Department of Mathematics

Maths 130 Assignment 3

Due: 4pm, Thursday 1 June, 2023

- Your assignment must be submitted online through Canvas by 4pm on the due date.
- If you are prevented from completing your assignment by circumstances beyond your control (e.g. illness), speak to or email the course coordinator as soon as possible.
- Make sure your results are presented in a logical manner using clear English, and that your ideas are easy to follow. You must show your working to obtain full marks.
- The total number of marks available is 40.

1. (8=4+4 marks) Consider the equation $f(x) = 0$, on $\mathbb{R}$, where

$$f(x) := 7x^3 + 2x - \sin x - 1.$$

(a) Prove that there is at least one solution to the equation.

(b) Use the mean value theorem to prove that there is only one solution.

2. [10 marks] Determine whether the function

$$h(x) = \frac{1}{x^2 - 4x}$$

has any absolute extrema on the interval $(0, 4)$. If there are find them and state where they occur, otherwise prove that there are none.

3. (15=3+3+3+3+3 marks) Consider the function of two real variables given by the formula

$$f(x, y) = \frac{1}{\sqrt{x^2 - 9y^2}}$$

(a) Determine the natural domain on f.

(b) Determine and sketch the level curves of f for the values $k = 0$, and $k = 1$.

(c) Compute the gradient ∇f at a general point (x, y) and then at the point $(4, 1)$.

(d) Compute the directional derivative of f at $(4, 1)$ in the direction of the vector $u = (9, 4)$.

(e) At the point $(4, 1)$ find the unit vector of a direction in which the function increases most rapidly. What is the directional derivative in that direction?

4. (17 =5+2+5+5 marks) Consider the integral

$$g(x) := \int_{-7}^{x} |t|^{\frac{1}{2}} \cdot e^{t^2}\ dt,$$

for each $x \in \mathbb{R}$. This defines a function $g : \mathbb{R} \to \mathbb{R}$.

(a) Without calculating it, explain why $g'(x)$ exists for all $x \in \mathbb{R}$.

(b) Compute $g'(x)$, naming any results/theorems that you use.

(c) Use the mean value theorem to prove that g is strictly increasing on $\mathbb{R}$.

(d) Use **proof by contradiction** to prove that the second derivative of h does not exist at 0.

$\boxed{\textbf{50 marks total}}$