined and will be based on various factors related to the type of assets selected and performance in relation to each other (Ma et al., 2020). This highlights the importance of comparative analysis that includes consideration of the specifics of the performance of BTC and ETH within the identified market regime to make more complex portfolio management decisions (Jiang et al., 2020). To that end, this paper engages in a comparative study of both BTC and ETH in the context of market regimes, and specifically, in the context of bull markets, where the diversification potential seems to be the most promising to gain a better understanding of the utility of the two in terms of portfolio optimization.

2.5 Identified Research Gaps

Although there has been an increasing literature on cryptocurrency portfolio diversification, few limitations remain. First, the majority of studies do not consider market regimes when they examine the performance of a crypto inclusive portfolio. However, it is known that correlations of cryptocurrencies are conditional on the market regime (Lahajnar & Rožanec, 2020). Second, most of the diversification analysis has been focused on BTC, and the economic contribution of ETFs with its utility-based risk-reward dynamics and the connection to the economic matter is relatively little studied (Ma et al., 2020). It must carry out more in-depth research, whereby BTC and ETH have different profiles and market dynamics (Jiang et al., 2020 ; Ma et al., 2020).

Additionally, the recent history of crypto and its integration with traditional finance at an accelerated rate is marked by an influx of institutional investor attention and spot ETFs on both BTC and ETH gaining regulatory acceptance (Jackon, 2024 ; Sergio & Wedemeier, 2025)/**.** Accordingly, the altering environment of crypto-equity correlation dynamics and diversification potential may not be exhaustively discussed in some of the prior studies(Wu, 2024).

Thus, this work seeks to address these gaps by proposing crypto inclusive portfolio performance analysis under the framework of a regime-sensitive model. The earlier works that employed Pearson correlations or sophisticated econometric models such as DCC-GARCH, copulas, and wavelets did not employ testing of differences in correlation across regimes. However, the study uses Spearman rank correlation and Fisher Z test and provides a powerful, non-parametric analysis of the hypothesis that the effects of diversification between bull and bear markets are significantly different, which is not sufficiently discussed in current literature.

Moreover, regime-based correlation analysis is combined with different performance measures such as dual-framework (MPT and PMPT) and the portfolio results are bootstrapped. This work, therefore, presents a rigorous and empirically based evaluation of whether BTC and ETH truly improve S&P 500 equity portfolios specifically in the bull market.

2.6 Conceptual Framework

According to the examined literature, the conceptual framework of **Figure 2.1** underlies the research based on the synthesis of main variables and the suggested relationship between them in a dual theoretical framework.

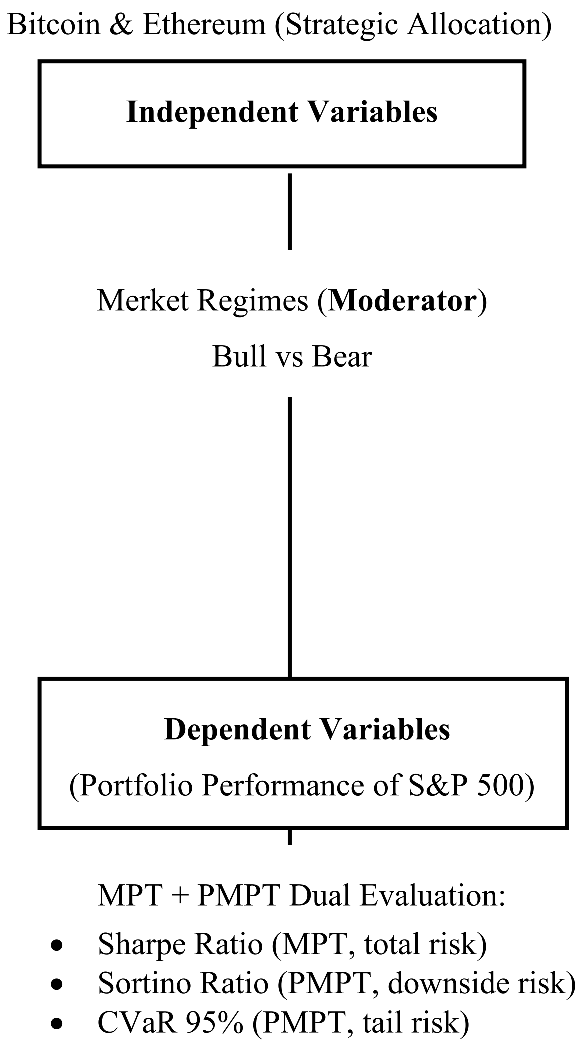

Figure 2.1

Conceptual Framework

Source: Author's own Creation

Figure 2.1 presents the conceptual framework of the hypotheses of this study and depicts how the variables work. The strategic allocation of BTC and ETH primarily 1~5%

serves as independent variables. This is not static and the moderator that would drive the amplification or reduction of diversification benefits is the market regimes (bull vs. bear). Dependence variables include the results of portfolio performance in terms of Sharpe Ratio (risk adjusted returns under MPT), Sortino Ratio (downside adjusted returns under MPT), and Conditional Value-at-Risk (tail risk under PMPT). This framework aims at examining BTC and ETH can be classified as diversification assets during bull markets as theorized by the combination of MPT and PMPT.

Chapter Three: Methodology

In this chapter, the empirical procedures and the analysis models employed to investigate the regime-dependent correlations and the portfolio performance analysis and the portfolio allocations of BTC, ETF and S&P 500 as shown in **Figure 3.1** are presented.

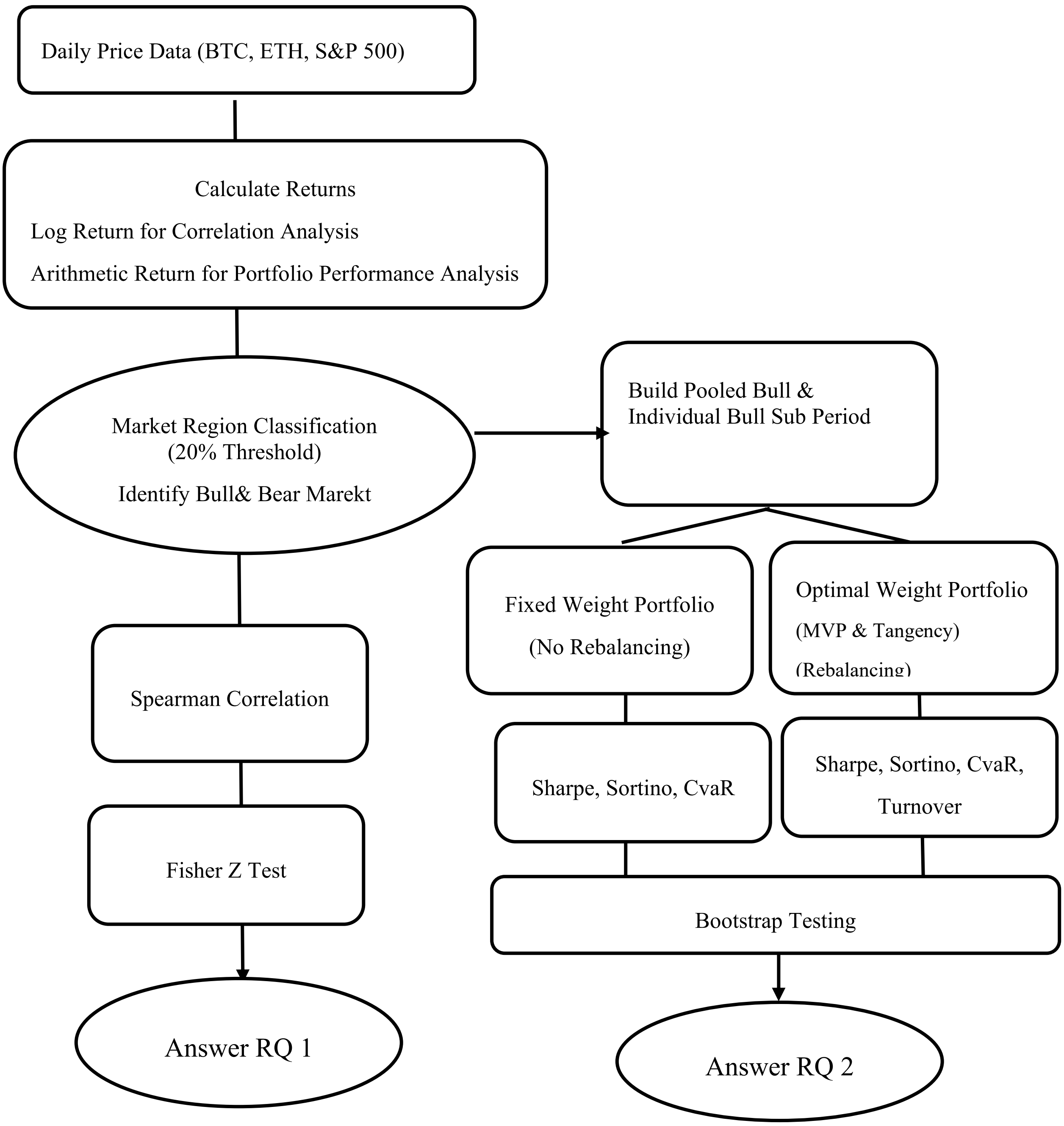

Figure 3.1

Research Methodology Flow Chart (Note. Author's own illustration)

3.1 Data Collection and Preparatory Analysis of Financial Time Series

This section describes the finer details of data collection and methodology adopted in this study. In order to guarantee the accuracy and strength of data, the primary data source was Refinitiv DataStream, a reliable service that offers people certain historical data records on finances. To ensure the trustworthiness of the data retrieved and improve its quality, the primary data was cross checked with Yahoo Finance. The dataset consists of 1566 observations per day of each of the financial assets in the 2019-2024 period. The five-year period selected is a solid foundation on which to perform sound statistical analysis regarding asset behavior in various market regimes.

3.1.1 Return Calculation and Initial Analysis

The Daily BTC, ETH and S&P 500 returns were transformed into daily returns to perform statistical analysis and simulate a portfolio. The mode of calculating the returns was chosen according to the purpose of the analysis.

1. **For Correlation Analysis:** Continuously compounded log returns were applied, and it was calculated as follows:

$$\boxed{R_t^{log} = \ln(P_t/P_{t-1})}$$

This change is typical of financial econometrics to evaluate relationships among time series since it is used to enhance symmetry and time-additivity of the distributions.

2. **For Constructing the Portfolio and Evaluating Its Performance**: Simple arithmetic returns were used, calculated as:

$$\boxed{R_t^{simple} = (P_t - P_{t-1})/P_{t-1}}$$

Arithmetic returns are the conventional basis for portfolio optimization and calculation of performance metrics (Sharpe ratio, CVaR).

The approach reinforces suitability as the log returns are robust inputs to correlation analysis when non-normal, whereas arithmetic returns are realistic simulations of portfolio performance. To demonstrate and get a more qualitative feel of some of the quantitative features of the assets, **Figure 3.2** displays the log return of the sample period.

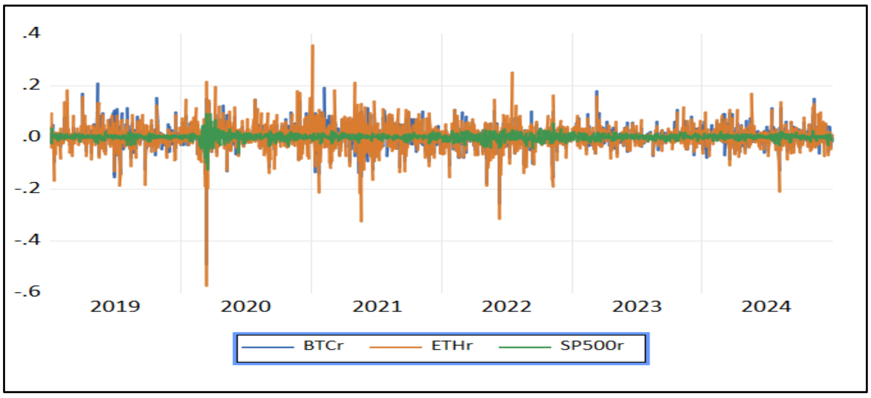

Figure 3.2

Time Series of Daily Log Returns for BTC, ETH, and the S&P 500

Source: EViews output of Daily log return. Data sourced from Refinitiv (2019–2024).

3.1.2 Descriptive Statistics

This sub-section outlines the most significant statistical features of the daily returns of BTC, ETH and the S&P 500 during the time 2019-2024 **(Table 3.1).** The time frame is characterized by various stages of the markets, such as sharp growth and turbulence, introduction of digital currencies, and new regulation systems.

Table 3.1

Summary Descriptive Statistics (Full Period: 2019~2024)

Index	Mean	Median	Standard Deviation	Kurtosis	Skewness	JB p-value	Minimum	Maximum
BTC	0.002	0.0012	0.0418	15.6028	-1.1105	<0.0001	-0.4940	0.2081
ETH	0.002	0.0008	0.0531	13.4161	-0.9011	<0.0001	-0.5756	0.3551
S&P 500	0.0005	0.0006	0.0125	15.9379	-0.8637	<0.0001	-0.1277	0.0897

Note. Author's own calculations using Python, verified with Excel output.

The data highlights that cryptocurrencies have significantly greater volatility and dispersion of returns as compared to traditional equity. The standard deviation of BTC is more than three times as large as the one of S&P 500, and the standard deviation of ETH is even more than 4.25 times higher than the equity standard deviation. Quantitatively, ETH has an average daily volatility of 5.31 per cent (versus 1.25 per cent in the case of the S&P 500). It seems that such volatility is a long-term characteristic of cryptocurrencies like BTC and ETH, which has been recorded in recent literature and in different market environments (Dudek et al., 2023 ; Chinazzo & Jeleskovic, 2024 ; Omrane et al., 2024).

Additionally, non-normality is unquestionably established in the return distributions of all the three assets. The Jarque-Brau test rejects the null hypothesis of normality in all series ($p = 0.0001$), in agreement with the extreme kurtosis values which are indicative of leptokurtic distributions with heavy tails. The empirical finding that the distribution of cryptocurrency returns is heavy tailed is a long-established empirical fact in the existing literature (Shanaev et al., 2023 ; Silva et al., 2024).

Further, our analysis confirms that these assets exhibit pronounced negative skewness. While some studies have documented positive skewness in BTC during certain windows of time and with different approaches(Ang et al., 2023), the literature on daily returns tends to identify negative skewness, especially in short time horizons (Takaishi, 2018).

Taken Together, the likelihood of large loss is far higher than the likelihood of large gain, meaning there is more risk of downside than upside possible. Such non-normal, heavy-tailed, negatively skewed properties of cryptocurrency returns make Spearman correlation methods the right tool to use in **Section 3.3** because traditional tools like the Pearson correlation method cannot work with such non-normal distributions(Bouteska et al., 2023).

Notably, the average returns of BTC and ETH are approximately four times higher compared to the risk-adjusted returns of S&P 500, although the risk-adjusted performance is quite different. Volatility/return ratio in ETH is 27% higher than that of BTC, and it is clear that the risk in ETH is higher than the reward. This initial estimation forms the basis of further regime-based correlation, and portfolio optimization of these digital assets, with different risk-return profiles.

3.1.3 Stationarity Test: Augmented Dickey–Fuller (ADF) test

The stationarity of the series of daily returns was examined formally to make the following analyses valid: the Augmented DickeyFuller (ADF) test (**Table 3.2**). Preliminary diagnostics (correlgrams, correlation analysis, preliminary check of VAR stability) were also performed. ADF test is a commonly known statistical test in financial econometrics that is used to find out whether a time series is stationary or needs to be different to achieve stationarity (Guo, 2023).In addition, the importance of stationarity tests is that when there is a non-stationarity, the results of spurious regression and unreliable conclusions are possible (Mushtaq, 2011).

Table 3.2

Unit Root Test/Augmented Dickey-Fuller (ADF) tests

Variable	t-statistic	p-value	Critical value (5%)	Stationary?
BTH	-41.7174	0. 0000	-2. 8631	✓ Yes
ETH	-42.3663	0. 0000	-2. 8631	✓ Yes
SP 500	-11.6380	0. 0000	-2. 5677	✓ Yes

Note. Output from EViews of the Augmented Dickey–Fuller (ADF) test using daily log returns.

The test statistics on both return series have a much larger value than the critical values at the 5% level and the p-values are less than 0.05 and thus reject the null hypothesis that two-return series have a unit root. The data is therefore considered to be stationary and can be subjected to further econometric analysis of the data, including correlation analysis.

3.2 Market Regime Classification

Following the classification criteria commonly used in financial industry practice such as Charles Schwab, and academic discussion, the S&P 500 market regimes were determined with a 20% boundary. According to this rule, a bear market is when there is a drop of more than 20% than the earlier high, and a bull market is when there is an increase of 20% or more since a trough (Hanna, 2017 ; Pagan & Sossounov, 2002). Although this may seem somewhat ad hoc, the S&P 500 is widely regarded as one of the best benchmarks for U.S. equities, and as such its bull and bear markets are relatively easy to identify and track, which improves the replicability of this study's regime classification.

Using this threshold rule, it was possible to identify five market phases occurring between January 2019 and December 2024, as shown in **Figure 3.3** and summarized in **Table 3.3.**

Figure 3.3

Output of 20% Threshold Method for S&P 500's Bull/Bear Markets using a 20% threshold

Note: Author's own calculations using Python. Data sourced from Refinitiv (2019–2024).

Table 3.3

Output of 20% Threshold Method for S&P 500 Bull/Bear Markets using a 20% threshold

No	Phase	Start	End	Start Price	End Price	Duration (Day)	Percentage Change
1	Bull	01/01/2019	19/02/2020	2506.85	3386.15	414	35.1
2	Bear	19/02/2020	23/03/2020	3386.15	2237.4	33	-33.9
3	Bull	23/03/2020	03/01/2022	2237.4	4796.56	651	114.4

4	Bear	03/01/2022	12/10/2022	4796.56	3577.03	282	-25.4
5	Bull	12/10/2022	31/12/2024	3577.03	5881.63	811	64.4

Note. Author's own calculation using Python

Each of the five market phases described in Table 2 corresponds to a certain set of economic conditions and reshaped the market behavior of the S&P 500 index and cryptocurrencies. Phase 1 (01/01/2019 19/02/2020): 414-day bull market, during which the economy has increased steadily by 35.1 per cent, indicates overall positive attitudes of investors until the COVID-19 pandemic spread around the world.

Phase 2 (19/02/2020 -23/03/2020) covered 33 days of a bear market, which fell by 33.9 per cent due to an unprecedented speedy market-selling process imposed by the pandemic and related economic lockdowns.

Phase 3 (23/03/2020 - 03/01/2022) lasted 377 days of a bull market, the longest in history, with excessive optimism among investors based on the next round of strong fiscal stimulus, exceedingly low interest rates, effective vaccine development, and economic reopening.

Phase 4 (03/01/2022 -12/10/2022) was 284 days long, with a 25.4% fall, or bear market, in a climate of more global monetary tightening, endemic inflation, and rising recession risks.

Phase 5 (12/10/2022 -31/12/2024) was the latest 80 days of a bull market with a 64.4 per cent gain as a result of falling inflation, global expansion, a new wave of investor confidence and a slight rise in investor sentiment. These clear market stages form the basic basis of the correlation dynamics and portfolio performance of cryptocurrencies under expansion and contraction periods.

3.3 Static Correlations Analysis

A two-step methodological framework (refer to **Figure 3.1**) was adopted to test the hypothesis of the relationship between BTC, ETH, and the S&P500 in different market regimes.

3.3.1 Step 1: Measuring Association with Spearman's Rank Correlation coefficient (ρ)

First, the correlation between the daily returns of the BTC, ETH and S&P 500 is calculated using Spearman's rank correlation coefficient (ρ) over Pearson's correlation. This choice can be justified by the fact that Pearson correlation assumes the data to be normally distributed (Alsaqr, 2021), which can provide biased or inflated results when used on non-normally, heavy-tailed financial instruments such as cryptocurrencies (Bishara & Hittner, 2012, 2014).The Spearman correlation is calculated using the following formula:

$$\boxed{\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}}$$

where d_i is the difference in ranks between paired observations, and n is the number of observations.

3.3.2 Step 2: Testing for Significance with Fisher's Z-Transformation

The fact that the correlation coefficients between bull and bear markets are different, is not evidence of regime dependence. A formal test of statistical significance was thus carried out to determine the significance of the differences in Spearman coefficients of correlation. The independent correlation coefficients were compared using Z-transformation by Fisher (Aert, 2023).

Such a transformation allows testing the differences between the correlation coefficients in a parametric way (Holbrook et al., 2021). It converts the otherwise skewed sampling distribution of Spearman rank correlation coefficients into a more or less normally distributed Z-score (Aert, 2023). It is defined as:

$$\boxed{Z = 0.5 \cdot \ln\left(\frac{1+\rho}{1-\rho}\right)}$$

The standard error of Z depends on sample size n and this explicitly accounts for differing durations in its standard formula. For instance, $n_{Bull} \neq n_{Bear}$. The test statistics (Z) scores for comparing two independent correlation coefficients with different sample size ($n_{Bull} \neq n_{Bear}$) are computed as:

$$\boxed{Z_{\text{test}} = \frac{Z_{\text{Bull}} - Z_{\text{Bear}}}{\sqrt{\frac{1}{n_{\text{Bull}}-3} + \frac{1}{n_{\text{Bear}}-3}}}}$$

The tests of statistical significance were performed at a traditional level of $\alpha = 0.05$. This process minimizes the risk of misinterpreting random correlations as substantive relationships dependent on market phase and therefore increases the validity of the results.

3.4 Portfolio Performance Analysis

To answer the second research hypothesis that a crypto-inclusive portfolio should perform better than U.S equity portfolio in bull markets, the analysis takes place in three steps using two different sets of data: (1) a single pooled bull-market period and (2) multiple bull-market sub-periods. The stages of the performance analysis are as under:

Step 1: Calculate Sharpe, Sortino, CvaR based on fixed crypto allocations.

Step 2: Calculate Sharpe, Sortino, CvaR and Turnover based on Optimal crypto allocations derived from applying Minimum-Variance Portfolio (MVP) and Tangency models.

Step 3: Apply Bootstrapping to determine if the observed differences in performance metrics are statistically significant.

3.4.1 Step 1: Fixed-Weight Portfolios Performance

This stage is the analysis of the fixed-weight portfolios, which are passive investment plans. There were constructed two types of portfolios. First, the U.S. equity portfolio at the baseline is chosen as the portfolio that is 100 percentage invested in the S&P 500 index, which is also used as a reference point against which other portfolios are compared. Later, portfolios were formed that were crypto-inclusive and comprised the S&P 500 with 1, 2, 3, 4, and 5 percent of investments in BTC, ETH, or both. These levels of allocation are supported by industry's best practice and literature. According to Crypto.Com (2024), the optimal range of crypto asset allocation is 1-5 percent. It is also evidenced by (Holovatiuk, 2020), who identified that allocations of around 1.9 per cent to 2.8 per cent in cryptocurrency positively turned around risk-adjusted returns without meaningfully raising the downside risk of portfolios. The performance of every fixed-weight portfolio was then compared against a set of measures, all of which have been described in the next section.

1. Sharpe Ratio

The Sharpe Ratio is a key figure in the family of foundational measures used in the Modern Portfolio Theory (MPT) and has become the most widely used industry standard in the assessment of risk-adjusted returns (Sharpe, 1994 ; Anelli, 2023). A higher value of the Sharpe ratio signifies superior compensation for the aggregate risk undertaken (Siegel et al., 2001). It is calculated as follows:

$$\boxed{Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}}$$

Where:

R_p = The annualized mean portfolio return.

R_f = The annualized time-varying risk-free rate (In a bid to be accurate with the computation, this research will use the risk-free rate as calculated by the 3 Month yield of the U.S. Treasury Bill (FRED series: DTB3), but then it will be converted into a daily rate to match the daily frequency of the portfolio returns)

σ_p = The annualized standard deviation of portfolio returns.

2. Sortino Ratio

The Sortino Ratio is used to capture the asymmetric distributions of returns that cryptocurrencies are known to have (high skewness and kurtosis, as demonstrated in **Table 3.1)**. This PMPT measure corrects the Sharpe Ratio and only punishes downside volatility (Nassar & Ephrem, 2020a, 2020b; Sortino & Price, 1994). It is calculated in the following way:

$$\boxed{SortinoRatio = \frac{R_p - MAR}{DD_{MAR}}}$$

Where:

R_p = The annualized mean portfolio return.

MAR = The Minimum Acceptable Return, set to 0. This is a common and conservative choice in academic studies that aims to assess absolute loss minimization without introducing subjective return targets (Holovatiuk, 2020).

DD_{MAR} = The downside deviation, representing the standard deviation of returns that fall below the MAR.

3.Conditional Value-at-Risk at 95% (CvaR)

CVaR is a measure of tail risk, which is one core aspect of the volatility of cryptocurrencies. It represents a more detailed evaluation of extreme downside risk because it estimates the expected average loss given returns larger than the selected VaR threshold

(Ding & Feinberg, 2022). Having a smaller CVaR value means that the portfolio is better resilient to extreme negative events, and thus it is preferable in the context of efficient tail risk management (Downing et al., 2015 ; Girach et al., 2021). At the 95 percent threshold, CVaR reflects the average worst 5 percent of portfolio returns and is defined as:

$$\boxed{CVaR_{95\%} = E[L|L > VaR_{95\%}}$$

Where:

- L represents the portfolio's daily losses.
- $VaR_{95}\%$ is the 95th percentile loss value.

3.4.2 Step 2: Optimal-Weight Portfolios

To provide the different investor risk preferences, we also construct two types of optimal portfolios: the Minimum-Variance Portfolio (MVP) for risk-averse investors and the Tangency Portfolio (TP) for return-seeking investors. This step, based on MPT and PMPT The lookback window used is one year (252 days) to estimate the inputs and therefore time-varying financial time series. This is optimized with a maximum allocation of 10 per cent to the collective weight of cryptocurrencies (BTC + ETH) to reduce the exposure to highly volatile instruments (Andrianto, 2017). The portfolios are rebalanced on a rigid calendar basis; monthly and quarterly. This rebalancing frequency causes portfolio turnover, which is the ratio of assets exchanged, an important factor in optimisation analysis since a high turnover raises transaction costs and may negate the advantages of rebalancing to optimum weights (DeMiguel et al., 2009).

Every transaction made in the course of rebalancing is associated with a 25 basis points (0.25 %) transaction cost which is charged on the asset value to be bought and sold. This amount is a realistic estimate and falls within the typical range of cryptocurrency trading fees (Holovatiuk, 2020). It is included because empirical research indicates the need to consider transaction costs when analysing and rebalancing cryptocurrency portfolios (Brauneis & Mestel, 2018). The results of each optimal portfolio are then measured in the same metrics as fixed-weight portfolios.

1.The Minimum-Variance Portfolio (MVP)

The MVP is constructed by solving for the weights that minimize portfolio variance, and it is defined as:

$$\boxed{\min_w w^T \Sigma w \quad \text{subject to} \quad \sum_{i=1}^{n} w_i = 1 \quad \text{and} \quad w_i \geq 0}$$

2.The Tangency Portfolio

Tangency is constructed by solving for the weights that maximize Sharpe Ratio. The optimization is defined as:

$$\boxed{\max_w \frac{w^T \mu - R_f}{\sqrt{w^T \Sigma w}} \quad \text{subject to} \quad \sum_{i=1}^{n} w_i = 1 \quad \text{and} \quad w_i \geq 0}$$

3.Turnover

Turnover ratio is calculated at each rebalancing date. The turnover for a single rebalancing event from old weights w_d to new optimal weights w_{new} is computed as:

$$\boxed{\text{Turnover}_{\text{event}} = \frac{1}{2} \sum_{i=1}^{n} |w_{\text{new},i} - w_{\text{old},i}|}$$

3.4.3 Step 3: Statistical Significance Testing (Bootstrapping)

This study uses the stationary bootstrap procedure of Politis and Romano (1994) to determine whether the differences in metrics of performance observed are statistically significant. A null distribution, based on 10,000 bootstrap resamples of the difference between the crypto-inclusive and the baseline S&P 500 portfolios is generated, per each performance metric. This is a one-tailed test, that is, testing an improvement in the performance metrics. As a result, the finding that the crypto-inclusive strategy outperforms the S&P 500 portfolios is considered significant at the 10 per cent level on the assumption that the whole 90 per cent confidence interval of the bootstrapped difference takes values greater than zero.

3.5 Validation of the Simulation Methodology

A portfolio simulation engine validation was done before running the formal analysis to ensure the appropriate functionality of the simulation system. The computation of the portfolio returns, including the rebalancing logic, transaction costs, and dynamic weights, was named as the core functionality to be validated. The process of validation consisted of three steps. First, a benchmark portfolio was created, in which 100% is invested in the S&P 500 index and 0% in cryptocurrencies. The same rebalancing rules and thresholds were applied to this portfolio, as to the crypto-inclusive portfolios.

Lastly, the series of returns that the simulation engine generated in relation to this benchmark portfolio was cross-checked against the actual daily returns of the S&P 500. The validation criterion stated that the return series had to perfectly coincide, but only by rounding off to floating-point accuracy and by a very small margin. Application was done using the numpy.allclose (strict epsilon) (atol = 1e-10).

The validation results showed that the simulated returns on the 100 percent S&P 500 portfolio were absolutely in agreement with the real historical returns, thus, confirming that the simulation system is working properly with regard to its return, transaction cost, and reinvestment algorithms. This check guarantees a very high level of confidence that the results achieved on the crypto-inclusive portfolios are valid and that there are no major mistakes made in the common framework.

Chapter Four: Main Empirical Results

This chapter reports on the primary empirical findings of the thesis based on the methodology of Chapter Three. The results are grouped into four general groups to provide logical development of simple correlations to the implication of portfolio performance to both fixed and optimal allocation strategies, and ultimately to their statistical validation.

4.1 Correlation Analysis Across Market Regimes

This part summarizes empirical evidence on the dynamic relationship between BTC, ETH, and the S&P 500 during different bull and bear market regimes that were detected between January 2019 and December 2024. The calculated Spearman correlation coefficients of all the identified market phases during the sample time are summarized (refer Table **4.1**).

Table 4.1

Spearman Correlation Coefficients by Market Regime |

Phase	Start	End	BTC-S&P500	ETH-S&P500	BTC-ETH
Bull	01/01/2019	19/02/2020	-0.047	-0.047	0.802
Bear	19/02/2020	23/03/2020	0.399	0.480	0.917
Bull	23/03/2020	03/01/2022	0.227	0.223	0.769
Bear	03/01/2022	12/10/2022	0.559	0.555	0.885
Bull	12/10/2022	31/12/2024	0.299	0.297	0.813

Source: Author's own calculations using Python, cross-checked with EViews

Regime based correlation analysis shows that cryptocurrencies exhibit different patterns compared to traditional assets and among themselves. These correlations during bear market regimes were never negative, but in most instances, they were moderate to strong.

The bear market that started after the beginning of the COVID-19 is a good example, where the correlation coefficients are 0.399 with BTC-S&P and 0.480 with ETH-S&P. This pronounced correlation during downturns indicates that the diversification that cryptocurrencies provide is most diminished in periods of market stress and is least available when it is most needed by investors (Allen, 2022)**.** Moreover, the bear market of 2022 showed an even stronger coupling of 0.559 for BTC-S&P500 and 0.555 for ETH-S&P-500, further supporting the idea of deepening integration of cryptocurrencies into the financial system during contractions (Wątorek et al., 2023).

By contrast, correlations of cryptocurrencies to the S&P 500 were generally weaker in bull markets than in bear markets but were still largely positive post-March 2020. The correlations in the 2019 bull market were astoundingly close to zero, at -0.047 in both BTC-S500 and ETF-S500, which confirmed the hypothesis of diversification at least, initially. This result confirms the rationale in more recent literature that the relationship between crypto assets and traditional financial assets is generally weaker when the market is in good health (Koutrouli et al., 2025).

On the other hand, BTC-ETH correlation was positive and high in all market regimes and was largely above 0.75. This proves that diversification benefits of holding the two largest cryptocurrencies in a portfolio are minimal over the period being considered. Instead, the diversification capability of these investors is relative to traditional assets in bull markets.

Figure 4.1 presents the time evolution of these Spearman correlations. It helps to distinguish the shift in the relationships between cryptocurrencies and the S&P500 across market regimes, as quantified in **Table 4.1.**

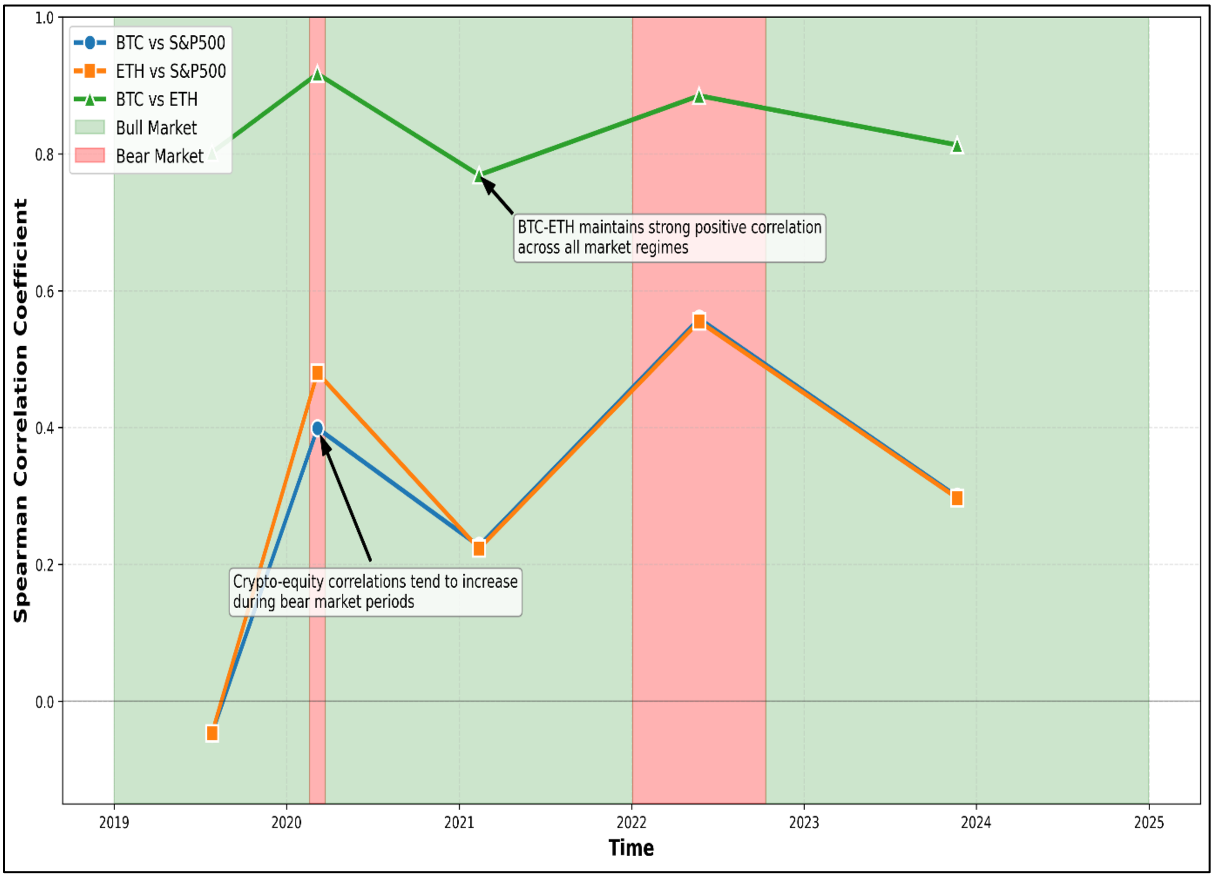

Figure 4.1

Evolution of Spearman Correlation Coefficients by Market Regime

Source: Author's own illustration using Python

4.1.1 Statistical Significance of Regime-Dependent Correlations

To provide a test of Hypothesis 1, which claims that BTC and ETF have much lower correlations with the S&P 500 in bull markets compared to bear markets, a Fisher Z test was used on the spearman correlation coefficients calculated. The Z -statistics and p -values of these comparisons are given in **Table 4.2.**

Table 4.2

Fisher's Z-Test Results for Changes in Correlation

Crypto Asset	ρ (Bull Market)	ρ (Bear Market)	Z-Statistic	p-value	Significance Level
BTC vs S&P 500	0.2004	0.5142	-5.0598	< 0.0001	0.1%
ETH vs S&P 500	0.1997	0.5195	-5.1706	< 0.0001	0.1%

Source: Author's own calculations using python

The first hypothesis is supported by the Fisher Z-test results summarized in **Table 4.2**. These indicate that in both cryptocurrency assets, the correlation with the S&P 500 was significantly smaller in bull markets than in bear markets; Z-statistics were significantly negative (e.g. -5.0598 with BTC), supporting the claim of a much weaker relationship in bull markets. This is further supported by the p-values which are all less than 0.0001. These findings have the following consequences as shown in **Figure 4.2.**

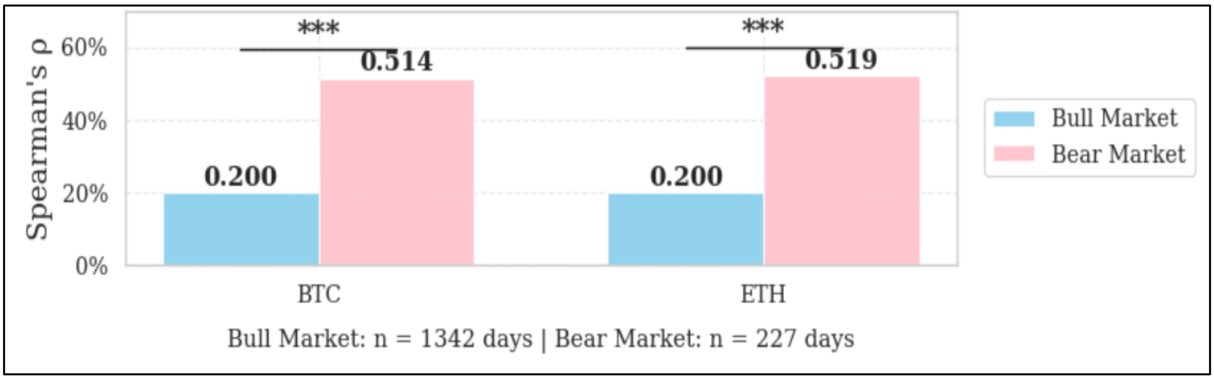

Figure 4.2

Fisher Z test results for changes in Correlations

Source: Author's own illustration using Python

Negative values that are significant at the 0.1% level ($p < 0.001$) are marked with statistical significance (asterisks) as in **Figure 4.2.** The figure supports Hypothesis I and is also the visual confirmation of the weaker cryptocurrency-equity relationships seen in bull markets.

4.2 Portfolio Performance Analysis for Pooled Bull Market Period

This section presents empirical results concerning performance of portfolios in the pooled bull market. It compares fixed-weight portfolios where crypto is held in constant amounts; and then it compares dynamically optimized portfolios (Minimum-Variance and Tangency) considering frequency of rebalancing and transaction costs.

4.2.1 Fixed weight Portfolio Performance

Fixed-weight portfolios are a type of passive investment, which keeps the proportions of BTC, ETH, and the S&P500 fixed over the given period of the bull market. Despite being cheap to manage, these portfolios are not sensitive to changing market dynamics. **Table 4.3** shows the detailed results.

Table 4.3

Fixed-Weight Portfolios (Pooled Bull Market Performance Metrics)

Portfolio	Sharpe Ratio	Sortino Ratio	CVaR 95%
100% S&P 500	1.7202	3.2743	0.0219
99% S&P 500, 1% BTC	1.7699	3.3860	0.0218
98% S&P 500, 2% BTC	1.8171	3.4928	0.0218
97% S&P 500, 3% BTC	1.8614	3.5944	0.0217
96% S&P 500, 4% BTC	1.9028	3.6904	0.0217

95% S&P 500, 5% BTC	1.9422	3.7846	0.0217
99% S&P 500, 1% ETH	1.7726	3.3903	0.0220
98% S&P 500, 2% ETH	1.8207	3.4971	0.0220
97% S&P 500, 3% ETH	1.8641	3.5940	0.0221
96% S&P 500, 4% ETH	1.9039	3.6861	0.0222
95% S&P 500, 5% ETH	1.9375	3.7647	0.0224
99% S&P 500, 0.5% BTC, 0.5% ETH	1.7715	3.3886	0.0219
98% S&P 500, 1% BTC, 1% ETH	1.8198	3.4968	0.0219
97% S&P 500, 1.5% BTC, 1.5% ETH	1.8648	3.5984	0.0219
96% S&P 500, 2% BTC, 2% ETH	1.9064	3.6930	0.0219
95% S&P 500, 2.5% BTC, 2.5% ETH	1.9442	3.7810	0.0220

Note. Author's Own Calculation using Python

As shown in **Table 4.3,** even small, fixed percentage allocations of BTC and ETH in an S&P 500 portfolio during bull markets boost risk-adjusted returns. All crypto-inclusive portfolios systematically exceeded the 100% S&P 500 baseline on all performance measures, as well as on Sharpe Ratio and Sortino Ratio. For instance, an increase in the BTC allocation by 1 percent increased Sharpe Ratio by 1.7202 to 1.7699 and Sortino Ratio by 3.2743 to 3.3860, which are tangible changes in the risk of the expected returns. The overall improvement in performance of the fixed-weight portfolios is also illustrated in **Figure 4.3.**

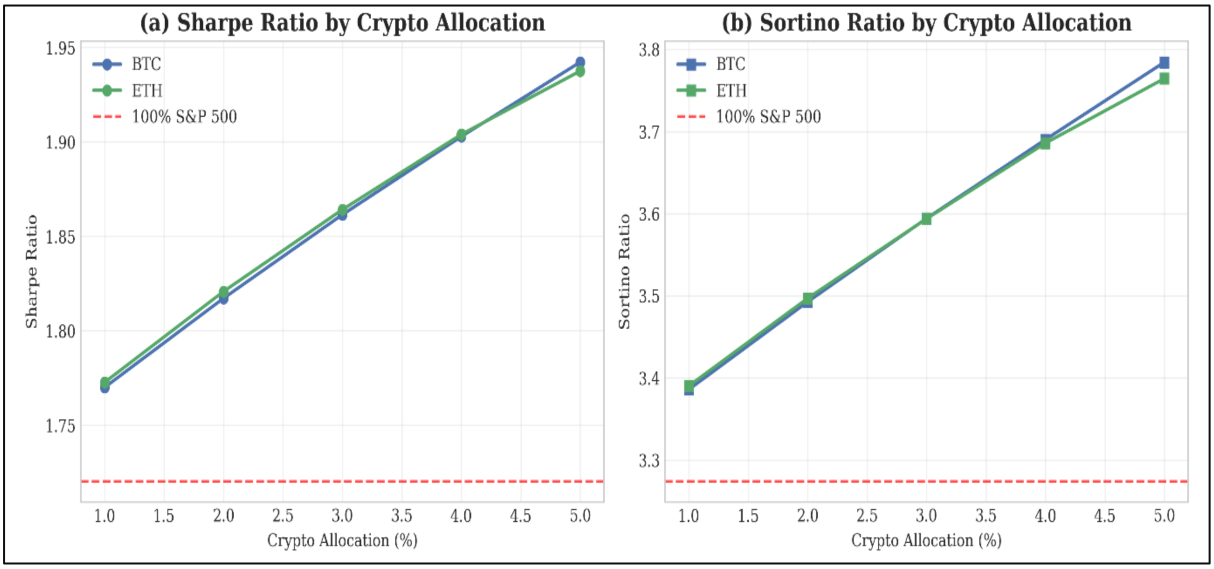

Figure 4.3

Enhancement of Risk-Adjusted Return with crypto-allocation (Pooled Bull Market Period)

Source: Author's own illustration using Python

Although both BTC and ETH were found to significantly improve risk-adjusted returns as indicated in **Table 4.3**, and **Figure 4.3**, a significant difference appeared in the effect of both on tail risk, as measured by Conditional Value-at-Risk (CVaR) at 95 percentiles. BTC-based portfolios invariably provided better CVaR than the fully invested S&P 500, evidence of a greater ability to withstand the worst downside losses in extended bull markets.

On the other hand, ETH-inclusive portfolios tended to have a small positive increment in CVaR at higher levels of allocation. This difference confirms the hypothesis that both digital assets increase the expected returns, but BTC exhibits a more consistent capacity to, in bullish market conditions, reduce extreme downside risk. The relatively larger CVaR of ETH can be explained by its greater sensitivity to volatility related to utility-based ecosystems and the increased pace at which decentralized finance models develop, as shown in **Figure 4.4.**

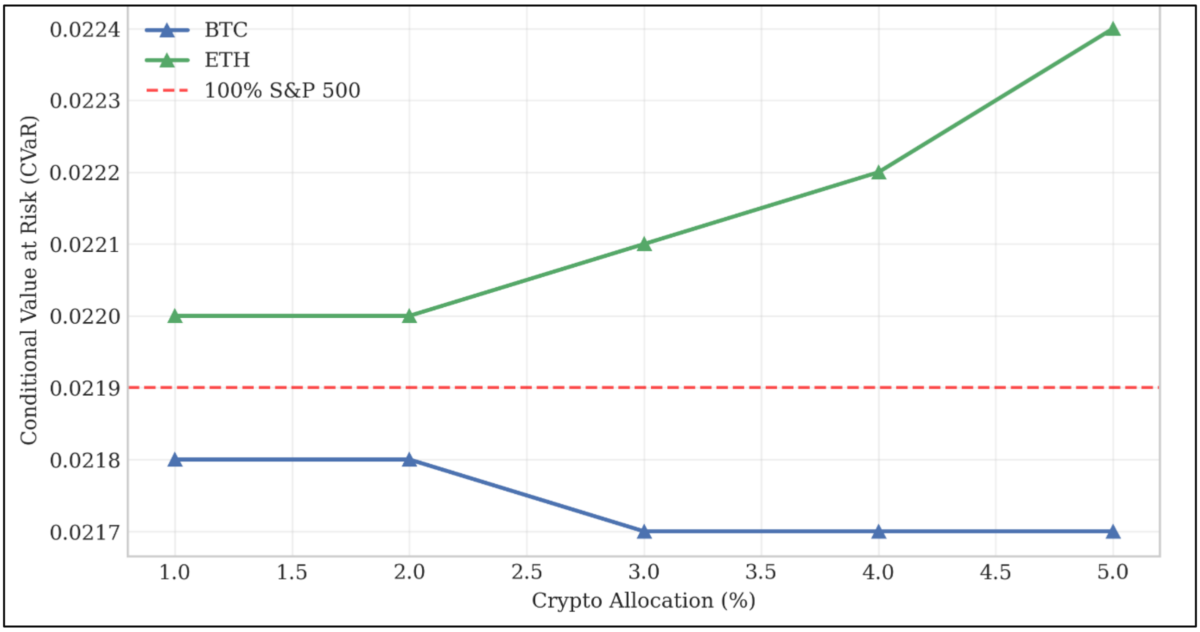

Figure 4.4

Impact on Tail Risk (CvaR 95%) with Crypto Allocation (Pooled Bull Market Period)

Source: Author's own illustration using Python

4.2.2 Optimized-Weight Portfolios Performance and the Impact of Rebalancing

In this subsection, the dynamically optimized portfolios are looked at, specifically, Minimum Variance Portfolio (MVP), and Tangency Portfolio (TP), their performance in the pooled bull market period is analyzed. It emphasizes practicality surrounding the right rebalancing frequency and its effect on the portfolio turnover. The dynamically optimized portfolios are more efficient compared with the fixed-weight counterparts relative to the 100% S&P 500 benchmark and all fixed-crypto portfolios summarized in **Table 4.3**. As shown in **Table 4.4** both MVP and TP when rebalanced monthly, achieve higher risk-adjusted returns.

Table 4.4

Optimized-Portfolio Performance Metrics (Pooled Bull Market)

Portfolio	Rebalancing Frequency	Sharpe ratio	Sortino ratio	CVaR 95%
MVP	Monthly	2.1126	4.1931	0.0223
Tangency (TP)	Monthly	2.1864	4.4194	0.0255
MVP	Quarterly	2.0047	3.9303	0.0226
Tangency (TP)	Quarterly	2.1026	4.1785	0.0255

Note: Author's own calculations.

Rebalancing monthly generates a Sharpe ratio of 2.1126 and a Sortino ratio of 4.1931 in the MVP, which validates the effectiveness of the volatility minimization approach in the adjustment process. Optimized to maximize the Sharpe ratio, the Tangency Portfolio surpasses both ratios with Sharpe ratios of 2.1864 and 4.4194, the most efficient portfolio with respect to risk and return on the frontier over the period considered.

However, there is a trade balance to the optimization of return efficiency. The highest CVaR of 0.0255 is observed in the Tangency Portfolio, which is greater than the CVaR of the S&P 500 benchmark, demonstrating that an aggressive returning maximization portfolio also distributes risk disproportionately to cryptocurrencies and thereby increases the likelihood of tail risk events. The improved risk-adjusted returns of these plans are graphically represented in **Figure 4.5.**

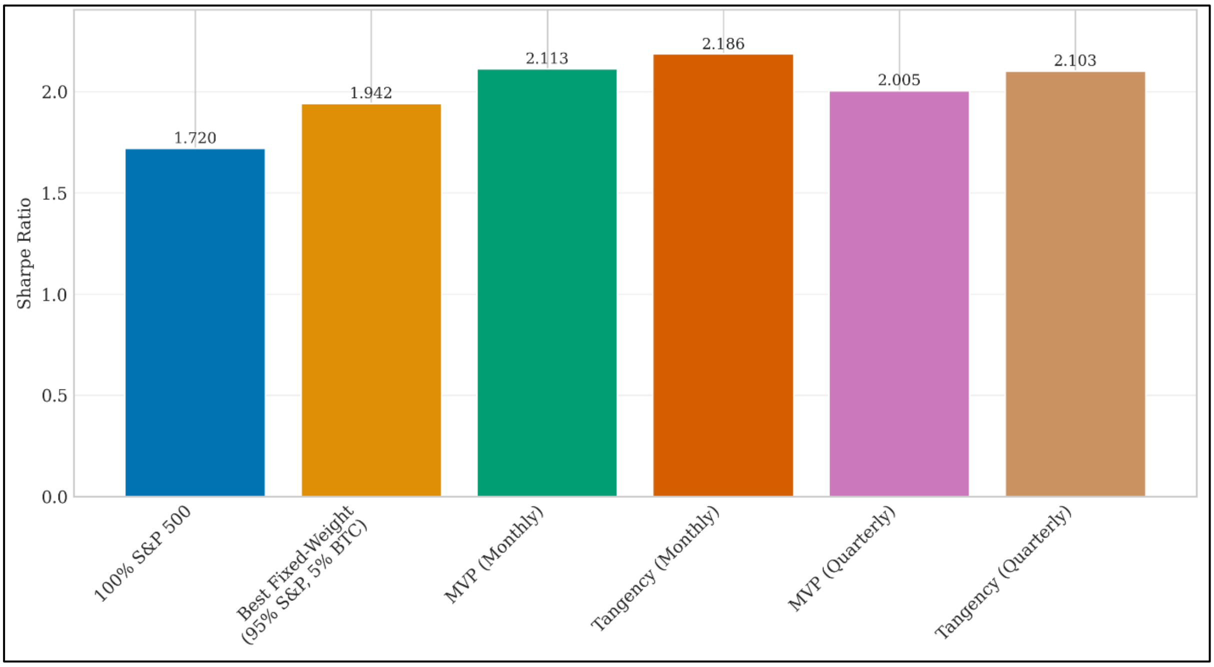

Figure 4.5

Sharpe Ratio Comparison of Optimized vs. Best Fixed weight Portfolios (Pooled Bull Market)

Source: Author's own illustration using python

Theoretical optimization of portfolios, however, should be compared with implementation costs. More often rebalancing will have transaction costs which can lead to net return erosion. Thus, the effects of monthly and quarterly rebalancing on performance and turnover of the portfolio have been analyzed.

The findings in **Tables 4.5** and **4.6** show that there is indeed a trade-off: in both portfolios, more frequent rebalancing provides a slightly better performance indicator at a significantly higher turnover cost.

Table 4.5

MVP Performance and Turnover by Rebalancing Frequency (Pooled Bull Market Period)

Rebalancing Frequency	**Sharpe ratio**	**Total Turnover**
Monthly	2.1126	31.78%
Quarterly	2.0047	14.47%

Note: Author's own calculations.

For the MVP, shifting from quarterly to monthly rebalancing improved the Sharpe ratio by approximately 5.4 percent but increased annual turnover by 119.6 percent (from 14.47 percent to 31.78 percent).

Table 4.6

Tangency Portfolio Performance and Turnover by Rebalancing Frequency (Pooled Bull Market Period)

Rebalancing Frequency	**Sharpe ratio**	**Total Turnover**
Monthly	2.1864	107.33%
Quarterly	2.1026	62.66%

Note: Author's own calculations.

The trade-off is more pronounced for the return-seeking Tangency Portfolio. The highest Sharpe ratio of 2.1864 was achieved with monthly rebalancing. However, the 4 percent improvement from the quarterly strategy is offset by a 71.3 percent increase in turnover (107.33 percent vs. 62.66 percent) as illustrated in **Figure 4.6.**

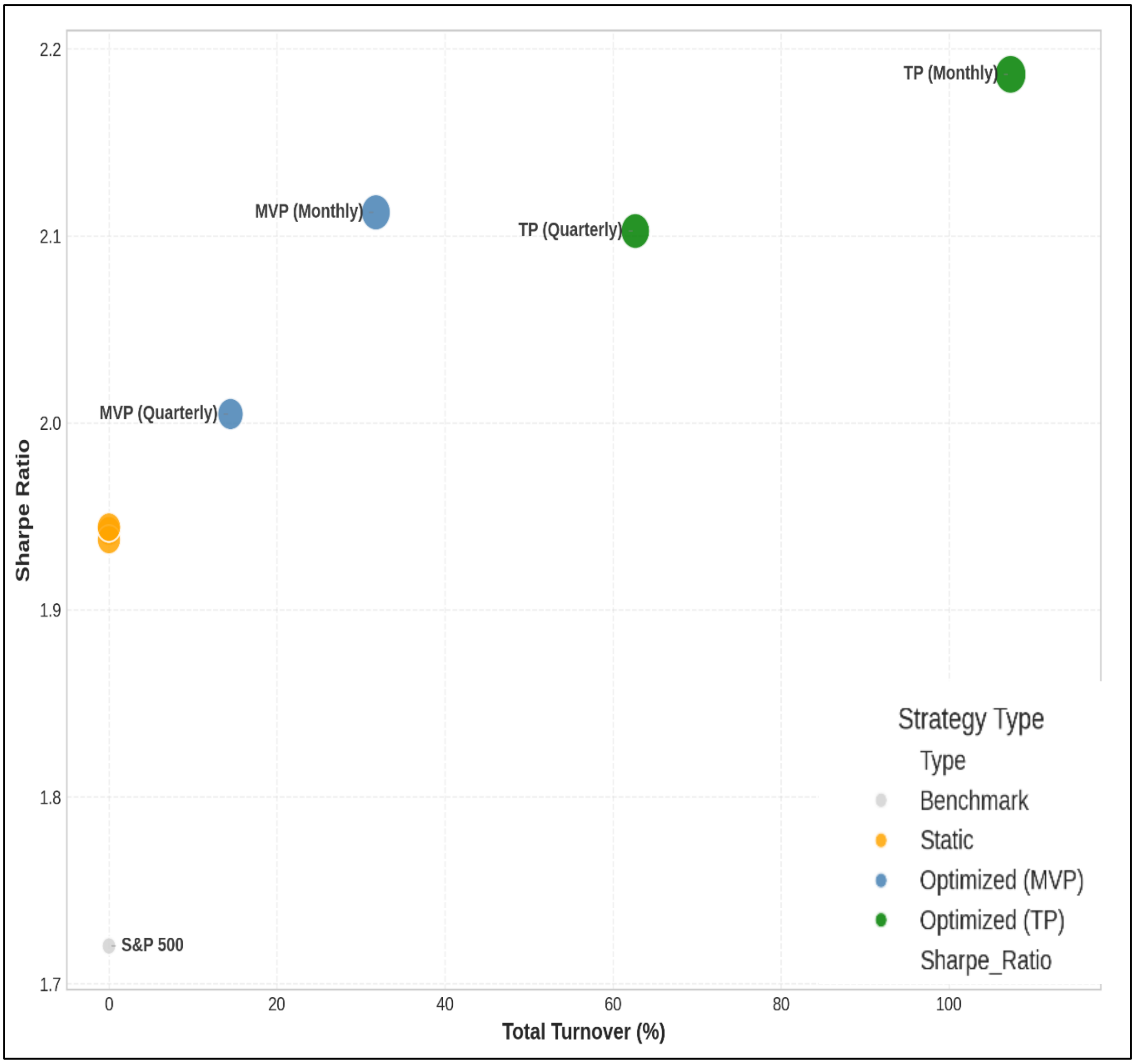

Figure 4.6

Trade-Off Between Risk-Adjusted Return (Sharpe) and Portfolio Turnover

Note: Author's Own Illustration using Python

Figure 4.6 provides several insights. The quarterly-rebalanced portfolios persistently outperform all static portfolios, and it demonstrates that the key advantage of dynamic optimization does not rely on hyper-frequent, high-cost trading.

The frequency of rebalancing is a trade-off between the costs and benefits that must be determined. For institutional investors with lower transaction costs, the marginal benefits of monthly rebalancing could be more enticing.

On the other hand, retail investors might gravitate towards the quarterly strategy, which is more economical and retains a significant portion of optimization benefits. Fixed-weight strategies for crypto allocation will eliminate the costs associated with rebalancing; however, these strategies forego the dynamic risk-return trade-off achieved through dynamic optimization.

In summary, strategically incorporating cryptocurrencies within a dynamically optimized portfolio framed during bull markets enhances robust diversification, reinforcing the argument for dynamic portfolio optimization even after accounting for turnover-related transaction costs.

4.3 Subperiod Analysis: Consistency and Dynamic Shifts

Although the pooled analysis revealed average benefits of including cryptocurrencies in the portfolios identified in Section 4.2.1 and 4.2.2, this section examines whether these benefits are maintained at three distinct stages of a bull market. Since such phases were discussed in Section 3.2, one can examine whether the benefits of portfolio diversification of BTC and ETH are consistent across different market conditions.

4.3.1 Fixed weight Portfolio Performance

The Sharpe Ratio is the main performance measure in this subperiod analysis as it would indicate risk-adjusted performance under the conventional MPT model. **Table 4.7** gives the results of fixed-weight portfolios.

Table 4.7

Summary of Sharpe Ratios Across Individual Bull Market Phases

Portfolio	**Phase 1 (01/01/2019 - 19/02/2020)**	**Phase 2 (23/03/2020 - 03/01/2022)**	**Phase 3 (12/10/2022 - 31/12/2024)**
100% S&P 500	2.0099	2.1992	1.2629
95% S&P 500, 5% BTC	2.4116	2.4065	1.4446
95% S&P 500, 5% ETH	2.2138	2.5646	1.3358

Source: Author's own calculations using Python

Note. A 5% allocation is shown as a representative example due to its consistent outperformance in the pooled analysis.

The most prominent finding in **Table 4.7** is that diversion benefit is enormous and consistent in three individual bull-markets. A 5 percent allocation of BTC or ETF increased the Sharpe Ratio of the broad portfolio in all periods relative to a 100 percent S&P 500 baseline. Surprisingly, under Phase 3, which was characterized by a relatively low baseline S&P 500 Sharpe Ratio of 1.2629, the portfolio that included 5 per cent BTC achieved a Sharpe Ratio of 1.4446, and the identical portfolio that included 5 per cent ETH achieved a Sharpe Ratio of 1.3358.

These findings suggest that the capability of cryptocurrencies to increase portfolio efficiency is not a temporal or period-based phenomenon but is an enduring phenomenon in bull markets. The dynamics of this behaviour are explained in Figure **4.7** below, and it shows the Sharpe Ratios of crypto-inclusive portfolios in subperiods of bull markets.

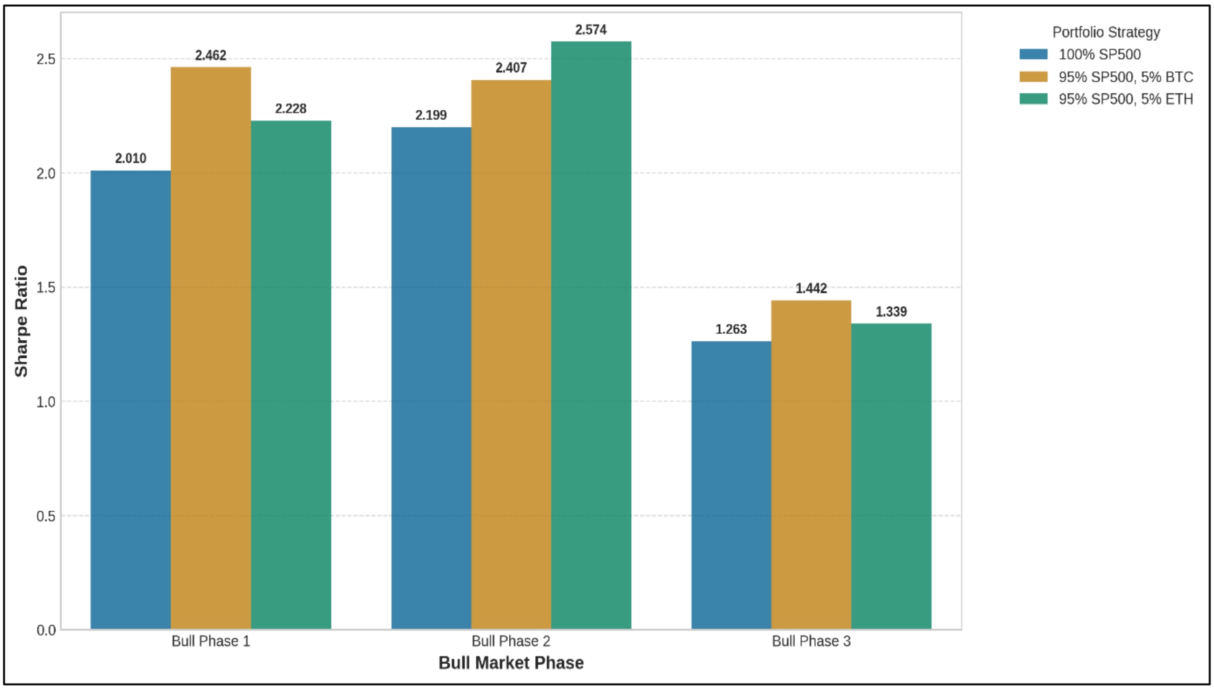

Figure 4.7

Sharpe Ratios of Crypto-Inclusive Portfolios Across Bull Market Phases

Source: Author's own illustration using Python

The total diversification advantage of adding a cryptocurrency to a portfolio has been consistent, but the magnitude of the benefit that BTC offers compared to ETH has exhibited regime-specific fluctuations. This kind of behavior helps to highlight the essence of leadership in the cryptocurrency market. These regime changes are shown graphically in **figure 4.7** which plots Crypto-Inclusive Portfolios Sharpe Ratios during different bull market periods. In the first Bull Phase, BTC has demonstrated the highest performance with a significantly greater Sharpe Ratio at the constant 5 percentage weight than ETH. Bull Phase two was the opposite where ETH replaced BTC as the ideal diversifier; a 5 percent position in ETH delivered a Sharpe Ratio of 2.5646, more than twice that of BTC (2.4065). Seeing Bull Phase 3, the leadership changed to BTC once more, and the highest ratio was regained with the same allocation. These repetitive superiority changes between the phases of bull support the idea that the most efficient cryptocurrency to diversify with is different. The investment professionals are therefore advised not to consider past performance in evaluating relative market environment and asset-specific developments.

4.3.2 Optimized-Weight Portfolios Performance and the Impact of Rebalancing

This section discusses the behavior of dynamic allocations applied to MVP and TP. In contrast to the constant-weight portfolios, MVP and TP are governed by a dynamic rebalancing mechanism. The results are presented in **Table 4.8.**

Table 4.8

Representative Dynamic Portfolio Allocations based on Monthly Rebalancing

Portfolio	**Bull Market Phase**	**S&P 500 Allocation**	**BTC Allocation**	**ETH Allocation**
MVP	Phase 1 (2019-2020)	~90%	~10%	0% - 5%
	Phase 2 (2020-2022)	~90%	~10%	0% -1%
	Phase 3 (2022-2024)	~90%	5~10%	0% - 5%
Tangency	Phase 1 (2019-2020)	85% - 90%	5% - 10%	0% - 5%
	Phase 2 (2020-2022)	80% - 90%	0% - 10%	~10%
	Phase 3 (2022-2024)	80% - 90%	5% - 10%	0% - 10%

Source: Author's own calculation using Python

Note: This table compiles the empirical allocation profiles derived from MVP and Tangency portfolios throughout each discrete bull market phase.

In Phase 1, the MVP and TP were skewed towards the S&P 500 and BTC, and ETH was allotted very little. That explains why BTC is the most dominant diversifier in the initial stages of the bull market. In Phase 2, the MVP continued to hold about 90 percent S&P 500, 10 percent BTC, and ETH was negligible. Conversely, the Tangency Portfolio tended to bump the 10 % limit on ETH, and it was easily able to reach the limit and change BTC allocation between 0 10 percent and decreasing equity exposure. Here is the evidence of the optimizer trying to make ETH perform better because the setting at this stage was the high-growth and technology-driven one.

In Phase 3, both portfolios reverted to BTC focus. The MVP had a BTC exposure of 5 to 10 percent, as well as ETH exposure closely limited at 0 to 5 percentage. The Tangency Portfolio is, in contrast, flexibly invested in BTC and ETH (0-10% each) and 80-90% equities. These distributions illustrate stabilizing markets and the re-emergence of an institutional interest in BTC over the latest bull cycle, without neglecting the diversification ability of ETH.

4.4 Statistical Significance of Performance Enhancement: Bootstrap Test

This section describes the differences in performance as identified through the bootstrapping methodology described in Chapter 3. The performance of a 5 % cryptocurrency allocation is shown in **Table 4.9.** These findings are a very strong indicator that strategically positioned cryptocurrency can significantly enhance the performance of a portfolio in the context of a shared bull market.

Table 4.9

Bootstrapping Results for Performance Metric Differences (5% Allocation)

Cryptocurrency	Metric	Observed Difference	90% CI (One-Tailed)	p-value	Significant?
BTC (BTC)	Sharpe Ratio	0.221	0.108	0.0048	Yes

	Sortino Ratio	0.505	0.234	0.0068	Yes
	CVaR 95%	-0.000145	-0.001	0.6636	No
ETH (ETH)	Sharpe Ratio	0.229	0.1001	0.01	Yes
	Sortino Ratio	0.514	0.2237	0.0112	Yes
	CVaR 95%	0.000469	0.00008	0.0965	Yes

Note: For each one-tailed test, the null hypothesis (H_0) posed is that the difference in metrics is less than or equal to zero. The observed positive difference indicates the crypto-inclusive portfolio outperformed the S&P 500 benchmark. Significance is evaluated at the 10% level ($\alpha = 0.10$).

The outcome of the bootstrapping analysis provides two valuable results with respect to the effect of cryptocurrency assets on the performance of the portfolio in bull markets.

4.4.1 Improvement of Risk-Adjusted Returns

Sharpe and Sortino ratios were found to be elevated in BTC and ETH holders. In each of the ratios, the 90 percent confidence interval is greater than zero and the p value of all ratios was less than 0.012. This is a support of the alternative hypothesis of improvement. Therefore, the risk-adjusted returns of cryptocurrency-mix portfolios were much higher, and crypto allocations were not too different between portfolios with and without crypto assets.

4.4.2 Difference in the Influence of Tail Risk

The findings also demonstrate the variations of the BTC and ETH effects on portfolio tail risk, CVaR. The slight decline in the portfolio CVaR in the case of BTC indicates that BTC-inclusive portfolios have a lower tail risk. Nevertheless, this effect is not important because p=0.664, corresponding to a 90 percent interval including zero obviously. Therefore, the evidence indicates that BTC has no effect on tail risk of the portfolio. Conversely, it seems that ETH allocations had a substantial effect on CVaR, reaching the 10 % mark ($p = 0.097$). The lower limit of the 90 percent confidence interval represents a positive value of +0.000008, which indicates that the increase in the anticipated growth to extreme losses would not be random. This contrast between BTC and ETH shows that, despite any potential application of both cryptocurrencies to enhance risk-adjusted returns, the effect of ETH is much stronger because it amplifies extreme downside risk in bull markets. Conversely, tail risk in BTC can and does enhance returns without significantly increasing tail risk, highlighting its role as a tail-risk-diversifying investment in such situations.

Chapter Five: Discussion & Conclusion

This chapter discusses the research results and how they relate to the existing literature regarding the diversification of cryptocurrency investments. It starts with a summary of the findings and discusses them elaborately while juxtaposing with the existing literature and highlighting the contributions this research makes. It then shifts to the practical and theoretical implications, critiquing the work and suggesting possible avenues for further research.

5.1 Summary of Key Findings

In order to address the methodological shortcomings of earlier studies on the subject, the current study will explore how BTC and ETH could be used in diversifying a benchmark S&P portfolio. There are two main findings of the results and analysis. To begin with, according to the Z -tests developed by Fisher, the consistency of both BTC and ETH with the S&P 500 Index, in bull markets, is significantly lower than bear markets. Second, strategic BTC or ETH allocations of 1 to 5 percent in either (or both) exhibit significant improvements in portfolio risk-adjusted returns (Sharpe ratio, Sortino ratio) in bull markets. An important difference is noted, in regard to tail risk: though no impact on Conditional Value-at-Risk (CVaR) is observed in BTC allocations, a small increase in CVaR is observed in ETH allocations, especially at the high allocation levels. Interestingly, the dynamically optimized portfolios are always superior to the fixed weight portfolios; including the rebalancing transaction costs, the dynamically optimized portfolios still have a better performance.

5.2 Discussion of Findings & Comparison with Literature

This section discusses the empirical results of the work in context of the wider discussion on the diversification of cryptocurrencies. It is discussed in three thematic sub-sections.

5.2.1 The End of "Regime Blindness": Addressing the Correlation Debate

The key value of the present work is its attempt to balance the conflicting evidence about the advantages of cryptocurrencies diversification, evidence that has been largely the result of an absence of market-regime sensitivity in previous studies. The existence of regime-dependent correlations—specifically, weaker cryptocurrency-equity correlations in bull markets and stronger correlations in bear markets—addresses this debate directly.

The results that we obtain in bull markets resonate with the initial optimistic sentiments of some studies regarding the low cryptocurrency-equity correlations. For instance, (Brière et al., 2015) emphasized the low correlation of BTC to any other asset class. It shows there are considerable diversification gains to be had in relation to the portfolio of BTC (Platanakis & Urquhart, 2019). It was also found that the correlation between cryptocurrencies and traditional financial assets was often negligible. These studies align with our findings within market upturn phases. (Aslanidis et al., 2019).

However, our results during bear markets tell a different story, aligning with the more cautious and recent findings. BTC was singled out as a “risky hazard” during the bear market triggered by COVID-19, which showed rising correlations with equities(Conlon & McGee, 2020). Watorek et al., (2022) strengthened this position by observing that, during periods of market turmoil, cryptocurrencies lost their safe haven status and became highly cross-correlated with traditional markets, especially since late 2021. Goodell & Goutte, (2021b) also concluded that BTC and ETH were poor diversifiers for equity during the COVID-19 period.

These contradictions in literature are the gaps that our study seeks to address by systematically analyzing market regimes. We determine that the two strands of research are indeed right, but only for certain market conditions. Cryptocurrencies, like our conclusive research tests, were fair diversifiers. Moreover, the ability of cryptocurrencies to enhance portfolio diversification is far more pronounced during bull markets, and this tendency wanes or disappears in bear markets. This underscores the need for a sensitive, regime-centric analytical framework.

5.2.2 Beyond Bitcoin: The Ethereum

Previous cryptocurrency-diversification literature has mostly concentrated on BTC and ETH remains relatively under-researched. This work fills that gap by using comparative analysis to determine both similarities and major differences between BTC and ETH as portfolio diversifiers.

As with BTC, ETH enhances the risk-adjusted returns of U.S. equity portfolios in bull market environments, which is why ETH-oriented research, including Ma et al. (2020) who found that ETH has a greater ability at enhancing diversification benefits compared to BTC, and Jiang et al. (2020) who found that ETH is an effective diversifier in the short run, is warranted. But another tail-risk analysis of CVaR showed a critical difference; BTC allocations do not have any effect on CVaR when measured through bootstrapping, whereas ETF allocations, especially at higher percentages, have a small positive effect on CVaR. This evidence indicates that the decision between BTC and ETH is not about the augmentation of returns per se, but rather a strategic choice that depends on the tail-risk profile of an investor. The change in the simplistic paradigm of BTC versus ETH to a more complex view of the risks involved in each case (particularly in tail events) is a qualitative change in the literature.

5.2.3 The Theoretical Contribution: Validating the PMPT Lens

This paper is a considerable theoretical contribution as it shows that PMPT measures are needed to conduct a thorough analysis of cryptocurrency portfolios. The Sharpe Ratio was used mainly in previous studies in the MPT framework. In comparison, PMPT measures like the Sortino Ratio and CVaR which are used in this case highlight the greater flexibility of PMPT, compared with MPT. According to Brauneis and Mestel (2018), the standard Markowitz mean-variance framework can be used to make improvements in crypto-inclusive portfolios, but improvements in Sharpe Ratio can be observed in general, PMPT metrics represent risk in a more detailed way. The Sortino Ratio which rewards only downside risk is more appropriate as a risk-adjusted performance metric to reflect the skewed and leptokurtic distribution of cryptocurrency returns. More to the point, a CVaR analysis would indicate separate tail-risk exposures to both BTC and ETH, which would otherwise be lost in the total volatility. As stated by Petukhina and Sprünken (2021), CVaR optimization has shown better results than normal benchmarks, thus making it significant in cryptocurrency portfolios. Similarly, Tenkam et al. (2022) support the adoption of CVaR and other advanced downside risk metrics as a portfolio optimisation tool in cryptocurrency. To this end, we support the use of PMPT metrics, namely, CVaR, in any future study involving cryptocurrency portfolios. Relying solely on MPT would be too simplistic and would provide a false image of risk compared to the extreme loss potential of these highly volatile digital assets.

5.3 Practical Implications

Investors with a goal to improve the performance of U. S. equity portfolios, specifically those tracking the S & P 500 throughout bull market cycles, can be recommended to allocate small tactical allocations (1-5-percent) to cryptocurrencies. BTC specifically seems to suit well an augmentation of returns without significantly raising tail risk and therefore would seem appropriate across a wide range of risk preferences. ETH also produces significant returns improvements but is better positioned to investors who take a more aggressive approach to tail risk. Investors must embrace a market-regime model and be ready to defend ETH and BTC as a fair-weather diversifier compared to equity portfolios. The analysis is biased towards dynamically revised portfolio plan to achieve optimum effectiveness, which is superior to non-dynamically revised weightings even where rebalancing is done on a quarterly basis. This also means that active optimisation strategies are still viable despite the dynamics of transactions and generate better risk-return results. Portfolio managers are advised to use strategic and dynamic regime-sensitive strategies when investing in cryptocurrencies. The results also show that the co-movement of cryptocurrencies with traditional finance is stronger in times of market stress, highlighting the necessity of regulation to better capture complex systemic risks and nonlinear market behaviour instead of dismissing these instruments as marginal to core finance.

5.4 Limitations and Future Research

Although this study is informative and incisive, there are several limitations to this study that can be subject to future inquiry. To begin with, the analysis is limited to BTC, ETH, and the S&P 500 only, thus excluding other cryptocurrencies (e.g., altcoins), as well as other traditional assets, such as bonds, commodities, and real estate, and international equity markets; therefore, the generalisability of the findings is limited. Further studies should expand the asset base and have more global vision. Second, the analysis addresses the range of 2019-2024, which, despite including many market cycles and growing institutional involvement in crypto (e.g., ETF approvals), might not represent other epochs or longer historical intervals. A longer span of time would enable to investigate tail-end behaviours, stability of correlation, and regime-specific dynamics over different periods of the market. Thirdly, the market segmentation used a 20 per cent heuristic definition of a threshold rule. Other segmentation approaches, like disaggregated volatility thresholds, economic fundamental thresholds, or machine-learned regime thresholds, would change regime classification and hence the relationships of interest. The stability of results to other regime-classification techniques should be tested in subsequent work. Fourthly, the transaction cost model used fixed set of costs. A more accurate understanding of the market impact, liquidity considerations, and varying exchange fees may be incorporated in the insights given. The maximum exposure to crypto assets in the study is set to 10 per cent; other exposure limits or optimised portfolio construction methods may be explored. With these constraints in mind, a future study ought to expand the regime-based model into a broader, multi-asset portfolio comprising not only bonds and gold but also alternative assets such as cryptocurrencies, in order to evaluate the maximum diversification advantages offered by cryptocurrencies. To identify more risk-reward profiles, it would also be interesting to consider a broader range of altcoins, not limited to Ether. In addition, more detailed and prompt allocation policies may be provided by using more sophisticated time-series models and machine-learning tools to predict dynamic regimes and optimise a portfolio. Lastly, out of sample forecasting would be used to assess the relevance and strength of the best allocation models in different market conditions.

5.5 Conclusion

To sum up this study, it has established regime sensitive evidence that confirms, if only partially, that BTC and ETH serve as viable assets for diversification within portfolios that include U.S. equities, but only during specific bullish conditions of the market. As the blindness that comes with relying on regimes, and the lack of a unified theory approach, is a common issue within the literature, this study seeks to make the contradictory notions within the literature more precise and provide relevant and actionable portfolio approach for the growing digital assets market. Our study also corroborates that while both BTC and ETH improve the risk and return ratio, BTC has a more attractive profile for the bullish tail risk. In a nutshell, BTC and ETH do not seem to have replaced the more conventional and accepted safe haven assets, but rather, seem to have developed a more nuanced and conditional position as diversifying assets for those investors that want to boost portfolio return during bullish periods using aggressive and smart portfolio management.

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