COMPSCI130 — Assignment 3

University of Auckland

Aidan Webster aweb904@aucklanduni.ac.nz

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Contents

1) Functions

a) Let $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d\}$. State with reason which of the rules defined below is (or is not) a function with domain X and codomain Y.

i) $f(4) = a,\ f(2) = d,\ f(1) = c,\ f(3) = b$

ii) $g(3) = a,\ g(4) = d,\ g(1) = \ c$

iii) $h(2) = 3,\ h(3) = 1,\ h(1) = 4,\ h(4) = 2$

iv) $i(4) = c,\ i(1) = b,\ i(2) = a,\ i(3) = d,\ i(4) = a$

2) Limits

a) Find the following limits. Explain your answer.

i) $\lim_{n\to\infty} \frac{2n}{n \log_2\left(\frac{1}{n}\right)}$

ii) $\lim_{n\to\infty} \frac{\sqrt{n^5+4n+5}-\sqrt{5n^6+3n}}{4n^3+81n+64}$

iii) $\lim_{n\to\infty} \frac{3^n+n^9}{7^n+n^2+5}$

iv) $\lim_{n\to\infty} \frac{4n+3}{25-5^{\frac{4n+3}{n^2+5n}+2}}$

3) Algorithms

a) Consider the following algorithm:

Input: A positive natural number n.

1. If $n \,\%\, 3 = 0$, output n and stop. Otherwise, go to Step 2.
2. If n is even, replace n with $n + 1$ and go back to 1. Otherwise, go to Step 3.
3. Replace n with $n + 2$ and go to Step 1.

Will this algorithm run forever? Explain why or why not.

b) Consider two algorithms, called **Algorithm A** and **Algorithm B** with the following runtimes:

$$\textbf{AlgorithmASteps}(n) = n^{10} + 4n + \log_2(n)$$
$$\textbf{AlgorithmBSteps}(n) = 5\log_2(n) + 2^n$$

i) Find the run times for each algorithm for $n = 2$ and $n = 8$.

ii) Which algorithm is more efficient when $n < 10$?

iii) Which algorithm is more efficient for very large values of n? Explain your answer.

4) Graphs

In lectures, we looked at complete bipartite graphs. Here, we will look at bipartite graphs that are not necessarily complete. A bipartite graph is a simple graph whose vertex set V can be split into two disjoint nonempty sets V_1 and V_2 for which $V_1 \cup V_2 = V$ so that every edge in the graph connects a vertex in V_1 with a vertex in V_2.

a) Draw two different bipartite graphs, each containing 7 vertices in one set and 2 vertices in another set.

b) Consider the following graph G: Is G bipartite? Justify your answer either by drawing G as a bipartite graph with two distinct sets of vertices, or by explaining why G cannot be bipartite.

c) San draws a graph and asks Mika if it is bipartite. Mika says the graph is not bipartite because it contains the subgraph, C_3. Is Mika correct? Can a graph with C_3 as a subgraph be bipartite? Explain your answer.

d) San draws another graph. This time he notices that the graph contains the subgraph C_4 and assumes that the graph he has drawn cannot be bipartite. Is he correct? Explain your answers.

e) In general, come up with a rule for determining whether a graph with a subgraph C_n is bipartite. Explain your answer.