Kinematics of Exclusive (not sure) Meson Electroproduction

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This document generated by ChatGPT-5, NEEDS a deep review and cross-check everything is correct.

1 Reaction Overview

We consider the exclusive electroproduction reaction

$$e(k) + p(P) \;\to\; e'(k') + \pi(p_\pi) + N'(P'),$$

in the one–photon exchange approximation. The incoming electron has energy E and scatters to E' at angle θ_e. The target proton is at rest in the laboratory.

Four-momenta

$$\begin{aligned}
k &= (E, \vec{k}) && \text{incoming electron,}\\
k' &= (E', \vec{k}') && \text{scattered electron,}\\
q &= k - k' = (\nu, \vec{q}) && \text{virtual photon,}\\
P &= (M_p, \vec{0}) && \text{target proton,}\\
p_\pi &= (E_\pi, \vec{p}_\pi) && \text{produced pion,}\\
P' &= \text{recoil nucleon.}
\end{aligned}$$

2 Basic Invariants

$$Q^2 \equiv -q^2 = 4EE' \sin^2 \frac{\theta_e}{2}, \tag{1}$$

$$\nu \equiv E - E', \tag{2}$$

$$|\vec{q}| = \sqrt{\nu^2 + Q^2}, \tag{3}$$

$$W^2 \equiv (P + q)^2 = M_p^2 + 2M_p\nu - Q^2. \tag{4}$$

3 Two-body CM kinematics

Treat the hadronic final state as the two-body system $\gamma^*(q) + p(P) \to \pi(p_\pi) + N'(P')$. In the hadronic CM (center-of-mass) frame the pion momentum magnitude is

$$p^* \equiv |\vec{p}_\pi^{\,*}| = \frac{1}{2W} \sqrt{[W^2 - (M_p + M_\pi)^2]\,[W^2 - (M_p - M_\pi)^2]}\,, \tag{5}$$

and the pion four-vector in the CM (choosing $\phi^* = 0$) is

$$p_\pi^{*\mu} = \big(E_\pi^*,\, p^* \sin\theta^*,\, 0,\, p^* \cos\theta^*\big), \qquad E_\pi^* = \sqrt{M_\pi^2 + (p^*)^2}.$$

4 Relation between t and θ^*

The Mandelstam variable t used in the code is

$$t = (q - p_\pi)^2.$$

In the CM frame t can be expressed (implicitly) through θ^*; the code searches for the θ^* that yields the desired t_{target}. Numerically one solves

$$t(\theta^*) = (q_{\text{lab}} - p_\pi^{\text{lab}}(\theta^*))^2 \stackrel{!}{=} t_{\text{target}},$$

where $p_\pi^{\text{lab}}(\theta^*)$ is obtained by boosting the CM pion along the CM velocity.

5 Boost and rotation steps (as implemented)

1. Compute kinematic invariants: $Q^2,\ \nu,\ W$.
2. Compute CM pion momentum p^* from W.
3. Build the pion four-vector in the CM: $p_\pi^{*\mu}$ (with chosen θ^* and $\phi^* = 0$).
4. Compute the virtual-photon 3-vector in the lab from electron kinematics:

   $$\vec{q}_{\text{lab}} = \vec{k} - \vec{k}' \approx \big(-E' \sin\theta_e,\, 0,\, E - E' \cos\theta_e\big),$$

   and its magnitude $|\vec{q}| = \sqrt{\nu^2 + Q^2}$.
5. The hadronic CM moves along $\vec{q}$ with velocity

   $$\boldsymbol{\beta}_{\text{CM}} = \frac{|\vec{q}|}{M_p + \nu}\, \hat{q}.$$

   The code performs a *vector* Lorentz boost of $p_\pi^{*\mu}$ by $\boldsymbol{\beta}_{\text{CM}}$ to obtain p_π^μ in the true laboratory coordinates.
6. Compute $t = (q - p_\pi)^2$ in the lab; iterate/solve for θ^* so that $t = t_{\text{target}}$.

6 Laboratory pion observables

Given the lab pion three-vector $\vec{p}_\pi$:

$$|\vec{p}_\pi| = \sqrt{p_x^2 + p_y^2 + p_z^2}, \tag{6}$$

$$p_{\pi,\perp}^{(\text{beam})} = \sqrt{p_x^2 + p_y^2}, \tag{7}$$

$$\theta_\pi^{\text{lab}} = \arccos\left(\frac{p_z}{|\vec{p}_\pi|}\right). \tag{8}$$

7 Alternative transverse definitions and the user formula

The code computes and compares several commonly used quantities:

- CM transverse momentum:
$$k_\pi^{(\mathrm{CM})} = p^* \sin\theta^*.$$
- Lab transverse wrt beam:
$$k_\pi^{(\mathrm{beam})} = p_{\pi,\perp}^{(\mathrm{beam})}.$$
- Lab transverse wrt $\vec{q}$:
$$k_\pi^{(q)} = |\vec{p}_\pi| \sin\theta_{pq}, \quad \text{with} \quad \cos\theta_{pq} = \frac{\vec{p}_\pi \cdot \vec{q}}{|\vec{p}_\pi||\vec{q}|}.$$
- Your requested formula (implemented exactly in the code):
$$k_\pi^{(\mathrm{user})} = \sqrt{|\vec{p}_\pi|^2 + |\vec{q}|^2 - 2\,|\vec{p}_\pi|\,|\vec{q}|\,\cos\theta_{pq}}\,.$$
Note this is algebraically equivalent to the magnitude of $|\vec{p}_\pi - \vec{q}|$.

8 Figures

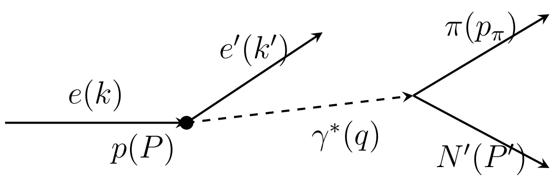

Figure 1: Reaction scheme for exclusive pion electroproduction in the one–photon exchange approximation.

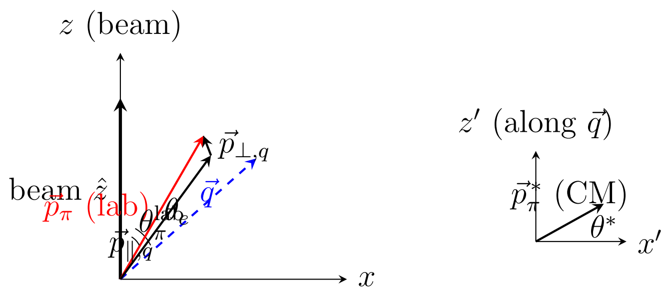

Figure 2: Kinematic geometry: beam axis z, virtual-photon direction $\vec{q}$, and lab pion momentum $\vec{p}_\pi$. The inset shows the q-aligned (CM-like) frame where the CM pion angle θ^* is defined.

9 Short derivation flow (how equations are chained in the code)

1. Inputs: $E, E_{\text{beam}}, Q^2, t_{\text{target}}$.
2. From E, E' compute ν and $|\vec{q}|$ and then W.
3. From W compute p^* (two-body CM momentum).
4. For a trial θ^* build $p_\pi^{*\mu}$ and boost to lab with $\boldsymbol{\beta}_{\text{CM}}$ along $\vec{q}$.
5. Rotate the q-aligned lab vectors into the beam lab (if necessary) so that pion components are given w.r.t. the beam $+\hat{z}$.
6. Compute $t(\theta^*) = (q - p_\pi)^2$; iterate on θ^* until $t(\theta^*) = t_{\text{target}}$.
7. Extract lab observables $p_\pi, \theta_\pi^{\text{lab}}, p_\perp$ and compute any chosen definition of k_π.