# RBE 500-F17-191 Foundations of Robotics



**Fall 2017 (Center)**

# FINAL EXAMINATION

Assigned: 11/14/17

Due: 12/13/17

# Submission Instructions:

The following is a take-home examination. It is expected that the answers you provide are the results of your own, individual work. Discussions about the answers or the approach for answering these specific problems with anyone other than the course staff are prohibited.

Be sure to explain and show all work. For questions involving calculations on data, explain the calculations used and the results obtained in prose – don’t just attach code or graphs and expect us to guess what the results mean! The way you get to an answer, and your demonstration of what that answer means is just as important, perhaps more important, than the answer itself!

Some of these problems involve writing Python code. Please annotate your code with appropriate comments. You don’t have to comment every line, but at least comment major sections.

Please include the answers (graphs, explanations, page scans, etc) in a single PDF file and attach the code in a separate zip file. So one PDF file and one zip file for the code. This will streamline the grading process for the TA’s because they can just leave comments on the PDF.

# Problems:

1. Prove that for any random variable X (The symbol means “is identical to”):
   1. (5 points)
   2. And if X has infinite range, . (5 points)
   3. And for any independent random variables X and Y

(5 points)

1. For independent, unbiased samples, of the random variable X. Show that:
   1. The expected value of the sample mean, . (5 points)
   2. The variance of the sample mean is

. (5 points)

1. You are given that X is an Random Variable with the PDF for all and otherwise:
   1. What is the value of that makes fX(x) a proper PDF? (5 points)
   2. Given your calculated value for , what is the E[X] and E[X2]? (5 points)
2. You’ve been spending far too much time in the lab getting a mobile robot to work. You go to the storage room to get some wheels for your robot. You find that there are two boxes of wheels. Box 1 contains 5 large wheels and 7 small wheels; Box 2 contains 3 large wheels and 12 small wheels. Frustrated at the lack of organization (and your lack of sleep) you decide to flip a fair coin to figure out which box to randomly grab a wheel from. If it’s heads, you pick from Box 1, if it’s tails, you pick from Box 2. You run the experiment and have selected a small wheel. What is the probability you selected the wheel from Box 2?
3. points)
4. The purpose of this problem is to illustrate the benefits of differential GPS. You are given GPS data for a master station in the file Master.csv and two Rover stations in the files Rover1.csv and Rover2.csv. In each file the first column is GPS time (in the format hhmmss where hh means hours, mm means minutes, and ss means seconds, UTC-Coordinate Universal Time, which rolls over to zero when it is midnight in Greenwich, England). The following columns are latitude and longitude. You can ignore the N’s and W’s. Latitude and longitude are in the format ddmm.mmm, which is interpreted as dd degrees and mm.mmm minutes. [ Hint: You might want to read the file “Converting GPS Code to Degrees.docx” in the Final folder.]
   1. Calculate the average locations in latitude and longitude in degrees of the Master station and the two rover stations. (5 points)
   2. Assuming these average positions are the true locations, calculate the errors in degrees by subtracting the corresponding truth value from each sample latitude and longitude. Convert each of these errors from degrees to meters. What are the standard deviations of the errors in latitude and longitude in meters for the Master and the Rovers? How are the standard deviations different for latitude than for longitude? . [Hint: You will have to look up the meters per degree for latitude and for longitude at these locations. You can use <http://www.csgnetwork.com/degreelenllavcalc.html> ] (8 points)
   3. How far is each Rover in meters from the Master? (4 points)
   4. Find the data which is aligned at the same GPS times for the Master and Rover\_1. For each corresponding sample time, correct the Rover\_1 latitude and longitude errors in meters by subtracting the Master latitude and longitude errors in meters. Compute the standard deviations for these corrected errors. Do the same for Rover\_2. How does this change the standard deviations of the rover position errors? (8 points)
5. The file “Lidar\_Data.zip” contains LIDAR data (in cm) taken every 45 degrees from two locations at 85 Prescott Street. The map of the relevant portion of the building (as it was when the data was collected) is included in this folder as “Selected P85 Map” in four different formats. (Use whichever you wish.) Assume that 0 degrees points directly to the North wall (the top of the map). Further, assume that each grid square is 60 x 60 cm. Using the average range measurements, estimate the grid position of the two robot locations. Please illustrate the robot locations on the map along with their average range measurements. (It is acceptable to print out a template of each robot’s average measurements and manually find the best position for each robot, but you must draw these onto the map and submit the marked map in the pdf of answers.) (15 points)
6. Using the methods described at the end of the file “RBE500\_17\_011\_02.pptx” (in Lecture 011) compute transformations between the body coordinate frame of a strapdown IMU and the navigation frame (which is aligned with the z axis pointing down along the local gravity vector.) Assume that the azimuth . Assume the accelerometer biases have been compensated and the average accelerometer vector measured in the body frame is

* 1. Starting with this force vector in body coordinates, compute the sines and cosines of the pitch and the roll and use them to compute the Direction Cosine Matrix . (5 points)
  2. Use to compute . (5 points)
  3. Using your previous results, compute the sines and cosines of the half angles and use the results to compute the quaternion that transforms from the navigation frame to the body frame and compute its conjugate quaternion , which transforms from the body frame to the navigation frame. (5 points)
  4. Using compute the direction cosine matrix , which transforms a vector represented in body coordinates to its representation in navigation coordinates, and compare it to the Direction Cosine Matrix that you computed in step (b) . (5 points)