In [1]:
```
%%javascript
IPython.OutputArea.prototype._should_scroll = function(lines) {
    return false;
}
```

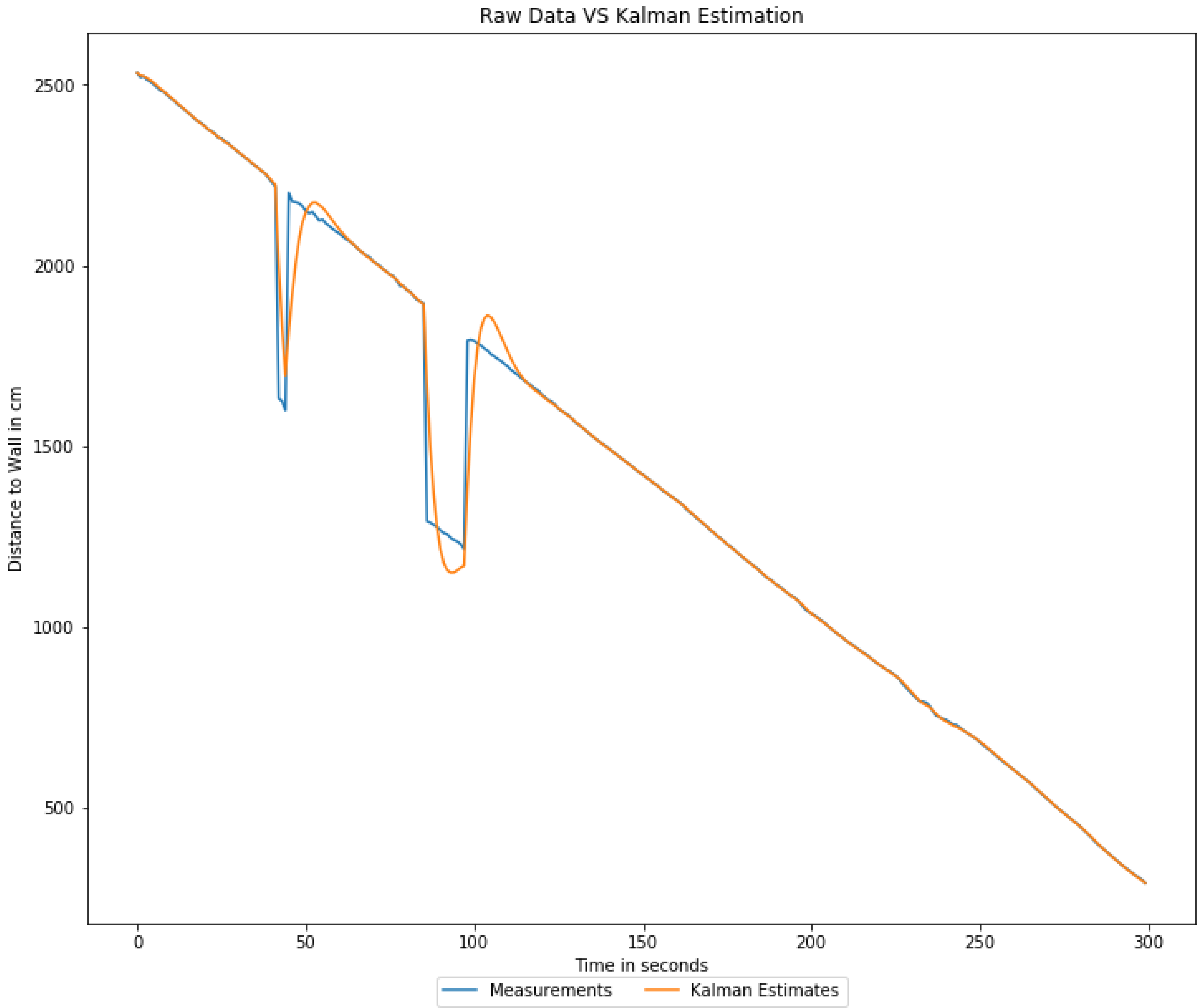
In [2]:
```
import matplotlib.pyplot as plt
import numpy as np

dt = 0.1

def draw_plot(measurements, mlabel=None, estimates=None, estlabel=None, title=None, xlabel=None, ylabel=None
):
    xvals = np.linspace(0, dt * len(measurements), len(measurements))
    plt.title(title, fontsize=12)

    xlabel and plt.xlabel("Time in seconds")
    ylabel and plt.ylabel("Distance to Wall in cm")

    ax = plt.subplot(111)

    ax.plot(measurements, label=mlabel)
    np.any(estimates) and estlabel and ax.plot(estimates, label=estlabel)

    box = ax.get_position()
    ax.set_position([box.x0, box.y0 + box.height * 0.1,
                     box.width, box.height * 1.1])

    ax.legend(loc='upper center', bbox_to_anchor=(.5, -0.05), ncol=3)
    plt.show()

def add_noise(data, size=20):
    noise = np.random.uniform(-1, 1, len(data)) * size
    return data + noise
```

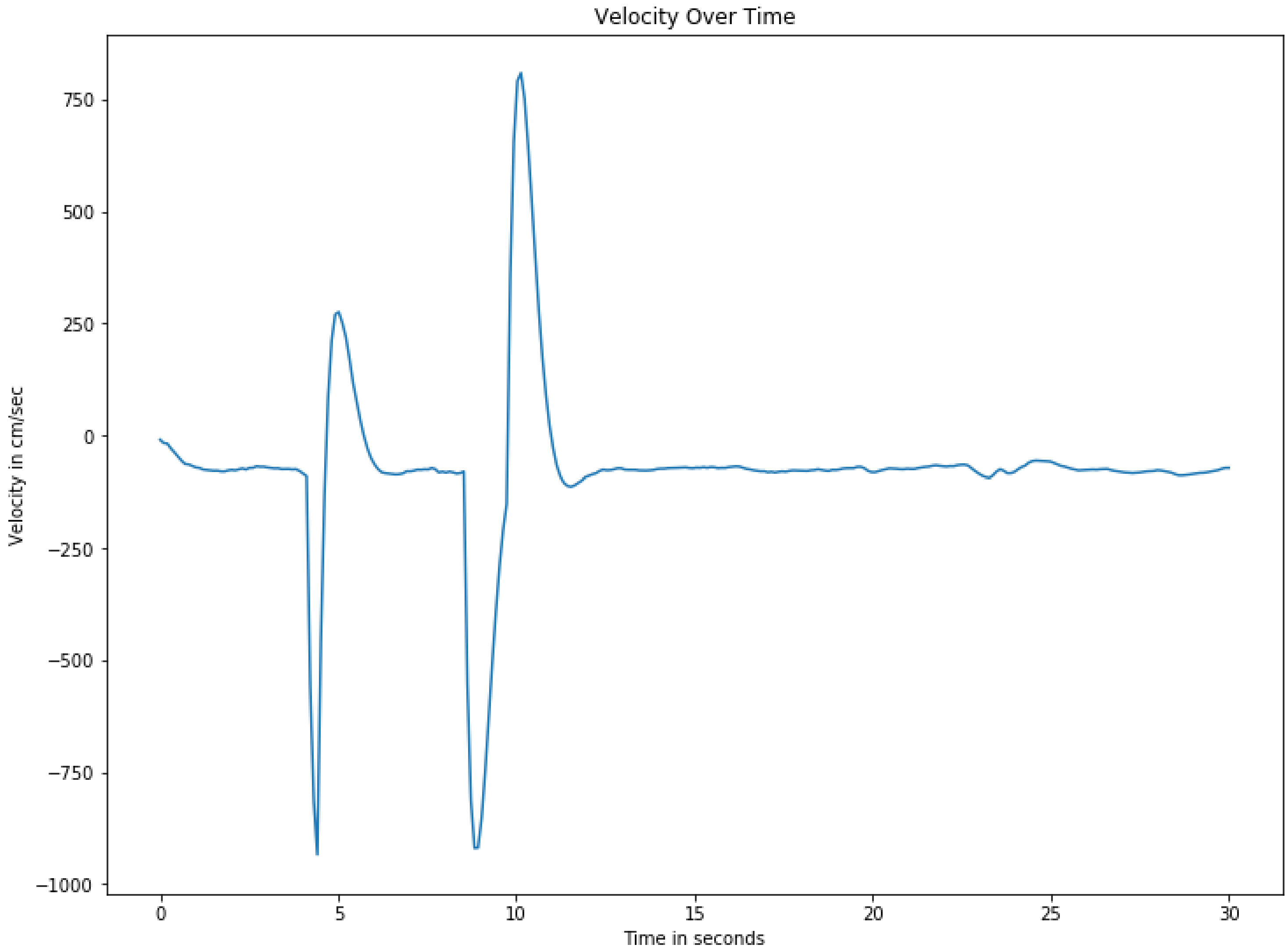
Part A

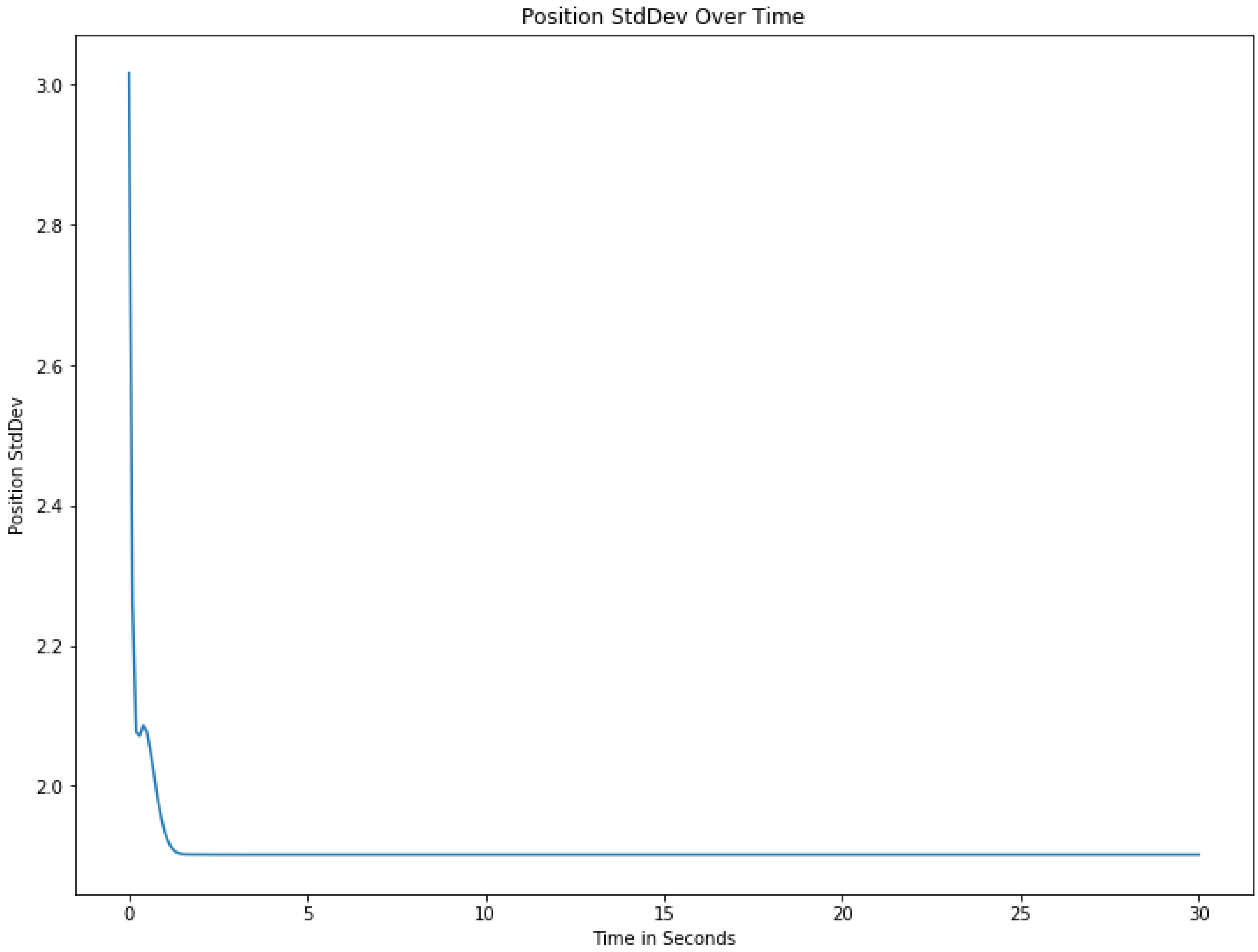
In [4]:
```
from kalman import predict, update, dot3
```

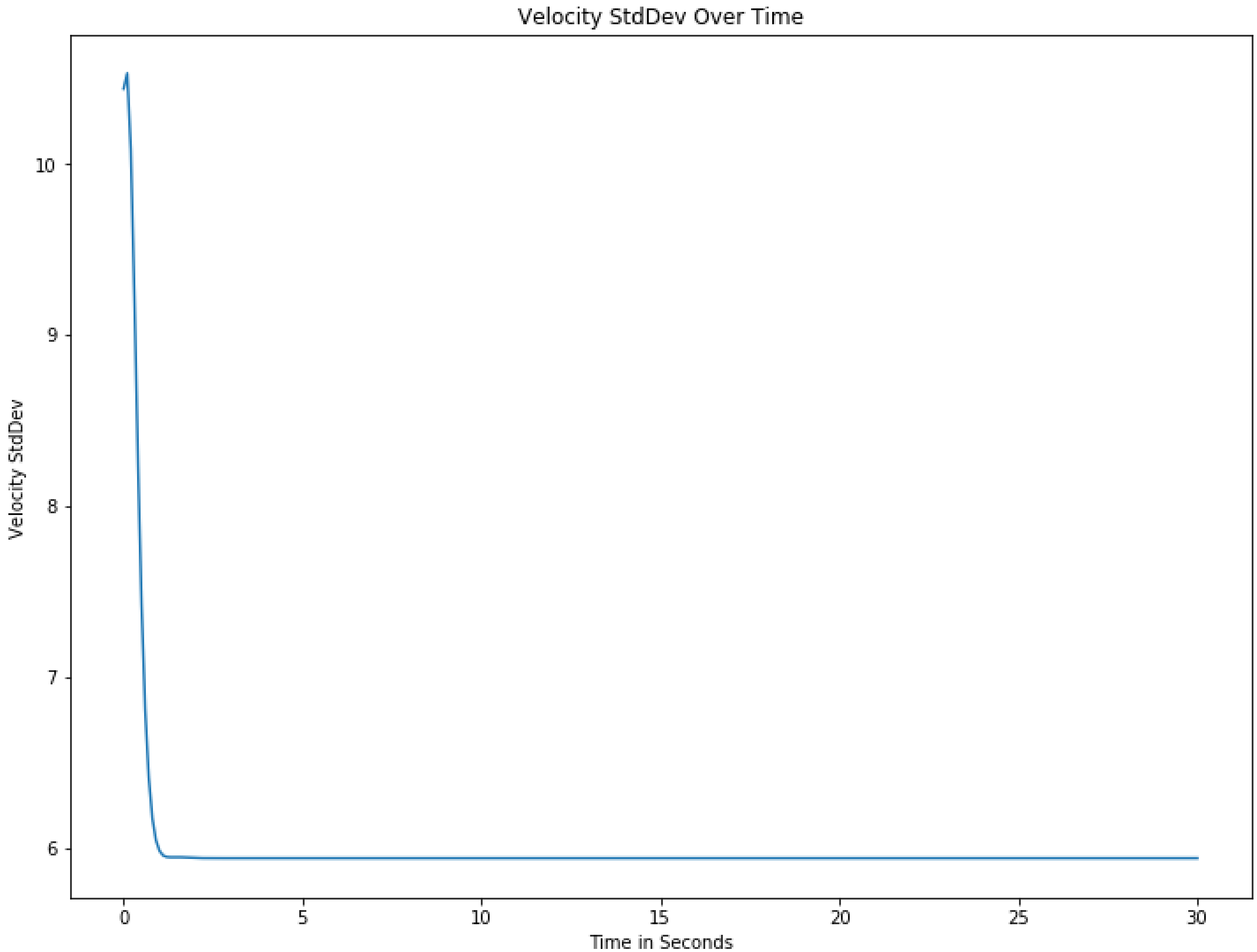
```
plt.rcParams["figure.figsize"] = [12, 9]

wall_file = "RBE500-F17-100ms-Constant-Vel.csv"

data = add_noise(np.loadtxt(wall_file, delimiter=","), 0)

# Setup initial variables and matrices
initial_pos = 2530
velocity = -10.0
variance = 10.0
dt = 0.1
F = np.array([[1., dt],
              [0., 1.]])
P = np.array([[100., 0],
              [0, 100.0]])
H = np.array([[1., 0.]])
R = np.array([[variance]])
Q = np.array([[0.1, 1], [1, 10.]])
x = np.array([initial_pos, velocity]).T

predicted_xs = []

def run(x, P, R, Q, dt, zs):
    # run the kalman filter and store the results
    xs, cov = [], []
    for z in zs:
        x, P = predict(x, P, F, Q)
        x, P = update(x, P, z, R, H)
        xs.append(x)
        cov.append(P)

    xs, cov = np.array(xs), np.array(cov)
    return xs, cov

time_values = np.linspace(0, dt * len(data), len(data))

est_x, est_P = run(x, P, R, Q, dt, data)
est_pos = [v[0] for v in est_x]
est_vel = [v[1] for v in est_x]
draw_plot(data,
          estimates=est_pos,
          title="Raw Data VS Kalman Estimation",
```

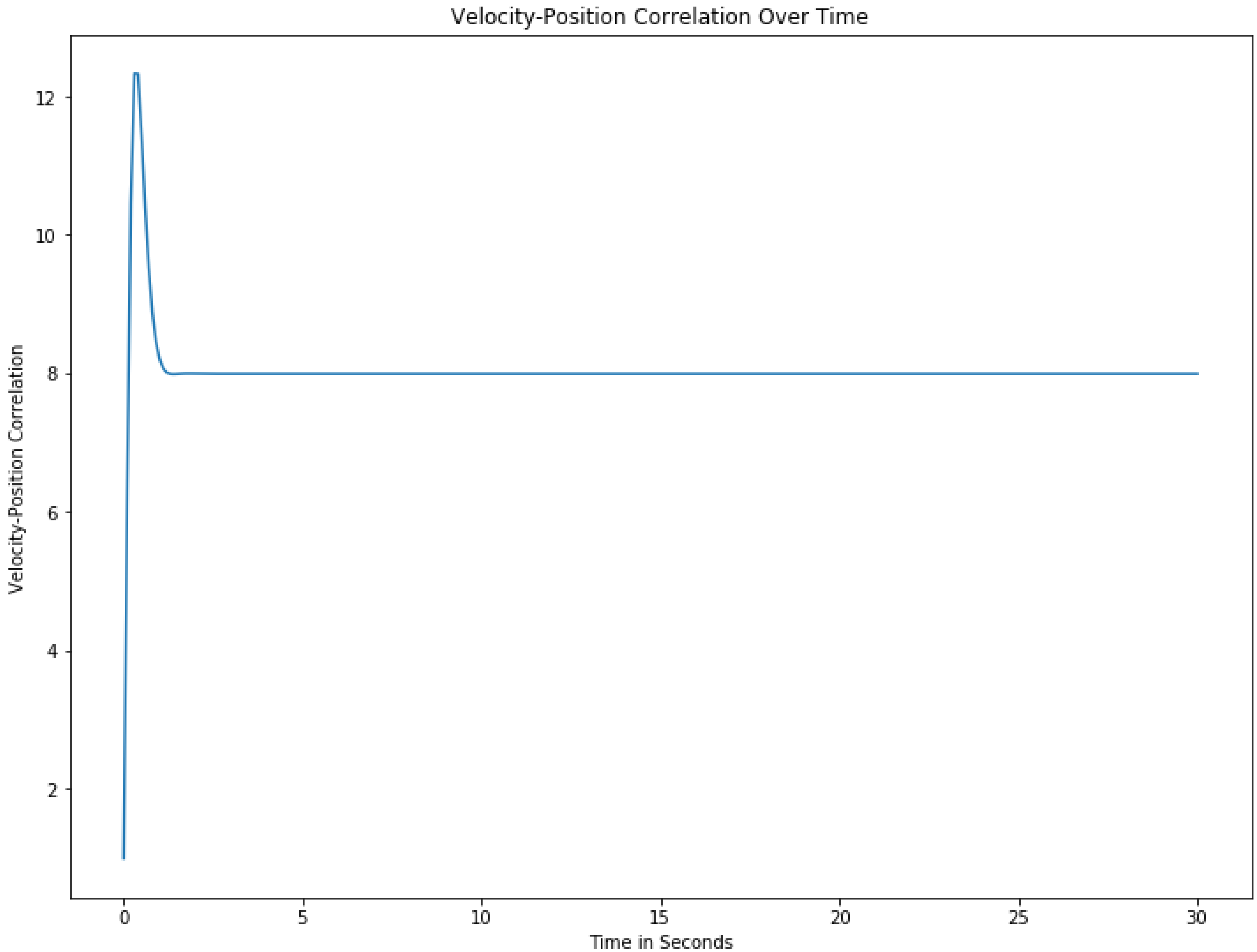
```
          mlabel="Measurements",
          estlabel="Kalman Estimates",
          xlabel="Time in Seconds",
          ylabel="Distance to Wall in CM")

plt.plot(time_values, est_vel)
plt.xlabel("Time in seconds")
plt.ylabel("Velocity in cm/sec")
plt.title("Velocity Over Time")
plt.show()

pos_std = [p[0][0]**0.5 for p in est_P]
vel_var = [p[1][1]**0.5 for p in est_P]
pos_vel_corr = [p[0][1] for p in est_P]
plt.plot(time_values, pos_std)
plt.ylabel("Position StdDev")
plt.xlabel("Time in Seconds")
plt.title("Position StdDev Over Time")
plt.show()

plt.plot(time_values, vel_var)
plt.ylabel("Velocity StdDev")
plt.xlabel("Time in Seconds")
plt.title("Velocity StdDev Over Time")
plt.show()

plt.plot(time_values, pos_vel_corr)
plt.ylabel("Velocity-Position Correlation")
plt.xlabel("Time in Seconds")
plt.title("Velocity-Position Correlation Over Time")
plt.show()
```


Raw Data VS Kalman Estimation
Distance to Wall in cm
2500
2000
1500
1000
500
0
50
100
150
200
250
300
Time in seconds
Measurements
Kalman Estimates


Velocity Over Time
Velocity in cm/sec
Time in seconds
750
500
250
0
−250
−500
−750
−1000
0
5
10
15
20
25
30


Position StdDev Over Time
Position StdDev
3.0
2.8
2.6
2.4
2.2
2.0
0
5
10
15
20
25
30
Time in Seconds


Velocity StdDev Over Time
Velocity StdDev
10
9
8
7
6
0
5
10
15
20
25
30
Time in Seconds

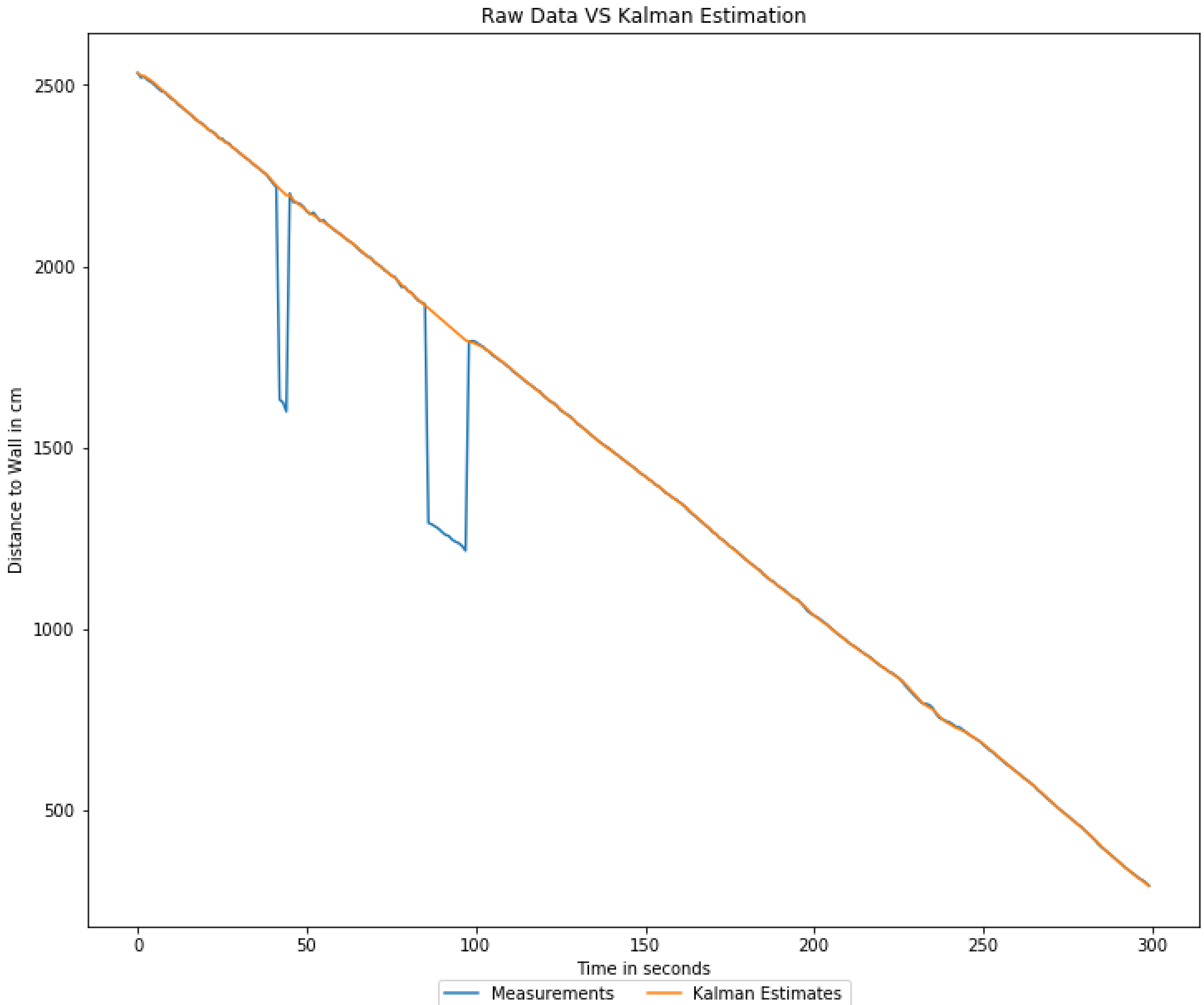

Velocity-Position Correlation Over Time
Velocity-Position Correlation
Time in Seconds
12
10
8
6
4
2
0
5
10
15
20
25
30

Part B

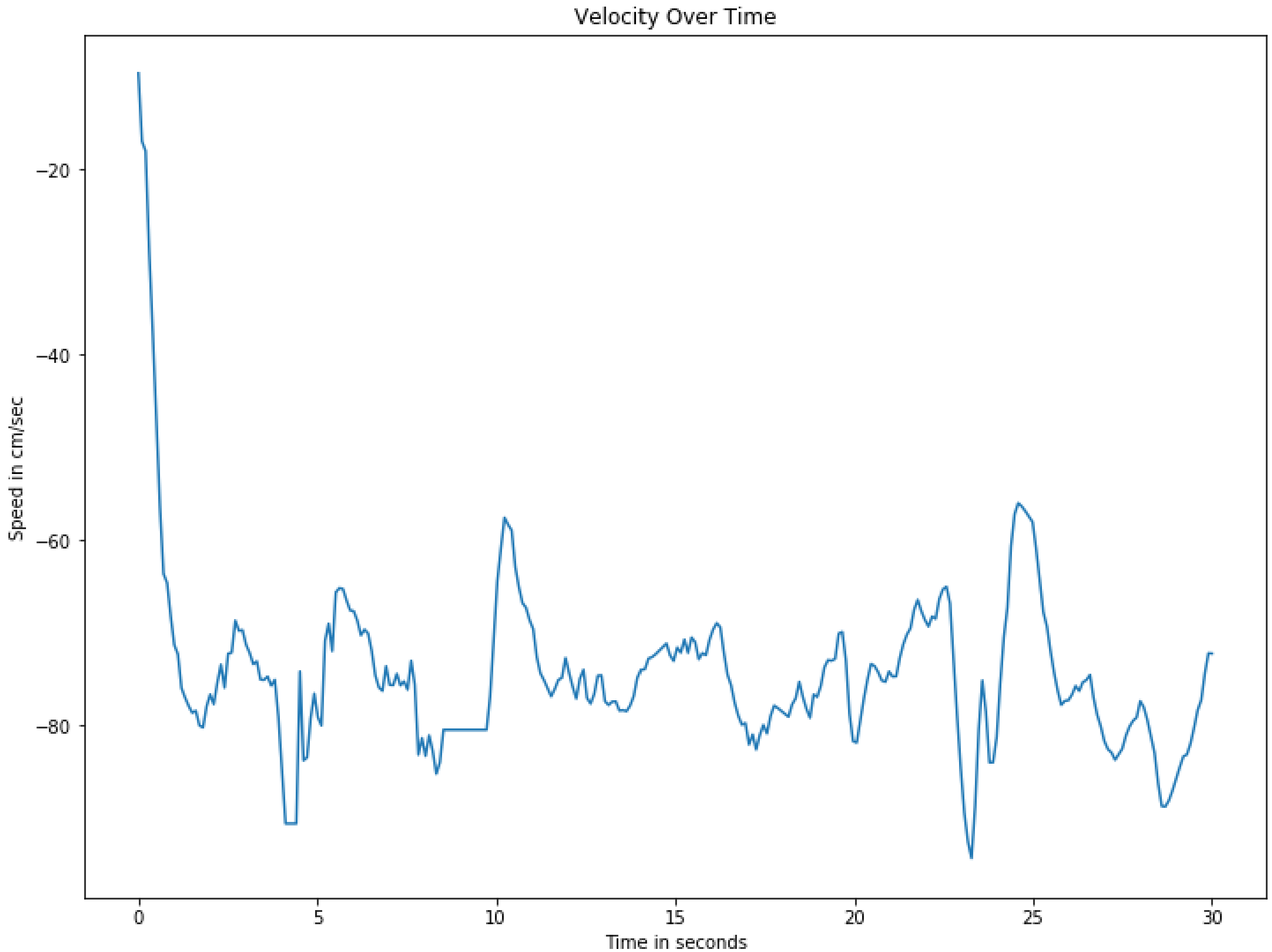
In [5]:
```
data = add_noise(np.loadtxt(wall_file, delimiter=","), 0)

# Setup initial variables and matrices
initial_pos = 2530
velocity = -10.0
```

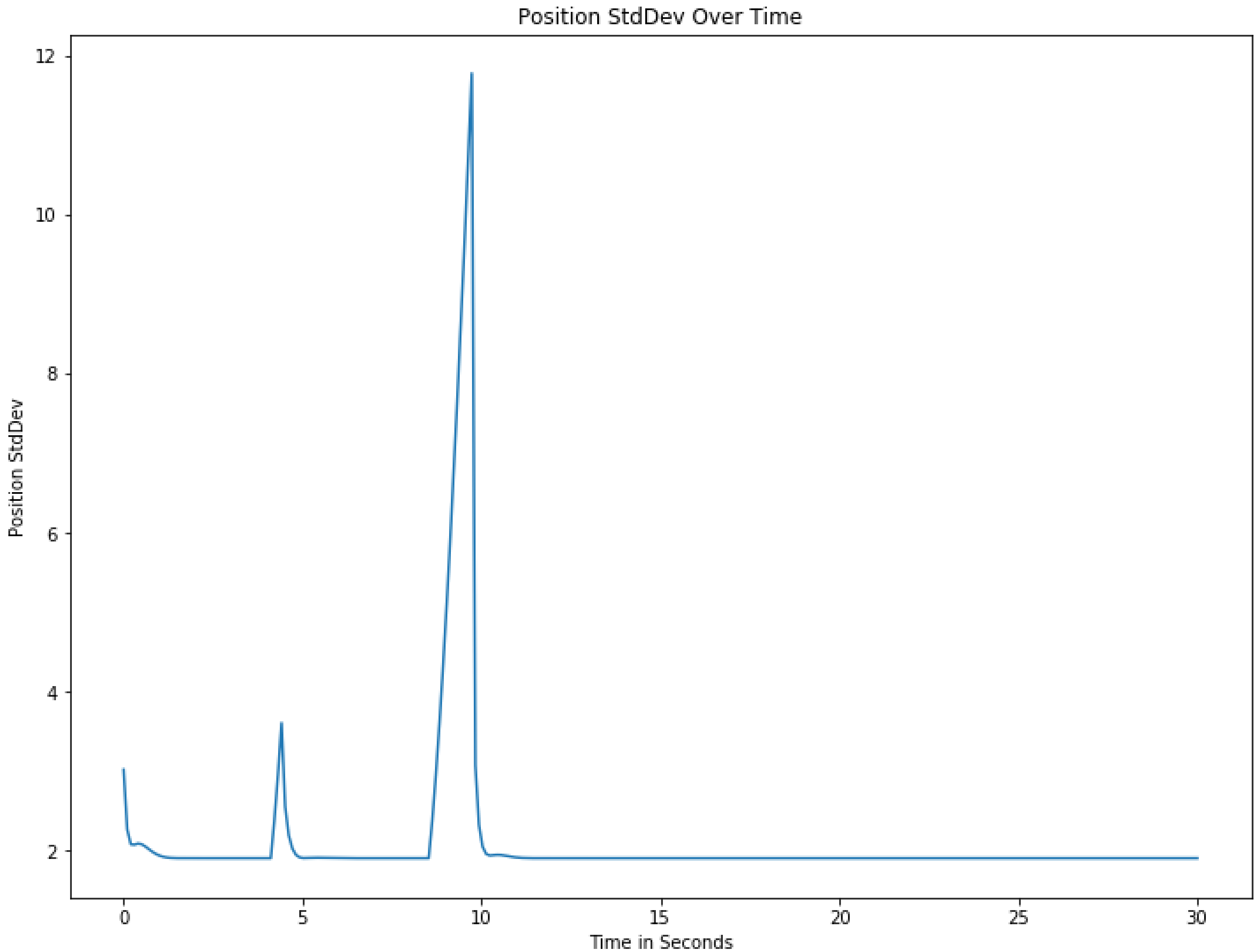
```
variance = 10.0
dt = 0.1
F = np.array([[1., dt],
              [0., 1.]])
P = np.array([[100., 0],
              [0, 100.0]])
H = np.array([[1., 0.]])
R = np.array([[variance]])
Q = np.array([[0.1, 1], [1, 10.]])
x = np.array([initial_pos, velocity]).T

predicted_xs = []

def run(x, P, R=0, Q=0, dt=0.1, zs=None):
    # run the kalman filter and store the results
    xs, cov = [], []
    for z in zs:
        x, P = predict(x, P, F, Q)
        S = dot3(H, P, H.T) + R
        n = z - np.dot(H, x)
        d = n*n / S

        if d < 9.0:
            x, P = update(x, P, z, R, H)

        xs.append(x)
        cov.append(P)

    xs, cov = np.array(xs), np.array(cov)
    return xs, cov

time_values = np.linspace(0, dt * len(data), len(data))

est_x, est_P = run(x, P, R, Q, dt, data)
est_pos = [v[0] for v in est_x]
draw_plot(data,
          estimates=est_pos,
          title="Raw Data VS Kalman Estimation",
          mlabel="Measurements",
          estlabel="Kalman Estimates",
          xlabel="Time in Seconds",
          ylabel="Distance to Wall in CM")
```

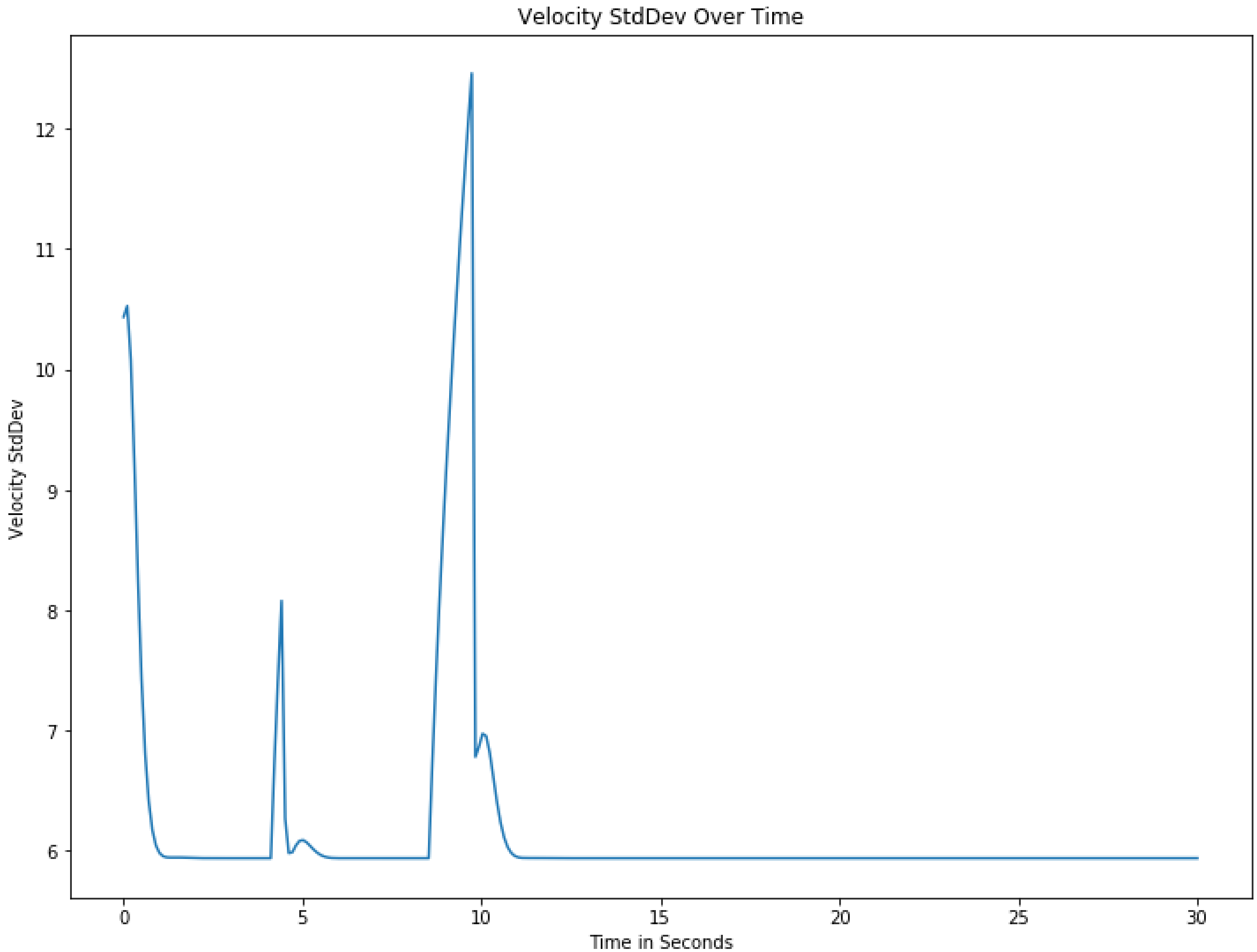
```
est_vel = [v[1] for v in est_x]
plt.plot(time_values, est_vel)
plt.xlabel("Time in seconds")
plt.ylabel("Speed in cm/sec")
plt.title("Velocity Over Time")
plt.show()

pos_std = [p[0][0]**0.5 for p in est_P]
vel_var = [p[1][1]**0.5 for p in est_P]
pos_vel_corr = [p[0][1] for p in est_P]
plt.plot(time_values, pos_std)
plt.ylabel("Position StdDev")
plt.xlabel("Time in Seconds")
plt.title("Position StdDev Over Time")
plt.show()

plt.plot(time_values, vel_var)
plt.ylabel("Velocity StdDev")
plt.xlabel("Time in Seconds")
plt.title("Velocity StdDev Over Time")
plt.show()

plt.plot(time_values, pos_vel_corr)
plt.ylabel("Velocity-Position Correlation")
plt.xlabel("Time in Seconds")
plt.title("Velocity-Position Correlation Over Time")
plt.show()
```

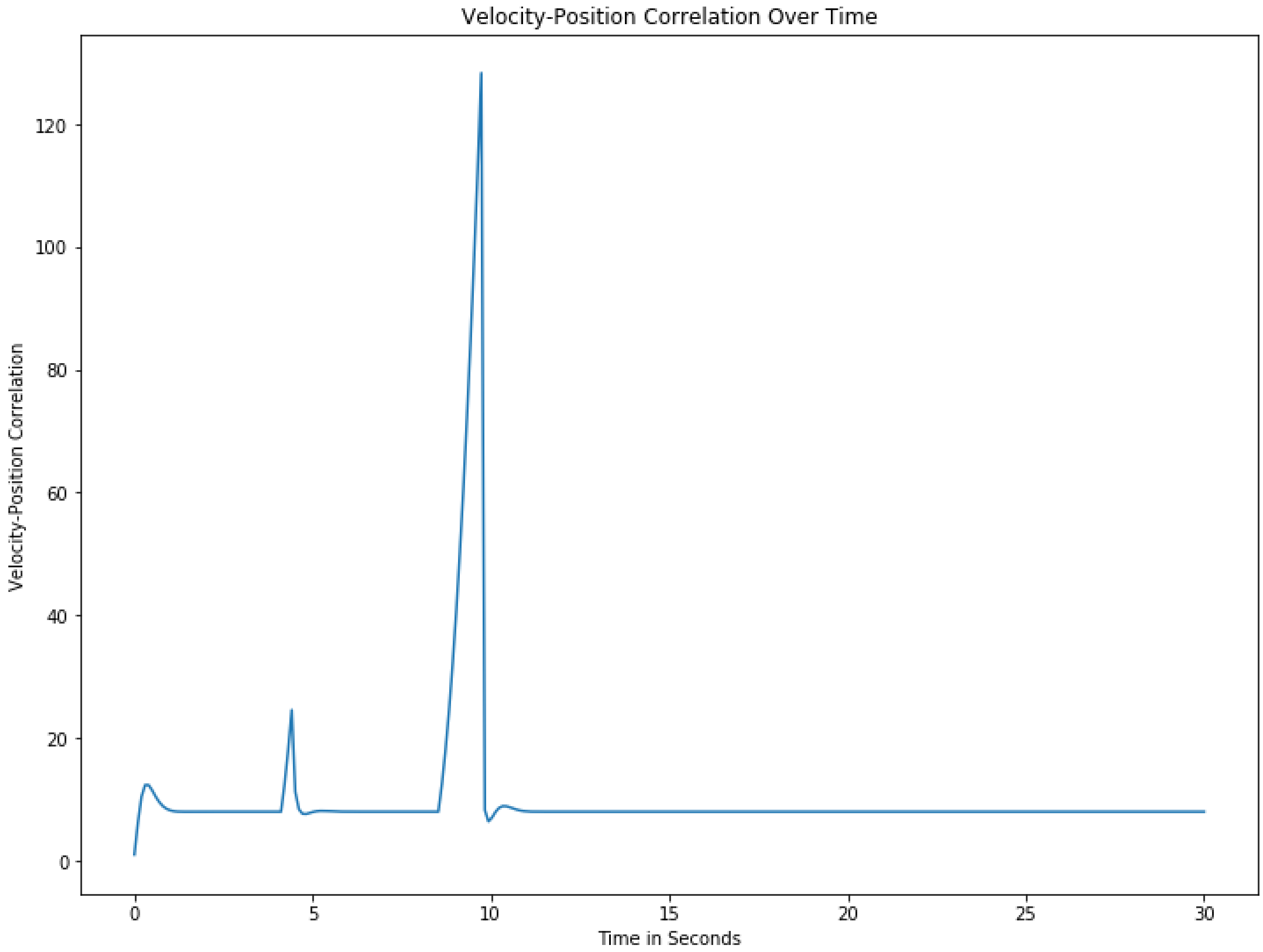


Raw Data VS Kalman Estimation
Distance to Wall in cm
2500
2000
1500
1000
500
0
50
100
150
200
250
300
Time in seconds
Measurements
Kalman Estimates


Velocity Over Time
Speed in cm/sec
Time in seconds
−20
−40
−60
−80
0
5
10
15
20
25
30


Position StdDev Over Time
Position StdDev
Time in Seconds
12
10
8
6
4
2
0
5
10
15
20
25
30


Velocity StdDev Over Time
Velocity StdDev
Time in Seconds
6
7
8
9
10
11
12
0
5
10
15
20
25
30

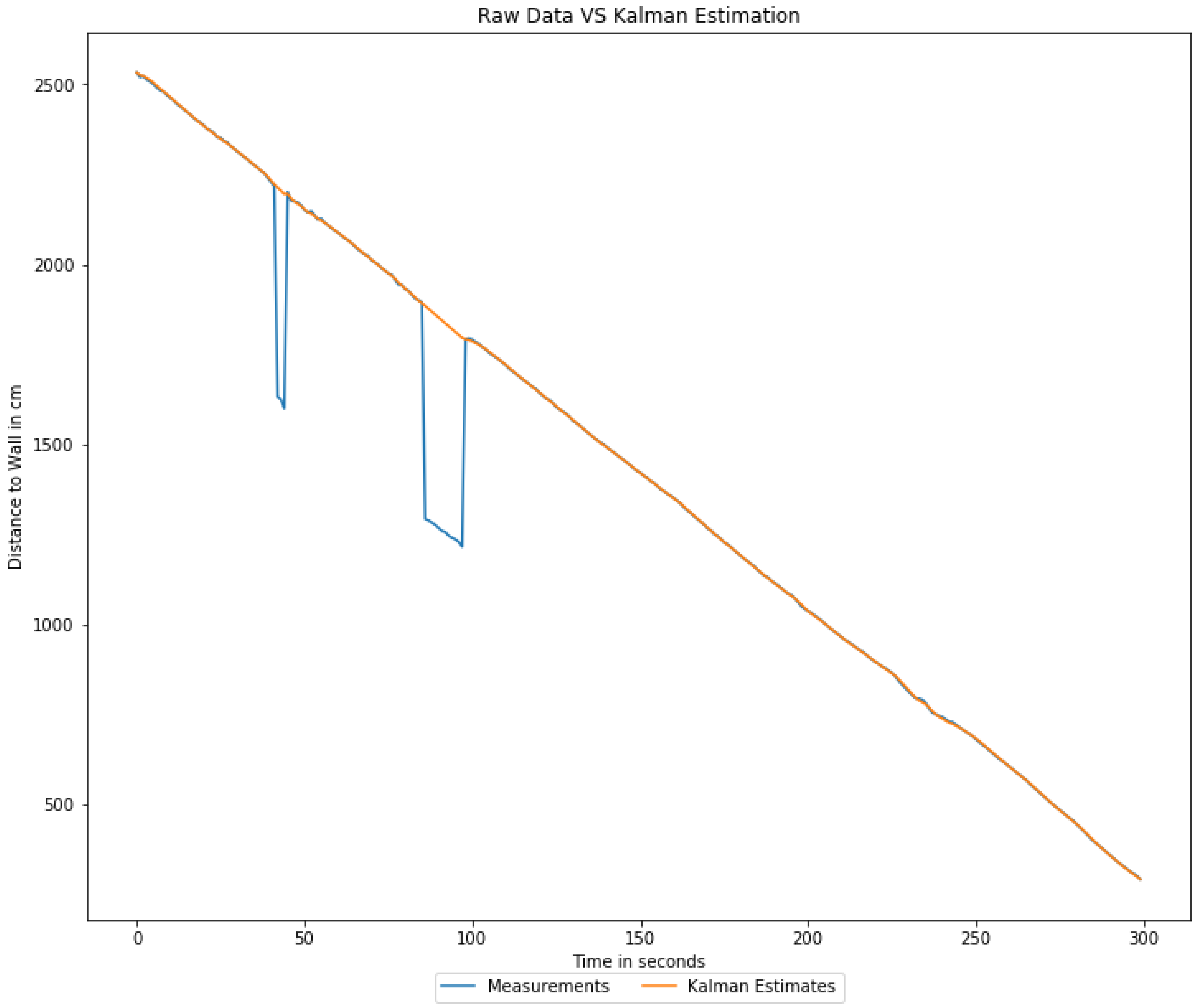

Velocity-Position Correlation Over Time
Velocity-Position Correlation
Time in Seconds
0
20
40
60
80
100
120
0
5
10
15
20
25
30

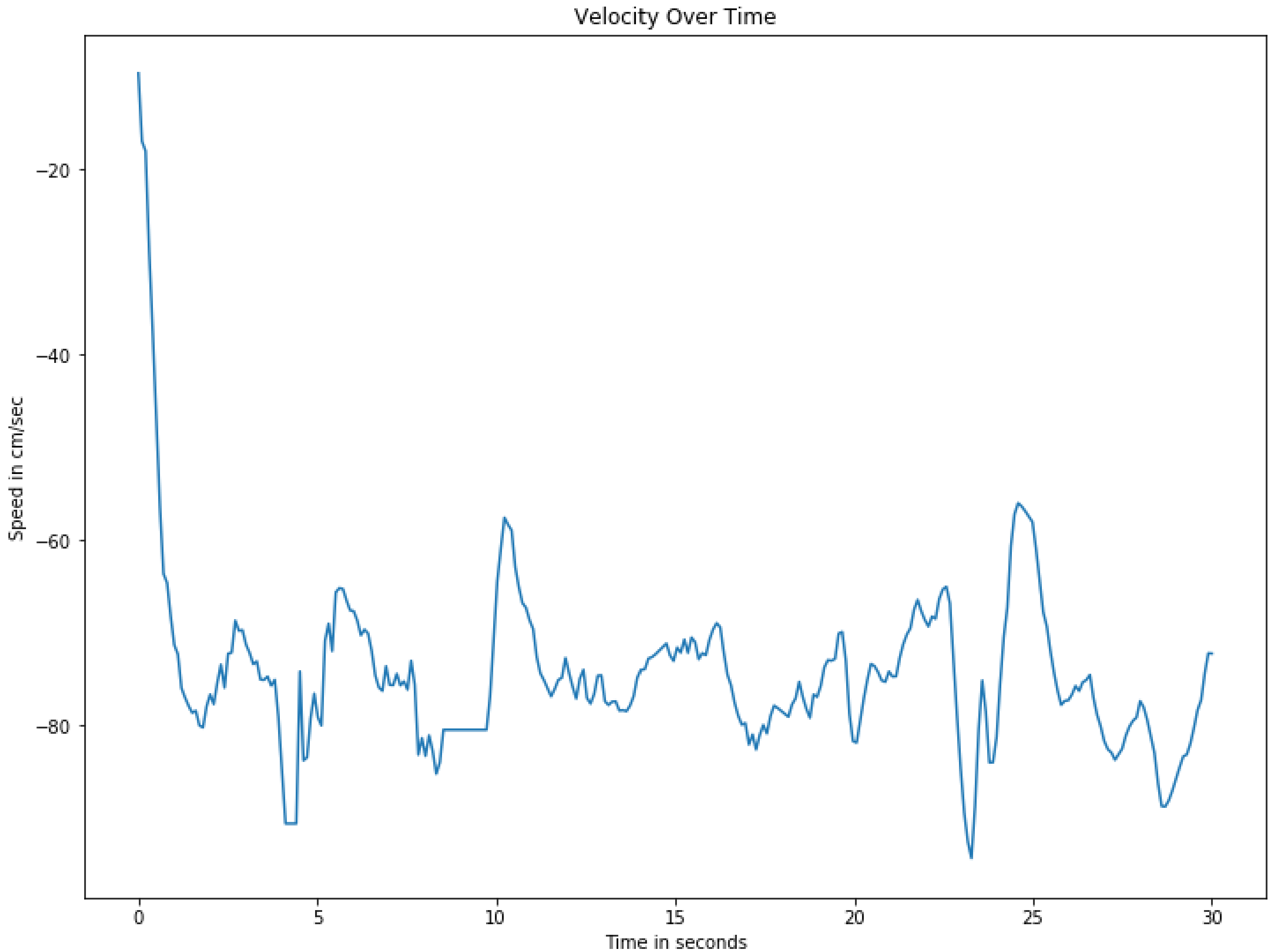
Part C

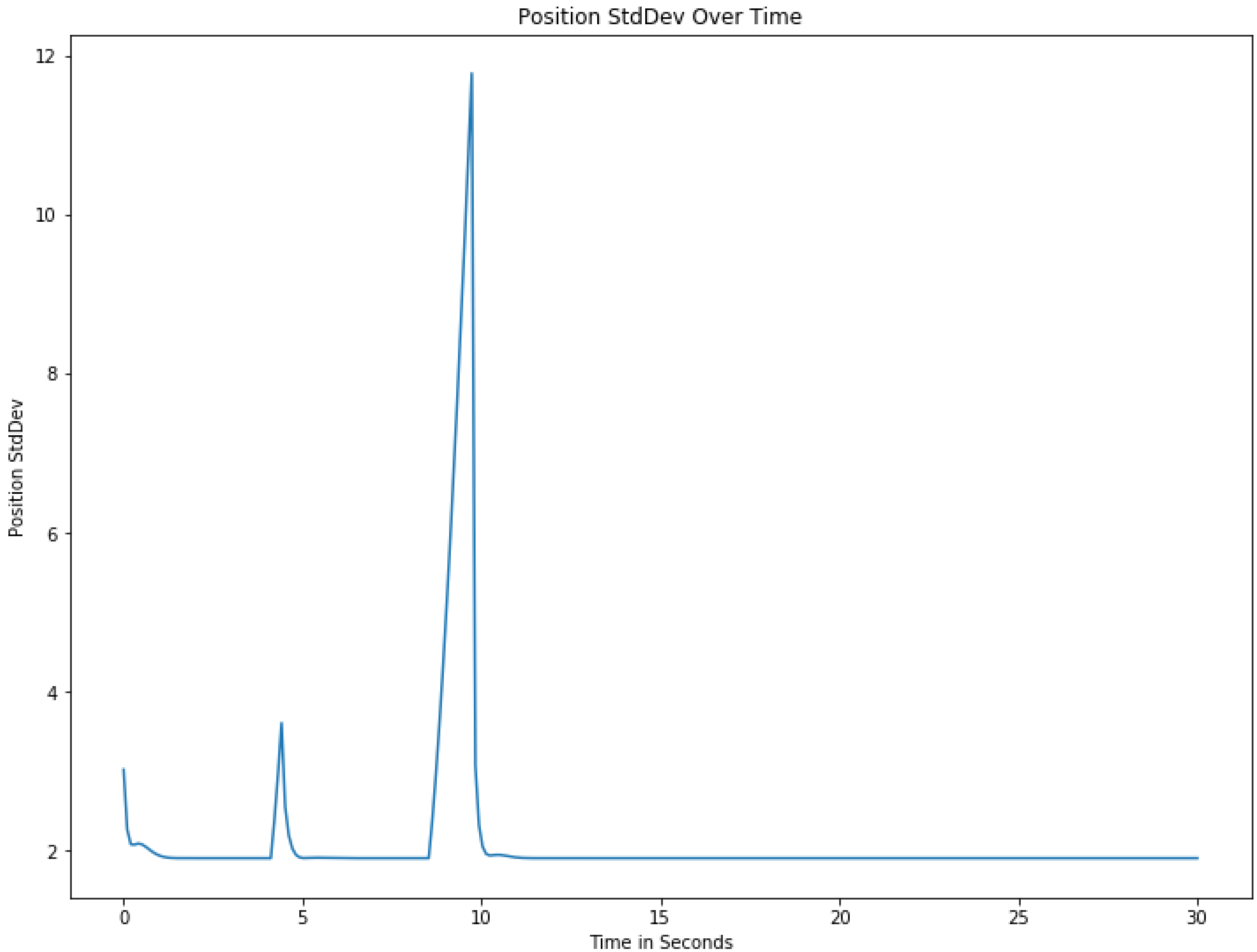
In [7]:
```
data = add_noise(np.loadtxt(wall_file, delimiter=","), 0)

def gaussian(x, mu, sig):
    return (1/(np.sqrt(2 * np.pi * np.power(sig, 2.)))) * np.exp(-np.power(x - mu, 2.) / (2 * np.power(sig,
 2.)))
```

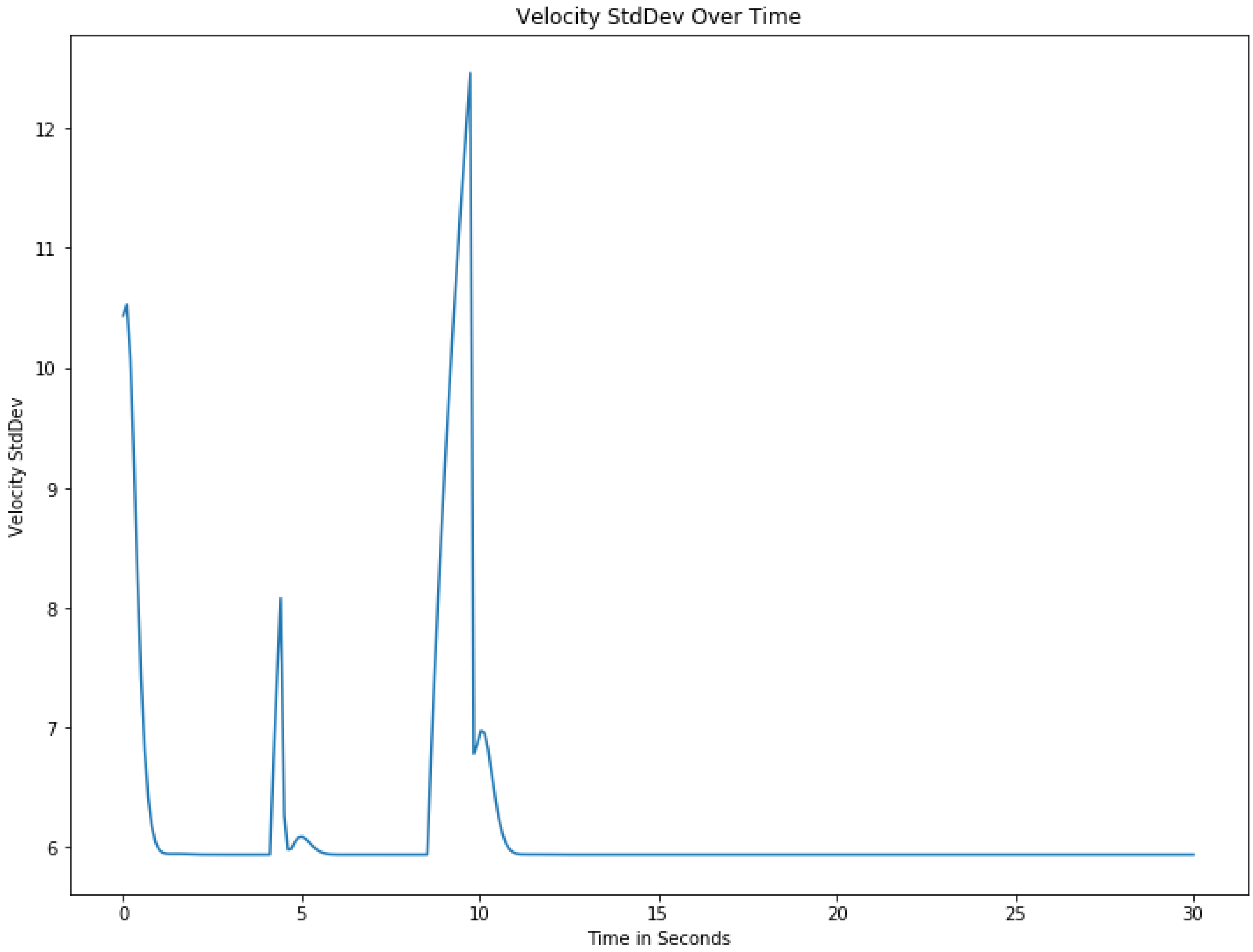
```python
# Setup initial variables and matrices
initial_pos = 2530
velocity = -10.0
variance = 10.0
dt = 0.1
F = np.array([[1., dt],
              [0., 1.]])
P = np.array([[100., 0],
              [0, 100.0]])
H = np.array([[1., 0.]])
R = np.array([[variance]])
Q = np.array([[0.1, 1], [1, 10.]])
x = np.array([initial_pos, velocity]).T
xs = []

object_c = 0.2
wall_c = 0.8
lamb = 0.0005

def object_pdf(z):
    return object_c * lamb * np.exp(-lamb * z)

def wall_pdf(z, wall_mean):
    return wall_c * gaussian(z, wall_mean, variance)

def run(x, P, R, Q, dt, zs):
    # run the kalman filter and store the results
    xs, cov = [], []
    for z in zs:
        x, P = predict(x, P, F, Q)
        prob_wall = wall_pdf(x[0], z)
        prob_obj = object_pdf(z)

        if prob_obj < prob_wall:
            x, P = update(x, P, z, R, H)

        xs.append(x)
        cov.append(P)

    xs, cov = np.array(xs), np.array(cov)
    return xs, cov
```

```
old_est_x = est_x
est_x, est_P = run(x, P, R, Q, dt, data)

est_pos = [v[0] for v in est_x]
draw_plot(data,
          estimates=est_pos,
          title="Raw Data VS Kalman Estimation",
          mlabel="Measurements",
          estlabel="Kalman Estimates",
          xlabel="Time in Seconds",
          ylabel="Distance to Wall in CM")

est_vel = [v[1] for v in est_x]
plt.plot(time_values, est_vel)
plt.xlabel("Time in seconds")
plt.ylabel("Speed in cm/sec")
plt.title("Velocity Over Time")
plt.show()

pos_std = [p[0][0]**0.5 for p in est_P]
vel_var = [p[1][1]**0.5 for p in est_P]
pos_vel_corr = [p[0][1] for p in est_P]
plt.plot(time_values, pos_std)
plt.ylabel("Position StdDev")
plt.xlabel("Time in Seconds")
plt.title("Position StdDev Over Time")
plt.show()

plt.plot(time_values, vel_var)
plt.ylabel("Velocity StdDev")
plt.xlabel("Time in Seconds")
plt.title("Velocity StdDev Over Time")
plt.show()

plt.plot(time_values, pos_vel_corr)
plt.ylabel("Velocity-Position Correlation")
plt.xlabel("Time in Seconds")
plt.title("Velocity-Position Correlation Over Time")
plt.show()
```


Raw Data VS Kalman Estimation
Distance to Wall in cm
2500
2000
1500
1000
500
0
50
100
150
200
250
300
Time in seconds
Measurements
Kalman Estimates


Velocity Over Time
−20
−40
−60
−80
Speed in cm/sec
0
5
10
15
20
25
30
Time in seconds


Position StdDev Over Time
Position StdDev
Time in Seconds
12
10
8
6
4
2
0
5
10
15
20
25
30


Velocity StdDev Over Time
Velocity StdDev
Time in Seconds
12
11
10
9
8
7
6
0
5
10
15
20
25
30