### Two-Way Tables

#### Gaston Sanchez

Creative Commons Attribution Share-Alike 4.0 International CC BY-SA

# Two-way tables, crosstables, contingency tables

#### Example

Suppose we observe 2 qualitative binary variables:

Gender

male, female

Condition

smoker, non-smoker

#### Example of crosstable 2x2

#### Table formed by crossing Gender and Condition

	smoker	non-smoker
male	20	35
female	15	40

#### Example of crosstable 2x2

Table formed by crossing Gender and Condition

		В	Bc
		smoker	non-smoker
A	male	20	35
Ac	female	15	40

note that these are absolute frequencies

#### Example of crosstable 2x2

#### Table formed by crossing Gender and Condition

		В	Bc	
		smoker	non-smoker	Total
A	male	20	35	55
Ac	female	15	40	<i>55</i>
	Total	35	75	110 grand total

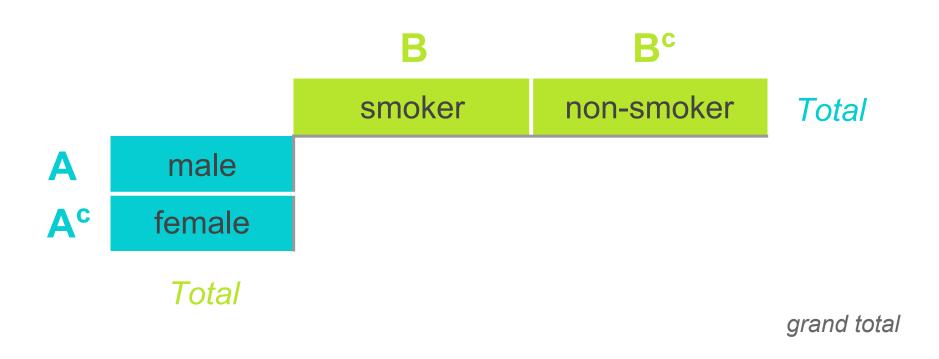
note that these are absolute frequencies

#### Crosstable 2x2: general case

	В	B <sup>c</sup>	Total
Α	A and B	A and B <sup>c</sup>	# A
Ac	A <sup>c</sup> and B	A <sup>c</sup> and B <sup>c</sup>	# A <sup>c</sup>
Total	# B	# B <sup>c</sup>	# N

# In order to get probabilities...

Table formed by crossing Gender and Condition



#### Table formed by crossing Gender and Condition

		В	Bc	
		smoker	non-smoker	Total
A	male	20/110	35/110	55/110
Ac	female	15/110	40/110	55/110
	Total	35/110	75/110	110/110 grand total

note that these are relative frequencies

#### Table formed by crossing Gender and Condition

		В	Bc	
		smoker	non-smoker	Total
A	male	0.1818	0.3181	0.5
Ac	female	0.1363	0.3636	0.5
	Total	0.3181	0.6818	<b>1.0</b> grand total

note that these are relative frequencies

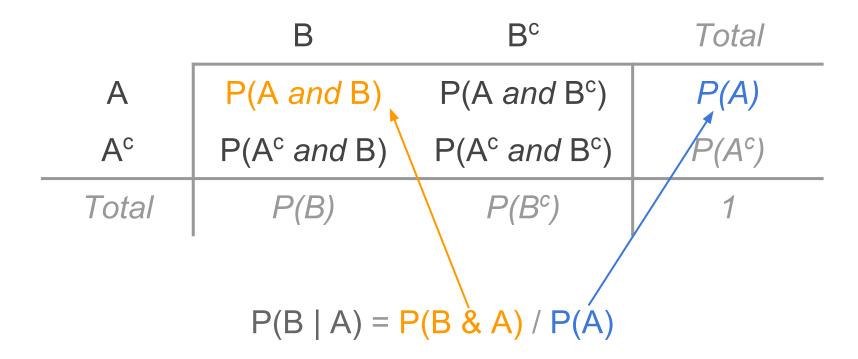
	В	B <sup>c</sup>	Total
Α	P(A and B)	P(A and B <sup>c</sup> )	P(A)
Ac	P(A <sup>c</sup> and B)	P(A <sup>c</sup> and B <sup>c</sup> )	P(A <sup>c</sup> )
Total	P(B)	$P(B^c)$	1

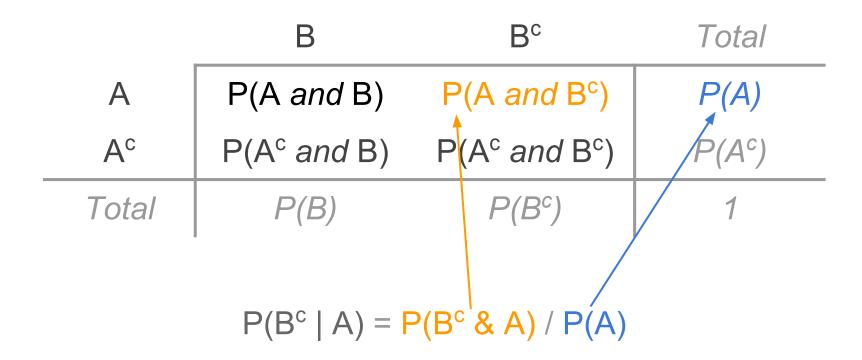
	В	B <sup>c</sup>	Total
Α	P(A and B)	P(A and B <sup>c</sup> )	P(A)
Ac	P(A <sup>c</sup> and B)	P(A <sup>c</sup> and B <sup>c</sup> )	$P(A^c)$
Total	P(B)	P(B <sup>c</sup> )	1
	$P(A \mid B) = P$	(A & B) / P(B)	

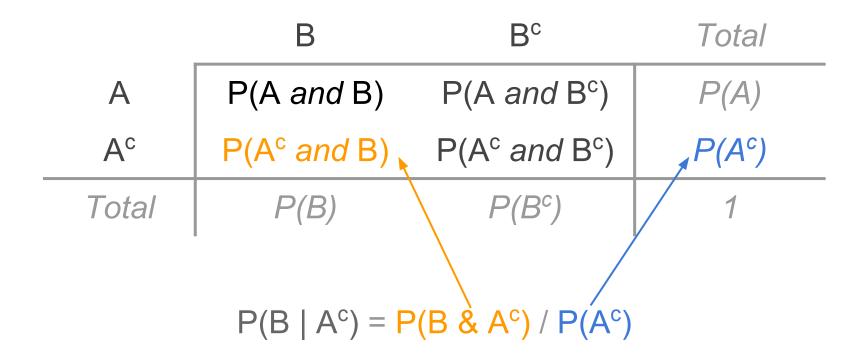
	В	B <sup>c</sup>	Total		
Α	P(A and B)	P(A and B <sup>c</sup> )	P(A)		
Ac	P(A <sup>c</sup> and B)	P(A <sup>c</sup> and B <sup>c</sup> )	$P(A^c)$		
Total	P(B)	P(B <sup>c</sup> )	1		
$P(A^c \mid B) = P(A^c \& B) / P(B)$					
	$\Gamma(A \mid D) - I$	$P(A \otimes D) / P(D)$			

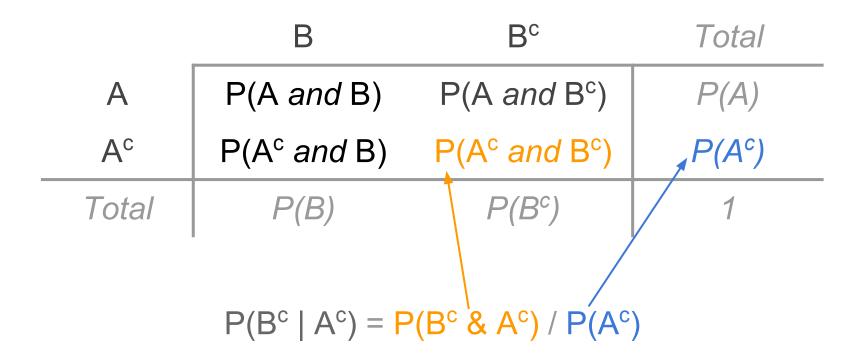
	В	B <sup>c</sup>	Total		
Α	P(A and B)	P(A and B <sup>c</sup> )	P(A)		
Ac	P(A <sup>c</sup> and B)	P(A <sup>c</sup> and B <sup>c</sup> )	$P(A^c)$		
Total	P(B)	$P(B^c)$	1		
$P(A \mid B^c) = P(A \& B^c) / P(B^c)$					

	В	B <sup>c</sup>	Total
Α	P(A and B)	P(A and B <sup>c</sup> )	P(A)
Ac	P(A <sup>c</sup> and B)	P(A <sup>c</sup> and B <sup>c</sup> )	P(A <sup>c</sup> )
Total	P(B)	$P(B^c)$	1
	$P(A^c \mid B^c) = P$	P(Ac & Bc) / P(Bc	)









# General Crosstable pxq

#### Crosstables pxq

#### We observe 2 qualitative

variables (nominal or ordinal)



$$A_1, A_2, A_3, ..., A_p$$



#### Crosstable *p*x*q*: general case

	B <sub>1</sub>	$B_2$	 $B_q$	Total
A <sub>1</sub>	A <sub>1</sub> and B <sub>1</sub>	$A_1$ and $B_2$	 $A_1$ and $B_q$	# A <sub>1</sub>
$A_2$	A <sub>2</sub> and B <sub>1</sub>	$A_2$ and $B_2$	 $A_2$ and $B_q$	# A <sub>2</sub>
$A_3$	$A_3$ and $B_1$	$A_3$ and $B_2$	 $A_3$ and $B_q$	# A <sub>3</sub>
$A_p$	$A_p$ and $B_1$	$A_{p}$ and $B_{2}$	 $A_{p}$ and $B_{q}$	# A <sub>p</sub>
Total	# B <sub>1</sub>	# B <sub>2</sub>	 $\# B_q$	# N

#### Example

### Crosstable for current enrollment in public and private schools by level of education

	Public	Private	Total
Elementary	20%	30%	50%
High School	15%	20%	35%
College	10%	5%	15%
Total	45%	55%	100%

Probability that a student randomly selected is enrolled in Elementary and High School?

#### P(enrolled in Elementary and HS) = ?

	Public	Private	Total
Elementary	20%	30%	50%
High School	15%	20%	35%
College	10%	5%	15%
Total	45%	55%	100%

Independent?
Mutually Exclusive?
None of the above?

#### P(enrolled in Elementary and HS) = ?

	Public	Private	Total
Elementary	20%	30%	50%
High School	15%	20%	35%
College	10%	5%	15%
Total	45%	55%	100%

#### mutually exclusive events

#### Keep in mind ...

# Mut. Exclusive events



## Independent events

typically has to do with outcomes of same experiment

typically has to do with outcomes of different experiments