**Problems:**

***Cumulative Distributions***

1. You purchase a wheel sensor for a robot that has a mean lifetime of 20 years. The actual lifetime follows a typical exponential product life relationship which, in this case, has a Cumulative Distribution Function (CDF) of

for all *t* ≥ 0. Using this equation, the probability that the wheel sensor lasts for at least 3 years is

.

* 1. If the wheel sensor has been working for 3 three years, what is the probability that it will fail sometime in the 4th year of use?

To calculate the failure probability of the sensor within its 4th year of use we need to find the area under the PDF curve between 3rd and 4th years.

***Bayes Rule***

1. A robot uses a range sensor that can measure ranges from 0 meters to 3 meters. For simplicity, assume that actual ranges are distributed uniformly in this interval. Unfortunately, the sensor can be faulty. When the sensor is faulty, it constantly outputs a range below 1 meter, regardless of the actual range in the sensor’s measurement cone. We know the prior probability for a sensor to be faulty is p = 0.01.

Suppose the robot queried its sensor N times, and every single time the measurement value is below 1 meter.

* 1. What is the posterior probability of a sensor fault, for N=1,2,…10?

F – faulty sensor

G – good sensor

B – reading below zero

A – reading above zero

P(B|F) = 1 --> probability of event below 1 meter reading given faulty sensor.

P(F) = 0.01 --> probability of faulty sensor. This is my prior.

P(B) = 1/3 --> Probability of getting below 1 meter reading is 1/3, given the range from 0 to 3 is uniformly distributed

P(G) = 0.99 --> Probability the sensor is good

P(B|G) = P(B) P(G) / P(G) = P(B) = 1/3 --> Probability of getting below 1 meter reading given good sensor.

Bayes Theorem:

P(F|B) =

Substituting and updating the values 10 times:

Reading 1 produces new prior 0.029411764705882356

Reading 2 produces new prior 0.08333333333333336

Reading 3 produces new prior 0.21428571428571433

Reading 4 produces new prior 0.4500000000000001

Reading 5 produces new prior 0.7105263157894738

Reading 6 produces new prior 0.8804347826086957

Reading 7 produces new prior 0.9566929133858268

Reading 8 produces new prior 0.9851351351351352

Reading 9 produces new prior 0.9949954504094632

Reading 10 produces new prior 0.9983262325015216

***Conditional Independence***

1. Assume that . This says that the random variables X and Y are conditionally independent given the knowledge that the random variable Z = *z*.
   1. Using the laws of probability prove that and

Rewriting using the intersection via conditioning:

Applying that with extra condition z yields:

Equating this result to the assumed definition in the problem:

Same operation can be repeated to obtain equality for

***Cumulative Distribution and Distribution Descriptors***

1. For some sensors, the error Probability Density Function (PDF) can be modeled as
   1. Determine the equation for the Cumulative Distribution Function (CDF).
   2. Determine the mean, median, and mode of this distribution (accurate to 5 digits of precision).
2. PDF is the derivate of CDF, so let’s find the integral of given PDF:
3. Mean: Apply E(x) formula:

*=*

*=*

1. Median: Integrate over the interval -1 to median(m) for :

Median = **-0.29289**

1. Mode: It is the highest point on the graph. PDF is a linear increasing function with maximum value of 2 at x = 0. Mode = **0.00000**

***Combining Measurements with Different Characteristics***

1. We have two measurements of a variable *x. The two are corrupted by independent, zero-mean noises. That is, we have*

and

Form a weighted linear combination of these measurements that is unbiased and minimizes its variance. In other words, form an estimate of *x*

such that

and the weights are chosen to minimize .

Substituting the z1 and z2 into x-hat:

Don’t know what to do here? What is E(x) equal to? Pure guessing at this point. Since x is a single variable, not a random variable its expected value is itself – x.

Equating this result to E[xhat]:

What do these give me? I don’t know.

To minimize finds its derivative and set it equal to 0. Solve:

At this point, should I take the derivative with respect to x, w1, w2?