**Homework 03**

1. Go back to the integral definitions of expected value and variance and prove:
   1. (5%)
   2. (5%)
   3. (5%)

Integral after “b” resolves to 1

* 1. If X, Y are independent): (5%)

Because X and Y are independent p(x, y) can be rewritten as p(x)p(y)

Rearranging terms

* 1. (10%)

Now replace “m” with E[x] and resolve integrals to their respective values:

This shows that variance, by definition, will always be smaller than and greater than 0. If E[X] = 0 then the equality part will hold.

* 1. (5%)

I used “a\*m” because

From this point on I will simply write integral sign and infinity limits will be assumed – too mouse clicky for me to add them.

* 1. (5%)

Resolve integrals:

* 1. (5%)

Now replace “m” with E[x] and resolve integrals to their respective values:

* 1. (10%)
  2. if X and Y are independent. (10%)

Resolve integrals to their definitions:

1. Suppose we are a mobile robot who lives on an infinitely long straight road. Our location *x* will simply be the position along this road. Now suppose that initially, we believe to be at location *xint = 1 000 m,* but we know that this estimate is uncertain. Based on this uncertainty, we model our initial belief by a Gaussian with variance.

To find out more about our location, we query a GPS receiver. The GPS gives us a measurement of our location . This GPS receiver is known to have a mean of the true location and an error variance of [Hint: Write out formulas symbolically first and then plug in the numbers.]

* 1. Write the probability density functions of the prior and

the measurement . (10%)

Prior:

Likelihood:

* 1. Using Bayes rule and the “completing the squares” technique, determine the posterior distribution and show that it is Gaussian. (25%)

[Hint: After applying Bayes’ rule, switch to working in terms of logarithms before “completing the squares”. If you don’t know what that means, read through the second presentation on Gaussian variables in the Module 03 (which we didn’t get to last time, but will next time.)]

Bayes Theorem:

Where “n” is the normalizing constant:

Calculate the numerator first:

Switching focus to the exponent:

Divide by scalar of :

**Let a = 200-1.8z**

Completing the square:

Breaking things apart a little:

Getting back to the exponent form:

Let “s” be a scaling factor, such as: