# Midterm Examination

Assigned: 11 October 2017

Due: 25 October 2017

Notes: The following is a take-home examination. It is expected that the answers you provide are the results of your own, individual work. Discussions about the answers or the approach for answering these specific problems with anyone other than the course staff are prohibited.

Be sure to explain/show all work. For questions involving calculations on data, explain the calculations used and the results obtained in prose – don’t just attach code or graphs and expect us to guess what the results mean! The way you get to an answer, and your demonstration of what that answer means is just as important, perhaps more important, than the answer itself!

**All coding must be done in Python. It is very important that you read and follow the submission instructions (in a separate file) or else you will lose points.**

**Problems:**

1. You went to the storeroom to get six IR ranging sensors for a project. You find a box of sensors with a note that says “box contains 8 sensors – 6 good, 2 bad.” You need 6 good sensors, so you’ll have to start testing sensors, one-by-one, to find the 6 good ones. What is the probability that the first two tests you do will identify the two bad sensors? **[5 points]**

This is an example of sampling without replacement, which can be solved by applying the product rule.

P[(pick#1=bad)&(pick#2=bad)]

= P(pick#1=bad)P(pick#2=bad |pick#1 =bad)

= (#bad/total#)x(#bad left/total# left)

= (2/8)x(1/7) = (1/28).

1. Assume that a given sensor has a one year warranty. The probability that a sensor that has no hidden defect will fail during the first year is 0.01. However, the manufacturing process is not perfect, and it is possible that a sensor that tests good at the factory has a hidden defect. The probability that a sensor having a hidden defect will fail in the first year is 0.10. The probability of a sensor being good is 0.98 and the probability of the sensor having a hidden defect is 0.02. What is the probability that a randomly selected sensor will fail during the first year? **[5 points]**

This is an example of total probability that uses the both the sum and product rules.

P[fail&(good or bad)]

= P(fail&good) +P(fail&bad)

= P(fail|(good)xP(good) + P(fail|bad)xP(bad)

= (0.01)x(0.98) + (0.1)x(0.02) = 0.0098 + 0.002 = 0.0118

1. You are given a continuous random variable, X, that has a uniform distribution between a value of 6 and a value of 8 and zero elsewhere.
   1. Plot the PDF and CDF for the random variable**. [2 points]**
   2. If Y = X2, what is the mean and standard deviation of Y? **[5 points]**

a)

Normalizing factor computation =

b)

**The mean of is**

**The standard deviation of y is**

1. You have two continuous random variables that are independent, uniform, and randomly distributed from 0 to 1.What is probability that Y < X? **[5 points]**

1. You have a continuous random variable, X, that has a Gaussian distribution with mean, and standard deviation.
   1. What is the probability distribution function (PDF) formula for X? **[2 points]**
   2. Let Y = X+1, what is the PDF formula for Y? **[3 points]**
   3. Given part b, Y and X are clearly correlated. What is the normalized correlation coefficient between Y and X? [Hint: Use the formulas for the relations between expected values and variances.] **[8 points]**

**a)**

**The probability distribution for a Gaussian is**

**b)**

**Adding a constant to a Gaussian shifts the mean by the constant, but does not change the variance. Therefore and , and**

**c) The normalized covariance of x and y is**

1. Suppose we have a system in which there are three different ranging sensors, producing three measurements z1, z2 and z3 of the same distance x. Each of these measurements are corrupted by zero mean Gaussian noise with variances , ,and respectively. These noises are uncorrelated with each other. Use the trick of completing the squares to show that the posterior distribution is Gaussian with variance

and mean

where each coefficient has the form

**[8 points]**

**Since the noises on these three measurements are uncorrelated and Gaussian, they are independent. Therefore, the joint distribution of the measurements is the product of their individual distributions**

**We are implicitly assuming a flat prior on x; therefore, Bayes rule becomes**

**Working with minus twice the log of this pdf gives**

**Here *const* is just a place to absorb terms that do not depend on *x*, and it will change value as needed to absorb those terms.**

**Expanding the squares**

**Let**

**.**

**Then we can write**

**Let**

**where then**

**Adding and subtracting the same term from *const* yields**

**Now we go back to the probability domain by**

**Here acts as a normalization constant and therefore must be**

**.**