MATHS 130 – Assignment 3

University of Auckland

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Question 1

Consider the equation $f(x) = 0$, on $\mathbb{R}$, where

$$f(x) := 7x^3 + 2x - \sin(x) - 1.$$

(a) Prove that there is at least one solution to the equation.
(b) Use the mean value theorem to prove that there is only one solution.

Question 2

Determine whether the function

$$h(x) = \frac{1}{x^2 - 4x}$$

has any absolute extrema on the interval $(0, 4)$. If there are, find them and state where they occur, otherwise prove that there are none.

Question 3

Consider the function of two real variables given by the formula

$$f(x,y) = \frac{1}{\sqrt{x^2-9y^2}}$$

(a) Determine the natural domain on f.
(b) Determine and sketch the level curves of f for the values $k = 0$, and $k = 1$.
(c) Compute the gradient ∇f at a general point (x, y) and then at the point $(4, 1)$.
(d) Compute the directional derivative of f at $(4, 1)$ in the direction of the vector $u = (9, 4)$.
(e) At the point $(4, 1)$ find the unit vector of a direction in which the function increases most rapidly. What is the directional derivative in that direction?

Question 4

Consider the integral

$$g(x) := \int_{-7}^{x} |t|^{\frac{1}{2}} \cdot e^{t^2} \, \mathrm{d}t$$

(a) Without calculating it, explain why $g'(x)$ exists for all $x \in \mathbb{R}$.
(b) Compute $g'(x)$, naming any results/theorems that you use.
(c) Use the mean value theorem to prove that g is strictly increasing on $\mathbb{R}$.
(d) Use **proof by contradiction** to prove that the second derivative of h does not exist at 0.