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# STL

## bitset

[C++ bitset 用法](https://www.cnblogs.com/magisk/p/8809922.html)

bitset

C++的 bitset 在 bitset 头文件中，它是一种类似数组的结构，它的每一个元素只能是０或１，每个元素仅用１bit空间。  
**bitset数组与vector数组区别**  
bitset声明数组:bitset<100> number[10]  
vector声明数组:vector number[10];  
**bitset<每个bitset元素的长度(没有占满前面全部自动补0)> 元素**  
**bitset内置转化函数：可将bitset转化为string,unsigned long,unsigned long long。**

构造

bitset<4> bitset1;　　//无参构造，长度为４，默认每一位为０  
  
 bitset<8> bitset2(12);　　//长度为８，二进制保存，前面用０补充  
  
 string s = "100101";  
 bitset<10> bitset3(s);　　//长度为10，前面用０补充  
   
 char s2[] = "10101";  
 bitset<13> bitset4(s2);　　//长度为13，前面用０补充  
  
 cout << bitset1 << endl;　　//0000  
 cout << bitset2 << endl;　　//00001100  
 cout << bitset3 << endl;　　//0000100101  
 cout << bitset4 << endl;　　//0000000010101

函数

bitset<8> foo ("10011011");  
  
 cout << foo.count() << endl;　　//5　　（count函数用来求bitset中1的位数，foo中共有５个１  
 cout << foo.size() << endl;　　 //8　　（size函数用来求bitset的大小，一共有８位  
  
 cout << foo.test(0) << endl;　　//true　　（test函数用来查下标处的元素是０还是１，并返回false或true，此处foo[0]为１，返回true  
 cout << foo.test(2) << endl;　　//false　　（同理，foo[2]为０，返回false  
  
 cout << foo.any() << endl;　　//true　　（any函数检查bitset中是否有１  
 cout << foo.none() << endl;　　//false　　（none函数检查bitset中是否没有１  
 cout << foo.all() << endl;　　//false　　（all函数检查bitset中是全部为１

[2019-2020 ICPC Asia Taipei-Hsinchu Regional Contest（H](https://blog.csdn.net/chitudexixi/article/details/109453360)

H

#include <bits/stdc++.h>  
#define ll long long  
using namespace std;  
int t,n,m;  
char str[1010];  
bitset<500> number[30];  
int main() {  
 ios::sync\_with\_stdio(false); cin.tie(0); cout.tie(0);  
 //freopen("test.in","r",stdin);  
 //freopen("test.out","w",stdout);  
 scanf("%d",&t);  
 while(t--)  
 {  
 scanf("%d %d",&n,&m);  
 for(int i=0;i<m;i++)  
 {  
 scanf("%s",str);  
 number[i]=bitset<500>(str);  
 }  
 int len=1<<m,ans=m+1;  
 for(int i=1;i<len;i++)  
 {  
 int t=i,s=0;  
 bitset<500> num(0);  
 for(int j=0;j<m&&t>0;j++)  
 {  
 if(t&1)   
 {  
 num=num|number[j];  
 s++;  
 }  
 t>>=1;  
 }  
 if(num.count()==n) ans=min(ans,s);  
 }  
 if(ans==m+1) printf("-1\n");  
 else printf("%d\n",ans);  
 }  
 return 0;  
}

# windows环境下的对拍

@echo off  
:loop  
 dataa.exe > data.txt  
 biaocheng.exe < data.txt > ac.txt  
 A.exe < data.txt > test.txt  
 fc ac.txt test.txt  
 if not errorlevel 1 goto loop  
pause  
goto loop

**其中要改的部分（标红辽）**：

@echo off  
:loop  
 dataa.exe > data.txt  
 $\color{red}{biaocheng.exe}$ < data.txt > ac.txt  
 $\color{red}{A.exe}$ < data.txt > test.txt  
 fc ac.txt test.txt  
 if not errorlevel 1 goto loop  
pause  
goto loop

文件以.bat作为后缀

将三个程序（数据生成文件（dataa），标程或暴力代码（biaocheng）, 要看的代码（A））放在同一目录下，

记得加 freopen

随机数记得加srand((int)time(0));

随机数生成code

#include <iostream>  
#include <cstdlib>  
#include <ctime>  
using namespace std;  
  
int main(){  
 freopen("data.txt", "w", stdout);  
   
 srand((int)time(0));  
 int T = rand() % 100000;  
 cout << T << endl;  
   
 for (int i = 0; i < T; i++){  
 cout << rand() % 100;  
 }  
}

rand() 似乎只有三万多，需要更大的数的话要乘一下

#

# 图论

## KM

#include<bits/stdc++.h>  
  
using namespace std;  
  
const int inf = 0x3f3f3f3f;  
const int maxN = 505;  
  
namespace KM {  
 int mp[maxN][maxN], link\_x[maxN], link\_y[maxN], N;  
 bool visx[maxN], visy[maxN];  
 int que[maxN << 1], top, fail, pre[maxN];  
 int hx[maxN], hy[maxN], slk[maxN];  
  
 inline int check(int i) {  
 visx[i] = true;  
 if (link\_x[i]) {  
 que[fail++] = link\_x[i];  
 return visy[link\_x[i]] = true;  
 }  
 while (i) {  
 link\_x[i] = pre[i];  
 swap(i, link\_y[pre[i]]);  
 }  
 return 0;  
 }  
  
 void bfs(int S) {  
 for (int i = 1; i <= N; i++) {  
 slk[i] = inf;  
 visx[i] = visy[i] = false;  
 }  
 top = 0;  
 fail = 1;  
 que[0] = S;  
 visy[S] = true;  
 while (true) {  
 int d;  
 while (top < fail) {  
 for (int i = 1, j = que[top++]; i <= N; i++) {  
 if (!visx[i] && slk[i] >= (d = hx[i] + hy[j] - mp[i][j])) {  
 pre[i] = j;  
 if (d) slk[i] = d;  
 else if (!check(i)) return;  
 }  
 }  
 }  
 d = inf;  
 for (int i = 1; i <= N; i++) {  
 if (!visx[i] && d > slk[i]) d = slk[i];  
 }  
 for (int i = 1; i <= N; i++) {  
 if (visx[i]) hx[i] += d;  
 else slk[i] -= d;  
 if (visy[i]) hy[i] -= d;  
 }  
 for (int i = 1; i <= N; i++) {  
 if (!visx[i] && !slk[i] && !check(i)) return;  
 }  
 }  
 }  
  
 void prework() {  
 for (int i = 1; i <= N; i++) {  
 link\_x[i] = link\_y[i] = 0;  
 visy[i] = false;  
 }  
 for (int i = 1; i <= N; i++) {  
 hx[i] = 0;  
 for (int j = 1; j <= N; j++) {  
 if (hx[i] < mp[i][j]) hx[i] = mp[i][j];  
 }  
 }  
 }  
  
 void init(int n) {  
 N = n;  
 top = fail = 0;  
 for (int i = 1; i <= N; i++) {  
 link\_x[i] = link\_y[i] = visx[i] = visy[i] = pre[i] = hx[i] = hy[i] = slk[i] = 0;  
 for (int j = 1; j <= N; j++) {  
 mp[i][j] = 0;  
 }  
 }  
 }  
}  
  
int main() {  
 ios::sync\_with\_stdio(false);  
 cin.tie(0);  
 cout.tie(0);  
  
 int n, m;  
 cin >> n >> m;  
 KM::init(max(n, m));  
 for (int i = 1; i <= n; i++) {  
 for (int j = 1; j <= m; j++) {  
 cin >> KM::mp[i][j];  
 }  
 }  
 KM::prework();  
 int ans = 0;  
 for (int i = 1; i <= KM::N; i++) KM::bfs(i);  
 for (int i = 1; i <= KM::N; i++) ans += KM::mp[i][KM::link\_x[i]];  
  
}

## prufer序列

#include <iostream>  
#include <cstdio>  
#include <cstring>  
#include <algorithm>  
  
using namespace std;  
  
const int N = 100010;  
  
int n, m;  
int f[N], d[N], p[N];  
  
void tree2prufer()  
{  
 for (int i = 1; i < n; i ++ )  
 {  
 scanf("%d", &f[i]);  
 d[f[i]] ++ ;  
 }  
  
 for (int i = 0, j = 1; i < n - 2; j ++ )  
 {  
 while (d[j]) j ++ ;  
 p[i ++ ] = f[j];  
 while (i < n - 2 && -- d[p[i - 1]] == 0 && p[i - 1] < j) p[i ++ ] = f[p[i - 1]];  
 }  
  
 for (int i = 0; i < n - 2; i ++ ) printf("%d ", p[i]);  
}  
  
void prufer2tree()  
{  
 for (int i = 1; i <= n - 2; i ++ )  
 {  
 scanf("%d", &p[i]);  
 d[p[i]] ++ ;  
 }  
 p[n - 1] = n;  
  
 for (int i = 1, j = 1; i < n; i ++, j ++ )  
 {  
 while (d[j]) j ++ ;  
 f[j] = p[i];  
 while (i < n - 1 && -- d[p[i]] == 0 && p[i] < j) f[p[i]] = p[i + 1], i ++ ;  
 }  
  
 for (int i = 1; i <= n - 1; i ++ ) printf("%d ", f[i]);  
}  
  
int main()  
{  
 scanf("%d%d", &n, &m);  
 if (m == 1) tree2prufer();  
 else prufer2tree();  
  
 return 0;  
}

## spfa最短路及负环

#include<bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
const int N = 1 << 20;  
struct edge {  
 ll to, len;  
};  
  
vector<edge> g[N];  
ll d[N], cnt[N], vis[N];  
  
bool spfa(ll s, ll n) {  
 queue<int> que;  
 for (int i = 1; i <= n; i++) { //防止不连通，全加进去  
 que.push(i);  
 vis[i] = 1;  
 }  
 while (!que.empty()) {  
 ll p = que.front();  
 que.pop();  
 vis[p] = 0;  
 for (auto x:g[p]) {  
 if (d[x.to] > d[p] + x.len) {  
 d[x.to] = d[p] + x.len;  
 cnt[x.to] = cnt[p] + 1;  
 if (!vis[x.to]) {  
 if (cnt[x.to] > n) return 0;  
 vis[x.to] = 1;  
 que.push(x.to);  
 }  
 }  
 }  
 }  
 return 1;  
}

## 一些定理

Hall定理：若二分图存在完美匹配，且大小为n，那么取任意1≤k≤n，均满足X集选出k个不同的点，它们连向Y集的点的个数不小于k。

## 二分图匹配（HK匈牙利匹配）

//大量使用了memset，但常数貌似很小？HDU6808跑了998ms（限制5000ms），然而这个代int main()不是HDU6808的  
#include<bits/stdc++.h>  
using namespace std;  
  
const int maxn=505;// 最大点数  
const int inf=0x3f3f3f3f;// 距离初始值  
struct HK\_Hungary{//这个板子从1开始，0点不能用,nx为左边点数，ny为右边点数  
 int nx,ny;//左右顶点数量  
 vector<int>bmap[maxn];  
 int cx[maxn];//cx[i]表示左集合i顶点所匹配的右集合的顶点序号  
 int cy[maxn]; //cy[i]表示右集合i顶点所匹配的左集合的顶点序号  
 int dx[maxn];  
 int dy[maxn];  
 int dis;  
 bool bmask[maxn];  
 void init(int a,int b){  
 nx=a,ny=b;  
 for(int i=0;i<=nx;i++){  
 bmap[i].clear();  
 }  
 }  
 void add\_edge(int u,int v){  
 bmap[u].push\_back(v);  
 }  
 bool searchpath(){//寻找 增广路径  
 queue<int>Q;  
 dis=inf;  
 memset(dx,-1,sizeof(dx));  
 memset(dy,-1,sizeof(dy));  
 for(int i=1;i<=nx;i++){//cx[i]表示左集合i顶点所匹配的右集合的顶点序号  
 if(cx[i]==-1){//将未遍历的节点 入队 并初始化次节点距离为0  
 Q.push(i);  
 dx[i]=0;  
 }  
 }//广度搜索增广路径  
 while(!Q.empty()){  
 int u=Q.front();  
 Q.pop();  
 if(dx[u]>dis) break;//取右侧节点  
 for(int i=0;i<bmap[u].size();i++){  
 int v=bmap[u][i];//右侧节点的增广路径的距离  
 if(dy[v]==-1){  
 dy[v]=dx[u]+1;//v对应的距离 为u对应距离加1  
 if(cy[v]==-1)dis=dy[v];  
 else{  
 dx[cy[v]]=dy[v]+1;  
 Q.push(cy[v]);  
 }  
 }  
 }  
 }  
 return dis!=inf;  
 }  
 int findpath(int u){//寻找路径 深度搜索  
 for(int i=0;i<bmap[u].size();i++){  
 int v=bmap[u][i];//如果该点没有被遍历过 并且距离为上一节点+1  
 if(!bmask[v]&&dy[v]==dx[u]+1){//对该点染色  
 bmask[v]=1;  
 if(cy[v]!=-1&&dy[v]==dis)continue;  
 if(cy[v]==-1||findpath(cy[v])){  
 cy[v]=u;cx[u]=v;  
 return 1;  
 }  
 }  
 }  
 return 0;  
 }  
 int MaxMatch(){//得到最大匹配的数目  
 int res=0;  
 memset(cx,-1,sizeof(cx));  
 memset(cy,-1,sizeof(cy));  
 while(searchpath()){  
 memset(bmask,0,sizeof(bmask));  
 for(int i=1;i<=nx;i++){  
 if(cx[i]==-1){  
 res+=findpath(i);  
 }  
 }  
 }  
 return res;  
 }  
}HK;  
  
int main(){  
 int nn,n,m;  
 cin>>nn;  
 while(nn--){  
 scanf("%d%d",&n,&m);  
 HK.init(n,m);//左端点和右端点数量  
 for(int i=1;i<=n;i++){  
 int snum;  
 cin>>snum;  
 int v;  
 for(int j=1;j<=snum;j++){  
 cin>>v;  
 HK.add\_edge(i,v);//连边  
 }  
 }  
 cout<<HK.MaxMatch()<<endl;//求最大匹配  
 }  
 return 0;  
}

## 带花树

/\*  
����һ�� n ���� m ���ߵ�����ͼ�����ͼ�����ƥ�䡣  
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\*/  
  
#include<bits/stdc++.h>  
using namespace std;  
#define I inline int  
#define V inline void  
#define FOR(i,a,b) for(int i=a;i<=b;i++)  
#define REP(u) for(int i=h[u],v;v=e[i].t,i;i=e[i].n)  
const int N=1e3+1,M=1e5+1;  
queue<int>q;  
int n,m,tot,qwq,ans;  
int h[N],lk[N],tag[N],fa[N],pre[N],dfn[N];  
struct edge{int t,n;}e[M];  
V link(int x,int y){lk[x]=y,lk[y]=x;}  
V add\_edge(int x,int y){  
 if(!lk[x]&&!lk[y])link(x,y),ans++;  
 e[++tot]=(edge){y,h[x]},h[x]=tot;  
 e[++tot]=(edge){x,h[y]},h[y]=tot;  
}  
V rev(int x){if(x)rev(x[pre][lk]),link(x,pre[x]);}  
I find(int x){return fa[x]==x?x:fa[x]=find(fa[x]);}  
I lca(int x,int y){  
 for(qwq++;;x=x[lk][pre],swap(x,y))  
 if(dfn[x=find(x)]==qwq)return x;  
 else if(x)dfn[x]=qwq;  
}  
V shrink(int x,int y,int p){  
 for(;find(x)!=p;x=pre[y]){  
 pre[x]=y,y=lk[x],fa[x]=fa[y]=p;  
 if(tag[y]==2)tag[y]=1,q.push(y);  
 }  
}  
I blossom(int u){  
 FOR(i,1,n)tag[i]=pre[i]=0,fa[i]=i;  
 tag[u]=1,q=queue<int>(),q.push(u);  
 for(int p;!q.empty();q.pop())REP(u=q.front())  
 if(tag[v]==1)  
 p=lca(u,v),shrink(u,v,p),shrink(v,u,p);  
 else if(!tag[v]){  
 pre[v]=u,tag[v]=2;  
 if(!lk[v])return rev(v),1;  
 else tag[lk[v]]=1,q.push(lk[v]);  
 }  
 return 0;  
}  
int main(){  
 scanf("%d%d",&n,&m);  
 for(int x,y;m--;add\_edge(x,y))scanf("%d%d",&x,&y);  
 FOR(i,1,n)ans+=!lk[i]&&blossom(i);  
 cout<<ans<<'\n';  
 FOR(i,1,n)cout<<lk[i]<<' ';  
 return 0;  
}

## 带花树2

// graph  
template <typename T>  
class graph {  
 public:  
 struct edge {  
 int from;  
 int to;  
 T cost;  
 };  
 vector<edge> edges;  
 vector<vector<int> > g;  
 int n;  
 graph(int \_n) : n(\_n) { g.resize(n); }  
 virtual int add(int from, int to, T cost) = 0;  
};  
  
// undirectedgraph  
template <typename T>  
class undirectedgraph : public graph<T> {  
 public:  
 using graph<T>::edges;  
 using graph<T>::g;  
 using graph<T>::n;  
  
 undirectedgraph(int \_n) : graph<T>(\_n) {}  
 int add(int from, int to, T cost = 1) {  
 assert(0 <= from && from < n && 0 <= to && to < n);  
 int id = (int)edges.size();  
 g[from].push\_back(id);  
 g[to].push\_back(id);  
 edges.push\_back({from, to, cost});  
 return id;  
 }  
};  
  
// blossom / find\_max\_unweighted\_matching  
template <typename T>  
vector<int> find\_max\_unweighted\_matching(const undirectedgraph<T> &g) {  
 std::mt19937 rng(chrono::steady\_clock::now().time\_since\_epoch().count());  
 vector<int> match(g.n, -1); // ƥ��  
 vector<int> aux(g.n, -1); // ʱ�����  
 vector<int> label(g.n); // "o" or "i"  
 vector<int> orig(g.n); // ����  
 vector<int> parent(g.n, -1); // ���ڵ�  
 queue<int> q;  
 int aux\_time = -1;  
  
 auto lca = [&](int v, int u) {  
 aux\_time++;  
 while (true) {  
 if (v != -1) {  
 if (aux[v] == aux\_time) { // �ҵ��ݷù��ĵ� Ҳ����LCA  
 return v;  
 }  
 aux[v] = aux\_time;  
 if (match[v] == -1) {  
 v = -1;  
 } else {  
 v = orig[parent[match[v]]]; // ��ƥ���ĸ��ڵ����Ѱ��  
 }  
 }  
 swap(v, u);  
 }  
 }; // lca  
  
 auto blossom = [&](int v, int u, int a) {  
 while (orig[v] != a) {  
 parent[v] = u;  
 u = match[v];  
 if (label[u] == 1) { // ��ʼ����Ϊ"o" ������·  
 label[u] = 0;  
 q.push(u);  
 }  
 orig[v] = orig[u] = a; // ����  
 v = parent[u];  
 }  
 }; // blossom  
  
 auto augment = [&](int v) {  
 while (v != -1) {  
 int pv = parent[v];  
 int next\_v = match[pv];  
 match[v] = pv;  
 match[pv] = v;  
 v = next\_v;  
 }  
 }; // augment  
  
 auto bfs = [&](int root) {  
 fill(label.begin(), label.end(), -1);  
 iota(orig.begin(), orig.end(), 0);  
 while (!q.empty()) {  
 q.pop();  
 }  
 q.push(root);  
 // ��ʼ����Ϊ "o", ������"0"����"o", "1"����"i"  
 label[root] = 0;  
 while (!q.empty()) {  
 int v = q.front();  
 q.pop();  
 for (int id : g.g[v]) {  
 auto &e = g.edges[id];  
 int u = e.from ^ e.to ^ v;  
 if (label[u] == -1) { // �ҵ�δ�ݷõ�  
 label[u] = 1; // ��� "i"  
 parent[u] = v;  
 if (match[u] == -1) { // �ҵ�δƥ���  
 augment(u); // Ѱ������·��  
 return true;  
 }  
 // �ҵ���ƥ��� ������ƥ��ĵ㶪��queue ���콻����  
 label[match[u]] = 0;  
 q.push(match[u]);  
 continue;  
 } else if (label[u] == 0 && orig[v] != orig[u]) {  
 // �ҵ��Ѱݷõ� �ұ��ͬΪ"o" �����ҵ�"��"  
 int a = lca(orig[v], orig[u]);  
 // ��LCA Ȼ������  
 blossom(u, v, a);  
 blossom(v, u, a);  
 }  
 }  
 }  
 return false;  
 }; // bfs  
  
 auto greedy = [&]() {  
 vector<int> order(g.n);  
 // ������� order  
 iota(order.begin(), order.end(), 0);  
 shuffle(order.begin(), order.end(), rng);  
  
 // ������ƥ��ĵ�ƥ��  
 for (int i : order) {  
 if (match[i] == -1) {  
 for (auto id : g.g[i]) {  
 auto &e = g.edges[id];  
 int to = e.from ^ e.to ^ i;  
 if (match[to] == -1) {  
 match[i] = to;  
 match[to] = i;  
 break;  
 }  
 }  
 }  
 }  
 }; // greedy  
  
 // һ��ʼ�����ƥ��  
 greedy();  
 // ��δƥ���������·  
 for (int i = 0; i < g.n; i++) {  
 if (match[i] == -1) {  
 bfs(i);  
 }  
 }  
 return match;  
}

## 强连通（kosaraju）

#include <bits/stdc++.h>  
using namespace std;  
struct SCC {  
 static const int MAXV = 100000;  
 int V;  
 vector<int> g[MAXV], rg[MAXV], vs;  
 bool used[MAXV];  
 int cmp[MAXV];  
  
 void add\_edge(int from, int to) {  
 g[from].push\_back(to);  
 rg[to].push\_back(from);  
 }  
  
 void dfs(int v) {  
 used[v] = 1;  
 for (int i = 0; i < g[v].size(); i++) {  
 if (!used[g[v][i]]) dfs(g[v][i]);  
 }  
 vs.push\_back(v);  
 }  
  
 void rdfs(int v, int k) {  
 used[v] = 1;  
 cmp[v] = k;  
 for (int i = 0; i < rg[v].size(); i++) {  
 if (!used[rg[v][i]]) rdfs(rg[v][i], k);  
 }  
 }  
  
 int solve() {  
 memset(used, 0, sizeof(used));  
 vs.clear();  
 for (int v = 1; v <= V; v++) {  
 if (!used[v]) dfs(v);  
 }  
 memset(used, 0, sizeof(used));  
 int k = 0;  
 for (int i = (int)vs.size() - 1; i >= 0; i--) {  
 if (!used[vs[i]]) rdfs(vs[i], ++k);  
 }  
 return k;  
 }  
  
 void init(int n) {  
 V = n;  
 vs.clear();  
 for (int i = 0; i < MAXV; i++) {  
 g[i].clear();  
 rg[i].clear();  
 used[i] = 0;  
 cmp[i] = 0;  
 }  
 }  
  
} scc;  
  
//记得调用init()

## 强连通（tarjan无vector）

#include <bits/stdc++.h>  
using namespace std;  
struct SCC {  
 static const int MAXN = 5000;  
 static const int MAXM = 2000000;  
 int dfs\_clock, edge\_cnt = 1, scc\_cnt;  
 int head[MAXN];  
 int dfn[MAXN], lowlink[MAXN];  
 int sccno[MAXN];  
 stack<int> s;  
  
 struct edge {  
 int v, next;  
 } e[MAXM];  
  
 void add\_edge(int u, int v) {  
 e[edge\_cnt].v = v;  
 e[edge\_cnt].next = head[u];  
 head[u] = edge\_cnt++;  
 }  
  
 void tarjan(int u) {  
 int v;  
 dfn[u] = lowlink[u] = ++dfs\_clock; //每次dfs，u的次序号增加1  
 s.push(u); //将u入栈  
 for (int i = head[u]; i != -1; i = e[i].next) //访问从u出发的边  
 {  
 v = e[i].v;  
 if (!dfn[v]) //如果v没被处理过  
 {  
 tarjan(v); // dfs(v)  
 lowlink[u] = min(lowlink[u], lowlink[v]);  
 } else if (!sccno[v])  
 lowlink[u] = min(lowlink[u], dfn[v]);  
 }  
 if (dfn[u] == lowlink[u]) {  
 scc\_cnt++;  
 do {  
 v = s.top();  
 s.pop();  
 sccno[v] = scc\_cnt;  
 } while (u != v);  
 }  
 }  
  
 int find\_scc(int n) {  
 for (int i = 1; i <= n; i++)  
 if (!dfn[i]) tarjan(i);  
 return scc\_cnt;  
 }  
  
 void init() {  
 scc\_cnt = dfs\_clock = 0;  
 edge\_cnt = 1; //不用初始化e数组，省时间  
 while (!s.empty()) s.pop();  
 memset(head, -1, sizeof(head));  
 memset(sccno, 0, sizeof(sccno));  
 memset(dfn, 0, sizeof(dfn));  
 memset(lowlink, 0, sizeof(lowlink));  
 }  
} scc;

## 强连通（tarjan）

#include <bits/stdc++.h>  
using namespace std;  
  
struct SCC {  
 static const int MAXN = 100000;  
 vector<int> g[MAXN];  
 int dfn[MAXN], lowlink[MAXN], sccno[MAXN], dfs\_clock, scc\_cnt;  
 stack<int> S;  
  
 void dfs(int u) {  
 dfn[u] = lowlink[u] = ++dfs\_clock;  
 S.push(u);  
 for (int i = 0; i < g[u].size(); i++) {  
 int v = g[u][i];  
 if (!dfn[v]) {  
 dfs(v);  
 lowlink[u] = min(lowlink[u], lowlink[v]);  
 } else if (!sccno[v]) {  
 lowlink[u] = min(lowlink[u], dfn[v]);  
 }  
 }  
 if (lowlink[u] == dfn[u]) {  
 ++scc\_cnt;  
 for (;;) {  
 int x = S.top();  
 S.pop();  
 sccno[x] = scc\_cnt;  
 if (x == u) break;  
 }  
 }  
 }  
  
 void solve(int n) {  
 dfs\_clock = scc\_cnt = 0;  
 memset(sccno, 0, sizeof(sccno));  
 memset(dfn, 0, sizeof(dfn));  
 memset(lowlink, 0, sizeof(lowlink));  
 for (int i = 1; i <= n; i++) {  
 if (!dfn[i]) dfs(i);  
 }  
 }  
} scc;  
  
// scc\_cnt为SCC计数器，sccno[i]为i所在SCC的编号  
// vector<int> g[MAXN]中加边  
//之后再补充init()

## 拓扑排序

#include <bits/stdc++.h>  
using namespace std;  
const int MAXN = 100000;  
  
int c[MAXN];  
int topo[MAXN], t, V;  
vector<int> g[MAXN];  
  
bool dfs(int u) {  
 c[u] = -1;  
 for (int i = 0; i < g[u].size(); i++) {  
 int v = g[u][i];  
 if (c[v] < 0)  
 return false;  
 else if (!c[v] && !dfs(v))  
 return false;  
 }  
 c[u] = 1;  
 topo[t--] = u;  
 return true;  
}  
  
bool toposort(int n) {  
 V = n;  
 t = n;  
 memset(c, 0, sizeof(c));  
 for (int u = 1; u <= V; u++)  
 if (!c[u] && !dfs(u)) return false;  
 return true;  
}

## 数链剖分

ll fa[N], son[N], dep[N], siz[N], dfn[N], rnk[N], top[N];  
ll dfscnt;  
vector<ll> g[N];  
ll tree[N << 1];  
ll lazy[N << 1];  
  
void dfs1(ll u, ll f, ll d) {  
 son[u] = -1;  
 siz[u] = 1;  
 fa[u] = f;  
 dep[u] = d;  
 for (auto v:g[u]) {  
 if (v == f) continue;  
 dfs1(v, u, d + 1);  
 siz[u] += siz[v];  
 if (son[u] == -1 || siz[v] > siz[son[u]]) son[u] = v;  
 }  
}  
  
void dfs2(ll u, ll t) {  
 dfn[u] = ++dfscnt;  
 rnk[dfscnt] = u;  
 top[u] = t;  
 if (son[u] == -1) return;  
 dfs2(son[u], t);  
 for (auto v:g[u]) {  
 if (v == son[u] || v == fa[u]) continue;  
 dfs2(v, v);  
 }  
}  
  
ll lca(ll a, ll b) {  
 while (top[a] != top[b]) {  
 if (dep[top[a]] < dep[top[b]]) swap(a, b);  
 a = fa[top[a]];  
 }  
 return dep[a] < dep[b] ? a : b;  
}  
  
void init() {  
 for (ll i = 0; i < N; i++) g[i].clear();  
 for (ll i = 0; i < (N << 1); i++) {  
 tree[i] = 0;  
 lazy[i] = 0;  
 }  
 dfscnt = 0;  
}  
  
  
void pushdown(ll k, ll l, ll r) {  
 if (k >= N || lazy[k] == 0) return;  
 ll len = (r - l + 1) / 2;  
 tree[k << 1] = tree[k << 1] + len \* lazy[k];  
 tree[k << 1 | 1] = tree[k << 1 | 1] + len \* lazy[k];  
 lazy[k << 1] = lazy[k << 1] + lazy[k];  
 lazy[k << 1 | 1] = lazy[k << 1 | 1] + lazy[k];  
 lazy[k] = 0;  
}  
  
ll merge\_range(ll a, ll b) {  
 ll ans = a + b;  
 return ans;  
}  
  
void change\_range(ll k, ll l, ll r, ll ql, ll qr, ll x) {  
 if (r < ql || qr < l)return;  
 if (ql <= l && r <= qr) {  
 tree[k] = tree[k] + x \* (r - l + 1);  
 lazy[k] = lazy[k] + x;  
 return;  
 }  
 pushdown(k, l, r);  
 ll mid = (l + r) >> 1;  
 change\_range(k << 1, l, mid, ql, qr, x);  
 change\_range(k << 1 | 1, mid + 1, r, ql, qr, x);  
 tree[k] = merge\_range(tree[k << 1], tree[k << 1 | 1]);  
}  
  
ll query\_range(ll k, ll l, ll r, ll ql, ll qr) {  
 if (r < ql || qr < l)return 0;  
 if (ql <= l && r <= qr) {  
 return tree[k];  
 }  
 pushdown(k, l, r);  
 ll mid = (l + r) >> 1;  
 ll lq = query\_range(k << 1, l, mid, ql, qr);  
 ll rq = query\_range(k << 1 | 1, mid + 1, r, ql, qr);  
 return merge\_range(lq, rq);  
}  
  
ll query\_path(ll a, ll b) {  
 ll sum = 0;  
 while (top[a] != top[b]) {  
 if (dep[top[a]] < dep[top[b]]) swap(a, b);  
 sum = sum + query\_range(1, 1, N, dfn[top[a]], dfn[a]);  
 //dfn[top[a]]~dfn[a]  
 a = fa[top[a]];  
 }  
 if (dep[a] > dep[b]) swap(a, b);  
 //点权  
 sum = sum + query\_range(1, 1, N, dfn[a], dfn[b]);  
 //边权  
 //if (a != b) sum = sum + query\_range(1, 1, N, dfn[a] + 1, dfn[b]);  
 //dfn[a]~dfn[b],x  
 return sum;  
}  
  
void change\_path(ll a, ll b, ll x) {  
 while (top[a] != top[b]) {  
 if (dep[top[a]] < dep[top[b]]) swap(a, b);  
 change\_range(1, 1, N, dfn[top[a]], dfn[a], x);  
 //dfn[top[a]]~dfn[a]  
 a = fa[top[a]];  
 }  
 if (dep[a] > dep[b]) swap(a, b);  
 //点权  
 change\_range(1, 1, N, dfn[a], dfn[b], x);  
 //边权  
 //if (a != b) change\_range(1, 1, N, dfn[a] + 1, dfn[b], x);  
 //dfn[a]~dfn[b],x  
}

## 最大流

#include <bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
  
struct Edge {  
 ll from, to, cap, flow;  
 Edge(ll a, ll b, ll c, ll d) : from(a), to(b), cap(c), flow(d) {}  
};  
  
struct Dinic {  
 static const ll maxn = 10000;  
 static const ll inf = 0x3f3f3f3f3f3f3f3f;  
 ll N, M, S, T;  
 vector<Edge> edges;  
 vector<ll> G[maxn];  
 bool vis[maxn];  
 ll d[maxn];  
 ll cur[maxn];  
  
 void AddEdge(ll from, ll to, ll cap) {  
 edges.push\_back(Edge(from, to, cap, 0));  
 edges.push\_back(Edge(to, from, 0, 0));  
 M = edges.size();  
 G[from].push\_back(M - 2);  
 G[to].push\_back(M - 1);  
 }  
  
 bool BFS() {  
 memset(vis, 0, sizeof(vis));  
 queue<ll> Q;  
 Q.push(S);  
 d[S] = 0;  
 vis[S] = 1;  
 while (!Q.empty()) {  
 ll x = Q.front();  
 Q.pop();  
 for (ll i = 0; i < G[x].size(); i++) {  
 Edge& e = edges[G[x][i]];  
 if (!vis[e.to] && e.cap > e.flow) {  
 vis[e.to] = 1;  
 d[e.to] = d[x] + 1;  
 Q.push(e.to);  
 }  
 }  
 }  
 return vis[T];  
 }  
  
 ll DFS(ll x, ll a) {  
 if (x == T || a == 0) return a;  
 ll flow = 0, f;  
 for (ll& i = cur[x]; i < G[x].size(); i++) {  
 Edge& e = edges[G[x][i]];  
 if (d[x] + 1 == d[e.to] &&  
 (f = DFS(e.to, min(a, e.cap - e.flow))) > 0) {  
 e.flow += f;  
 edges[G[x][i] ^ 1].flow -= f;  
 flow += f;  
 a -= f;  
 if (a == 0) break;  
 }  
 }  
 return flow;  
 }  
  
 ll Maxflow(ll S, ll T) {  
 this->S = S, this->T = T;  
 ll flow = 0;  
 while (BFS()) {  
 memset(cur, 0, sizeof(cur));  
 flow += DFS(S, inf);  
 }  
 return flow;  
 }  
} MF;  
  
//有源汇上下界最大流，跑完可行流后，s-t的最大流即为答案  
  
//有源汇上下届最小流，不连无穷边，s-t跑最大流，再加上t-s无穷边，再跑最大流，无穷边流量为答案  
  
//最大权闭合子图  
//构造一个新的流网络，建一个源点s和汇点t，从s向原图中所有点权为正数的点建一条容量为点权的边，  
//从点权为负数的点向t建一条容量为点权绝对值的边，原图中各点建的边都建成容量为正无穷的边。  
//然后求从s到t的最小割，再用所有点权为正的权值之和减去最小割，就是我们要求的最大权值和了。  
  
//最大密度子图  
//01分数规划  
//addedge(S,V,m),addedge(E,1),addedge(V,T,2\*g-deg(v)+m)  
//h(g)=n\*m-maxflow(S,T)

## 最大流（double）

#include <iostream>  
#include <cstring>  
#include <algorithm>  
  
using namespace std;  
  
struct Dinic {  
 static constexpr int N = 10010, M = 100010, INF = 1e8;  
 static constexpr double eps = 1e-8;   
// int n, m, S, T;  
 int S, T;  
 int h[N], e[M], ne[M], idx;  
 double f[M];  
 int q[N], d[N], cur[N]; // d ��ʾ��Դ�㿪ʼ�ߵ��õ��·�������бߵ���������Сֵ   
   
 void AddEdge(int a, int b, double c)  
 {  
 e[idx] = b, f[idx] = c, ne[idx] = h[a], h[a] = idx ++ ;  
 e[idx] = a, f[idx] = 0, ne[idx] = h[b], h[b] = idx ++ ;  
 }  
   
 bool bfs()  
 {  
 int hh = 0, tt = 0;  
 memset(d, -1, sizeof d);  
 q[0] = S, d[S] = 0, cur[S] = h[S];  
 while (hh <= tt)  
 {  
 int t = q[hh ++ ];  
 for (int i = h[t]; ~i; i = ne[i])  
 {  
 int ver = e[i];  
 if (d[ver] == -1 && f[i] > 0)  
 {  
 d[ver] = d[t] + 1;  
 cur[ver] = h[ver];  
 if (ver == T) return true;  
 q[ ++ tt] = ver;  
 }  
 }  
 }  
 return false;  
 }  
   
 double find(int u, double limit)  
 {  
 if (u == T) return limit;  
 double flow = 0;  
 for (int i = cur[u]; ~i && flow < limit; i = ne[i])  
 {  
 cur[u] = i;  
 int ver = e[i];  
 if (d[ver] == d[u] + 1 && f[i] > 0)  
 {  
 double t = find(ver, min(f[i], limit - flow));  
 if (t < eps) d[ver] = -1;  
 f[i] -= t, f[i ^ 1] += t, flow += t;  
 }  
 }  
 return flow;  
 }  
   
 double Maxflow(int S, int T)  
 {  
 this->S = S, this->T = T;  
 double r = 0, flow;  
 while (bfs()) while (flow = find(S, INF)) r += flow;  
 return r;  
 }   
 void init() ////////   
 {  
 memset(h, -1, sizeof h);  
 idx = 0;   
 }  
} MF;  
  
// ?��init

## 最小费用最大流

#include <bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
  
struct Edge {  
 ll from, to, cap, flow, cost;  
 Edge(ll u, ll v, ll c, ll f, ll w):from(u), to(v), cap(c), flow(f), cost(w) {}  
};  
  
struct MCMF {  
 static const ll maxn = 6000;  
 static const ll INF = 0x3f3f3f3f3f3f3f;  
 ll n, m;  
 vector<Edge> edges;  
 vector<ll> G[maxn];  
 ll inq[maxn];  
 ll d[maxn];  
 ll p[maxn];  
 ll a[maxn];  
  
 void init(ll n) {  
 this->n = n;  
 for (ll i = 1; i <= n; i++) G[i].clear();  
 edges.clear();  
 }  
  
 void add\_edge(ll from, ll to, ll cap, ll cost) {  
 from++,to++;//原板子无法使用0点，故修改  
 edges.push\_back(Edge(from, to, cap, 0, cost));  
 edges.push\_back(Edge(to, from, 0, 0, -cost));  
 m = edges.size();  
 G[from].push\_back(m - 2);  
 G[to].push\_back(m - 1);  
 }  
  
 bool BellmanFord(ll s, ll t, ll& flow, ll& cost) {  
 for (ll i = 1; i <= n; ++i) d[i] = INF;  
 memset(inq, 0, sizeof(inq));  
 d[s] = 0, inq[s] = 1, p[s] = 0, a[s] = INF;  
 queue<ll> Q;  
 Q.push(s);  
 while (!Q.empty()) {  
 ll u = Q.front();  
 Q.pop();  
 inq[u] = 0;  
 for (ll i = 0; i < G[u].size(); ++i) {  
 Edge& e = edges[G[u][i]];  
 if (e.cap > e.flow && d[e.to] > d[u] + e.cost) {  
 d[e.to] = d[u] + e.cost;  
 p[e.to] = G[u][i];  
 a[e.to] = min(a[u], e.cap - e.flow);  
 if (!inq[e.to]) {  
 Q.push(e.to);  
 inq[e.to] = 1;  
 }  
 }  
 }  
 }  
 if (d[t] == INF) return false;  
 flow += a[t];  
 cost += (ll)d[t] \* (ll)a[t];  
 for (ll u = t; u != s; u = edges[p[u]].from) {  
 edges[p[u]].flow += a[t];  
 edges[p[u] ^ 1].flow -= a[t];  
 }  
 return true;  
 }  
  
 //需要保证初始网络中没有负权圈  
 ll MincostMaxflow(ll s, ll t, ll& cost) {  
 s++,t++;//原板子无法使用0点，故修改  
 ll flow = 0;  
 cost = 0;  
 while (BellmanFord(s, t, flow, cost));  
 return flow;  
 }  
} mcmf; // 若固定流量k，增广时在flow+a>=k的时候只增广k-flow单位的流量，然后终止程序  
//下标从0开始

## 最小路径覆盖

对于有向无环图（DAG）

定义：在一个有向图中，找出最少的路径，使得这些路径经过了所有的点。

最小路径覆盖分为**最小不相交路径覆盖**和**最小可相交路径覆盖**。

**最小不相交路径覆盖**：每一条路径经过的顶点各不相同。

**最小可相交路径覆盖**：每一条路径经过的顶点可以相同。

**DAG的最小不相交路径覆盖**：

把原图的每个点v拆成和两个点，如果有一条有向边 , 就加边, 这样就得到一个二分图，最小路径覆盖=原图的节点数-新图的最大匹配数

**DAG的最小可相交路径覆盖**：

先用floyd求出原图的传递闭包，即若a到b有路径，则加边, 转化为最小不相交路径覆盖问题

## 最近公共祖先（倍增）

#include <algorithm>  
#include <cstdio>  
#include <cstring>  
#include <iostream>  
using namespace std;  
const int MAX = 600000;  
  
struct edge {  
 int t, nex;  
} e[MAX << 1];  
int head[MAX], tot;  
  
int depth[MAX], fa[MAX][22], lg[MAX];  
  
void add\_edge(int x, int y) {  
 e[++tot].t = y;  
 e[tot].nex = head[x];  
 head[x] = tot;  
  
 e[++tot].t = x;  
 e[tot].nex = head[y];  
 head[y] = tot;  
}  
  
void dfs(int now, int fath) {  
 fa[now][0] = fath;  
 depth[now] = depth[fath] + 1;  
 for (int i = 1; i <= lg[depth[now]]; ++i)  
 fa[now][i] = fa[fa[now][i - 1]][i - 1];  
 for (int i = head[now]; i; i = e[i].nex)  
 if (e[i].t != fath) dfs(e[i].t, now);  
}  
  
int lca(int x, int y) {  
 if (depth[x] < depth[y]) swap(x, y);  
 while (depth[x] > depth[y]) x = fa[x][lg[depth[x] - depth[y]] - 1];  
 if (x == y) return x;  
 for (int k = lg[depth[x]] - 1; k >= 0; --k)  
 if (fa[x][k] != fa[y][k]) x = fa[x][k], y = fa[y][k];  
 return fa[x][0];  
}  
  
void init(int n, int root) {  
 for (int i = 1; i <= n; ++i) lg[i] = lg[i - 1] + (1 << lg[i - 1] == i);  
 dfs(root, 0);  
}

## 最近公共祖先（线段树）

#include <bits/stdc++.h>  
using namespace std;  
int n, m, root;  
const int MAX\_N = 500005;  
const int MAX = 1 << 20;  
vector<int> g[MAX\_N];  
vector<int> vs;  
pair<int, int> tree[MAX \* 2 + 10];  
int fir[MAX\_N];  
int fa[MAX\_N];  
int dep[MAX\_N];  
void dfs(int k, int p, int d) {  
 fa[k] = p;  
 dep[k] = d;  
 vs.push\_back(k);  
 for (int i = 0; i < g[k].size(); i++) {  
 if (g[k][i] != p) {  
 dfs(g[k][i], k, d + 1);  
 vs.push\_back(k);  
 }  
 }  
}  
void build(int k) {  
 if (k >= MAX) return;  
 build(k << 1);  
 build(k << 1 | 1);  
 tree[k] = min(tree[k << 1], tree[k << 1 | 1]);  
}  
pair<int, int> query(int k, int s, int e, int l, int r) {  
 if (e < l || r < s) return pair<int, int>(INT\_MAX, 0);  
 if (l <= s && e <= r) return tree[k];  
 return min(query(k << 1, s, (s + e) >> 1, l, r),  
 query(k << 1 | 1, ((s + e) >> 1) + 1, e, l, r));  
}  
void init() {  
 dfs(root, root, 0);  
 for (int i = 0; i < MAX \* 2 + 10; i++) tree[i] = pair<int, int>(INT\_MAX, 0);  
 for (int i = MAX; i < MAX + vs.size(); i++)  
 tree[i] = pair<int, int>(dep[vs[i - MAX]], vs[i - MAX]);  
 for (int i = 0; i < vs.size(); i++) {  
 if (fir[vs[i]] == 0) fir[vs[i]] = i + 1;  
 }  
 build(1);  
}  
int lca(int a, int b) {  
 return query(1, 1, MAX, min(fir[a], fir[b]), max(fir[a], fir[b])).second;  
}  
int main() {  
 scanf("%d%d%d", &n, &m, &root);  
 for (int i = 1; i < n; i++) {  
 int a, b;  
 scanf("%d%d", &a, &b);  
 g[a].push\_back(b);  
 g[b].push\_back(a);  
 }  
 init();  
 for (int i = 1; i <= m; i++) {  
 int a, b;  
 scanf("%d%d", &a, &b);  
 printf("%d\n", lca(a, b));  
 }  
}

## 有源汇上下界最大小流

#include <bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
  
struct Edge {  
 ll from, to, cap, flow, mn;  
 Edge(ll a, ll b, ll c, ll d, ll e) : from(a), to(b), cap(c), flow(d), mn(e) {}  
};  
  
ll n, m;  
  
struct Dinic {  
 static const ll maxn = 50010; // 点的大小，记得改  
 static const ll inf = 0x3f3f3f3f3f3f3f3f;  
 ll N, M, S, T;  
 vector<Edge> edges;  
 vector<ll> G[maxn];  
 bool vis[maxn];  
 ll d[maxn];  
 ll cur[maxn];  
  
 void AddEdge(ll from, ll to, ll cap, ll c) {  
 edges.push\_back(Edge(from, to, cap, 0, c));  
 edges.push\_back(Edge(to, from, 0, 0, c));  
 M = edges.size();  
 G[from].push\_back(M - 2);  
 G[to].push\_back(M - 1);  
 }  
  
 bool BFS() {  
 memset(vis, 0, sizeof(vis));  
 queue<ll> Q;  
 Q.push(S);  
 d[S] = 0;  
 vis[S] = 1;  
 while (!Q.empty()) {  
 ll x = Q.front();  
 Q.pop();  
 for (ll i = 0; i < G[x].size(); i++) {  
 Edge& e = edges[G[x][i]];  
 if (!vis[e.to] && e.cap > e.flow) {  
 vis[e.to] = 1;  
 d[e.to] = d[x] + 1;  
 Q.push(e.to);  
 }  
 }  
 }  
 return vis[T];  
 }  
  
 ll DFS(ll x, ll a) {  
 if (x == T || a == 0) return a;  
 ll flow = 0, f;  
 for (ll& i = cur[x]; i < G[x].size(); i++) {  
 Edge& e = edges[G[x][i]];  
 if (d[x] + 1 == d[e.to] &&  
 (f = DFS(e.to, min(a, e.cap - e.flow))) > 0) {  
 e.flow += f;  
 edges[G[x][i] ^ 1].flow -= f;  
 flow += f;  
 a -= f;  
 if (a == 0) break;  
 }  
 }  
 return flow;  
 }  
  
 void deleteEdge(ll u, ll v) {  
 ll siz = edges.size();  
 for(ll i = 0; i < siz; ++ i) {  
 if(edges[i].from == u && edges[i].to == v) {  
 edges[i].cap = edges[i].flow = 0;  
 edges[i ^ 1].cap = edges[i ^ 1].flow = 0;   
 break;   
 }  
  
 }  
  
 }  
  
 ll getValue() {  
 return edges[2 \* m].flow;  
 }   
  
 ll Maxflow(ll S, ll T) {  
 this->S = S, this->T = T;  
 ll flow = 0;  
 while (BFS()) {  
 memset(cur, 0, sizeof(cur));  
 flow += DFS(S, inf);  
 }  
 return flow;  
 }  
} MF;  
  
int main() {  
 ll s, t;  
 cin >> n >> m >> s >> t;  
 // n个点，m条边，给的源点汇点  
  
 ll mp[50010] = {0}; // 点的大小，记得改  
 for(ll i = 1; i <= m; ++ i) {  
 ll a, b, c, d; // 从a到b有一条下界c上界d的边  
 cin >> a >> b >> c >> d;  
 mp[b] += c;  
 mp[a] -= c;  
 MF.AddEdge(a, b, d - c, c);  
 }  
 MF.AddEdge(t, s, 1e18, 0); //  
 ll tot = 0;  
 for(ll i = 1; i <= n; ++ i) {  
 if(mp[i] > 0) {  
 tot += mp[i];  
 MF.AddEdge(0, i , mp[i], 0);  
 }  
 else {  
 MF.AddEdge(i, n + 1, -mp[i], 0);  
 }  
 }  
  
 if( MF.Maxflow(0, n + 1) != tot) {   
 cout << "No Solution" << endl;  
 }   
 else {  
 ll res = MF.getValue(); // 从t到s边的流量  
 MF.deleteEdge(t, s);  
 //cout << res + MF.Maxflow(s, t) << endl; // 最大流  
 cout << res - MF.Maxflow(t, s) << endl; // 最小流  
 }  
  
 return 0;  
}

## 朱刘算法

#include <iostream>  
#include <cstring>  
#include <cstdio>  
#include <algorithm>  
#include <cmath>  
  
#define x first  
#define y second  
  
using namespace std;  
  
typedef pair<double, double> PDD;  
  
const int N = 110;  
const double INF = 1e8;  
  
int n, m;  
PDD q[N];  
bool g[N][N];  
double d[N][N], bd[N][N];  
int pre[N], bpre[N];  
int dfn[N], low[N], ts, stk[N], top;  
int id[N], cnt;  
bool st[N], ins[N];  
  
void dfs(int u) {  
 st[u] = true;  
 for (int i = 1; i <= n; i++)  
 if (g[u][i] && !st[i])  
 dfs(i);  
}  
  
bool check\_con() {  
 memset(st, 0, sizeof st);  
 dfs(1);  
 for (int i = 1; i <= n; i++)  
 if (!st[i])  
 return false;  
 return true;  
}  
  
double get\_dist(int a, int b) {  
 double dx = q[a].x - q[b].x;  
 double dy = q[a].y - q[b].y;  
 return sqrt(dx \* dx + dy \* dy);  
}  
  
void tarjan(int u) {  
 dfn[u] = low[u] = ++ts;  
 stk[++top] = u, ins[u] = true;  
  
 int j = pre[u];  
 if (!dfn[j]) {  
 tarjan(j);  
 low[u] = min(low[u], low[j]);  
 } else if (ins[j]) low[u] = min(low[u], dfn[j]);  
  
 if (low[u] == dfn[u]) {  
 int y;  
 ++cnt;  
 do {  
 y = stk[top--], ins[y] = false, id[y] = cnt;  
 } while (y != u);  
 }  
}  
  
double work() {  
 double res = 0;  
 for (int i = 1; i <= n; i++)  
 for (int j = 1; j <= n; j++)  
 if (g[i][j]) d[i][j] = get\_dist(i, j);  
 else d[i][j] = INF;  
  
 while (true) {  
 for (int i = 1; i <= n; i++) {  
 pre[i] = i;  
 for (int j = 1; j <= n; j++)  
 if (d[pre[i]][i] > d[j][i])  
 pre[i] = j;  
 }  
  
 memset(dfn, 0, sizeof dfn);  
 ts = cnt = 0;  
 for (int i = 1; i <= n; i++)  
 if (!dfn[i])  
 tarjan(i);  
  
 if (cnt == n) {  
 for (int i = 2; i <= n; i++) res += d[pre[i]][i];  
 break;  
 }  
  
 for (int i = 2; i <= n; i++)  
 if (id[pre[i]] == id[i])  
 res += d[pre[i]][i];  
  
 for (int i = 1; i <= cnt; i++)  
 for (int j = 1; j <= cnt; j++)  
 bd[i][j] = INF;  
  
 for (int i = 1; i <= n; i++)  
 for (int j = 1; j <= n; j++)  
 if (d[i][j] < INF && id[i] != id[j]) {  
 int a = id[i], b = id[j];  
 if (id[pre[j]] == id[j]) bd[a][b] = min(bd[a][b], d[i][j] - d[pre[j]][j]);  
 else bd[a][b] = min(bd[a][b], d[i][j]);  
 }  
  
 n = cnt;  
 memcpy(d, bd, sizeof d);  
 }  
  
 return res;  
}  
  
int main() {  
 while (~scanf("%d%d", &n, &m)) {  
 for (int i = 1; i <= n; i++) scanf("%lf%lf", &q[i].x, &q[i].y);  
  
 memset(g, 0, sizeof g);  
 while (m--) {  
 int a, b;  
 scanf("%d%d", &a, &b);  
 if (a != b && b != 1) g[a][b] = true;  
 }  
  
 if (!check\_con()) puts("poor snoopy");  
 else printf("%.2lf\n", work());  
 }  
  
 return 0;  
}

## 树上启发式合并

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const int N = 2e5 + 10;  
  
int vis[N], now;  
  
vector<int> g[N];  
int fa[N], son[N], siz[N], ans[N];  
  
void insert(int pos) {  
 vis[pos] = 1;  
 now = now + 1 - vis[pos - 1] - vis[pos + 1];  
}  
  
void remove(int pos) {  
 vis[pos] = 0;  
 now = now - 1 + vis[pos - 1] + vis[pos + 1];  
}  
  
void dfs1(ll u, ll f) {  
 siz[u] = 1;  
 fa[u] = f;  
 son[u] = -1;  
 for (auto v:g[u]) {  
 if (v == f) continue;  
 dfs1(v, u);  
 siz[u] += siz[v];  
 if (son[u] == -1 || siz[v] > siz[son[u]]) son[u] = v;  
 }  
}  
  
void add(int u, int exc, int op) {  
 if (op) insert(u);  
 else remove(u);  
 for (auto x:g[u]) {  
 if (x == fa[u] || x == exc) continue;  
 add(x, exc, op);  
 }  
}  
  
void dfs(ll u, ll opt) {  
 for (auto x:g[u]) {  
 if (x == fa[u] || x == son[u]) continue;  
 dfs(x, 0);  
 }  
 if (son[u] != -1) dfs(son[u], 1);  
 add(u, son[u], 1);  
 ans[u] = now;  
 if (!opt) {  
 add(u, 0, 0);  
 }  
}  
  
  
int main() {  
 ios::sync\_with\_stdio(false),  
 cin.tie(nullptr),  
 cout.tie(nullptr);  
 int t;  
 cin >> t;  
 int test = 0;  
 while (t--) {  
 int n;  
 cin >> n;  
 for (int i = 1; i < n; i++) {  
 int a, b;  
 cin >> a >> b;  
 g[a].push\_back(b);  
 g[b].push\_back(a);  
 }  
 cout << "Case #" << ++test << ": ";  
 dfs1(1, -1);  
 dfs(1, 0);  
 for (int i = 1; i <= n; i++) {  
 if (i != 1) cout << ' ';  
 cout << ans[i];  
 }  
 cout << endl;  
 for (int i = 1; i <= n; i++) g[i].clear();  
 }  
}

## 树分治

#include <bits/stdc++.h>  
using namespace std;  
const int MAXN = 10005;  
const int INF = 1000000000;  
struct edge {  
 int to, length;  
 edge() {}  
 edge(int a, int b) : to(a), length(b) {}  
};  
  
  
vector<edge> g[MAXN];  
  
bool centroid[MAXN];  
int subtree\_size[MAXN];  
  
int ans;  
  
//计算子树大小  
int compute\_subtree\_size(int v, int p) {  
 int c = 1;  
 for (int i = 0; i < g[v].size(); i++) {  
 int w = g[v][i].to;  
 if (w == p || centroid[w]) continue;  
 c += compute\_subtree\_size(w, v);  
 }  
 subtree\_size[v] = c;  
 return c;  
}  
  
//查找重心，t为连通分量大小  
// pair（最大子树顶点数，顶点编号）  
pair<int, int> search\_centroid(int v, int p, int t) {  
 pair<int, int> res = pair<int, int>(INF, -1);  
 int s = 1, m = 0;  
 for (int i = 0; i < g[v].size(); i++) {  
 int w = g[v][i].to;  
 if (w == p || centroid[w]) continue;  
 res = min(res, search\_centroid(w, v, t));  
 m = max(m, subtree\_size[w]);  
 s += subtree\_size[w];  
 }  
 m = max(m, t - s);  
 res = min(res, pair<int, int>(m, v));  
 return res;  
}  
  
void init(int n) {  
 memset(centroid, 0, sizeof(centroid));  
 memset(subtree\_size, 0, sizeof(subtree\_size));  
 for (int i = 0; i <= n; i++) g[i].clear();  
 ans = 0;  
}  
  
int solve(int u) {  
 compute\_subtree\_size(u, -1);  
 int s = search\_centroid(u, -1, subtree\_size[u]).second;  
 centroid[s] = 1;  
 for (int i = 0; i < g[s].size(); i++) {  
 int v = g[s][i].to;  
 if (centroid[v]) continue;  
 /\*solve()\*/  
 }  
 /\*do something\*/  
 centroid[s] = 0;  
 return ans;  
}

## 欧拉回路

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const int N = 1e6 + 10;  
  
  
int stk[N], top;  
struct edge {  
 int to, idx;  
};  
  
vector<edge> g[N];  
  
namespace Euler1 { //无向图欧拉回路  
 bool vis[N];  
 int cur[N];  
  
 void dfs(int u, const int &w) {  
 vis[abs(w)] = true;  
 for (int &i = cur[u]; i < g[u].size();) {  
 int idx = g[u][i].idx, v = g[u][i].to;  
 i++;  
 if (!vis[abs(idx)]) dfs(v, idx);  
 }  
 stk[++top] = w;  
 }  
  
 bool solve(int n) {  
 // init();  
 for (int i = 0; i <= n; i++) cur[i] = 0;  
 for (int i = 0; i <= n; i++) vis[i] = 0;  
 // calculate degree  
 for (int i = 1; i <= n; i++) {  
 if (g[i].size() & 1) return false;  
 }  
 // Hierholzer  
 for (int i = 1; i <= n; i++)  
 if (!g[i].empty()) {  
 dfs(i, 0);  
 break;  
 }  
 return true;  
 }  
} // namespace Euler1  
  
namespace Euler2 { // 有向图欧拉回路  
 int deg[N], cur[N];  
  
 void dfs(int u, const int &w) {  
 for (int &i = cur[u]; i < g[u].size();) {  
 int idx = g[u][i].idx, v = g[u][i].to;  
 i++;  
 dfs(v, idx);  
 }  
 stk[++top] = w;  
 }  
  
 bool solve(int n) {  
 // init  
 for (int i = 0; i <= n; i++) deg[i] = 0;  
 for (int i = 0; i <= n; i++) cur[i] = 0;  
 // calculate degree  
 for (int i = 1; i <= n; ++i) {  
 for (auto x: g[i]) deg[i]++, deg[x.to]--;  
 }  
 for (int i = 1; i <= n; ++i)  
 if (deg[i]) return false;  
 // Hierholzer  
 for (int i = 1; i <= n; ++i)  
 if (!g[i].empty()) {  
 dfs(i, 0);  
 break;  
 }  
 return true;  
 }  
} // namespace Euler2  
  
int main() {  
 int t, n, m;  
 cin >> t >> n >> m;  
 for (int u, v, i = 1; i <= m; i++) {  
 cin >> u >> v;  
 g[u].push\_back({v, i});  
 if (t == 1) g[v].push\_back({u, -i});  
 }  
 // solve  
 bool flag = t == 1 ? Euler1::solve(n) : Euler2::solve(n);  
 // output  
 if (!flag || (m > 0 && top - 1 < m))  
 puts("NO");  
 else {  
 puts("YES");  
 for (int i = top - 1; i > 0; --i) printf("%d%c", stk[i], " \n"[i == 1]);  
 }  
 return 0;  
}

## 点分树

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const ll N = 2e5 + 10;  
  
ll age[N];  
struct edge {  
 ll to, val;  
};  
  
struct father {  
 ll u, num;  
 ll dist;  
};  
  
struct son {  
 ll age, dist;  
  
 bool operator<(const son &s) const {  
 return age < s.age;  
 }  
};  
  
vector<father> f[N];  
vector<vector<son> > s[N];  
vector<edge> g[N];  
bool st[N];  
ll siz[N];  
  
ll getsiz(ll u, ll fa) {  
 if (st[u]) return 0;  
 siz[u] = 1;  
 for (auto x:g[u]) {  
 if (x.to == fa) continue;  
 if (st[x.to]) continue;  
 siz[u] += getsiz(x.to, u);  
 }  
 return siz[u];  
}  
  
void getwc(ll u, ll fa, ll tot, ll &wc) {  
 if (st[u]) return;  
 ll mmax = 0, sum = 1;  
 for (auto x:g[u]) {  
 if (x.to == fa) continue;  
 if (st[x.to]) continue;  
 getwc(x.to, u, tot, wc);  
 mmax = max(mmax, siz[x.to]);  
 sum += siz[x.to];  
 }  
 mmax = max(mmax, tot - sum);  
 if (2 \* mmax <= tot) wc = u;  
}  
  
void getdist(ll u, ll fa, ll now, ll rt, ll kth, vector<son> &v) {  
 if (st[u]) return;  
 f[u].push\_back({rt, kth, now});  
 v.push\_back({age[u], now});  
 for (auto x:g[u]) {  
 if (x.to == fa || st[x.to]) continue;  
 getdist(x.to, u, now + x.val, rt, kth, v);  
 }  
}  
  
void calc(ll u) {  
 if (st[u]) return;  
 getwc(u, -1, getsiz(u, -1), u);  
  
 st[u] = 1;  
  
 for (auto x: g[u]) {  
 if (st[x.to]) continue;  
 s[u].push\_back(vector<son>(0));  
 auto &v = s[u].back();  
 v.push\_back({-0x3f3f3f3f, 0});  
 v.push\_back({0x3f3f3f3f, 0});  
 getdist(x.to, u, x.val, u, (ll) s[u].size() - 1, v);  
 sort(v.begin(), v.end(), [](son a, son b) { return a.age < b.age; });  
 for (ll i = 1; i < v.size(); i++) {  
 v[i].dist += v[i - 1].dist;  
 }  
 }  
 for (auto x:g[u]) {  
 calc(x.to);  
 }  
}  
  
ll query(ll u, ll l, ll r) {  
 ll ans = 0;  
 for (auto x:f[u]) {  
 if (l <= age[x.u] && age[x.u] <= r) ans += x.dist;  
 for (ll i = 0; i < s[x.u].size(); i++) {  
 if (i == x.num) continue;  
 auto &v = s[x.u][i];  
 ll btn = lower\_bound(v.begin(), v.end(), (son) {l, 0}) - v.begin() - 1;  
 ll top = upper\_bound(v.begin(), v.end(), (son) {r, 0}) - v.begin() - 1;  
 ans += v[top].dist - v[btn].dist;  
 ans += (top - btn) \* x.dist;  
 }  
 }  
 for (auto v:s[u]) {  
 ll btn = lower\_bound(v.begin(), v.end(), (son) {l, 0}) - v.begin() - 1;  
 ll top = upper\_bound(v.begin(), v.end(), (son) {r, 0}) - v.begin() - 1;  
 ans += v[top].dist - v[btn].dist;  
  
 }  
 return ans;  
}  
  
signed main() {  
 ios::sync\_with\_stdio(false);  
 cin.tie(nullptr);  
 cout.tie(nullptr);  
  
 ll n, q, a;  
 cin >> n >> q >> a;  
 for (ll i = 1; i <= n; i++) cin >> age[i];  
 for (ll i = 1; i < n; i++) {  
 ll x, y, z;  
 cin >> x >> y >> z;  
 g[x].push\_back({y, z});  
 g[y].push\_back({x, z});  
 }  
  
 calc(1);  
  
 ll ans = 0;  
 while (q--) {  
 ll u, l, r;  
 cin >> u >> l >> r;  
 l = (l + ans) % a;  
 r = (r + ans) % a;  
 if (l > r) swap(l, r);  
 ans = query(u, l, r);  
 cout << ans << endl;  
 }  
}

## 点双连通分量+缩点建图

#include <bits/stdc++.h>  
  
#define ll long long  
using namespace std;  
const int N = 10010;  
const int M = 10010 \* 4;  
int head[N];  
int ver[M];  
int Next[M];  
int tot, n, m;  
  
void add(int x, int y) {  
 ver[++tot] = y;  
 Next[tot] = head[x];  
 head[x] = tot;  
}  
  
int root;  
vector<int> dcc[N];  
int stackk[N];  
int dfn[N], low[N];  
int num = 0;//ʱ���  
int top;//stackk  
int cnt = 0;//��ͨ����Ŀ  
bool cut[N];//����ж�  
void tarjan(int x) {  
 dfn[x] = low[x] = ++num;  
 stackk[++top] = x;  
 if (x == root && head[x] == 0) {  
 dcc[++cnt].push\_back(x);//cnt��ͨ����  
 return;  
 }  
 int flag = 0;  
 for (int i = head[x]; i; i = Next[i]) {  
 int y = ver[i];  
 if (!dfn[y]) {  
 tarjan(y);  
 low[x] = min(low[x], low[y]);  
 if (low[y] >= dfn[x]) {  
 flag++;  
 if (x != root || flag > 1)cut[x] = true;  
 cnt++;  
 int z;  
 do//������Ԫ����xһ�𹹳�һ����ͨ��(����˵���������еĽڵ�+���?)  
 {  
 z = stackk[top--];  
 dcc[cnt].push\_back(z);  
 } while (z != y);  
 dcc[cnt].push\_back(x);  
 }  
 } else low[x] = min(low[x], dfn[y]);  
 }  
}  
  
int tot2 = 1;  
int new\_id[N];  
  
int hc[N];  
int vc[M];  
int nc[M];  
  
void add\_c(int x, int y) {  
 vc[++tot2] = y;  
 nc[tot2] = hc[x];  
 hc[x] = tot2;  
}  
  
int main() {  
 while (cin >> n >> m) {  
 tot = 1;//������^������ʸ��ߵ��յ�  
 for (int i = 1; i <= m; ++i) {  
 int x, y;  
 cin >> x >> y;  
 if (x == y)continue;  
 add(x, y);  
 add(y, x);  
 }  
 for (int i = 1; i <= n; ++i) {  
 if (!dfn[i])root = i, tarjan(i);  
 }  
 /\*for(int i=1;i<=n;++i)  
 if(cut[i])printf("%d ",i);\*/  
 //��������ͬʱ��V-DCC  
 //�������ÿ����ͨ���еĵ�  
 for (int i = 1; i <= cnt; ++i) {  
 for (int j = 0; j < dcc[i].size(); ++j)cout << i << " " << dcc[i][j] << endl;  
 }  
  
 //����  
 tot2 = 1;  
 int num2 = cnt;  
 for (int i = 1; i <= n; ++i) {  
 if (cut[i])new\_id[i] = ++num2;//���������±��,�൱��ÿ����㵥����Ϊһ����ͨ��  
 }  
 for (int i = 1; i <= cnt; ++i) {  
 for (int j = 0; j < dcc[i].size(); ++j) {  
 int x = dcc[i][j];  
 if (cut[x])//һ����ͨ��������ֻ��һ����㣬ͨ������ǰ���Щ��ͨ����������;  
 {  
 add\_c(i, new\_id[x]);  
 add\_c(new\_id[x], i);  
 } else new\_id[x] = i;//������ֻ����һ����ͨ��  
 }  
 }  
  
 //���������ͼ�и���֮����ڽӹ�ϵ���ٴ�ע��^���ŵ�ʹ�ã�i��2��ʼ��ÿ�μ�2��<tot2����<=��  
 for (int i = 2; i < tot2; i += 2)  
 cout << vc[i ^ 1] << " " << vc[i] << endl;  
  
  
 }  
 return 0;  
}  
  
/\*  
 \* tot2Ϊ��������2��ʼ����  
 \* num2Ϊ����֮��ĵ���  
 \* ��˫���㣬�����Խ��Խ�࣬ע��N��С  
\*/

## 虚树

ll fa[N], son[N], dep[N], siz[N], dfn[N], rnk[N], top[N];  
ll dfscnt;  
vector<ll> g[N];  
ll mmin[N];  
  
void dfs1(ll u, ll f, ll d) {  
 son[u] = -1;  
 siz[u] = 1;  
 fa[u] = f;  
 dep[u] = d;  
 for (auto v:g[u]) {  
 if (v == f) continue;  
 dfs1(v, u, d + 1);  
 siz[u] += siz[v];  
 if (son[u] == -1 || siz[v] > siz[son[u]]) son[u] = v;  
 }  
}  
  
void dfs2(ll u, ll t) {  
 dfn[u] = ++dfscnt;  
 rnk[dfscnt] = u;  
 top[u] = t;  
 if (son[u] == -1) return;  
 dfs2(son[u], t);  
 for (auto v:g[u]) {  
 if (v == son[u] || v == fa[u]) continue;  
 dfs2(v, v);  
 }  
}  
  
ll lca(ll a, ll b) {  
 while (top[a] != top[b]) {  
 if (dep[top[a]] < dep[top[b]]) swap(a, b);  
 a = fa[top[a]];  
 }  
 return dep[a] < dep[b] ? a : b;  
}  
  
struct edge {  
 ll s, t, v;  
};  
edge e[N];  
  
vector<int> vg[N];  
int sta[N], tot;  
int h[N];  
  
void build(int \*H, int num) {  
 sort(H + 1, H + 1 + num, [](int a, int b) { return dfn[a] < dfn[b]; });  
 sta[tot = 1] = 1, vg[1].clear();// 1 号节点入栈，清空 1 号节点对应的邻接表，设置邻接表边数为 1  
 for (int i = 1, l; i <= num; ++i) {  
 if (H[i] == 1) continue; //如果 1 号节点是关键节点就不要重复添加  
 l = lca(H[i], sta[tot]); //计算当前节点与栈顶节点的 LCA  
 if (l != sta[tot]) { //如果 LCA 和栈顶元素不同，则说明当前节点不再当前栈所存的链上  
 while (dfn[l] < dfn[sta[tot - 1]]) {//当次大节点的 Dfs 序大于 LCA 的 Dfs 序  
 vg[sta[tot - 1]].push\_back(sta[tot]);  
 vg[sta[tot]].push\_back(sta[tot - 1]);  
 tot--;  
 } //把与当前节点所在的链不重合的链连接掉并且弹出  
 if (dfn[l] > dfn[sta[tot - 1]]) { //如果 LCA 不等于次大节点（这里的大于其实和不等于没有区别）  
 vg[l].clear();  
 vg[l].push\_back(sta[tot]);  
 vg[sta[tot]].push\_back(l);  
 sta[tot] = l;//说明 LCA 是第一次入栈，清空其邻接表，连边后弹出栈顶元素，并将 LCA 入栈  
 } else {  
 vg[l].push\_back(sta[tot]);  
 vg[sta[tot]].push\_back(l);  
 tot--; //说明 LCA 就是次大节点，直接弹出栈顶元素  
 }  
 }  
 vg[H[i]].clear();  
 sta[++tot] = H[i];  
 //当前节点必然是第一次入栈，清空邻接表并入栈  
 }  
 for (int i = 1; i < tot; ++i) {  
 vg[sta[i]].push\_back(sta[i + 1]);  
 vg[sta[i + 1]].push\_back(sta[i]);  
 } //剩余的最后一条链连接一下  
 return;  
}

# 多项式

## fft

const double Pi = acos(-1.0);  
  
struct Complex {  
 double x, y;  
  
 Complex(double xx = 0, double yy = 0) { x = xx, y = yy; }  
} a[N], b[N];  
  
Complex operator+(Complex \_a, Complex \_b) { return Complex(\_a.x + \_b.x, \_a.y + \_b.y); }  
  
Complex operator-(Complex \_a, Complex \_b) { return Complex(\_a.x - \_b.x, \_a.y - \_b.y); }  
  
Complex operator\*(Complex \_a, Complex \_b) {  
 return Complex(\_a.x \* \_b.x - \_a.y \* \_b.y, \_a.x \* \_b.y + \_a.y \* \_b.x);  
} //不懂的看复数的运算那部分  
  
int L, r[N];  
int limit = 1;  
  
void fft(Complex \*A, int type) {  
 for (int i = 0; i < limit; i++)  
 if (i < r[i]) swap(A[i], A[r[i]]); //求出要迭代的序列  
 for (int mid = 1; mid < limit; mid <<= 1) { //待合并区间的长度的一半  
 Complex Wn(cos(Pi / mid), type \* sin(Pi / mid)); //单位根  
 for (int R = mid << 1, j = 0; j < limit; j += R) { //R是区间的长度，j表示前已经到哪个位置了  
 Complex w(1, 0); //幂  
 for (int k = 0; k < mid; k++, w = w \* Wn) { //枚举左半部分  
 Complex x = A[j + k], y = w \* A[j + mid + k]; //蝴蝶效应  
 A[j + k] = x + y;  
 A[j + mid + k] = x - y;  
 }  
 }  
 }  
}  
  
void FFT(int n, int m) {  
 limit = 1;  
 L = 0;  
 while (limit <= n + m) limit <<= 1, L++;  
 for (int i = 0; i < limit; i++) r[i] = (r[i >> 1] >> 1) | ((i & 1) << (L - 1));  
 // 在原序列中 i 与 i/2 的关系是 ： i可以看做是i/2的二进制上的每一位左移一位得来  
 // 那么在反转后的数组中就需要右移一位，同时特殊处理一下奇数  
 fft(a, 1), fft(b, 1);  
 for (int i = 0; i <= limit; i++) a[i] = a[i] \* b[i];  
 fft(a, -1);  
 for (int i = 0; i <= n + m; i++) a[i].x /= limit;  
}

## ntt

const ll mod = 998244353, G = 3, Gi = 332748118;  
  
int limit = 1, L, r[N];  
ll a[N], b[N];  
  
ll qpow(ll \_a, ll \_b) {  
 ll ans = 1;  
 while (\_b) {  
 if (\_b & 1) ans = (ans \* \_a) % mod;  
 \_b >>= 1;  
 \_a = (\_a \* \_a) % mod;  
 }  
 return ans;  
}  
  
void ntt(ll \*A, int type) {  
 auto swap = [](ll &\_a, ll &\_b) {  
 \_a ^= \_b, \_b ^= \_a, \_a ^= \_b;  
 };  
 for (int i = 0; i < limit; i++)  
 if (i < r[i]) swap(A[i], A[r[i]]);  
 for (int mid = 1; mid < limit; mid <<= 1) {  
 ll Wn = qpow(type == 1 ? G : Gi, (mod - 1) / (mid << 1));  
 for (int j = 0; j < limit; j += (mid << 1)) {  
 ll w = 1;  
 for (int k = 0; k < mid; k++, w = (w \* Wn) % mod) {  
 int x = A[j + k], y = w \* A[j + k + mid] % mod;  
 A[j + k] = (x + y) % mod,  
 A[j + k + mid] = (x - y + mod) % mod;  
 }  
 }  
 }  
}  
  
void NTT(int n, int m) {   
 limit = 1;  
 L = 0;  
 while (limit <= n + m) limit <<= 1, L++;  
 for (int i = 0; i < limit; i++) r[i] = (r[i >> 1] >> 1) | ((i & 1) << (L - 1));  
 ntt(a, 1), ntt(b, 1);  
 for (int i = 0; i < limit; i++) a[i] = (a[i] \* b[i]) % mod;  
 ntt(a, -1);  
 ll inv = qpow(limit, mod - 2);  
 for (int i = 0; i <= n + m; i++) a[i] = a[i] \* inv % mod;  
}

## 多项式全家桶

//#pragma GCC optimize(2)  
#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
  
const ll N = 3000007;  
const ll p = 998244353, gg = 3, ig = 332738118, img = 86583718;  
const ll mod = 998244353;  
  
ll qpow(ll a, ll b) {  
 ll res = 1;  
 while (b) {  
 if (b & 1) res = 1ll \* res \* a % mod;  
 a = 1ll \* a \* a % mod;  
 b >>= 1;  
 }  
 return res;  
}  
  
namespace Poly {  
#define mul(x, y) (1ll \* x \* y >= mod ? 1ll \* x \* y % mod : 1ll \* x \* y)  
#define minus(x, y) (1ll \* x - y < 0 ? 1ll \* x - y + mod : 1ll \* x - y)  
#define plus(x, y) (1ll \* x + y >= mod ? 1ll \* x + y - mod : 1ll \* x + y)  
#define ck(x) (x >= mod ? x - mod : x)//取模运算太慢了  
  
 typedef vector<ll> poly;  
 const ll G = 3;//根据具体的模数而定，原根可不一定不一样！！！  
 //一般模数的原根为 2 3 5 7 10 6  
 const ll inv\_G = qpow(G, mod - 2);  
 ll RR[N], inv[N];  
  
 void init() {  
 inv[0] = inv[1] = 1;  
 for (ll i = 2; i < N; ++i)  
 inv[i] = 1ll \* inv[mod % i] \* (mod - mod / i) % mod;  
 }  
  
 ll NTT\_init(ll n) {//快速数论变换预处理  
 ll limit = 1, L = 0;  
 while (limit <= n) limit <<= 1, L++;  
 for (ll i = 0; i < limit; ++i)  
 RR[i] = (RR[i >> 1] >> 1) | ((i & 1) << (L - 1));  
 return limit;  
 }  
  
// 省空间用  
 ll deer[2][N];  
  
 void NTT(poly &A, ll type, ll limit) {//快速数论变换  
 A.resize(limit);  
 for (ll i = 0; i < limit; ++i)  
 if (i < RR[i])  
 swap(A[i], A[RR[i]]);  
 for (ll mid = 2, j = 1; mid <= limit; mid <<= 1, ++j) {  
 ll len = mid >> 1;  
  
// 省空间用  
 ll buf1 = qpow(G, (mod - 1) / (1 << j));  
 ll buf0 = qpow(inv\_G, (mod - 1) / (1 << j));  
 deer[0][0] = deer[1][0] = 1;  
 for (ll i = 1; i < (1 << j); ++i) {  
 deer[0][i] = 1ll \* deer[0][i - 1] \* buf0 % mod;//逆  
 deer[1][i] = 1ll \* deer[1][i - 1] \* buf1 % mod;  
 }  
  
 for (ll pos = 0; pos < limit; pos += mid) {  
// ll \*wn = deer[type][j];  
// 省空间用  
 ll \*wn = deer[type];  
 for (ll i = pos; i < pos + len; ++i, ++wn) {  
 ll tmp = 1ll \* (\*wn) \* A[i + len] % mod;  
 A[i + len] = ck(A[i] - tmp + mod);  
 A[i] = ck(A[i] + tmp);  
 }  
 }  
 }  
 if (type == 0) {  
 for (ll i = 0; i < limit; ++i)  
 A[i] = 1ll \* A[i] \* inv[limit] % mod;  
 }  
 }  
  
 poly poly\_mul(poly A, poly B) {//多项式乘法  
 ll deg = A.size() + B.size() - 1;  
 ll limit = NTT\_init(deg);  
 poly C(limit);  
 NTT(A, 1, limit);  
 NTT(B, 1, limit);  
 for (ll i = 0; i < limit; ++i)  
 C[i] = 1ll \* A[i] \* B[i] % mod;  
 NTT(C, 0, limit);  
 C.resize(deg);  
 return C;  
 }  
  
 poly poly\_inv(poly &f, ll deg) {//多项式求逆  
 if (deg == 1)  
 return poly(1, qpow(f[0], mod - 2));  
  
 poly A(f.begin(), f.begin() + deg);  
 poly B = poly\_inv(f, (deg + 1) >> 1);  
 ll limit = NTT\_init(deg << 1);  
 NTT(A, 1, limit), NTT(B, 1, limit);  
 for (ll i = 0; i < limit; ++i)  
 A[i] = B[i] \* (2 - 1ll \* A[i] \* B[i] % mod + mod) % mod;  
 NTT(A, 0, limit);  
 A.resize(deg);  
 return A;  
 }  
  
 poly poly\_dev(poly f) {//多项式求导  
 ll n = f.size();  
 for (ll i = 1; i < n; ++i) f[i - 1] = 1ll \* f[i] \* i % mod;  
 return f.resize(n - 1), f;//f[0] = 0，这里直接扔了,从1开始  
 }  
  
 poly poly\_idev(poly f) {//多项式求积分  
 ll n = f.size();  
 for (ll i = n - 1; i; --i) f[i] = 1ll \* f[i - 1] \* inv[i] % mod;  
 return f[0] = 0, f;  
 }  
  
 poly poly\_ln(poly f, ll deg) {//多项式求对数  
 poly A = poly\_idev(poly\_mul(poly\_dev(f), poly\_inv(f, deg)));  
 return A.resize(deg), A;  
 }  
  
 poly poly\_exp(poly &f, ll deg) {//多项式求指数  
 if (deg == 1)  
 return poly(1, 1);  
  
 poly B = poly\_exp(f, (deg + 1) >> 1);  
 B.resize(deg);  
 poly lnB = poly\_ln(B, deg);  
 for (ll i = 0; i < deg; ++i)  
 lnB[i] = ck(f[i] - lnB[i] + mod);  
  
 ll limit = NTT\_init(deg << 1);//n -> n^2  
 NTT(B, 1, limit), NTT(lnB, 1, limit);  
 for (ll i = 0; i < limit; ++i)  
 B[i] = 1ll \* B[i] \* (1 + lnB[i]) % mod;  
 NTT(B, 0, limit);  
 B.resize(deg);  
 return B;  
 }  
  
 poly poly\_sqrt(poly &f, ll deg) {//多项式开方  
 if (deg == 1) return poly(1, 1);  
 poly A(f.begin(), f.begin() + deg);  
 poly B = poly\_sqrt(f, (deg + 1) >> 1);  
 poly IB = poly\_inv(B, deg);  
 ll limit = NTT\_init(deg << 1);  
 NTT(A, 1, limit), NTT(IB, 1, limit);  
 for (ll i = 0; i < limit; ++i)  
 A[i] = 1ll \* A[i] \* IB[i] % mod;  
 NTT(A, 0, limit);  
 for (ll i = 0; i < deg; ++i)  
 A[i] = 1ll \* (A[i] + B[i]) \* inv[2] % mod;  
 A.resize(deg);  
 return A;  
 }  
  
 poly poly\_pow(poly f, ll k) {//多项式快速幂  
 if (f.size() == 1) {  
 f[0] = qpow(f[0], k);  
 return f;  
 }  
 f = poly\_ln(f, f.size());  
 for (auto &x: f) x = 1ll \* x \* k % mod;  
 return poly\_exp(f, f.size());  
 }  
  
 poly poly\_cos(poly f, ll deg) {//多项式三角函数（cos）  
 poly A(f.begin(), f.begin() + deg);  
 poly B(deg), C(deg);  
 for (ll i = 0; i < deg; ++i)  
 A[i] = 1ll \* A[i] \* img % mod;  
  
 B = poly\_exp(A, deg);  
 C = poly\_inv(B, deg);  
 ll inv2 = qpow(2, mod - 2);  
 for (ll i = 0; i < deg; ++i)  
 A[i] = 1ll \* (1ll \* B[i] + C[i]) % mod \* inv2 % mod;  
 return A;  
 }  
  
 poly poly\_sin(poly f, ll deg) {//多项式三角函数（sin）  
 poly A(f.begin(), f.begin() + deg);  
 poly B(deg), C(deg);  
 for (ll i = 0; i < deg; ++i)  
 A[i] = 1ll \* A[i] \* img % mod;  
  
 B = poly\_exp(A, deg);  
 C = poly\_inv(B, deg);  
 ll inv2i = qpow(img << 1, mod - 2);  
 for (ll i = 0; i < deg; ++i)  
 A[i] = 1ll \* (1ll \* B[i] - C[i] + mod) % mod \* inv2i % mod;  
 return A;  
 }  
  
 poly poly\_arcsin(poly f, ll deg) {  
 poly A(f.size()), B(f.size()), C(f.size());  
 A = poly\_dev(f);  
 B = poly\_mul(f, f);  
 for (ll i = 0; i < deg; ++i)  
 B[i] = minus(mod, B[i]);  
 B[0] = plus(B[0], 1);  
 C = poly\_sqrt(B, deg);  
 C = poly\_inv(C, deg);  
 C = poly\_mul(A, C);  
 C = poly\_idev(C);  
 return C;  
 }  
  
 poly poly\_arctan(poly f, ll deg) {  
 poly A(f.size()), B(f.size()), C(f.size());  
 A = poly\_dev(f);  
 B = poly\_mul(f, f);  
 B[0] = plus(B[0], 1);  
 C = poly\_inv(B, deg);  
 C = poly\_mul(A, C);  
 C = poly\_idev(C);  
 return C;  
 }  
}  
  
using namespace Poly;  
  
signed main() {  
 ios::sync\_with\_stdio(false);  
 cin.tie(0);  
 cout.tie(0);  
  
 ll n, k;  
 cin >> n >> k;  
 Poly::init();  
 vector<ll> v(n);  
 for (ll i = 0; i < n; i++) cin >> v[i];  
 auto res = Poly::poly\_pow(v, k);  
 for (auto x: res) cout << x << ' ';  
 cout << endl;  
}

# 字符串

## AC自动机

#include <bits/stdc++.h>  
using namespace std;  
struct AC {  
 static const int maxnode = 200005;  
 static const int sigma\_size = 26;  
 char T[maxnode];  
 int ch[maxnode][sigma\_size];  
 int val[maxnode], fail[maxnode], last[maxnode];  
 int sz;  
 vector<pair<int, int> > ans;  
  
 void init() {  
 sz = 1;  
 memset(ch[0], 0, sizeof(ch[0]));  
 ans.clear();  
 }  
  
 int idx(const char &c) { return c - 'a'; }  
  
 void insert(string s, int v) {  
 int u = 0, n = s.length();  
 for (int i = 0; i < n; i++) {  
 int c = idx(s[i]);  
 if (!ch[u][c]) {  
 memset(ch[sz], 0, sizeof(ch[sz]));  
 val[sz] = 0;  
 ch[u][c] = sz++;  
 }  
 u = ch[u][c];  
 }  
 val[u] = v;  
 }  
  
 void get\_fail() {  
 queue<int> que;  
 fail[0] = 0;  
 for (int c = 0; c < sigma\_size; c++) {  
 int u = ch[0][c];  
 if (u) {  
 fail[u] = 0;  
 que.push(u);  
 last[u] = 0;  
 }  
 }  
 while (!que.empty()) {  
 int r = que.front();  
 que.pop();  
 for (int c = 0; c < sigma\_size; c++) {  
 int u = ch[r][c];  
 if (!u) continue;  
 que.push(u);  
 int v = fail[r];  
 while (v && !ch[v][c]) v = fail[v];  
 fail[u] = ch[v][c];  
 last[u] = val[fail[u]] ? fail[u] : last[fail[u]];  
 }  
 }  
 }  
  
 void print(int j) {  
 if (j) {  
 ans.push\_back(pair<int, int>(j, val[j]));  
 print(last[j]);  
 }  
 }  
  
 void find() {  
 int n = strlen(T);  
 int j = 0;  
 for (int i = 0; i < n; i++) {  
 int c = idx(T[i]);  
 while (j && !ch[j][c]) j = fail[j];  
 j = ch[j][c];  
 if (val[j])  
 print(j);  
 else if (last[j])  
 print(last[j]);  
 }  
 }  
} ac; //字符串下标从0开始

## KMP 2

#include <bits/stdc++.h>  
using namespace std;  
struct KMP {  
 static const int MAXN = 1000010;  
 char T[MAXN], P[MAXN];  
 int fail[MAXN];  
 vector<int> ans;  
  
 void init() { ans.clear(); }  
  
 void get\_fail() {  
 int m = strlen(P);  
 fail[0] = fail[1] = 0;  
 for (int i = 1; i < m; i++) {  
 int j = fail[i];  
 while (j && P[i] != P[j]) j = fail[j];  
 fail[i + 1] = (P[i] == P[j] ? j + 1 : 0);  
 }  
 }  
  
 void find() {  
 int n = strlen(T), m = strlen(P);  
 get\_fail();  
 int j = 0;  
 for (int i = 0; i < n; i++) {  
 while (j && P[j] != T[i]) j = fail[j];  
 if (P[j] == T[i]) j++;  
 if (j == m) ans.push\_back(i - m + 1);  
 }  
 }  
} kmp; //P为模式串，下标从0开始，输入后直接调用find()

## kmp

//next数组等价于前缀函数  
#include<bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
  
int kmp(char \*s1,int \*p1,char \*s2=0,int \*p2=0){//必须先求s1的next数组，即kmp(s1,p1);再kmp(s1,p1,s2,p2);  
 int n=strlen(s1);  
 if(p2==0){  
 p1[0]=0;  
 for(int i=1;s1[i]!='\0';i++){  
 int j=p1[i-1];  
 while(j>0&&s1[i]!=s1[j])j=p1[j-1];  
 if(s1[i]==s1[j])j++;  
 p1[i]=j;  
 }  
 }  
 else{  
 for(int i=0;s2[i]!='\0';i++){  
 int j=i==0?0:p2[i-1];  
 while(j>0&&s2[i]!=s1[j])j=p1[j-1];  
 if(s2[i]==s1[j])j++;  
 p2[i]=j;  
 if(j==n)return i-n+2;//返回位置  
 }  
 }  
 return 0;  
}  
int main(){  
 char s1[15],s2[105];  
 int p1[15],p2[105];  
 cin>>s1>>s2;  
 kmp(s1,p1);  
 cout<<kmp(s1,p1,s2,p2)<<endl;  
 return 0;  
}

## regex

| 元字符 | 描述 |
| --- | --- |
| \ | 将下一个字符标记符、或一个向后引用、或一个八进制转义符。例如，“\n”匹配\n。“\n”匹配换行符。序列“\”匹配“\”而“(”则匹配“(”。即相当于多种编程语言中都有的“转义字符”的概念。 |
| ^ | 匹配输入字行首。如果设置了RegExp对象的Multiline属性，^也匹配“\n”或“\r”之后的位置。 |
| $ | 匹配输入行尾。如果设置了RegExp对象的Multiline属性，$也匹配“\n”或“\r”之前的位置。 |
| \* | 匹配前面的子表达式任意次。例如，zo*能匹配“z”，也能匹配“zo”以及“zoo”。*等价于{0,}。 |
| + | 匹配前面的子表达式一次或多次(大于等于1次）。例如，“zo+”能匹配“zo”以及“zoo”，但不能匹配“z”。+等价于{1,}。 |
| ? | 匹配前面的子表达式零次或一次。例如，“do(es)?”可以匹配“do”或“does”。?等价于{0,1}。 |
| {*n*} | *n*是一个非负整数。匹配确定的*n*次。例如，“o{2}”不能匹配“Bob”中的“o”，但是能匹配“food”中的两个o。 |
| {*n*,} | *n*是一个非负整数。至少匹配*n*次。例如，“o{2,}”不能匹配“Bob”中的“o”，但能匹配“foooood”中的所有o。“o{1,}”等价于“o+”。“o{0,}”则等价于“o\*”。 |
| {*n*,*m*} | *m*和*n*均为非负整数，其中*n*<=*m*。最少匹配*n*次且最多匹配*m*次。例如，“o{1,3}”将匹配“fooooood”中的前三个o为一组，后三个o为一组。“o{0,1}”等价于“o?”。请注意在逗号和两个数之间不能有空格。 |
| ? | 当该字符紧跟在任何一个其他限制符（*,+,?，{*n*}，{*n*,}，{*n*,*m\*}）后面时，匹配模式是非贪婪的。非贪婪模式尽可能少地匹配所搜索的字符串，而默认的贪婪模式则尽可能多地匹配所搜索的字符串。例如，对于字符串“oooo”，“o+”将尽可能多地匹配“o”，得到结果[“oooo”]，而“o+?”将尽可能少地匹配“o”，得到结果 ['o', 'o', 'o', 'o'] |
| .点 | 匹配除“\n”和"\r"之外的任何单个字符。要匹配包括“\n”和"\r"在内的任何字符，请使用像“[\s\S]”的模式。 |
| (pattern) | 匹配pattern并获取这一匹配。所获取的匹配可以从产生的Matches集合得到，在VBScript中使用SubMatches集合，在JScript中则使用9属性。要匹配圆括号字符，请使用“(”或“)”。 |
| (?:pattern) | 非获取匹配，匹配pattern但不获取匹配结果，不进行存储供以后使用。这在使用或字符“(|)”来组合一个模式的各个部分时很有用。例如“industr(?:y|ies)”就是一个比“industry|industries”更简略的表达式。 |
| (?=pattern) | 非获取匹配，正向肯定预查，在任何匹配pattern的字符串开始处匹配查找字符串，该匹配不需要获取供以后使用。例如，“Windows(?=95|98|NT|2000)”能匹配“Windows2000”中的“Windows”，但不能匹配“Windows3.1”中的“Windows”。预查不消耗字符，也就是说，在一个匹配发生后，在最后一次匹配之后立即开始下一次匹配的搜索，而不是从包含预查的字符之后开始。 |
| (?!pattern) | 非获取匹配，正向否定预查，在任何不匹配pattern的字符串开始处匹配查找字符串，该匹配不需要获取供以后使用。例如“Windows(?!95|98|NT|2000)”能匹配“Windows3.1”中的“Windows”，但不能匹配“Windows2000”中的“Windows”。 |
| (?<=pattern) | 非获取匹配，反向肯定预查，与正向肯定预查类似，只是方向相反。例如，“(?<=95|98|NT|2000)Windows”能匹配“2000Windows”中的“Windows”，但不能匹配“3.1Windows”中的“Windows”。\*python的正则表达式没有完全按照正则表达式规范实现，所以一些高级特性建议使用其他语言如java、scala等 |
| (?<!pattern) | 非获取匹配，反向否定预查，与正向否定预查类似，只是方向相反。例如“(?<!95|98|NT|2000)Windows”能匹配“3.1Windows”中的“Windows”，但不能匹配“2000Windows”中的“Windows”。\*python的正则表达式没有完全按照正则表达式规范实现，所以一些高级特性建议使用其他语言如java、scala等 |
| x|y | 匹配x或y。例如，“z|food”能匹配“z”或“food”(此处请谨慎)。“[z|f]ood”则匹配“zood”或“food”。 |
| [xyz] | 字符集合。匹配所包含的任意一个字符。例如，“[abc]”可以匹配“plain”中的“a”。 |
| [^xyz] | 负值字符集合。匹配未包含的任意字符。例如，“abc”可以匹配“plain”中的“plin”任一字符。 |
| [a-z] | 字符范围。匹配指定范围内的任意字符。例如，“[a-z]”可以匹配“a”到“z”范围内的任意小写字母字符。注意:只有连字符在字符组内部时,并且出现在两个字符之间时,才能表示字符的范围; 如果出字符组的开头,则只能表示连字符本身. |
| [^a-z] | 负值字符范围。匹配任何不在指定范围内的任意字符。例如，“a-z”可以匹配任何不在“a”到“z”范围内的任意字符。 |
| \b | 匹配一个单词的边界，也就是指单词和空格间的位置（即正则表达式的“匹配”有两种概念，一种是匹配字符，一种是匹配位置，这里的\b就是匹配位置的）。例如，“er\b”可以匹配“never”中的“er”，但不能匹配“verb”中的“er”；“\b1*”可以匹配“1*23”中的“1*”，但不能匹配“21*3”中的“1\_”。 |
| \B | 匹配非单词边界。“er\B”能匹配“verb”中的“er”，但不能匹配“never”中的“er”。 |
| \cx | 匹配由x指明的控制字符。例如，\cM匹配一个Control-M或回车符。x的值必须为A-Z或a-z之一。否则，将c视为一个原义的“c”字符。 |
| \d | 匹配一个数字字符。等价于[0-9]。grep 要加上-P，perl正则支持 |
| \D | 匹配一个非数字字符。等价于0-9。grep要加上-P，perl正则支持 |
| \f | 匹配一个换页符。等价于\x0c和\cL。 |
| \n | 匹配一个换行符。等价于\x0a和\cJ。 |
| \r | 匹配一个回车符。等价于\x0d和\cM。 |
| \s | 匹配任何不可见字符，包括空格、制表符、换页符等等。等价于[ \f\n\r\t\v]。 |
| \S | 匹配任何可见字符。等价于 \f\n\r\t\v。 |
| \t | 匹配一个制表符。等价于\x09和\cI。 |
| \v | 匹配一个垂直制表符。等价于\x0b和\cK。 |
| \w | 匹配包括下划线的任何单词字符。类似但不等价于“[A-Za-z0-9\_]”，这里的"单词"字符使用Unicode字符集。 |
| \W | 匹配任何非单词字符。等价于“A-Za-z0-9\_”。 |
| \x*n* | 匹配*n*，其中*n*为十六进制转义值。十六进制转义值必须为确定的两个数字长。例如，“\x41”匹配“A”。“\x041”则等价于“\x04&1”。正则表达式中可以使用ASCII编码。 |
| \*num\* | 匹配*num*，其中*num*是一个正整数。对所获取的匹配的引用。例如，“(.)\1”匹配两个连续的相同字符。 |
| \*n\* | 标识一个八进制转义值或一个向后引用。如果\*n*之前至少*n*个获取的子表达式，则*n*为向后引用。否则，如果*n*为八进制数字（0-7），则*n\*为一个八进制转义值。 |
| \*nm\* | 标识一个八进制转义值或一个向后引用。如果\*nm*之前至少有*nm*个获得子表达式，则*nm*为向后引用。如果\*nm*之前至少有*n*个获取，则*n*为一个后跟文字*m*的向后引用。如果前面的条件都不满足，若*n*和*m*均为八进制数字（0-7），则\*nm*将匹配八进制转义值*nm\*。 |
| \*nml\* | 如果*n*为八进制数字（0-7），且*m*和*l*均为八进制数字（0-7），则匹配八进制转义值*nml*。 |
| \u*n* | 匹配*n*，其中*n*是一个用四个十六进制数字表示的Unicode字符。例如，\u00A9匹配版权符号（©）。 |
| \p{P} | 小写 p 是 property 的意思，表示 Unicode 属性，用于 Unicode 正表达式的前缀。中括号内的“P”表示Unicode 字符集七个字符属性之一：标点字符。其他六个属性：L：字母；M：标记符号（一般不会单独出现）；Z：分隔符（比如空格、换行等）；S：符号（比如数学符号、货币符号等）；N：数字（比如阿拉伯数字、罗马数字等）；C：其他字符。*\*注：此语法部分语言不支持，例：javascript。* |
| <> | 匹配词（word）的开始（<）和结束（>）。例如正则表达式<the>能够匹配字符串"for the wise"中的"the"，但是不能匹配字符串"otherwise"中的"the"。注意：这个元字符不是所有的软件都支持的。 |
| ( ) | 将( 和 ) 之间的表达式定义为“组”（group），并且将匹配这个表达式的字符保存到一个临时区域（一个正则表达式中最多可以保存9个），它们可以用 \1 到\9 的符号来引用。 |
| | | 将两个匹配条件进行逻辑“或”（or）运算。例如正则表达式(him|her) 匹配"it belongs to him"和"it belongs to her"，但是不能匹配"it belongs to them."。注意：这个元字符不是所有的软件都支持的。 |

## Trie

#include <bits/stdc++.h>  
using namespace std;  
struct Trie {  
 static const int maxnode = 200005;  
 static const int sigma\_size = 26;  
 int ch[maxnode][sigma\_size];  
 int val[maxnode];  
 int sz;  
  
 Trie() {  
 sz = 1;  
 memset(ch[0], 0, sizeof(ch[0]));  
 }  
  
 int idx(const char &c) { return c - 'a'; }  
  
 void insert(string s, int v) {  
 int u = 0, n = s.length();  
 for (int i = 0; i < n; i++) {  
 int c = idx(s[i]);  
 if (!ch[u][c]) {  
 memset(ch[sz], 0, sizeof(ch[sz]));  
 val[sz] = 0;  
 ch[u][c] = sz++;  
 }  
 u = ch[u][c];  
 }  
 val[u] = v;  
 }  
  
 int find(string s) {  
 int u = 0, n = s.length();  
 for (int i = 0; i < n; i++) {  
 int c = idx(s[i]);  
 if (!ch[u][c]) return 0;  
 u = ch[u][c];  
 }  
 return val[u];  
 }  
} trie;

## 可持久化字典树

struct Trie01 {  
 static const int maxnode = 2000005;  
 static const int sigma\_size = 2;  
 int ch[maxnode << 5][sigma\_size], val[maxnode << 5];  
 int rt[maxnode];  
 int sz;  
  
 Trie01() {  
 sz = 0;  
 memset(ch[0], 0, sizeof(ch[0]));  
 }  
  
 void insert(int &now, int pre, int v) {  
 now = ++sz;  
 for (int i = 30; i >= 0; i--) {  
 int k = ((v >> i) & 1);  
 ch[now][k] = ++sz;  
 ch[now][k ^ 1] = ch[pre][k ^ 1];  
 val[ch[now][k]] = val[ch[pre][k]] + 1;  
 now = ch[now][k];  
 pre = ch[pre][k];  
 }  
 }  
} trie;

## 后缀数组

#include <bits/stdc++.h>  
using namespace std;  
struct SuffixArray {  
 static const int MAXN = 1100000;  
 char s[MAXN];  
 int sa[MAXN], t[MAXN], t1[MAXN], c[MAXN], ra[MAXN], height[MAXN], m;  
 inline void init() { memset(this, 0, sizeof(SuffixArray)); }  
  
 inline void get\_sa(int n) {  
 m = 256;  
 int \*x = t, \*y = t1;  
 for (int i = 1; i <= m; i++) c[i] = 0;  
 for (int i = 1; i <= n; i++) c[x[i] = s[i]]++;  
 for (int i = 1; i <= m; i++) c[i] += c[i - 1];  
 for (int i = n; i >= 1; i--) sa[c[x[i]]--] = i;  
 for (int k = 1; k <= n; k <<= 1) {  
 int p = 0;  
 for (int i = n - k + 1; i <= n; i++) y[++p] = i;  
 for (int i = 1; i <= n; i++)  
 if (sa[i] > k) y[++p] = sa[i] - k;  
 for (int i = 1; i <= m; i++) c[i] = 0;  
 for (int i = 1; i <= n; i++) c[x[y[i]]]++;  
 for (int i = 1; i <= m; i++) c[i] += c[i - 1];  
 for (int i = n; i >= 1; i--) sa[c[x[y[i]]]--] = y[i];  
 std::swap(x, y);  
 p = x[sa[1]] = 1;  
 for (int i = 2; i <= n; i++) {  
 x[sa[i]] = (y[sa[i - 1]] == y[sa[i]] &&  
 y[sa[i - 1] + k] == y[sa[i] + k])  
 ? p  
 : ++p;  
 }  
 if (p >= n) break;  
 m = p;  
 }  
 }  
  
 inline void get\_height(int n) {  
 int i, j, k = 0;  
 for (int i = 1; i <= n; i++) ra[sa[i]] = i;  
 for (int i = 1; i <= n; i++) {  
 if (k) k--;  
 int j = sa[ra[i] - 1];  
 while (s[i + k] == s[j + k]) k++;  
 height[ra[i]] = k;  
 }  
 }  
  
} SA; //字符串下标从一开始

## 后缀自动机

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const int N = 2e6 + 10;  
  
int tot = 1, last = 1;  
struct Node {  
 int len, fa;  
 int ch[26];  
} node[N];  
char str[N];  
ll f[N], ans;  
int h[N], e[N], ne[N], idx;  
  
void extend(int c) {  
 int p = last, np = last = ++tot;  
 f[tot] = 1;  
 node[np].len = node[p].len + 1;  
 for (; p && !node[p].ch[c]; p = node[p].fa) node[p].ch[c] = np;  
 if (!p) node[np].fa = 1;  
 else {  
 int q = node[p].ch[c];  
 if (node[q].len == node[p].len + 1) node[np].fa = q;  
 else {  
 int nq = ++tot;  
 node[nq] = node[q], node[nq].len = node[p].len + 1;  
 node[q].fa = node[np].fa = nq;  
 for (; p && node[p].ch[c] == q; p = node[p].fa) node[p].ch[c] = nq;  
 }  
 }  
}  
  
void add(int a, int b) {  
 e[idx] = b, ne[idx] = h[a], h[a] = idx++;  
}  
  
void dfs(int u) {  
 for (int i = h[u]; ~i; i = ne[i]) {  
 dfs(e[i]);  
 f[u] += f[e[i]];  
 }  
 if (f[u] > 1) ans = max(ans, f[u] \* node[u].len);  
}  
  
int main() {  
 scanf("%s", str);  
 for (int i = 0; str[i]; i++) extend(str[i] - 'a');  
 memset(h, -1, sizeof h);  
 for (int i = 2; i <= tot; i++) add(node[i].fa, i);  
 dfs(1);  
 printf("%lld\n", ans);  
  
 return 0;  
}

## 回文自动机

#include <bits/stdc++.h>  
  
using namespace std;  
const int maxn = 300000 + 5;  
  
namespace pam {  
 int sz, tot, last;  
 int cnt[maxn], ch[maxn][26], len[maxn], fail[maxn];  
 char s[maxn];  
  
 int node(int l) { // 建立一个新节点，长度为 l  
 sz++;  
 memset(ch[sz], 0, sizeof(ch[sz]));  
 len[sz] = l;  
 fail[sz] = cnt[sz] = 0;  
 return sz;  
 }  
  
 void clear() { // 初始化  
 sz = -1;  
 last = 0;  
 s[tot = 0] = '$';  
 node(0);  
 node(-1);  
 fail[0] = 1;  
 }  
  
 int getfail(int x) { // 找后缀回文  
 while (s[tot - len[x] - 1] != s[tot]) x = fail[x];  
 return x;  
 }  
  
 void insert(char c) { // 建树  
 s[++tot] = c;  
 int now = getfail(last);  
 if (!ch[now][c - 'a']) {  
 int x = node(len[now] + 2);  
 fail[x] = ch[getfail(fail[now])][c - 'a'];  
 ch[now][c - 'a'] = x;  
 }  
 last = ch[now][c - 'a'];  
 cnt[last]++;  
 }  
  
 long long solve() {  
 long long ans = 0;  
 for (int i = sz; i >= 0; i--) {  
 cnt[fail[i]] += cnt[i];  
 }  
 for (int i = 1; i <= sz; i++) { // 更新答案  
 ans = max(ans, 1ll \* len[i] \* cnt[i]);  
 }  
 return ans;  
 }  
} // namespace pam  
  
char s[maxn];  
  
int main() {  
 pam::clear();  
 scanf("%s", s + 1);  
 for (int i = 1; s[i]; i++) {  
 pam::insert(s[i]);  
 }  
 printf("%lld\n", pam::solve());  
 return 0;  
}

## 马拉车

#include <bits/stdc++.h>  
using namespace std;  
const int maxn = 100005;  
char s[maxn];  
char s\_new[maxn \* 2];  
int p[maxn \* 2];  
  
int Manacher(char\* a, int l) {  
 s\_new[0] = '$';  
 s\_new[1] = '#';  
 int len = 2;  
 for (int i = 0; i < l; i++) {  
 s\_new[len++] = a[i];  
 s\_new[len++] = '#';  
 }  
 s\_new[len] = '\0';  
 int id;  
 int mx = 0;  
 int mmax = 0;  
  
 for (int i = 1; i < len; i++) {  
 p[i] = i < mx ? min(p[2 \* id - i], mx - i) : 1;  
 while (s\_new[i + p[i]] == s\_new[i - p[i]]) p[i]++;  
 if (mx < i + p[i]) {  
 id = i;  
 mx = i + p[i];  
 }  
 mmax = max(mmax, p[i] - 1);  
 }  
 return mmax;  
}  
  
int main() {  
 cin >> s;  
 cout << Manacher(s, strlen(s));  
}

# 数据结构

## CDQ分治

/\*  
处理三维偏序问题，  
每个node的三维不能完全相等，完全相等的话加权做  
\*/  
  
#include <iostream>  
#include <cstring>  
#include <algorithm>  
  
using namespace std;  
  
const int N = 100010, M = 200010;  
  
int n, m;  
struct Data  
{  
 int a, b, c, s, res;  
  
 bool operator< (const Data& t) const  
 {  
 if (a != t.a) return a < t.a;  
 if (b != t.b) return b < t.b;  
 return c < t.c;  
 }  
 bool operator== (const Data& t) const  
 {  
 return a == t.a && b == t.b && c == t.c;  
 }  
}q[N], w[N];  
int tr[M], ans[N];  
  
int lowbit(int x)  
{  
 return x & -x;  
}  
  
void add(int x, int v)  
{  
 for (int i = x; i < M; i += lowbit(i)) tr[i] += v;  
}  
  
int query(int x)  
{  
 int res = 0;  
 for (int i = x; i; i -= lowbit(i)) res += tr[i];  
 return res;  
}  
  
void merge\_sort(int l, int r)  
{  
 if (l >= r) return;  
 int mid = l + r >> 1;  
 merge\_sort(l, mid), merge\_sort(mid + 1, r);  
 int i = l, j = mid + 1, k = 0;  
 while (i <= mid && j <= r)  
 if (q[i].b <= q[j].b) add(q[i].c, q[i].s), w[k ++ ] = q[i ++ ];  
 else q[j].res += query(q[j].c), w[k ++ ] = q[j ++ ];  
 while (i <= mid) add(q[i].c, q[i].s), w[k ++ ] = q[i ++ ];  
 while (j <= r) q[j].res += query(q[j].c), w[k ++ ] = q[j ++ ];  
 for (i = l; i <= mid; i ++ ) add(q[i].c, -q[i].s);  
 for (i = l, j = 0; j < k; i ++, j ++ ) q[i] = w[j];  
}  
  
int main()  
{  
 scanf("%d%d", &n, &m);  
 for (int i = 0; i < n; i ++ )  
 {  
 int a, b, c;  
 scanf("%d%d%d", &a, &b, &c);  
 q[i] = {a, b, c, 1};  
 }  
 sort(q, q + n);  
  
 int k = 1;  
 for (int i = 1; i < n; i ++ )  
 if (q[i] == q[k - 1]) q[k - 1].s ++ ;  
 else q[k ++ ] = q[i];  
  
 merge\_sort(0, k - 1);  
 for (int i = 0; i < k; i ++ )  
 ans[q[i].res + q[i].s - 1] += q[i].s;  
  
 for (int i = 0; i < n; i ++ ) printf("%d\n", ans[i]);  
  
 return 0;  
}

## kruskal重构树

int pa[N];  
  
void init(int n) {  
 for (int i = 0; i <= n; i++) {  
 pa[i] = i;  
 }  
}  
  
int find(int a) {  
 return pa[a] == a ? a : pa[a] = find(pa[a]);  
}  
  
struct edge {  
 int from, to, l;  
};  
  
int w[N];  
edge e[N];  
vector<int> g[N];  
  
int kruskal(int n, int m) {  
 int kcnt = n;  
 init(n);  
 sort(e + 1, e + 1 + m, [](edge a, edge b) { return a.l < b.l; });  
 for (int i = 1; i <= m; i++) {  
 int u = find(e[i].from);  
 int v = find(e[i].to);  
 if (u == v) continue;  
 w[++kcnt] = e[i].l;  
 pa[kcnt] = pa[u] = pa[v] = kcnt;  
 g[u].push\_back(kcnt);  
 g[v].push\_back(kcnt);  
 g[kcnt].push\_back(u);  
 g[kcnt].push\_back(v);  
 }  
 return kcnt;  
}

## LCT

ll ch[N][2], f[N], sum[N], val[N], tag[N], siz[N], siz2[N];  
  
inline void pushup(ll p) {  
 sum[p] = sum[ch[p][0]] ^ sum[ch[p][1]] ^ val[p];  
 siz[p] = siz[ch[p][0]] + siz[ch[p][1]] + 1 + siz2[p];  
}  
  
inline void pushdown(ll p) {  
 if (tag[p]) {  
 if (ch[p][0]) swap(ch[ch[p][0]][0], ch[ch[p][0]][1]), tag[ch[p][0]] ^= 1;  
 if (ch[p][1]) swap(ch[ch[p][1]][0], ch[ch[p][1]][1]), tag[ch[p][1]] ^= 1;  
 tag[p] = 0;  
 }  
}  
  
ll getch(ll x) { return ch[f[x]][1] == x; }  
  
bool isroot(ll x) { return ch[f[x]][0] != x && ch[f[x]][1] != x; }  
  
inline void rotate(ll x) {  
 ll y = f[x], z = f[y], k = getch(x);  
 if (!isroot(y)) ch[z][ch[z][1] == y] = x;  
 // 上面这句一定要写在前面，普通的Splay是不用的，因为 isRoot (后面会讲)  
 ch[y][k] = ch[x][!k], f[ch[x][!k]] = y;  
 ch[x][!k] = y, f[y] = x, f[x] = z;  
 pushup(y), pushup(x);  
}  
  
// 从上到下一层一层 pushDown 即可  
void update(ll p) {  
 if (!isroot(p)) update(f[p]);  
 pushdown(p);  
}  
  
inline void splay(ll x) {  
 update(x); // 马上就能看到啦。 在  
 // Splay之前要把旋转会经过的路径上的点都PushDown  
 for (ll fa; fa = f[x], !isroot(x); rotate(x)) {  
 if (!isroot(fa)) rotate(getch(fa) == getch(x) ? fa : x);  
 }  
}  
  
// 回顾一下代码  
inline void access(ll x) {  
 for (ll p = 0; x; p = x, x = f[x]) {  
 splay(x), siz2[x] += siz[ch[x][1]] - siz[p], ch[x][1] = p, pushup(x);  
 }  
}  
  
inline void makeroot(ll p) {  
 access(p);  
 splay(p);  
 swap(ch[p][0], ch[p][1]);  
 tag[p] ^= 1;  
}  
  
inline void split(ll a, ll b) {  
 makeroot(a);  
 access(b);  
 splay(b);  
}  
  
  
inline ll find(ll p) {  
 access(p), splay(p);  
 while (ch[p][0]) pushdown(p), p = ch[p][0];  
 splay(p);  
 return p;  
}  
  
inline void link(ll x, ll y) {  
 makeroot(y);  
 makeroot(x);  
 if (find(y) != x) {  
 f[x] = y;  
 siz2[y] += siz[x];  
 }  
}  
  
inline void cut(ll x, ll y) {  
 makeroot(x);  
 if (find(y) == x && f[y] == x) {  
 ch[x][1] = f[y] = 0;  
 pushup(x);  
 }  
}  
  
void init(int n) {  
 for (int i = 1; i <= n; i++) siz[i] = 1;  
}

## Splay

ll ch[N][2], f[N], sum[N], val[N], tag[N], siz[N];  
  
inline void pushup(ll p) {  
 sum[p] = sum[ch[p][0]] ^ sum[ch[p][1]] ^ val[p];  
 siz[p] = siz[ch[p][0]] + siz[ch[p][1]] + 1;  
}  
  
inline void pushdown(ll p) {  
 if (tag[p]) {  
 if (ch[p][0]) swap(ch[ch[p][0]][0], ch[ch[p][0]][1]), tag[ch[p][0]] ^= 1;  
 if (ch[p][1]) swap(ch[ch[p][1]][0], ch[ch[p][1]][1]), tag[ch[p][1]] ^= 1;  
 tag[p] = 0;  
 }  
}  
  
ll getch(ll x) { return ch[f[x]][1] == x; }  
  
bool isroot(ll x) { return ch[f[x]][0] != x && ch[f[x]][1] != x; }  
  
inline void rotate(ll x) {  
 ll y = f[x], z = f[y], k = getch(x);  
 if (!isroot(y)) ch[z][ch[z][1] == y] = x;  
 // 上面这句一定要写在前面，普通的Splay是不用的，因为 isRoot (后面会讲)  
 ch[y][k] = ch[x][!k], f[ch[x][!k]] = y;  
 ch[x][!k] = y, f[y] = x, f[x] = z;  
 pushup(y), pushup(x);  
}  
  
// 从上到下一层一层 pushDown 即可  
void update(ll p) {  
 if (!isroot(p)) update(f[p]);  
 pushdown(p);  
}  
  
inline void splay(ll x) {  
 update(x); // 马上就能看到啦。 在  
 // Splay之前要把旋转会经过的路径上的点都PushDown  
 for (ll fa; fa = f[x], !isroot(x); rotate(x)) {  
 if (!isroot(fa)) rotate(getch(fa) == getch(x) ? fa : x);  
 }  
}  
  
// 回顾一下代码  
inline void access(ll x) {  
 for (ll p = 0; x; p = x, x = f[x]) {  
 splay(x), ch[x][1] = p, pushup(x);  
 }  
}  
  
inline void makeroot(ll p) {  
 access(p);  
 splay(p);  
 swap(ch[p][0], ch[p][1]);  
 tag[p] ^= 1;  
}  
  
inline void split(ll a, ll b) {  
 makeroot(a);  
 access(b);  
 splay(b);  
}  
  
  
inline ll find(ll p) {  
 access(p), splay(p);  
 while (ch[p][0]) pushdown(p), p = ch[p][0];  
 splay(p);  
 return p;  
}  
  
inline void link(ll x, ll y) {  
 makeroot(x);  
 if (find(y) != x) f[x] = y;  
}  
  
inline void cut(ll x, ll y) {  
 makeroot(x);  
 if (find(y) == x && f[y] == x) {  
 ch[x][1] = f[y] = 0;  
 pushup(x);  
 }  
}

## ST表

#include <bits/stdc++.h>  
  
using namespace std;  
const int logn = 21;  
const int N = 2000001;  
int f[N][logn + 1], lg[N + 1];  
  
void pre() {  
 lg[1] = 0;  
 for (int i = 2; i < N; i++) {  
 lg[i] = lg[i / 2] + 1;  
 }  
}  
  
int main() {  
 ios::sync\_with\_stdio(false);  
 int n, m;  
 cin >> n >> m;  
 for (int i = 1; i <= n; i++) cin >> f[i][0];  
 pre();  
 for (int j = 1; j <= logn; j++)  
 for (int i = 1; i + (1 << j) - 1 <= n; i++)  
 f[i][j] = max(f[i][j - 1], f[i + (1 << (j - 1))][j - 1]);  
 for (int i = 1; i <= m; i++) {  
 int x, y;  
 cin >> x >> y;  
 int s = lg[y - x + 1];  
 printf("%d\n", max(f[x][s], f[y - (1 << s) + 1][s]));  
 }  
 return 0;  
}

## Treap

#include <bits/stdc++.h>  
using namespace std;  
struct node {  
 node\* ch[2];  
 int r;  
 int v;  
 int cmp(int const& a) const {  
 if (v == a) return -a;  
 return a > v ? 1 : 0;  
 }  
};  
void rotate(node\*& a, int d) {  
 node\* k = a->ch[d ^ 1];  
 a->ch[d ^ 1] = k->ch[d];  
 k->ch[d] = a;  
 a = k;  
}  
void insert(node\*& a, int x) {  
 if (a == NULL) {  
 a = new node;  
 a->ch[0] = a->ch[1] = NULL;  
 a->v = x;  
 a->r = rand();  
 } else {  
 int d = a->cmp(x);  
 insert(a->ch[d], x);  
 if (a->ch[d]->r > a->r) rotate(a, d ^ 1);  
 }  
}  
void remove(node\*& a, int x) {  
 int d = a->cmp(x);  
 if (d == -1) {  
 if (a->ch[0] == NULL)  
 a = a->ch[1];  
 else if (a->ch[1] == NULL)  
 a = a->ch[0];  
 else {  
 int d2 = a->ch[1]->r > a->ch[0]->r ? 0 : 1;  
 rotate(a, d2);  
 remove(a->ch[d2], x);  
 }  
 } else {  
 remove(a->ch[d], x);  
 }  
}  
int find(node\*& a, int x) {  
 if (a == NULL)  
 return 0;  
 else if (a->v == x)  
 return 1;  
 else {  
 int d = a->cmp(x);  
 return find(a->ch[d], x);  
 }  
}  
int main() {  
 node\* a = NULL;  
 int k, l;  
 while (cin >> k >> l) {  
 if (k == 1)  
 insert(a, l);  
 else if (k == 2)  
 remove(a, l);  
 else {  
 cout << find(a, l) << endl;  
 }  
 }  
}

## y总Splay Plus

#include <iostream>  
#include <cstdio>  
#include <cstring>  
#include <algorithm>  
  
using namespace std;  
  
const int N = 500010, INF = 1e9;  
  
int n, m;  
struct Node  
{  
 int s[2], p, v;  
 int rev, same;  
 int size, sum, ms, ls, rs;  
  
 void init(int \_v, int \_p)  
 {  
 s[0] = s[1] = 0, p = \_p, v = \_v;  
 rev = same = 0;  
 size = 1, sum = ms = v;  
 ls = rs = max(v, 0);  
 }  
}tr[N];  
int root, nodes[N], tt;  
int w[N];  
  
void pushup(int x)  
{  
 auto &u = tr[x], &l = tr[u.s[0]], &r = tr[u.s[1]];  
 u.size = l.size + r.size + 1;  
 u.sum = l.sum + r.sum + u.v;  
 u.ls = max(l.ls, l.sum + u.v + r.ls);  
 u.rs = max(r.rs, r.sum + u.v + l.rs);  
 u.ms = max(max(l.ms, r.ms), l.rs + u.v + r.ls);  
}  
  
void pushdown(int x)  
{  
 auto &u = tr[x], &l = tr[u.s[0]], &r = tr[u.s[1]];  
 if (u.same)  
 {  
 u.same = u.rev = 0;  
 if (u.s[0]) l.same = 1, l.v = u.v, l.sum = l.v \* l.size;  
 if (u.s[1]) r.same = 1, r.v = u.v, r.sum = r.v \* r.size;  
 if (u.v > 0)  
 {  
 if (u.s[0]) l.ms = l.ls = l.rs = l.sum;  
 if (u.s[1]) r.ms = r.ls = r.rs = r.sum;  
 }  
 else  
 {  
 if (u.s[0]) l.ms = l.v, l.ls = l.rs = 0;  
 if (u.s[1]) r.ms = r.v, r.ls = r.rs = 0;  
 }  
 }  
 else if (u.rev)  
 {  
 u.rev = 0, l.rev ^= 1, r.rev ^= 1;  
 swap(l.ls, l.rs), swap(r.ls, r.rs);  
 swap(l.s[0], l.s[1]), swap(r.s[0], r.s[1]);  
 }  
}  
  
void rotate(int x)  
{  
 int y = tr[x].p, z = tr[y].p;  
 int k = tr[y].s[1] == x;  
 tr[z].s[tr[z].s[1] == y] = x, tr[x].p = z;  
 tr[y].s[k] = tr[x].s[k ^ 1], tr[tr[x].s[k ^ 1]].p = y;  
 tr[x].s[k ^ 1] = y, tr[y].p = x;  
 pushup(y), pushup(x);  
}  
  
void splay(int x, int k)  
{  
 while (tr[x].p != k)  
 {  
 int y = tr[x].p, z = tr[y].p;  
 if (z != k)  
 if ((tr[y].s[1] == x) ^ (tr[z].s[1] == y)) rotate(x);  
 else rotate(y);  
 rotate(x);  
 }  
 if (!k) root = x;  
}  
  
int get\_k(int k)  
{  
 int u = root;  
 while (u)  
 {  
 pushdown(u);  
 if (tr[tr[u].s[0]].size >= k) u = tr[u].s[0];  
 else if (tr[tr[u].s[0]].size + 1 == k) return u;  
 else k -= tr[tr[u].s[0]].size + 1, u = tr[u].s[1];  
 }  
}  
  
int build(int l, int r, int p)  
{  
 int mid = l + r >> 1;  
 int u = nodes[tt -- ];  
 tr[u].init(w[mid], p);  
 if (l < mid) tr[u].s[0] = build(l, mid - 1, u);  
 if (mid < r) tr[u].s[1] = build(mid + 1, r, u);  
 pushup(u);  
 return u;  
}  
  
void dfs(int u)  
{  
 if (tr[u].s[0]) dfs(tr[u].s[0]);  
 if (tr[u].s[1]) dfs(tr[u].s[1]);  
 nodes[ ++ tt] = u;  
}  
  
int main()  
{  
 for (int i = 1; i < N; i ++ ) nodes[ ++ tt] = i;  
 scanf("%d%d", &n, &m);  
 tr[0].ms = w[0] = w[n + 1] = -INF;  
 for (int i = 1; i <= n; i ++ ) scanf("%d", &w[i]);  
 root = build(0, n + 1, 0);  
  
 char op[20];  
 while (m -- )  
 {  
 scanf("%s", op);  
 if (!strcmp(op, "INSERT"))  
 {  
 int posi, tot;  
 scanf("%d%d", &posi, &tot);  
 for (int i = 0; i < tot; i ++ ) scanf("%d", &w[i]);  
 int l = get\_k(posi + 1), r = get\_k(posi + 2);  
 splay(l, 0), splay(r, l);  
 int u = build(0, tot - 1, r);  
 tr[r].s[0] = u;  
 pushup(r), pushup(l);  
 }  
 else if (!strcmp(op, "DELETE"))  
 {  
 int posi, tot;  
 scanf("%d%d", &posi, &tot);  
 int l = get\_k(posi), r = get\_k(posi + tot + 1);  
 splay(l, 0), splay(r, l);  
 dfs(tr[r].s[0]);  
 tr[r].s[0] = 0;  
 pushup(r), pushup(l);  
 }  
 else if (!strcmp(op, "MAKE-SAME"))  
 {  
 int posi, tot, c;  
 scanf("%d%d%d", &posi, &tot, &c);  
 int l = get\_k(posi), r = get\_k(posi + tot + 1);  
 splay(l, 0), splay(r, l);  
 auto& son = tr[tr[r].s[0]];  
 son.same = 1, son.v = c, son.sum = c \* son.size;  
 if (c > 0) son.ms = son.ls = son.rs = son.sum;  
 else son.ms = c, son.ls = son.rs = 0;  
 pushup(r), pushup(l);  
 }  
 else if (!strcmp(op, "REVERSE"))  
 {  
 int posi, tot;  
 scanf("%d%d", &posi, &tot);  
 int l = get\_k(posi), r = get\_k(posi + tot + 1);  
 splay(l, 0), splay(r, l);  
 auto& son = tr[tr[r].s[0]];  
 son.rev ^= 1;  
 swap(son.ls, son.rs);  
 swap(son.s[0], son.s[1]);  
 pushup(r), pushup(l);  
 }  
 else if (!strcmp(op, "GET-SUM"))  
 {  
 int posi, tot;  
 scanf("%d%d", &posi, &tot);  
 int l = get\_k(posi), r = get\_k(posi + tot + 1);  
 splay(l, 0), splay(r, l);  
 printf("%d\n", tr[tr[r].s[0]].sum);  
 }  
 else printf("%d\n", tr[root].ms);  
 }  
  
 return 0;  
}

## y总Splay

#include <bits/stdc++.h>  
  
using namespace std;  
const int N = 1e6 + 10;  
struct node {  
 int p, v, s[2];  
 int siz, tag;  
 void init(int \_v, int \_p) {  
 v = \_v, p = \_p;  
 siz = 1;  
 }  
};  
node tr[N];  
int root, idx;  
  
void pushup(int x) { tr[x].siz = tr[tr[x].s[0]].siz + tr[tr[x].s[1]].siz + 1; }  
  
void pushdown(int x) {  
 if (tr[x].tag) {  
 swap(tr[x].s[0], tr[x].s[1]);  
 tr[tr[x].s[0]].tag ^= 1;  
 tr[tr[x].s[1]].tag ^= 1;  
 tr[x].tag = 0;  
 }  
}  
void rotate(int x) {  
 pushdown(x);   
 int y = tr[x].p, z = tr[y].p;  
 int k = tr[y].s[1] == x;  
 tr[y].s[k] = tr[x].s[k ^ 1], tr[tr[y].s[k]].p = y;  
 tr[x].s[k ^ 1] = y, tr[y].p = x;  
 tr[z].s[tr[z].s[1] == y] = x, tr[x].p = z;  
 pushup(y), pushup(x);  
}  
  
void splay(int x, int k) {  
 while (tr[x].p != k) {  
 int y = tr[x].p, z = tr[y].p;  
 if (z != k) {  
 if ((tr[z].s[1] == y) ^ (tr[y].s[1] == x)) {  
 rotate(x);  
 } else {  
 rotate(y);  
 }  
 }  
 rotate(x);  
 }  
 if (!k) root = x;  
}  
  
void insert(int v) {  
 int u = root, p = 0;  
 while (u) p = u, u = tr[u].s[v > tr[u].v];  
 u = ++idx;  
 if (p) tr[p].s[v > tr[p].v] = u;  
 tr[u].init(v, p);  
 splay(u, 0);  
}  
  
int getk(int k) {  
 int u = root;  
 while (1) {  
 pushdown(u);  
 if (k <= tr[tr[u].s[0]].siz) {  
 u = tr[u].s[0];  
 } else if (k == tr[tr[u].s[0]].siz + 1) {  
 splay(u, 0);  
 return u;  
 } else {  
 k -= tr[tr[u].s[0]].siz + 1, u = tr[u].s[1];  
 }  
 }  
}  
  
int n, m;  
void output(int u) {  
 if (u == 0) return;  
 pushdown(u);  
 output(tr[u].s[0]);  
 if (1 <= tr[u].v && tr[u].v <= n) cout << tr[u].v << ' ';  
 output(tr[u].s[1]);  
}  
  
int main() {  
 ios::sync\_with\_stdio(0), cin.tie(0), cout.tie(0);  
 cin >> n >> m;  
 for (int i = 0; i <= n + 1; i++) insert(i);  
 while (m--) {  
 int a, b;  
 cin >> a >> b;  
 int id1 = getk(a), id2 = getk(b + 2);  
 splay(id1, 0), splay(id2, id1);  
 tr[tr[id2].s[0]].tag ^= 1;  
 }  
 output(root);  
}

## 主席树

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const ll N = 1 << 20;  
  
ll ch[N << 5][2], rt[N], tot;  
ll val[N << 5];  
  
ll update(ll a, ll b) {  
 return a + b;  
}  
  
ll build(ll l, ll r) { // 建树  
 ll p = ++tot;  
 if (l == r) {  
 //初始化  
 val[p] = 0;  
 return p;  
 }  
 ll mid = (l + r) >> 1;  
 ch[p][0] = build(l, mid);  
 ch[p][1] = build(mid + 1, r);  
 val[p] = update(val[ch[p][0]], val[ch[p][1]]);  
 return p; // 返回该子树的根节点  
}  
  
ll modify(ll pre, ll l, ll r, ll pos, ll v) { // 插入操作  
 ll now = ++tot;  
 ch[now][0] = ch[pre][0], ch[now][1] = ch[pre][1];  
 if (l == r) {  
 val[now] = val[pre] + v;  
 return now;  
 }  
 ll mid = (l + r) >> 1;  
 if (pos <= mid)  
 ch[now][0] = modify(ch[now][0], l, mid, pos, v);  
 else  
 ch[now][1] = modify(ch[now][1], mid + 1, r, pos, v);  
 val[now] = update(val[ch[now][0]], val[ch[now][1]]);  
 return now;  
}  
  
ll kth(ll pre, ll now, ll l, ll r, ll k) { // 查询操作  
 ll mid = (l + r) >> 1;  
 ll x = val[ch[now][0]] - val[ch[pre][0]]; // 通过区间减法得到左儿子的信息  
 if (l == r) return l;  
 if (k <= x) // 说明在左儿子中  
 return kth(ch[pre][0], ch[now][0], l, mid, k);  
 else // 说明在右儿子中  
 return kth(ch[pre][1], ch[now][1], mid + 1, r, k - x);  
}  
  
ll query(ll pre, ll now, ll l, ll r, ll ql, ll qr) { // 查询操作  
 if (ql <= l && r <= qr) {  
 return val[now] - val[pre];  
 }  
 if (qr < l || r < ql) {  
 return 0;  
 }  
 ll mid = (l + r) >> 1;  
 ll lv = query(ch[pre][0], ch[now][0], l, mid, ql, qr);  
 ll rv = query(ch[pre][1], ch[now][1], mid + 1, r, ql, qr);  
 return update(lv, rv);  
}  
//修改查询记得用rt[]!!!

## 仙人掌

/\*  
 仙人掌:任意一条边至多只出现在一条简单回路的无向连通图称为仙人掌。  
 转化为圆方树，然后根据树的算法来做一些问题，注意区分圆点和方点  
 这题:求带环（环和环之间无公共边）无向图两点间的最短路径  
 \*/  
  
#include <iostream>  
#include <cstring>  
#include <algorithm>  
  
using namespace std;  
  
const int N = 12010, M = N \* 3;  
  
int n, m, Q, new\_n;  
int h1[N], h2[N], e[M], w[M], ne[M], idx;  
int dfn[N], low[N], cnt;  
int s[N], stot[N], fu[N], fw[N];  
int fa[N][14], depth[N], d[N];  
int A, B;  
  
void add(int h[], int a, int b, int c)  
{  
 e[idx] = b, w[idx] = c, ne[idx] = h[a], h[a] = idx ++ ;  
}  
  
void build\_circle(int x, int y, int z)  
{  
 int sum = z;  
 for (int k = y; k != x; k = fu[k])  
 {  
 s[k] = sum;  
 sum += fw[k];  
 }  
 s[x] = stot[x] = sum;  
 add(h2, x, ++ new\_n, 0);  
 for (int k = y; k != x; k = fu[k])  
 {  
 stot[k] = sum;  
 add(h2, new\_n, k, min(s[k], sum - s[k]));  
 }  
}  
  
void tarjan(int u, int from)  
{  
 dfn[u] = low[u] = ++ cnt;  
 for (int i = h1[u]; ~i; i = ne[i])  
 {  
 int j = e[i];  
 if (!dfn[j])  
 {  
 fu[j] = u, fw[j] = w[i];  
 tarjan(j, i);  
 low[u] = min(low[u], low[j]);  
 if (dfn[u] < low[j]) add(h2, u, j, w[i]);  
 }  
 else if (i != (from ^ 1)) low[u] = min(low[u], dfn[j]);  
 }  
 for (int i = h1[u]; ~i; i = ne[i])  
 {  
 int j = e[i];  
 if (dfn[u] < dfn[j] && fu[j] != u)  
 build\_circle(u, j, w[i]);  
 }  
}  
  
void dfs\_lca(int u, int father)  
{  
 depth[u] = depth[father] + 1;  
 fa[u][0] = father;  
 for (int k = 1; k <= 13; k ++ )  
 fa[u][k] = fa[fa[u][k - 1]][k - 1];  
 for (int i = h2[u]; ~i; i = ne[i])  
 {  
 int j = e[i];  
 d[j] = d[u] + w[i];  
 dfs\_lca(j, u);  
 }  
}  
  
int lca(int a, int b)  
{  
 if (depth[a] < depth[b]) swap(a, b);  
 for (int k = 13; k >= 0; k -- )  
 if (depth[fa[a][k]] >= depth[b])  
 a = fa[a][k];  
 if (a == b) return a;  
 for (int k = 13; k >= 0; k -- )  
 if (fa[a][k] != fa[b][k])  
 {  
 a = fa[a][k];  
 b = fa[b][k];  
 }  
 A = a, B = b;  
 return fa[a][0];  
}  
  
int main()  
{  
 scanf("%d%d%d", &n, &m, &Q);  
 new\_n = n;  
 memset(h1, -1, sizeof h1);  
 memset(h2, -1, sizeof h2);  
 while (m -- )  
 {  
 int a, b, c;  
 scanf("%d%d%d", &a, &b, &c);  
 add(h1, a, b, c), add(h1, b, a, c);  
 }  
 tarjan(1, -1);  
 dfs\_lca(1, 0);  
  
 while (Q -- )  
 {  
 int a, b;  
 scanf("%d%d", &a, &b);  
 int p = lca(a, b);  
 if (p <= n) printf("%d\n", d[a] + d[b] - d[p] \* 2);  
 else  
 {  
 int da = d[a] - d[A], db = d[b] - d[B];  
 int l = abs(s[A] - s[B]);  
 int dm = min(l, stot[A] - l);  
 printf("%d\n", da + dm + db);  
 }  
 }  
  
 return 0;  
}

## 区间max

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const int N = 1 << 20;  
  
struct node {  
 int mmax, semax, cnt;  
 ll sum;  
};  
  
node tree[N << 1];  
int init[N << 1];  
  
node merge\_range(node a, node b) {  
 node ans;  
 ans.sum = a.sum + b.sum;  
 if (a.mmax == b.mmax) {  
 ans.mmax = a.mmax;  
 ans.cnt = a.cnt + b.cnt;  
 ans.semax = max(a.semax, b.semax);  
 } else {  
 if (a.mmax < b.mmax) swap(a, b);  
 ans.mmax = a.mmax;  
 ans.cnt = a.cnt;  
 ans.semax = max(a.semax, b.mmax);  
 }  
 return ans;  
}  
  
void build(int k, int l, int r) {  
 if (l == r) {  
 tree[k] = {init[l], -1, 1, init[l]};  
 return;  
 }  
 int mid = (l + r) >> 1;  
 build(k << 1, l, mid);  
 build(k << 1 | 1, mid + 1, r);  
 tree[k] = merge\_range(tree[k << 1], tree[k << 1 | 1]);  
}  
  
  
void pushdown(int k, int l, int r) {  
 if (l == r) return;  
 if (tree[k].mmax < tree[k << 1].mmax) {  
 tree[k << 1].sum -= 1LL \* (tree[k << 1].mmax - tree[k].mmax) \* tree[k << 1].cnt;  
 tree[k << 1].mmax = tree[k].mmax;  
 }  
 if (tree[k].mmax < tree[k << 1 | 1].mmax) {  
 tree[k << 1 | 1].sum -= 1LL \* (tree[k << 1 | 1].mmax - tree[k].mmax) \* tree[k << 1 | 1].cnt;  
 tree[k << 1 | 1].mmax = tree[k].mmax;  
 }  
}  
  
  
node query(int k, int l, int r, int ql, int qr) {  
 if (qr < l || r < ql) return {0, -1, 1, 0};  
 if (ql <= l && r <= qr) {  
 return tree[k];  
 }  
 pushdown(k, l, r);  
 int mid = (l + r) >> 1;  
 node lq = query(k << 1, l, mid, ql, qr);  
 node rq = query(k << 1 | 1, mid + 1, r, ql, qr);  
 return merge\_range(lq, rq);  
}  
  
void modify(int k, int l, int r, int ql, int qr, int x) {  
 if (qr < l || r < ql) return;  
 if (ql <= l && r <= qr && tree[k].semax < x) {  
 if (x < tree[k].mmax) {  
 tree[k].sum -= 1LL \* (tree[k].mmax - x) \* tree[k].cnt;  
 tree[k].mmax = x;  
 }  
 return;  
 }  
 pushdown(k, l, r);  
 int mid = (l + r) >> 1;  
 modify(k << 1, l, mid, ql, qr, x);  
 modify(k << 1 | 1, mid + 1, r, ql, qr, x);  
 tree[k] = merge\_range(tree[k << 1], tree[k << 1 | 1]);  
}  
  
  
signed main() {  
// freopen("data.txt", "r", stdin);  
// freopen("test1.txt", "w", stdout);  
 int t;  
 scanf("%d", &t);  
 while (t--) {  
 int n, q;  
 scanf("%d%d", &n, &q);  
 for (int i = 1; i <= n; i++) scanf("%d", &init[i]);  
 build(1, 1, n);  
 while (q--) {  
 int x, y, op, val;  
 scanf("%d%d%d", &op, &x, &y);  
 if (op == 0) {  
 scanf("%d", &val);  
 modify(1, 1, n, x, y, val);  
 } else if (op == 1) {  
 node ans = query(1, 1, n, x, y);  
 printf("%d\n", ans.mmax);  
 } else {  
 node ans = query(1, 1, n, x, y);  
 printf("%lld\n", ans.sum);  
 }  
 }  
 }  
}

## 回滚莫队

/\*  
离线，询问按左端点升序为第一关键字，右端点升序为第二关键字  
对于都在块内的点直接暴力，否则跨块：  
若当前左端点所属的块与上一个不同，则将左端点初始为当前块的右端点+1，右端点初始为当前块的右端点  
左端点每次暴力，右端点单调  
\*/  
  
#include <iostream>  
#include <cstring>  
#include <cstdio>  
#include <algorithm>  
#include <cmath>  
#include <vector>  
  
using namespace std;  
  
typedef long long LL;  
const int N = 100010;  
  
int n, m, len;  
int w[N], cnt[N];  
LL ans[N];  
struct Query  
{  
 int id, l, r;  
}q[N];  
vector<int> nums;  
  
int get(int x)  
{  
 return x / len;  
}  
  
bool cmp(const Query& a, const Query& b)  
{  
 int i = get(a.l), j = get(b.l);  
 if (i != j) return i < j;  
 return a.r < b.r;  
}  
  
void add(int x, LL& res)  
{  
 cnt[x] ++ ;  
 res = max(res, (LL)cnt[x] \* nums[x]);  
}  
  
int main()  
{  
 scanf("%d%d", &n, &m);  
 len = sqrt(n);  
 for (int i = 1; i <= n; i ++ ) scanf("%d", &w[i]), nums.push\_back(w[i]);  
 sort(nums.begin(), nums.end());  
 nums.erase(unique(nums.begin(), nums.end()), nums.end());  
 for (int i = 1; i <= n; i ++ )  
 w[i] = lower\_bound(nums.begin(), nums.end(), w[i]) - nums.begin();  
  
 for (int i = 0; i < m; i ++ )  
 {  
 int l, r;  
 scanf("%d%d", &l, &r);  
 q[i] = {i, l, r};  
 }  
 sort(q, q + m, cmp);  
  
 for (int x = 0; x < m;)  
 {  
 int y = x;  
 while (y < m && get(q[y].l) == get(q[x].l)) y ++ ;  
 int right = get(q[x].l) \* len + len - 1;  
  
 // 暴力求块内的询问  
 while (x < y && q[x].r <= right)  
 {  
 LL res = 0;  
 int id = q[x].id, l = q[x].l, r = q[x].r;  
 for (int k = l; k <= r; k ++ ) add(w[k], res);  
 ans[id] = res;  
 for (int k = l; k <= r; k ++ ) cnt[w[k]] -- ;  
 x ++ ;  
 }  
  
 // 求块外的询问  
 LL res = 0;  
 int i = right, j = right + 1;  
 while (x < y)  
 {  
 int id = q[x].id, l = q[x].l, r = q[x].r;  
 while (i < r) add(w[ ++ i], res);  
 LL backup = res;  
 while (j > l) add(w[ -- j], res);  
 ans[id] = res;  
 while (j < right + 1) cnt[w[j ++ ]] -- ;  
 res = backup;  
 x ++ ;  
 }  
 memset(cnt, 0, sizeof cnt);  
 }  
  
 for (int i = 0; i < m; i ++ ) printf("%lld\n", ans[i]);  
 return 0;  
}

## 带修莫队

#include <bits/stdc++.h>  
using namespace std;  
  
const int N = 10010;  
  
int a[N], cnt[1000010], ans[N];  
  
int len, mq, mc;  
  
struct Query {  
 int id, l, r, t;  
} q[N];  
  
struct Modify {  
 int p, c;  
} c[N];  
  
int getNum(int x) {  
 return x / len;  
}  
  
// l所在块的编号，r所在块的编号，t升序  
  
bool cmp(const Query& a, const Query& b) {  
 if(getNum(a.l) == getNum(b.l) && getNum(a.r) == getNum(b.r)) {  
 return a.t < b.t;  
 }   
 if(getNum(a.l) == getNum(b.l)) return a.r < b.r;  
 return a.l < b.l;   
}  
  
void add(int x, int& res) {  
 if (!cnt[x]) res ++ ;  
 cnt[x] ++ ;  
}  
  
void del(int x, int& res) {  
 cnt[x] -- ;  
 if (!cnt[x]) res -- ;  
}  
  
  
int main() {  
 ios::sync\_with\_stdio(0); cin.tie(0); cout.tie(0);  
   
 int n, m;  
 cin >> n >> m;  
 char op;  
 int x, y;  
 for(int i = 1; i <= n; ++ i) {  
 cin >> a[i];  
 }  
 for(int i = 1; i <= m; ++ i) {  
 cin >> op >> x >> y;  
 if (op == 'Q') q[++ mq] = {mq, x, y, mc};  
 else c[ ++ mc] = {x, y};  
 }  
   
 ///  
 len = cbrt((double)n \* mc) + 1;  
 sort(q + 1, q + mq + 1, cmp);  
   
 int i = 1, j = 0, t = 0, res = 0;  
 for(int k = 1; k <= mq; ++ k) {  
 int id = q[k].id, l = q[k].l, r = q[k].r, tm = q[k].t;  
 while(j < r) add(a[++ j], res);  
 while(j > r) del(a[j --], res);  
 while(i < l) del(a[i ++], res);  
 while(i > l) add(a[-- i], res);  
 while(t < tm) {  
 ++ t;  
 if(c[t].p >= i && c[t].p <= j) {  
 del(a[c[t].p], res);  
 add(c[t].c, res);  
 }  
 swap(a[c[t].p], c[t].c);  
 }  
 while(t > tm) {  
 if(c[t].p >= i && c[t].p <= j) {  
 del(a[c[t].p], res);  
 add(c[t].c, res);  
 }  
 swap(a[c[t].p], c[t].c);  
 -- t;  
 }  
 ans[id] = res;  
 }  
   
 for(int i = 1; i <= mq; ++ i) {  
 cout << ans[i] << endl;  
 }  
}

## 普通莫队

#include <bits/stdc++.h>  
using namespace std;  
  
const int N = 1e6 + 10, M = 1e6 + 10;  
int a[N];  
  
struct node {   
 int id, l, r;  
} mp[M];  
  
int len;  
int ans[M], cnt[1000010];  
  
int getNum(int l) {  
 return l / len;  
}  
  
//左指针的分块，右指针的大小  
bool cmp (const node &a, const node & b) {  
 if(getNum(a.l) == getNum(b.l)) return a.r < b.r;  
 return a.l < b.l;  
}  
/\* 奇偶优化  
struct node {  
 int l, r, id;  
 bool operator<(const node &x) const {  
 if (l / unit != x.l / unit) return l < x.l;  
 if ((l / unit) & 1)  
 return r < x.r; // 注意这里和下面一行不能写小于（大于）等于  
 return r > x.r;  
 }  
};  
\*/  
  
void add(int x, int& res) {  
 if(cnt[x] == 0) res++;  
 cnt[x] ++;  
}  
  
void del(int x, int& res) {  
 cnt[x] --;  
 if(cnt[x] == 0) res --;  
}  
  
int main() {  
 ios::sync\_with\_stdio(0); cin.tie(0); cout.tie(0);  
   
 int n;  
 cin >> n;  
 for(int i = 1; i <= n; ++ i) {  
 cin >> a[i];  
 }  
 int m;  
 cin >> m;  
 len = sqrt((double)n \* n / m);  
 for(int i = 1; i <= m; ++ i) {  
 mp[i].id = i;  
 cin >> mp[i].l >> mp[i].r;  
 }  
 sort(mp + 1, mp + m + 1, cmp);  
   
 //离线处理询问   
 int res = 0, i = 0, j = 0;  
 for(int k = 1; k <= m; ++ k) {  
 int id = mp[k].id, l = mp[k].l, r = mp[k].r;  
 while(j < r) add(a[++j], res);  
 while(j > r) del(a[j--], res);  
 while(i < l) del(a[i++], res);  
 while(i > l) add(a[--i], res);  
 ans[id] = res;  
 }  
   
 for(int i = 1; i <= m; ++ i) {  
 cout << ans[i] << endl;  
 }  
 return 0;  
}

## 树状数组（fenwick）

template <typename T>  
struct fenwick {  
 vector<T> fenw;  
 int n;  
  
 fenwick(int \_n) : n(\_n) {  
 fenw.resize(n);  
 }  
  
 void clear(){  
 fenw.clear();  
 fenw.resize(n);  
 }  
  
 void modify(int x, T v) {  
 while (x < n) {  
 fenw[x] += v;  
 //if(fenw[x]>=mod)fenw[x]-=mod;  
 x |= (x + 1);  
 }  
 }  
  
 T get(int x) {  
 T v{};  
 while (x >= 0) {  
 v += fenw[x];  
 //if(v>=mod)v-=mod;  
 x = (x & (x + 1)) - 1;  
 }  
 return v;  
 }  
  
 T gets(int l,int r){  
 T res=get(r)-get(l-1);  
 //if(res<0)res+=mod;  
 return res;  
 }  
};

## 线段树合并分裂

ll nodetot, recycnt, bac[N << 5], ch[N << 5][2], rt[N];  
ll val[N << 5];  
  
ll newnod() { return (recycnt ? bac[recycnt--] : ++nodetot); }  
  
void recyc(ll p) {  
 bac[++recycnt] = p, ch[p][0] = ch[p][1] = val[p] = 0;  
 return;  
}  
  
void pushdown(ll p) {  
  
}  
  
void pushup(ll p) {  
 val[p] = 0;  
 if (ch[p][0]) val[p] += val[ch[p][0]];  
 if (ch[p][1]) val[p] += val[ch[p][1]];  
}  
  
void modify(ll &p, ll l, ll r, ll pos, ll v) {  
 if (!p) { p = newnod(); }  
 if (l == r) {  
 val[p] += v;  
 return;  
 }  
 ll mid = (l + r) >> 1;  
// pushdown(p);  
 if (pos <= mid) { modify(ch[p][0], l, mid, pos, v); }  
 else { modify(ch[p][1], mid + 1, r, pos, v); }  
 pushup(p);  
 return;  
}  
  
ll query(ll p, ll l, ll r, ll xl, ll xr) {  
 if (xr < l || r < xl) { return 0; }  
 if (xl <= l && r <= xr) { return val[p]; }  
 ll mid = (l + r) >> 1;  
// pushdown(p);  
 return query(ch[p][0], l, mid, xl, xr) + query(ch[p][1], mid + 1, r, xl, xr);  
}  
  
ll kth(ll p, ll l, ll r, ll k) {  
 if (l == r) { return l; }  
 ll mid = (l + r) >> 1;  
// pushdown(p);  
 if (val[ch[p][0]] >= k) { return kth(ch[p][0], l, mid, k); }  
 else { return kth(ch[p][1], mid + 1, r, k - val[ch[p][0]]); }  
}  
  
ll merge(ll x, ll y, ll l, ll r) {  
 if (!x || !y) {  
 return x + y;  
 } // 只有一边有点，不用合并  
 ll p = newnod(); // 创建一个新结点 p  
 if (l == r) { // 边界（某些时候可以省略，见下面一个代码）  
 val[p] = val[x] + val[y];  
 return p;  
 }  
// pushdown(x), pushdown(y);  
 ll mid = (l + r) >> 1;  
 ch[p][0] = merge(ch[x][0], ch[y][0], l, mid);  
 ch[p][1] = merge(ch[x][1], ch[y][1], mid + 1, r);  
 recyc(x), recyc(y); // 垃圾回收  
 pushup(p); // pushup  
 return p;  
}  
  
void split(ll x, ll &y, ll k) {  
 if (x == 0) return;  
 y = newnod();  
 ll v = val[ch[x][0]];  
// pushdown(x);  
 if (k > v) { split(ch[x][1], ch[y][1], k - v); }  
 else { swap(ch[x][1], ch[y][1]); }  
 if (k < v) { split(ch[x][0], ch[y][0], k); }  
 val[y] = val[x] - k;  
 val[x] = k;  
 return;  
}

## 舞蹈链（多重覆盖）

#include <bits/stdc++.h>  
using namespace std;  
struct DLX {  
 static const int maxn = 1000; //列的上限  
 static const int maxr = 1000; //解的上限  
 static const int maxnode = 5000; //总结点数上限  
 static const int INF = 1000000000;  
 int n, sz;  
 int S[maxn];  
  
 int row[maxnode], col[maxnode];  
 int L[maxnode], R[maxnode], U[maxnode], D[maxnode];  
  
 int ansd, ans[maxr];  
  
 int vis[maxnode];  
  
 void init(int n) {  
 this->n = n;  
  
 //虚拟节点  
 for (int i = 0; i <= n; i++) {  
 U[i] = i;  
 D[i] = i;  
 L[i] = i - 1;  
 R[i] = i + 1;  
 }  
 R[n] = 0;  
 L[0] = n;  
  
 sz = n + 1;  
 memset(S, 0, sizeof(S));  
 }  
  
 void addRow(int r, vector<int> columns) {  
 int first = sz;  
 for (int i = 0; i < columns.size(); i++) {  
 int c = columns[i];  
 L[sz] = sz - 1;  
 R[sz] = sz + 1;  
 D[sz] = c;  
 U[sz] = U[c];  
 D[U[c]] = sz;  
 U[c] = sz;  
 row[sz] = r;  
 col[sz] = c;  
 S[c]++;  
 sz++;  
 }  
 R[sz - 1] = first;  
 L[first] = sz - 1;  
 }  
#define FOR(i, A, s) for (int i = A[s]; i != s; i = A[i])  
 void remove(int c) {  
 FOR(i, D, c) { L[R[i]] = L[i], R[L[i]] = R[i]; }  
 }  
  
 void restore(int c) {  
 FOR(i, U, c) { L[R[i]] = i, R[L[i]] = i; }  
 }  
 int f\_check() //精确覆盖区估算剪枝  
 {  
 /\*  
 强剪枝。这个  
 剪枝利用的思想是A\*搜索中的估价函数。即，对于当前的递归深度K下的矩阵，估计其最好情况下（即最少还需要多少步）才能出解。也就是，如果将能够覆盖当  
 前列的所有行全部选中，去掉这些行能够覆盖到的列，将这个操作作为一层深度。重复此操作直到所有列全部出解的深度是多少。如果当前深度加上这个估价函数返  
 回值，其和已然不能更优（也就是已经超过当前最优解），则直接返回，不必再搜。  
 \*/  
  
 int ret = 0;  
 FOR(c, R, 0) vis[c] = true;  
 FOR(c, R, 0)  
 if (vis[c]) {  
 ret++;  
 vis[c] = false;  
 FOR(i, D, c)  
 FOR(j, R, i) vis[col[j]] = false;  
 }  
 return ret;  
 }  
 // d为递归深度  
 void dfs(int d, vector<int>& v) {  
 if (d + f\_check() >= ansd) return;  
 if (R[0] == 0) {  
 if (d < ansd) {  
 ansd = d;  
 v.clear();  
 for (int i = 0; i < ansd; i++) {  
 v.push\_back(ans[i]);  
 }  
 } //找到解  
 return; //记录解的长度  
 }  
  
 //找到S最小的列c  
 int c = R[0];  
 FOR(i, R, 0)  
 if (S[i] < S[c])  
 c = i; //第一个未删除的列  
 //删除第c列  
 FOR(i, D, c) { //用结点i所在的行能覆盖的所有其他列  
 ans[d] = row[i];  
 remove(i);  
 FOR(j, R, i) remove(j); //删除结点i所在的能覆的所有其他列  
 dfs(d + 1, v);  
 FOR(j, L, i) restore(j);  
 restore(i); //恢复结点i所在的行能覆盖的所有其他列  
 } //恢复第c列  
 }  
  
 bool solve(vector<int>& v) {  
 v.clear();  
 ansd = INF;  
 dfs(0, v);  
 return !v.empty();  
 }  
};  
//使用时init初始化，vector中存入r行结点列表用addRow加行，solve(ans)后答案按行的选择在ans中  
DLX dlx;  
int main() {  
 int n, m;  
 cin >> n >> m;  
 dlx.init(m);  
 for (int i = 1; i <= n; i++) {  
 vector<int> v;  
 for (int j = 1; j <= m; j++) {  
 int a;  
 cin >> a;  
 if (a == 1) v.push\_back(j);  
 }  
 dlx.addRow(i, v);  
 }  
 vector<int> ans;  
 dlx.solve(ans);  
 for (int i = 0; i < ans.size(); i++) cout << ans[i];  
}

## 舞蹈链（精确覆盖）

#include <bits/stdc++.h>  
using namespace std;  
struct DLX {  
 static const int maxn = 1000; //列的上限  
 static const int maxr = 1000; //解的上限  
 static const int maxnode = 5000; //总结点数上限  
 int n, sz;  
 int S[maxn];  
  
 int row[maxnode], col[maxnode];  
 int L[maxnode], R[maxnode], U[maxnode], D[maxnode];  
  
 int ansd, ans[maxr];  
  
 void init(int n) {  
 this->n = n;  
  
 //虚拟节点  
 for (int i = 0; i <= n; i++) {  
 U[i] = i;  
 D[i] = i;  
 L[i] = i - 1;  
 R[i] = i + 1;  
 }  
 R[n] = 0;  
 L[0] = n;  
  
 sz = n + 1;  
 memset(S, 0, sizeof(S));  
 }  
  
 void addRow(int r, vector<int> columns) {  
 int first = sz;  
 for (int i = 0; i < columns.size(); i++) {  
 int c = columns[i];  
 L[sz] = sz - 1;  
 R[sz] = sz + 1;  
 D[sz] = c;  
 U[sz] = U[c];  
 D[U[c]] = sz;  
 U[c] = sz;  
 row[sz] = r;  
 col[sz] = c;  
 S[c]++;  
 sz++;  
 }  
 R[sz - 1] = first;  
 L[first] = sz - 1;  
 }  
#define FOR(i, A, s) for (int i = A[s]; i != s; i = A[i])  
 void remove(int c) {  
 L[R[c]] = L[c];  
 R[L[c]] = R[c];  
 FOR(i, D, c)  
 FOR(j, R, i) {  
 U[D[j]] = U[j];  
 D[U[j]] = D[j];  
 --S[col[j]];  
 }  
 }  
  
 void restore(int c) {  
 FOR(i, U, c)  
 FOR(j, L, i) {  
 ++S[col[j]];  
 U[D[j]] = j;  
 D[U[j]] = j;  
 }  
 L[R[c]] = c;  
 R[L[c]] = c;  
 }  
  
 // d为递归深度  
 bool dfs(int d) {  
 if (R[0] == 0) {  
 ansd = d; //找到解  
 return true; //记录解的长度  
 }  
  
 //找到S最小的列c  
 int c = R[0];  
 FOR(i, R, 0) if (S[i] < S[c]) c = i; //第一个未删除的列  
  
 remove(c); //删除第c列  
 FOR(i, D, c) { //用结点i所在的行能覆盖的所有其他列  
 ans[d] = row[i];  
 FOR(j, R, i) remove(col[j]); //删除结点i所在的能覆的所有其他列  
 if (dfs(d + 1)) return true;  
 FOR(j, L, i) restore(col[j]); //恢复结点i所在的行能覆盖的所有其他列  
 }  
 restore(c); //恢复第c列  
  
 return false;  
 }  
  
 bool solve(vector<int>& v) {  
 v.clear();  
 if (!dfs(0)) return false;  
 for (int i = 0; i < ansd; i++) v.push\_back(ans[i]);  
 return true;  
 }  
};  
//使用时init初始化，vector中存入r行结点列表用addRow加行，solve(ans)后答案按行的选择在ans中

# 数论

BSGS 扩展BSGS

## BSGS

求 (a,p) = 1的最小的t

对 建立hash表，枚举x看是否有解

#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
  
unordered\_map<int , int> mp;  
  
int bsgs(int a, int p, int b) {  
   
 if (1 % p == b % p) return 0; // 特判0是不是解  
 mp.clear();  
   
 int k = sqrt(p) + 1;  
   
 for(int i = 0, j = b % p; i < k; ++ i, j = (ll)j \* a % p) {  
 mp[j] = i;  
 }  
   
 int ak = 1;  
 for(int i = 0; i < k; ++i) {  
 ak = (ll)ak \* a % p;  
 }  
   
 for(int i = 1, j = ak % p; i <= k; ++ i, j = (ll)j \* ak % p) {  
 if(mp.count(j)) return (ll)i \* k - mp[j];  
 }  
   
 return -1;  
}  
  
int main() {  
 ios::sync\_with\_stdio(0);  
 cin.tie(0); cout.tie(0);  
   
 int a, p, b;  
 while(cin >> a >> p >> b, a | p | b) {  
 int res;  
 res = bsgs(a, p, b);  
 if(res == -1) {  
 cout << "No Solution\n";   
 }  
 else {  
 cout << res << endl;  
 }  
 }  
   
 return 0;  
}

## 扩展BSGS

求 的最小的t

当

无解

， 两边同时除以d，

#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
  
unordered\_map<ll, ll> mp;  
  
ll bsgs(ll a, ll p, ll b) {  
   
 if(1 % p == b % p) return 0; // 特判0是不是解  
 mp.clear();   
   
 ll k = sqrt(p) + 1;  
   
 for(ll i = 0, j = b % p; i < k; ++i, j = (ll)j \* a % p) {  
 mp[j] = i;  
 }  
   
 ll ak = 1;  
 for(ll i = 0; i < k; ++i) {  
 ak = (ll) ak \* a % p;  
 }  
   
 for(ll i = 1, j = ak % p;i <= k; ++i, j = (ll)j \* ak % p) {  
 if(mp.count(j)) return (ll) i \* k - mp[j];  
 }  
   
 return -1;  
}  
  
ll gcd(ll x, ll y) {  
 return x % y == 0 ? y : gcd(y, x % y);   
}  
  
void extgcd(ll a,ll b,ll& d,ll& x,ll& y){  
 if(!b){  
 d = a; x = 1; y = 0;  
 }  
 else{   
 extgcd(b, a%b, d, y, x);   
 y -= x \* (a / b);   
 }  
}  
  
ll inverse(ll a,ll n){  
 ll d,x,y;  
 extgcd(a,n,d,x,y);  
 return d == 1 ? (x + n) % n : -1;  
  
}  
  
int main() {  
 ll a, p, b;  
   
 while(cin >> a >> p >> b, a | p | b) {  
 ll d = gcd(a, p);  
 if(d == 1) {  
 ll res = bsgs(a, p, b);  
 if(res == -1) {  
 cout << "No Solution\n";  
 }  
 else {  
 cout << res << endl;  
 }  
 }  
 else {  
 if(b % d != 0) {  
 cout << "No Solution\n";  
 continue;  
 }  
 else {  
 p = p / d;  
 b = (b / d) \* inverse(a / d, p);  
 ll res = bsgs(a, p, b);  
 if(res == -1) {  
 cout << "No Solution\n";  
 }  
 else {  
 cout << res + 1 << endl;  
 }  
 }   
 }  
 }   
   
 return 0;  
}

burnside&polya

## burnside引理

burnside ：用 表示在置换下不变的元素的个数，L表示本质不同的方案数（等价类）：

定理：, 该定理的一个重要研究对象是群的元素个数，其中 是K不动置换类，设G是1,2，.. n 的置换群，若k是1到n中某个元素，则G中使K保持不变的置换的全体，为. 是等价类，设G是1,2，.. n 的置换群，若k是1到n中某个元素，k在G的作用下的轨迹，为, 即k在G的作用下能变化成的所有元素的集合

每个置换的不动点的平均值就是不同方案数

任何一个置换都可以拆解成若干个循环置换

## polya定理

polya: 设G是p个对象的一个置换群，用m种颜色涂染p个对象，则不同染色的方案为：

其中 为置换为置换的循环节数

浅证：

每个置换的不动点有公式可以求

(不同循环直接完全独立)

## Cipolla

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
  
ll mod;  
ll I\_mul\_I; // 虚数单位的平方  
  
struct Complex {  
 ll real, imag;  
  
 Complex(ll real = 0, ll imag = 0) : real(real), imag(imag) {}  
};  
  
inline bool operator==(Complex x, Complex y) {  
 return x.real == y.real and x.imag == y.imag;  
}  
  
inline Complex operator\*(Complex x, Complex y) {  
 return Complex((x.real \* y.real + I\_mul\_I \* x.imag % mod \* y.imag) % mod,  
 (x.imag \* y.real + x.real \* y.imag) % mod);  
}  
  
Complex power(Complex x, ll k) {  
 Complex res = 1;  
 while (k) {  
 if (k & 1) res = res \* x;  
 x = x \* x;  
 k >>= 1;  
 }  
 return res;  
}  
  
bool check\_if\_residue(ll x) {  
 return power(x, (mod - 1) >> 1) == 1;  
}  
  
void solve(ll n, ll &x0, ll &x1) {  
  
 ll a = rand() % mod;  
 while (!a or check\_if\_residue((a \* a + mod - n) % mod))  
 a = rand() % mod;  
 I\_mul\_I = (a \* a + mod - n) % mod;  
 x0 = ll(power(Complex(a, 1), (mod + 1) >> 1).real);  
 x1 = mod - x0;  
}  
  
signed main() {  
 ios::sync\_with\_stdio(false);  
 cin.tie(nullptr);  
 cout.tie(nullptr);  
  
 ll t;  
 cin >> t;  
 while (t--) {  
 ll n;  
 cin >> n >> mod;  
 if (n == 0) {  
 cout << 0 << endl;  
 continue;  
 }  
 if (!check\_if\_residue(n)) {  
 cout << "Hola!" << endl;  
 continue;  
 }  
 ll x0, x1;  
 solve(n, x0, x1);  
 if (x0 > x1) swap(x0, x1);  
 cout << x0 << ' ' << x1 << endl;  
 }  
}

## exgcd

ll ex\_gcd(ll a, ll b, ll &x, ll &y) {  
 if (b == 0) {  
 x = 1;  
 y = 0;  
 return a;  
 }  
 ll d = ex\_gcd(b, a % b, x, y);  
 ll temp = x;  
 x = y;  
 y = temp - a / b \* y;  
 return d;  
}

## FFT

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const int N = 1e7 + 10;  
  
  
const double Pi = acos(-1.0);  
  
struct Complex {  
 double x, y;  
  
 Complex(double xx = 0, double yy = 0) { x = xx, y = yy; }  
} a[N], b[N];  
  
Complex operator+(Complex \_a, Complex \_b) { return Complex(\_a.x + \_b.x, \_a.y + \_b.y); }  
  
Complex operator-(Complex \_a, Complex \_b) { return Complex(\_a.x - \_b.x, \_a.y - \_b.y); }  
  
Complex operator\*(Complex \_a, Complex \_b) {  
 return Complex(\_a.x \* \_b.x - \_a.y \* \_b.y, \_a.x \* \_b.y + \_a.y \* \_b.x);  
} //不懂的看复数的运算那部分  
  
int L, r[N];  
int limit = 1;  
  
void fft(Complex \*A, int type) {  
 for (int i = 0; i < limit; i++)  
 if (i < r[i]) swap(A[i], A[r[i]]); //求出要迭代的序列  
 for (int mid = 1; mid < limit; mid <<= 1) { //待合并区间的长度的一半  
 Complex Wn(cos(Pi / mid), type \* sin(Pi / mid)); //单位根  
 for (int R = mid << 1, j = 0; j < limit; j += R) { //R是区间的长度，j表示前已经到哪个位置了  
 Complex w(1, 0); //幂  
 for (int k = 0; k < mid; k++, w = w \* Wn) { //枚举左半部分  
 Complex x = A[j + k], y = w \* A[j + mid + k]; //蝴蝶效应  
 A[j + k] = x + y;  
 A[j + mid + k] = x - y;  
  
 }  
 }  
 }  
}  
  
void FFT(int n, int m) {  
 limit = 1;  
 L = 0;  
 while (limit <= n + m) limit <<= 1, L++;  
 for (int i = 0; i < limit; i++) r[i] = (r[i >> 1] >> 1) | ((i & 1) << (L - 1));  
 // 在原序列中 i 与 i/2 的关系是 ： i可以看做是i/2的二进制上的每一位左移一位得来  
 // 那么在反转后的数组中就需要右移一位，同时特殊处理一下奇数  
 fft(a, 1), fft(b, 1);  
 for (int i = 0; i <= limit; i++) a[i] = a[i] \* b[i];  
 fft(a, -1);  
 for (int i = 0; i <= n + m; i++) a[i].x /= limit;  
}  
  
int main() {  
 int n, m;  
 cin >> n >> m;  
 for (int i = 0; i <= n; i++) cin >> a[i].x;  
 for (int i = 0; i <= m; i++) cin >> b[i].x;  
 FFT(n, m);  
 for (int i = 0; i <= n + m; i++) cout << (int) (a[i].x + 0.5) << ' ';  
 return 0;  
}

## FWT

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const int mod = 998244353;  
  
void add(int &x, int y) {  
 (x += y) >= mod && (x -= mod);  
}  
  
void sub(int &x, int y) {  
 (x -= y) < 0 && (x += mod);  
}  
  
namespace FWT {  
 int extend(int n) {  
 int N = 1;  
 for (; N < n; N <<= 1);  
 return N;  
 }  
  
 void FWTor(std::vector<int> &a, bool rev) {  
 int n = a.size();  
 for (int l = 2, m = 1; l <= n; l <<= 1, m <<= 1) {  
 for (int j = 0; j < n; j += l)  
 for (int i = 0; i < m; i++) {  
 if (!rev) add(a[i + j + m], a[i + j]);  
 else sub(a[i + j + m], a[i + j]);  
 }  
 }  
 }  
  
 void FWTand(std::vector<int> &a, bool rev) {  
 int n = a.size();  
 for (int l = 2, m = 1; l <= n; l <<= 1, m <<= 1) {  
 for (int j = 0; j < n; j += l)  
 for (int i = 0; i < m; i++) {  
 if (!rev) add(a[i + j], a[i + j + m]);  
 else sub(a[i + j], a[i + j + m]);  
 }  
 }  
 }  
  
 void FWTxor(std::vector<int> &a, bool rev) {  
 int n = a.size(), inv2 = (mod + 1) >> 1;  
 for (int l = 2, m = 1; l <= n; l <<= 1, m <<= 1) {  
 for (int j = 0; j < n; j += l)  
 for (int i = 0; i < m; i++) {  
 int x = a[i + j], y = a[i + j + m];  
 if (!rev) {  
 a[i + j] = (x + y) % mod;  
 a[i + j + m] = (x - y + mod) % mod;  
 } else {  
 a[i + j] = 1LL \* (x + y) \* inv2 % mod;  
 a[i + j + m] = 1LL \* (x - y + mod) \* inv2 % mod;  
 }  
 }  
 }  
 }  
  
 std::vector<int> Or(std::vector<int> a1, std::vector<int> a2) {  
 int n = std::max(a1.size(), a2.size()), N = extend(n);  
 a1.resize(N), FWTor(a1, false);  
 a2.resize(N), FWTor(a2, false);  
 std::vector<int> A(N);  
 for (int i = 0; i < N; i++) A[i] = 1LL \* a1[i] \* a2[i] % mod;  
 FWTor(A, true);  
 return A;  
 }  
  
 std::vector<int> And(std::vector<int> a1, std::vector<int> a2) {  
 int n = std::max(a1.size(), a2.size()), N = extend(n);  
 a1.resize(N), FWTand(a1, false);  
 a2.resize(N), FWTand(a2, false);  
 std::vector<int> A(N);  
 for (int i = 0; i < N; i++) A[i] = 1LL \* a1[i] \* a2[i] % mod;  
 FWTand(A, true);  
 return A;  
 }  
  
 std::vector<int> Xor(std::vector<int> a1, std::vector<int> a2) {  
 int n = std::max(a1.size(), a2.size()), N = extend(n);  
 a1.resize(N), FWTxor(a1, false);  
 a2.resize(N), FWTxor(a2, false);  
 std::vector<int> A(N);  
 for (int i = 0; i < N; i++) A[i] = 1LL \* a1[i] \* a2[i] % mod;  
 FWTxor(A, true);  
 return A;  
 }  
};  
  
int main() {  
 int n;  
 scanf("%d", &n);  
 n = (1 << n);  
 std::vector<int> a1(n), a2(n);  
 for (int i = 0; i < n; i++) scanf("%d", &a1[i]);  
 for (int i = 0; i < n; i++) scanf("%d", &a2[i]);  
 std::vector<int> A;  
 A = FWT::Or(a1, a2);  
 for (int i = 0; i < n; i++) {  
 printf("%d%c", A[i], " \n"[i == n - 1]);  
 }  
 A = FWT::And(a1, a2);  
 for (int i = 0; i < n; i++) {  
 printf("%d%c", A[i], " \n"[i == n - 1]);  
 }  
 A = FWT::Xor(a1, a2);  
 for (int i = 0; i < n; i++) {  
 printf("%d%c", A[i], " \n"[i == n - 1]);  
 }  
 return 0;  
}

## lucas求组合数

#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
  
ll p;  
  
const int maxn = 1e5 + 10;  
  
ll qpow(ll x, ll n){  
 ll res = 1;  
 while(n){  
 if(n & 1) res = (res \* x) % p;  
 x = (x \* x) % p;  
 n >>= 1;  
 }  
   
 return res;  
}  
  
ll C(ll up, ll down){  
 if(up > down) return 0;  
 ll res = 1;  
  
// for(int i = up + 1; i <= down; ++ i){  
// res = (res \* i) % p;  
// }  
// for(int i = 1; i <= down - up; ++ i){  
// res = (res \* qpow(i, p - 2)) % p;   
// }  
  
 for(int i = 1, j = down; i <= up; ++ i, -- j){  
 res = (res \* j) % p;  
 res = (res \* qpow(i, p - 2)) % p;  
 }  
   
 return res;  
}  
  
  
ll lucas(ll up, ll down){  
 if(up < p && down < p) return C(up, down);  
 return C(up % p, down % p) \* lucas(up / p, down / p) % p;   
}  
  
int main(){  
 ios::sync\_with\_stdio(0); cin.tie(0); cout.tie(0);  
   
 int T;  
 cin >> T;  
 while (T --){  
 ll down, up;  
 cin >> down >> up >> p;  
   
 cout << lucas(up, down) % p << endl;  
 }  
   
 return 0;  
}

## min\_25筛

/\*  
https://loj.ac/p/6053  
筛积性函数f的前缀和  
f(1)=1  
f(p^e)=f xor e  
n<=1e10，LOJ 347ms本地1100ms  
\*/  
#include<bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
const ll mod=1e9+7,inv3=333333336;  
const int N=1e5+5;//开到sqrt(n)即可  
  
ll prime[N],sp0[N],sp1[N],sp2[N],g0[N<<1],g1[N<<1],g2[N<<1];  
ll pnum,min25n,sqrn,w[N<<1],ind1[N],ind2[N];  
bool notp[N];  
  
void pre() { //预处理，线性筛  
 notp[1]=1;  
 for(int i=1; i<N; i++) {  
 if(!notp[i]) {  
 prime[++pnum]=i;  
 sp0[pnum]=(sp0[pnum-1]+1)%mod;//p^0前缀和（p指质数），可以按需增删，下标意义为第pnum个质数的前缀和，而g的实际下标意义为w之前的前缀和，两者有所区别  
 sp1[pnum]=(sp1[pnum-1]+i)%mod;//p^1前缀和  
 sp2[pnum]=(sp2[pnum-1]+1ll\*i\*i)%mod;//p^2前缀和  
 }  
 for(int j=1; j<=pnum&&prime[j]\*i<N; j++) {  
 notp[i\*prime[j]]=1;  
 if(i%prime[j]==0)break;  
 }  
 }  
}  
  
void min25(ll n) {  
 ll tot=0;  
 min25n=n;  
 sqrn=sqrt(n);  
 for(ll i=1; i<=n; i=n/(n/i)+1) {  
 w[++tot]=n/i;//实际下标  
 ll x=w[tot]%mod;  
 g0[tot]=x-1;//x^0前缀和  
 g1[tot]=x\*(x+1)/2%mod-1;//x^1前缀和  
 g2[tot]=x\*(x+1)/2%mod\*(2\*x+1)%mod\*inv3%mod-1;//x^2前缀和  
 if(n/i<=sqrn)ind1[n/i]=tot;//离散下标  
 else ind2[n/(n/i)]=tot;//离散下标  
 }  
 for(int i=1; i<=pnum; i++) {//扩展埃氏筛，筛质数部分前缀和  
 for(int j=1; j<=tot&&prime[i]\*prime[i]<=w[j]; j++) {  
 int id=w[j]/prime[i]<=sqrn?ind1[w[j]/prime[i]]:ind2[n/(w[j]/prime[i])];  
 g0[j]-=(g0[id]-sp0[i-1]+mod)%mod;  
 g1[j]-=prime[i]\*(g1[id]-sp1[i-1]+mod)%mod;  
 g2[j]-=prime[i]\*prime[i]%mod\*(g2[id]-sp2[i-1]+mod)%mod;  
 g0[j]%=mod,g1[j]%=mod,g2[j]%=mod;  
 if(g0[j]<0)g0[j]+=mod;  
 if(g1[j]<0)g1[j]+=mod;  
 if(g2[j]<0)g2[j]+=mod;  
 }  
 }  
}  
  
//该前缀和不计算f(1)，需要自行加上  
ll S(ll x,int y) {//x以内最小质因子大于第y个因子的前缀和  
 if(prime[y]>=x)return 0;  
 int id=x<=sqrn?ind1[x]:ind2[min25n/x];  
 ll ans=(((g1[id]-g0[id])-(sp1[y]-sp0[y]))%mod+mod)%mod;//x以内大于第y个因子的质数部分前缀和  
 if(x>=2&&y<1)ans=(ans+2)%mod;//特判包含f(2)的情况  
 for(int i=y+1; i<=pnum&&prime[i]\*prime[i]<=x; i++) {//筛合数部分前缀和  
 ll pe=prime[i];  
 for(int e=1; pe<=x; e++,pe=pe\*prime[i]) {  
 ll fpe=prime[i]^e;//f(p^e)  
 ans=(ans+fpe%mod\*(S(x/pe,i)+(e!=1)))%mod;  
 }  
 }  
 return ans%mod;  
}  
  
int main() {  
 pre();//预处理一次即可  
 ll n;  
 scanf("%lld",&n);  
 min25(n);//每个不同的n都要调用一次该函数，再调用S(n,0)  
 printf("%lld\n",S(n,0)+1);//加上f(1)  
 return 0;  
}

## NTT

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
  
const int N = 4e6 + 10;  
const ll mod = 998244353, G = 3, Gi = 332748118;  
  
int limit = 1, L, r[N];  
ll a[N], b[N];  
  
ll qpow(ll \_a, ll \_b) {  
 ll ans = 1;  
 while (\_b) {  
 if (\_b & 1) ans = (ans \* \_a) % mod;  
 \_b >>= 1;  
 \_a = (\_a \* \_a) % mod;  
 }  
 return ans;  
}  
  
void ntt(ll \*A, int type) {  
 auto swap = [](ll &\_a, ll &\_b) {  
 \_a ^= \_b, \_b ^= \_a, \_a ^= \_b;  
 };  
 for (int i = 0; i < limit; i++)  
 if (i < r[i]) swap(A[i], A[r[i]]);  
 for (int mid = 1; mid < limit; mid <<= 1) {  
 ll Wn = qpow(type == 1 ? G : Gi, (mod - 1) / (mid << 1));  
 for (int j = 0; j < limit; j += (mid << 1)) {  
 ll w = 1;  
 for (int k = 0; k < mid; k++, w = (w \* Wn) % mod) {  
 int x = A[j + k], y = w \* A[j + k + mid] % mod;  
 A[j + k] = (x + y) % mod,  
 A[j + k + mid] = (x - y + mod) % mod;  
 }  
 }  
 }  
}  
  
void NTT(int n, int m) {  
 limit = 1;  
 L = 0;  
 while (limit <= n + m) limit <<= 1, L++;  
 for (int i = 0; i < limit; i++) r[i] = (r[i >> 1] >> 1) | ((i & 1) << (L - 1));  
 ntt(a, 1), ntt(b, 1);  
 for (int i = 0; i < limit; i++) a[i] = (a[i] \* b[i]) % mod;  
 ntt(a, -1);  
 ll inv = qpow(limit, mod - 2);  
 for (int i = 0; i <= n + m; i++) a[i] = a[i] \* inv % mod;  
}  
  
int main() {  
 int n, m;  
 cin >> n >> m;  
 for (int i = 0; i <= n; i++) {  
 cin >> a[i];  
 a[i] = (a[i] + mod) % mod;  
 }  
 for (int i = 0; i <= m; i++) {  
 cin >> b[i];  
 b[i] = (b[i] + mod) % mod;  
 }  
 NTT(n, m);  
 for (int i = 0; i <= n + m; i++) cout << a[i] << ' ';  
}  
  
/\*  
#include<cstdio>  
#include<cctype>  
#include<cstring>  
#include<cmath>  
namespace fast\_IO  
{  
 const int IN\_LEN=10000000,OUT\_LEN=10000000;  
 char ibuf[IN\_LEN],obuf[OUT\_LEN],\*ih=ibuf+IN\_LEN,\*oh=obuf,\*lastin=ibuf+IN\_LEN,\*lastout=obuf+OUT\_LEN-1;  
 inline char getchar\_(){return (ih==lastin)&&(lastin=(ih=ibuf)+fread(ibuf,1,IN\_LEN,stdin),ih==lastin)?EOF:\*ih++;}  
 inline void putchar\_(const char x){if(oh==lastout)fwrite(obuf,1,oh-obuf,stdout),oh=obuf;\*oh++=x;}  
 inline void flush(){fwrite(obuf,1,oh-obuf,stdout);}  
}  
using namespace fast\_IO;  
#define getchar() getchar\_()  
#define putchar(x) putchar\_((x))  
typedef long long LL;  
#define rg register  
template <typename T> inline T max(const T a,const T b){return a>b?a:b;}  
template <typename T> inline T min(const T a,const T b){return a<b?a:b;}  
template <typename T> inline T mind(T&a,const T b){a=a<b?a:b;}  
template <typename T> inline T maxd(T&a,const T b){a=a>b?a:b;}  
template <typename T> inline T abs(const T a){return a>0?a:-a;}  
template <typename T> inline void swap(T&a,T&b){T c=a;a=b;b=c;}  
template <typename T> inline void swap(T\*a,T\*b){T c=a;a=b;b=c;}  
template <typename T> inline T gcd(const T a,const T b){if(!b)return a;return gcd(b,a%b);}  
template <typename T> inline T square(const T x){return x\*x;};  
template <typename T> inline void read(T&x)  
{  
 char cu=getchar();x=0;bool fla=0;  
 while(!isdigit(cu)){if(cu=='-')fla=1;cu=getchar();}  
 while(isdigit(cu))x=x\*10+cu-'0',cu=getchar();  
 if(fla)x=-x;   
}  
template <typename T> void printe(const T x)  
{  
 if(x>=10)printe(x/10);  
 putchar(x%10+'0');  
}  
template <typename T> inline void print(const T x)  
{  
 if(x<0)putchar('-'),printe(-x);  
 else printe(x);  
}  
const int maxn=262145;  
int n,m;  
struct Ntt  
{  
 LL mod,a[maxn],b[maxn];;  
 inline LL pow(LL x,LL y)  
 {  
 rg LL res=1;  
 for(;y;y>>=1,x=x\*x%mod)if(y&1)res=res\*x%mod;  
 return res;  
 }  
 int lenth,Reverse[maxn];  
 inline void init(const int x)  
 {  
 rg int tim=0;lenth=1;  
 while(lenth<=x)lenth<<=1,tim++;  
 for(rg int i=0;i<lenth;i++)Reverse[i]=(Reverse[i>>1]>>1)|((i&1)<<(tim-1));  
 }  
 inline void NTT(LL\*A,const int fla)  
 {  
 for(rg int i=0;i<lenth;i++)if(i<Reverse[i])swap(A[i],A[Reverse[i]]);  
 for(rg int i=1;i<lenth;i<<=1)  
 {  
 LL w=pow(3,(mod-1)/i/2);  
 if(fla==-1)w=pow(w,mod-2);  
 for(rg int j=0;j<lenth;j+=(i<<1))  
 {  
 LL K=1;  
 for(rg int k=0;k<i;k++,K=K\*w%mod)  
 {  
 const LL x=A[j+k],y=A[j+k+i]\*K%mod;  
 A[j+k]=(x+y)%mod;  
 A[j+k+i]=(mod+x-y)%mod;  
 }  
 }  
 }  
 if(fla==-1)  
 {  
 const int inv=pow(lenth,mod-2);  
 for(rg int i=0;i<lenth;i++)A[i]=A[i]\*inv%mod;  
 }   
 }  
}Q[3];  
LL EXgcd(const LL a,const LL b,LL &x,LL &y)   
{   
 if(!b)  
 {  
 x=1,y=0;  
 return a;   
 }  
 const LL res=EXgcd(b,a%b,y,x);  
 y-=a/b\*x;  
 return res;  
}  
inline LL msc(LL a,LL b,LL mod)  
{  
 LL v=(a\*b-(LL)((long double)a/mod\*b+1e-8)\*mod);  
 return v<0?v+mod:v;  
}  
int N,a[3],p[3];  
LL CRT()  
{   
 LL P=1,sum=0;   
 for(rg int i=1;i<=N;i++)P\*=p[i];  
 for(rg int i=1;i<=N;i++)   
 {  
 const LL m=P/p[i];  
 LL x,y;  
 EXgcd(p[i],m,x,y);  
 sum=(sum+msc(msc(y,m,P),a[i],P))%P;  
 }  
 return sum;  
}  
int P;  
int main()  
{  
 read(n),read(m),read(P);  
 Q[0].mod=469762049,Q[0].init(n+m);  
 Q[1].mod=998244353,Q[1].init(n+m);  
 Q[2].mod=1004535809,Q[2].init(n+m);  
 for(rg int i=0;i<=n;i++)read(Q[0].a[i]),Q[2].a[i]=Q[1].a[i]=Q[0].a[i];  
 for(rg int i=0;i<=m;i++)read(Q[0].b[i]),Q[2].b[i]=Q[1].b[i]=Q[0].b[i];  
 Q[0].NTT(Q[0].a,1),Q[0].NTT(Q[0].b,1);  
 Q[1].NTT(Q[1].a,1),Q[1].NTT(Q[1].b,1);  
 Q[2].NTT(Q[2].a,1),Q[2].NTT(Q[2].b,1);  
 for(rg int i=0;i<Q[0].lenth;i++)  
 Q[0].a[i]=(LL)Q[0].a[i]\*Q[0].b[i]%Q[0].mod,  
 Q[1].a[i]=(LL)Q[1].a[i]\*Q[1].b[i]%Q[1].mod,  
 Q[2].a[i]=(LL)Q[2].a[i]\*Q[2].b[i]%Q[2].mod;  
 Q[0].NTT(Q[0].a,-1);  
 Q[1].NTT(Q[1].a,-1);  
 Q[2].NTT(Q[2].a,-1);  
 N=2,p[1]=Q[0].mod,p[2]=Q[1].mod;  
 const int INV=Q[2].pow(Q[0].mod,Q[2].mod-2)\*Q[2].pow(Q[1].mod,Q[2].mod-2)%Q[2].mod;  
 for(rg int i=0;i<=n+m;i++)  
 {  
 a[1]=Q[0].a[i],a[2]=Q[1].a[i];  
 const LL ans1=CRT();  
 const LL ans2=((Q[2].a[i]-ans1)%Q[2].mod+Q[2].mod)%Q[2].mod\*INV%Q[2].mod;  
 print((ans2\*Q[0].mod%P\*Q[1].mod%P+ans1)%P),putchar(' ');  
 }  
 return flush(),0;  
}  
  
\*/

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相关证明可以看：https://oi-wiki.org/math/number-theory/powerful-number/

## PN筛

**积性函数**：对于所有互质的和，总有，则称为积性函数。

**PN(power number)**：对于正整数，记的质因数分解为。 是 PN 当且仅当。（1也是PN）

性质1：所有 PN 都可以表示成的形式，因为大于等于2的数总能分解成的形式。

**性质2： 以内的 PN 至多有个，可以通过dfs枚举下一个质数的次数在时间内找到这些PN。**

已知为积性函数，求

通过PN筛求解的一般过程：

**1.构造积性函数满足，且能够快速求出（或者快速预处理个有效值）**

2.构造（其实不用真的构造）满足，且能够快速求出，由可得也是积性函数

**3.其实满足即可，也可以移项递推**

3.5.如果你在程序里递推复杂度会是的，但是实测跑不满，耗时较少。

**4.dfs求出小于等于n的PN的同时计算和答案，**

复杂度与计算和的复杂度有关（有时需要预处理），两者均为时，总复杂度为

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，求,

构造积性函数,即，并求出

这里先递推找规律：

|  |  |  |  |
| --- | --- | --- | --- |
| 0 |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |

容易发现时

也可以直接推式子：

时：

预处理逆元（或者记忆化h函数后）可以求出。

dfs出以内的所有PN，在dfs过程中，对于每个PN——,将累加到答案上即可。

实际代码时要注意的点：

1.质数需要处理到，可以多处理一点例如处理到

2.h函数只会用到PN数处的值，预处理/记忆化时，只需要存的，之类的可以直接在dfs中通过积性函数性质运算。

3.满足且的都是PN，所以其数量是不超过的。

4.可以按质数的下标（即第几个质数） 和存来省空间。

5.对于同一个n，G的有效取值会有个，即和，有时需要预处理。

6.这题部分乘法要先转\_\_int128再乘再取模。

PN筛

///2022杭电多校5-1002  
#include<bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
typedef pair<int,int> pii;  
  
const int inf=0x3f3f3f3f,N=1e7+9;  
const ll mod=4179340454199820289;  
  
const int PMAX=N,PN=N;//PN开到n以内P的最大数量可以省空间  
int prime[PN],pcnt;//prime[0]=1,prime[1]=2  
bool notp[PMAX];//motp[1]=1,notp[2]=0,notp[4]=1  
void Prime(){  
 pcnt=0;  
 for(int i=2;i<PMAX;i++){  
 if(!notp[i])prime[++pcnt]=i;  
 for(int j=1;j<=pcnt&&i\*prime[j]<PMAX;j++){  
 notp[i\*prime[j]]=1;  
 if(i%prime[j]==0)break;  
 }  
 }  
 notp[1]=1;  
}  
  
ll qpow(ll a,ll b){  
 ll ans=1;  
 while(b){  
 if(b&1)ans=(\_\_int128)ans\*a%mod;  
 a=(\_\_int128)a\*a%mod;  
 b>>=1;  
 }  
 return ans;  
}  
  
struct PNS{//修改G，f，g即可使用，不同的n只需要初始化ans和n。  
 ll ans,n;  
 ll G(ll x){  
 return (\_\_int128)x\*(x+1)/2%mod;  
 }  
 //f和g均只在h函数中使用，如果可以直接公式求h函数则不用定义这两个函数  
 ll f(int pid,ll c){  
 return (\_\_int128)qpow(prime[pid],c)\*qpow(c,mod-2)%mod;  
 }  
 //g函数不用记忆化，实际调用次数很少  
 ll g(int pid,ll c){  
 return qpow(prime[pid],c);  
 }  
  
 vector<ll>vh[PN];//这里要确保c>2时调用h(pid,c)前调用过h(pid,c-1)  
 ll h(int pid,ll c){  
 if(c==0)return 1;  
 if(c==1)return 0;  
 if(c-2>=(ll)vh[pid].size()){//n=1e12、1e13、1e14时会进80070、230567、670121次，跑不满根号n，所以递推的耗时也是不高的。  
 //vh[pid].push\_back((ll)((-(\_\_int128)qpow(prime[pid],c)\*qpow(c\*(c-1),mod-2)%mod+mod)%mod));  
 //递推h函数，需要配合f函数和g函数一起使用  
 ll ans=f(pid,c);  
 for(ll i=1;i<=c;i++){  
 ans=((ans-(\_\_int128)g(pid,i)\*h(pid,c-i))%mod+mod)%mod;  
 }  
 vh[pid].push\_back(ans);  
 }  
 return vh[pid][c-2];  
 }  
 void dfs(ll prod,ll hprod,int pid){  
 ans=(ans+(\_\_int128)hprod\*G(n/prod))%mod;  
 for(int i=pid;i<=pcnt;i++){  
 if(prod>n/prime[i]/prime[i])break;  
 for(ll c=2,x=prod\*prime[i]\*prime[i];x<=n;c++,x\*=prime[i]){  
 dfs(x,(\_\_int128)hprod\*h(i,c)%mod,i+1);  
 if(x>n/prime[i])break;  
 }  
 }  
 }  
}pns;  
  
int main(){  
 #ifdef ONLINE\_JUDGE  
 //std::ios::sync\_with\_stdio(false);  
 #else  
 freopen("1002.in","r",stdin);  
 //freopen("out.txt","w",stdout);  
 #endif  
 Prime();  
 int t;  
 scanf("%d",&t);  
 while(t--){  
 pns.ans=0;  
 scanf("%lld",&pns.n);  
 pns.dfs(1,1,1);  
 //printf("%lld\n",pns.ans);  
 printf("%lld\n",(ll)((\_\_int128)pns.ans\*qpow(pns.n,mod-2)%mod));  
 }  
 return 0;  
}

## pn筛\_zyx

#include <bits/stdc++.h>  
  
using namespace std;  
#define de(x) cout << #x << " = " << x << endl  
#define dd(x) cout << #x << " = " << x << " "  
typedef long long ll;  
const \_\_int128 N = 1e6 + 10;  
const \_\_int128 M = 41;  
const \_\_int128 mod = 4179340454199820289ll;  
const \_\_int128 inv2 = 2089670227099910145ll;  
  
\_\_int128 isp[N], pri[N], pcnt;  
vector<\_\_int128> h[N];  
  
void getPrime() {  
 fill(isp + 2, isp + N, 1);  
 for (\_\_int128 i = 2; i < N; i++) {  
 if (isp[i]) {  
 pri[pcnt++] = i;  
 }  
 for (\_\_int128 j = 0; j < pcnt && i \* pri[j] < N; j++) {  
 isp[i \* pri[j]] = 0;  
 if (i % pri[j] == 0) break;  
 }  
 }  
}  
  
\_\_int128 qpow(\_\_int128 x, \_\_int128 y) {  
 \_\_int128 ans = 1;  
 while (y) {  
 if (y & 1) ans = ans \* x % mod;  
 x = x \* x % mod;  
 y >>= 1;  
 }  
 return ans;  
}  
  
\_\_int128 g(\_\_int128 x) {  
 return x;  
}  
  
\_\_int128 G(\_\_int128 x) {  
 return x \* (x + 1) % mod \* inv2 % mod;  
}  
  
\_\_int128 f(\_\_int128 x, \_\_int128 c) {  
 return x \* qpow(c, mod - 2) % mod;  
}  
  
\_\_int128 ans;  
ll n;  
  
void dfs(\_\_int128 deep, \_\_int128 hpn, \_\_int128 pn, bool flag) {  
 if (flag) {  
 ans = (ans + hpn \* G(n / pn)) % mod;  
 }  
 if (deep >= pcnt) return;  
 if (pri[deep] \* pri[deep] \* pn > n) return;  
 dfs(deep + 1, hpn, pn, false);  
 for (\_\_int128 i = 2, pi = pri[deep] \* pri[deep] % mod; pn \* pi <= n; i++, pi = pi \* pri[deep] % mod) {  
 dfs(deep + 1, hpn \* h[deep][i] % mod, pn \* pi, true);  
 }  
}  
  
signed main() {  
 ios::sync\_with\_stdio(0);  
 getPrime();  
 for (\_\_int128 pid = 0; pid < pcnt; pid++) {  
 h[pid].push\_back(1);  
 h[pid].push\_back(0);  
 \_\_int128 invp = qpow(pri[pid], mod - 2);  
 for (\_\_int128 c = 2, pc = pri[pid] \* pri[pid]; c < M && pc <= 1e12; c++, pc = pc \* pri[pid]) {  
 h[pid].push\_back(f(pri[pid], c));//唯一f使用，传入参数类型自定义  
 \_\_int128 pci = qpow(pri[pid], c);  
 for (\_\_int128 i = 1, pi = pri[pid]; i <= c; i++, pi = pi \* pri[pid] % mod) {  
 pci = pci \* invp % mod;  
 h[pid][c] = (h[pid][c] - g(pi) \* h[pid][c - i] % mod + mod) % mod;  
 }  
 }  
 }  
  
 ll t;  
 cin >> t;  
 while (t--) {  
 cin >> n;  
 ans = G(n);  
 dfs(0, 1, 1, false);  
 cout << (ll) ans << endl;  
 }  
}

## Pollard\_Rho+Miller-Robin

typedef long long ll;  
namespace Miller\_Rabin {  
 const ll Pcnt = 12;  
 const ll p[Pcnt] = {2, 3, 5, 7, 11, 13, 17, 19, 61, 2333, 4567, 24251};  
  
 ll pow(ll a, ll b, ll p) {  
 ll ans = 1;  
 for (; b; a = (\_\_int128) a \* a % p, b >>= 1)if (b & 1)ans = (\_\_int128) ans \* a % p;  
 return ans;  
 }  
  
 bool check(ll x, ll p) {  
 if (x % p == 0 || pow(p % x, x - 1, x) ^ 1)return true;  
 ll t, k = x - 1;  
 while ((k ^ 1) & 1) {  
 t = pow(p % x, k >>= 1, x);  
 if (t ^ 1 && t ^ x - 1)return true;  
 if (!(t ^ x - 1))return false;  
 }  
 return false;  
 }  
  
 inline bool MR(ll x) { //用这个  
 if (x < 2)return false;  
 for (int i = 0; i ^ Pcnt; ++i) {  
 if (!(x ^ p[i]))return true;  
 if (check(x, p[i]))return false;  
 }  
 return true;  
 }  
}  
namespace Pollard\_Rho {  
#define Rand(x) (1ll\*rand()\*rand()%(x)+1)  
  
 ll gcd(const ll a, const ll b) { return b ? gcd(b, a % b) : a; }  
  
 ll mul(const ll x, const ll y, const ll X) {  
 ll k = (1.0L \* x \* y) / (1.0L \* X) - 1, t = (\_\_int128) x \* y - (\_\_int128) k \* X;  
 while (t < 0)t += X;  
 return t;  
 }  
  
 ll PR(const ll x, const ll y) {  
 int t = 0, k = 1;  
 ll v0 = Rand(x - 1), v = v0, d, s = 1;  
 while (true) {  
 v = (mul(v, v, x) + y) % x, s = mul(s, abs(v - v0), x);  
 if (!(v ^ v0) || !s)return x;  
 if (++t == k) {  
 if ((d = gcd(s, x)) ^ 1)return d;  
 v0 = v, k <<= 1;  
 }  
 }  
 }  
  
 void Resolve(ll x, ll &ans) {  
 if (!(x ^ 1) || x <= ans)return;  
 if (Miller\_Rabin::MR(x)) {  
 if (ans < x)ans = x;  
 return;  
 }  
 ll y = x;  
 while ((y = PR(x, Rand(x))) == x);  
 while (!(x % y))x /= y;  
 Resolve(x, ans);  
 Resolve(y, ans);  
 }  
  
 long long check(ll x) { //用这个，素数返回本身  
 ll ans = 0;  
 Resolve(x, ans);  
 return ans;  
 }  
}

## prufer

Prufer 序列 (Prufer code)，这是一种将带标号的树用一个唯一的整数序列表示的方法。

Prufer 序列可以将一个带标号n个结点的树用中的个整数表示。你也可以把它理解为完全图的生成树与数列之间的双射。

显然你不会想不开拿这玩意儿去维护树结构。这玩意儿常用组合计数问题上。

线性建立prufer

Prufer 是这样建立的：每次选择一个编号最小的叶结点并删掉它，然后在序列中记录下它连接到的那个结点。重复n - 2次后就只剩下两个结点，算法结束。

线性构造的本质就是维护一个指针指向我们将要删除的结点。首先发现，叶结点数是非严格单调递减的。要么删一个，要么删一个得一个。

于是我们考虑这样一个过程：维护一个指针p 。初始时 p指向编号最小的叶结点。同时我们维护每个结点的度数，方便我们知道在删除结点的时侯是否产生新的叶结点。操作如下：

1. 删除 指向的结点，并检查是否产生新的叶结点。
2. 如果产生新的叶结点，假设编号为x ，我们比较 p, x的大小关系。如果 x>p，那么不做其他操作；否则就立刻删除 x，然后检查删除x 后是否产生新的叶结点，重复 2步骤，直到未产生新节点或者新节点的编号>p 。
3. 让指针 p 自增直到遇到一个未被删除叶结点为止；

循环上述操作n - 2 次，就完成了序列的构造。

// 从原文摘的代码，同样以 0 为起点  
vector<vector<int>> adj;  
vector<int> parent;  
  
void dfs(int v) {  
 for (int u : adj[v]) {  
 if (u != parent[v]) parent[u] = v, dfs(u);  
 }  
}  
  
vector<int> pruefer\_code() {  
 int n = adj.size();  
 parent.resize(n), parent[n - 1] = -1;  
 dfs(n - 1);  
  
 int ptr = -1;  
 vector<int> degree(n);  
 for (int i = 0; i < n; i++) {  
 degree[i] = adj[i].size();  
 if (degree[i] == 1 && ptr == -1) ptr = i;  
 }  
  
 vector<int> code(n - 2);  
 int leaf = ptr;  
 for (int i = 0; i < n - 2; i++) {  
 int next = parent[leaf];  
 code[i] = next;  
 if (--degree[next] == 1 && next < ptr) {  
 leaf = next;  
 } else {  
 ptr++;  
 while (degree[ptr] != 1) ptr++;  
 leaf = ptr;  
 }  
 }  
 return code;  
}

性质

1. 在构造完 Prufer 序列后原树中会剩下两个结点，其中一个一定是编号最大的点 。
2. 每个结点在序列中出现的次数是其度数减1 。（没有出现的就是叶结点）

线性prufer转化成树

同线性构造 Prufer 序列的方法。在删度数的时侯会产生新的叶结点，于是判断这个叶结点与指针p的大小关系，如果更小就优先考虑它

// 原文摘代码  
vector<pair<int, int>> pruefer\_decode(vector<int> const& code) {  
 int n = code.size() + 2;  
 vector<int> degree(n, 1);  
 for (int i : code) degree[i]++;  
  
 int ptr = 0;  
 while (degree[ptr] != 1) ptr++;  
 int leaf = ptr;  
  
 vector<pair<int, int>> edges;  
 for (int v : code) {  
 edges.emplace\_back(leaf, v);  
 if (--degree[v] == 1 && v < ptr) {  
 leaf = v;  
 } else {  
 ptr++;  
 while (degree[ptr] != 1) ptr++;  
 leaf = ptr;  
 }  
 }  
 edges.emplace\_back(leaf, n - 1);  
 return edges;  
}

cayley公式

完全图 有 棵生成树。

用 Prufer 序列证:任意一个长度为n - 2的值域 [1, n] 的整数序列都可以通过 Prufer 序列双射对应一个生成树，于是方案数就是 。

图连通方案数

一个n个点m条边的带标号无向图有k个连通块。我们希望添加k - 1条边使得整个图连通。求方案数。

设表示每个连通块的数量。我们对k个连通块构造 Prufer 序列，然后你发现这并不是普通的 Prufer 序列。因为每个连通块的连接方法很多。不能直接淦就设啊。于是设为第 i个连通块的度数。由于度数之和是边数的两倍，于是 。则对于给定的d 序列构造 Prufer 序列的方案数是

$$\tbinom{k - 2}{d\_1 - 1, d\_2 - 1, \dots, d\_k - 1} = \frac{(k - 2)!}{(d\_1 - 1)!(d\_2 - 1)! \dots (d\_k - 1)!}$$

对于第i个连通块，它的连接方式有种，因此对于给定d序列使图连通的方案数是

$$\tbinom{k - 2}{d\_1 - 1, d\_2 - 1, \dots, d\_k - 1} \prod\_{i = 1}^{k}s\_i^{d\_i}$$

现在我们要枚举d序列，式子变成

$$\sum\_{d\_i\geq 1. \sum\_{i = 1}^{k} d\_i = 2k - 2} \tbinom{k - 2}{d\_1 - 1, d2 - 1, \dots ,d\_k - 1} \prod\_{i = 1}^{k}s\_i^{d\_i}$$

根据多元二项式定理

$$(x\_1+\dots+x\_m)^{p}= \sum\_{c\_i \geq 0, \sum\_{i = 1}^{m} c\_i = p} \tbinom{p}{C\_1, C\_2, \dots , C\_m} \prod\_{i= 1}^{m}x\_i^{C\_i}$$

对原式换元，设 ，显然有

$$\Rightarrow \sum\_{e\_i\ge 0, \sum\_{i= 1}^{k} e\_i = k - 2} \tbinom{k -2}{e\_1, e\_2, \dots, e\_k} \prod\_{i = 1}^{k}s\_i ^{e \_i+ 1} \\
化简 \Rightarrow (s\_1 + s\_2 + \dots + s\_k)^{k - 2} \prod\_{i = 1}^{k}s\_i \\
\Rightarrow n^{k - 2} \prod\_{i = 1}^{k} s\_i$$

## 中国剩余定理

#include<cstdio>  
  
using namespace std;  
typedef long long ll;  
  
ll n;  
ll a[100010], b[100010];  
  
ll mul(ll A, ll B, ll mod) //快速乘取余 模板  
{  
 ll ans = 0;  
 while (B > 0) {  
 if (B & 1) ans = (ans + A % mod) % mod;  
 A = (A + A) % mod;  
 B >>= 1;  
 }  
 return ans;  
}  
  
ll exgcd(ll A, ll B, ll &x, ll &y) //扩展欧几里得 模板  
{  
 if (!B) {  
 x = 1, y = 0;  
 return A;  
 }  
 ll d = exgcd(B, A % B, x, y);  
 ll tmp = x;  
 x = y, y = tmp - A / B \* y;  
 return d;  
}  
  
ll lcm(ll A, ll B) //求最小公倍数  
{  
 ll xxx, yyy;  
 ll g = exgcd(A, B, xxx, yyy);  
 return (A / g \* B);  
}  
  
ll excrt() //重点:求解同余方程组  
{  
 ll x, y;  
 ll M = b[1], ans = a[1]; //赋初值  
 //M为前k-1个数的最小公倍数，ans为前k-1个方程的通解  
 for (int i = 2; i <= n; i++) {  
 ll A = M, B = b[i];  
 ll C = (a[i] - ans % B + B) % B; //代表同余方程 ax≡c(mod b) 中a,b,c  
  
 ll g = exgcd(A, B, x, y);  
 //求得A,B的最大公约数，与同余方程ax≡gcd(a,b)(mod b)的解，  
  
 if (C % g) return -1; //无解的情况  
  
 x = mul(x, C / g, B); //求得x的值,x即t  
 ans += x \* M; //获得前k个方程的通解  
 M = lcm(M, B); //更改M的值  
 ans = (ans % M + M) % M;  
 }  
 return ans;  
}  
  
int main() {  
 scanf("%lld", &n);  
 for (int i = 1; i <= n; i++)  
 scanf("%lld%lld", &b[i], &a[i]);  
 ll ans = excrt();  
 printf("%lld", ans);  
}

## 二次剩余

解的数量

对于 能满足n是mod p的二次剩余的n一共有个（不包括0），非二次剩余为个

勒让德符号

欧拉判别准则

若n是二次剩余，当且仅当

若n是非二次剩余，当且仅当

Cipolla

找到一个数a满足是 **非二次剩余** ，至于为什么要找满足非二次剩余的数，在下文会给出解释。 这里通过生成随机数再检验的方法来实现，由于非二次剩余的数量为 ，接近 ，所以期望约 2 次就可以找到这个数。

建立一个＂复数域＂，并不是实际意义上的复数域，而是根据复数域的概念建立的一个类似的域。 在复数中 ，这里定义 ，于是就可以将所有的数表达为 的形式，这里的 和 都是模意义下的数，类似复数中的实部和虚部。

在有了 i和 a后可以直接得到答案， 的解为。

#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
int t;  
ll n, p;  
ll w;  
  
struct num { //建立一个复数域  
  
 ll x, y;  
};  
  
num mul(num a, num b, ll p) { //复数乘法  
 num ans = {0, 0};  
 ans.x = ((a.x \* b.x % p + a.y \* b.y % p \* w % p) % p + p) % p;  
 ans.y = ((a.x \* b.y % p + a.y \* b.x % p) % p + p) % p;  
 return ans;  
}  
  
ll binpow\_real(ll a, ll b, ll p) { //实部快速幂  
 ll ans = 1;  
 while (b) {  
 if (b & 1) ans = ans \* a % p;  
 a = a \* a % p;  
 b >>= 1;  
 }  
 return ans % p;  
}  
  
ll binpow\_imag(num a, ll b, ll p) { //虚部快速幂  
 num ans = {1, 0};  
 while (b) {  
 if (b & 1) ans = mul(ans, a, p);  
 a = mul(a, a, p);  
 b >>= 1;  
 }  
 return ans.x % p;  
}  
  
ll cipolla(ll n, ll p) {  
 n %= p;  
 if (p == 2) return n;  
 if (binpow\_real(n, (p - 1) / 2, p) == p - 1) return -1;  
 ll a;  
 while (1) { //生成随机数再检验找到满足非二次剩余的a  
 a = rand() % p;  
 w = ((a \* a % p - n) % p + p) % p;  
 if (binpow\_real(w, (p - 1) / 2, p) == p - 1) break;  
 }  
 num x = {a, 1};  
 return binpow\_imag(x, (p + 1) / 2, p);  
}

## 勾股数圆上格点数

## 勾股数

1.任何一个勾股数(a,b,c)内的三个数同时乘以一个正整数n得到的新数组(na, nb, nc)仍然是勾股数，

于是找abc互质的勾股数

一，当a为大于1的奇数2n+1时，

（把a拆成两个连续的自然数）

二，当a为大于4的偶数2n时，

（只想得到互质的数的话：a=4n，

公式1

a=2mnt

b=（m²-n²）t

c=（m²+n²）t

（t是倍数）

**完全公式**

a=m，b=(m^2 / k - k) / 2，c=(m^2 / k + k) / 2 ①

其中m ≥3

⒈ 当m确定为任意一个 ≥3的奇数时，k={1，m^2的所有小于m的因子}

⒉ 当m确定为任意一个 ≥4的偶数时，k={m^2 / 2的所有小于m的偶数因子}

## 高斯整数/高斯素数

[3B1B的视频](https://www.bilibili.com/video/av12131743/)

[洛谷某题](https://www.luogu.com.cn/problem/P2508)

二维平面转化为复数平面，

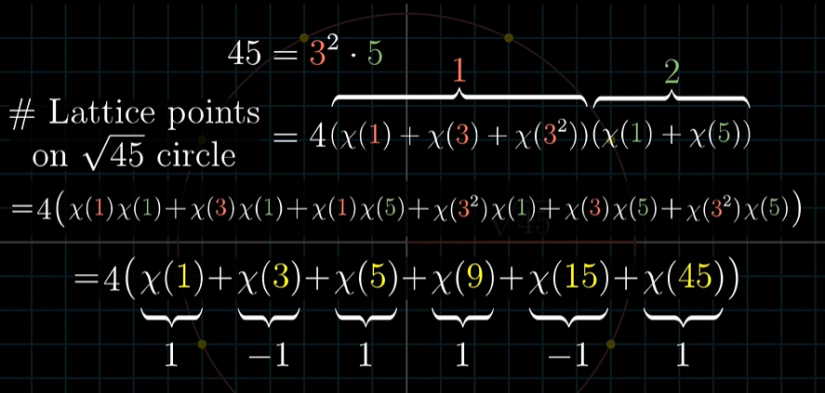
4n+1的素数，都能分解成高斯素数，4n+3的素数，他们本身就是高斯素数，2特殊

（乘以1， -1， i，-i 四个

半径为 的圆上的格点数，先将n分解质因数，对每个不是高斯素数的数分解成共轭的高斯素数，分配数比指数多1，指数是偶数的话，有一种方法分配，不然就没有格点

2 = (1+ i)(1 + i) ，但是这对数格点数没有影响，因为要乘-i。

它是一个周期函数，同时是一个积性函数，



再来看这个问题，

$$45 = 3^2 \times 5 \\
半径为 \sqrt{45} 圆上格点数问题 = 4 \times (f(1)+f(3)+f(3^2)) \times(f(1)+f(5))\\
=4 \times (f(1)+f(3)+f(5)+f(9)+f(15)+f(45))$$

最后转化为45的所有约数

$$f(x) = \begin{cases}
1 ,x 为素数 x = 4n+1 \\
-1, x为素数 x = 4n+3 \\
0, x为偶数\\
\end{cases}\\
半径为\sqrt { n}的圆上的格点数（二维坐标轴xy都为整数的点）是4 \times \sum\_{d|n}f(d)$$

## 博弈拾遗

## SG定理：

mex(minimal excludant)运算，表示最小的不属于这个集合的非负整数。例如mex{0,1,2,4}=3、mex{2,3,5}=0、mex{}=0。  
Sprague-Grundy定理（SG定理）：游戏和的SG函数等于各个游戏SG函数的Nim和。这样就可以将每一个子游戏分而治之，从而简化了问题。而Bouton定理就是Sprague-Grundy定理在Nim游戏中的直接应用，因为单堆的Nim游戏 SG函数满足 SG(x) = x。

## Nimk：

普通的NIM游戏是在n堆石子中每次选一堆，取任意个石子，而NIMK游戏是在n堆石子中每次选择k堆，1<=k<=n，从这k堆中每堆里都取出任意数目的石子，取的石子数可以不同，其他规则相同。  
对于普通的NIM游戏，我们采取的是对每堆的SG值进行异或，异或其实就是对每一个SG值二进制位上的数求和然后模2，比如说3^5就是011+101=112，然后对每一位都模2就变成了110，所以3^5=6。而NIMK游戏和NIM游戏的区别就在于模的不是2，如果是取k堆，就模k+1，所以取1堆的普通NIM游戏是模2。当k=2时,3^5→011+101=112，对每一位都模3之后三位二进制位上对应的数仍然是1，1，2。那么当且仅当每一位二进制位上的数都是0的时候，先手必败，否则先手必胜。

## anti\_nim

**描述**

和最普通的Nim游戏相同，不过是取走最后一个石子的人输。

**先手必胜条件**

以下两个条件满足其一即可：

1. 所有堆的石子个数=1，且异或和=0（其实这里就是有偶数堆的意思）。
2. 至少存在一堆石子个数>1，且异或和≠0。

## 卡特兰

‘卡特兰数1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012,...

~~C~~*~~{n}=\frac{4n-2}{n+1}C~~*~~{n-1}(C\_0=1)~~

超级卡特兰数1, 1, 3, 11, 45, 197, 903, 4279, 20793, 103049,...（从第0项开始）

大施罗德数(OEIS A006318)1, 2, 6, 22, 90, 394, 1806, 8558, 41586, 206098,...

超级卡特兰数的两倍（除第一项）

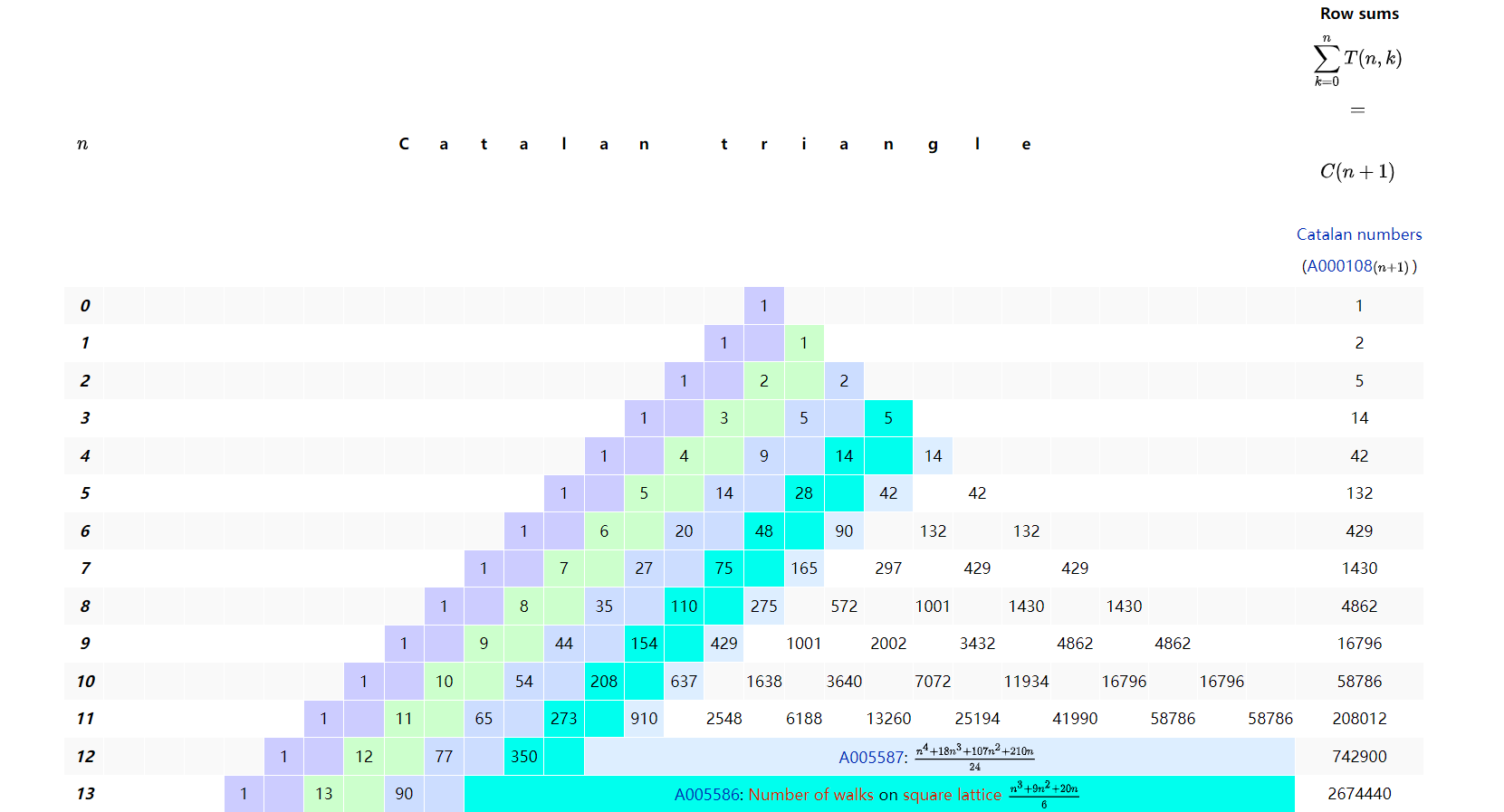
## 卡特兰三角

卡特兰三角

**卡特兰数**：由n个X和n个Y组成的一个序列中，满足**所有前缀中Y出现的次数不超过X出现的次数**的序列的个数

**卡特兰三角**：由n个X和k个Y组成的一个序列，满足**所有前缀中Y出现的次数-X出现的次数小于m**的序列的个数

卡特兰三角（OEIS）：



## 原根

#include<bits/stdc++.h>  
  
using namespace std;  
  
ll qpow(ll n, ll m, ll p) {  
 ll ans = 1;  
 n %= p;  
 for (; m; m >>= 1, n = n \* n % p)  
 if (m & 1)ans = ans \* n % p;  
 return ans;  
}  
  
ll pri[N], tot;  
  
ll getRoot(ll p) //求质数p的最小原根  
{  
 tot = 0;  
 ll n = p - 1, sq = sqrt(p + 0.5);  
 for (ll i = 2; i <= sq; i++)  
 if (n % i == 0) {  
 pri[tot++] = i;  
 while (n % i == 0)n /= i;  
 }  
 if (n > 1)pri[tot++] = n;  
 for (ll g = 2; g <= p - 1; g++) //试探每一个g是否原根  
 {  
 ll flag = 1;  
 for (ll i = 0; i < tot; i++)  
 if (qpow(g, (p - 1) / pri[i], p) == 1) {  
 flag = 0;  
 break;  
 }  
 if (flag)return g;  
 }  
 return -1; //没有原根  
}

## 快速幂

ll qpow(ll a, ll b) {  
 ll ans = 1;  
 while (b) {  
 if (b & 1) ans = (ans \* a) % mod;  
 a = (a \* a) % mod;  
 b >>= 1;  
 }  
 return ans;  
}

## 扩展欧拉定理

用于在底数与模数不互质的情况下将质数降将至与模数同阶大小，从而使用快速幂

$$a^c = \begin{cases}
a^{c \mod \phi(m)} , \gcd(a, m) = 1 \\
a^c, gcd(a, m)\not = 1 \and c < \phi(m) \\
a^{c \mod \phi(m) + \phi(m)}, gcd(a, m)\not = 1 \and c \ge \phi(m)
\end{cases}$$

证明以及引理：

**欧拉定理**：

证明欧拉：记 为第i个与m互质的数，则小于m的范围内共有 个这样的数

: {p*i}{x*i}$ 两两不同余

所有 的模m的集合与 相等 他们的积模m相等

**扩展欧拉**：

$$a^c = \begin{cases}
a^{c \mod \phi(m)} , \gcd(a, m) = 1 \\
a^c, gcd(a, m)\not = 1 \and c < \phi(m) \\
a^{c \mod \phi(m) + \phi(m)}, gcd(a, m)\not = 1 \and c \ge \phi(m)
\end{cases}$$

证明扩展欧拉(3)：

1. , P为质数
2. $\exist a, b, x, y , s.t. x^a \times y^b = k, 都有 a， b\le \phi(k) $
3. $\exist r \le c ,s.t. a^{\phi(m)+r} \equiv a ^r (\mod m)$

证明其中3：, 其中

又 是一个积性函数，故

两边同乘以

根据2， 又 ，得证

## 扩欧求逆元

#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
  
void extgcd(ll a,ll b,ll& d,ll& x,ll& y){  
 if(!b){ d=a; x=1; y=0;}  
 else{ extgcd(b,a%b,d,y,x); y-=x\*(a/b); }  
}  
  
ll inverse(ll a,ll n){  
 ll d,x,y;  
 extgcd(a,n,d,x,y);  
 return d==1?(x+n)%n:-1;  
}  
  
int main(){  
 int x, y;  
 //cin >> x >> y;  
 while(1){  
 cin >> x >> y;  
 cout << inverse(x, y) << endl;  
 }   
 //cout << inverse(x, y) << endl;  
}

## 数学知识

数学知识的一些范围（？

1 ~ n 的质数个数

1 ~ 2e9 中拥有最多约数个数的数拥有的约数个数

约1600

n个不同的点可以构成 棵不同的树

判断一个数是否为11的倍数

奇偶位置上的数位和的差是否为11的倍数

平方前缀和

立方前缀和

## 库默尔定理

设m,n为正整数，p为素数，则 含p的幂次等于m+n在p进制下的进位次数

## 原根存在定理

一个数m存在原根当且仅当, 其中p为奇素数，

## 整除分块（向上向下取整）

int x;  
scanf("%d",&x);  
int ans1=0,ans2=0;  
//向下取整  
for(int l=1,r;l<=x;l=r+1){  
 int m=x/l;  
 r=x/m;  
 ans1+=(r-l+1)\*m;  
}  
//向上取整  
int R=1e5;  
for(int l=1,r;l<=R;l=r+1){  
 int m=(x+l-1)/l;  
 r=m!=1?(x-1)/(m-1):R;  
 ans2+=(r-l+1)\*m;  
}

## 格雷码

int gray\_encode(int num) {  
 return num ^ (num >> 1);  
}  
  
int gray\_decode(int num) {  
 int head;  
 if (!num) return 0;  
 head = 1 << int(log(num) / log(2));  
 return head + gray\_decode((num ^ head) ^ (head >> 1));  
}

## 欧拉筛（素数）

#include <bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
const int N = 1000005;  
int phi[N], prime[N], cnt;  
bool st[N];  
  
void get\_eulers() {  
 phi[1] = 1;  
 for (int i = 2; i < N; i++) {  
 if (!st[i]) {  
 prime[cnt++] = i;  
 phi[i] = i - 1;  
 }  
 for (int j = 0; prime[j] \* i < N; j++) {  
 st[prime[j] \* i] = 1;  
 if (i % prime[j] == 0) {  
 phi[prime[j] \* i] = phi[i] \* prime[j];  
 break;  
 }  
 phi[prime[j] \* i] = phi[i] \* (prime[j] - 1);  
 }  
 }  
}  
  
int main() {  
 get\_eulers();  
 ll n;  
 cin >> n;  
 ll ans = 0;  
 for (int i = 1; i <= n; i++) ans += phi[i];  
 cout << ans;  
}

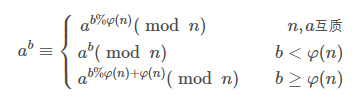
## 欧拉筛（莫比乌斯）

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const int N = 1e5 + 10;  
  
bool vis[N];  
ll prime[N], mu[N];  
  
void init\_mu() {  
 ll cnt = 0;  
 mu[1] = 1;  
 for (ll i = 2; i < N; i++) {  
 if (!vis[i]) {  
 prime[cnt++] = i;  
 mu[i] = -1;  
 }  
 for (ll j = 0; j < cnt && i \* prime[j] < N; j++) {  
 vis[i \* prime[j]] = 1;  
 if (i % prime[j] == 0) {  
 mu[i \* prime[j]] = 0;  
 break;  
 } else { mu[i \* prime[j]] = -mu[i]; }  
 }  
 }  
}  
  
int main() {  
 init\_mu();  
}

## 欧拉降幂

~~不知道它有什么用毕竟已经有快速幂了~~

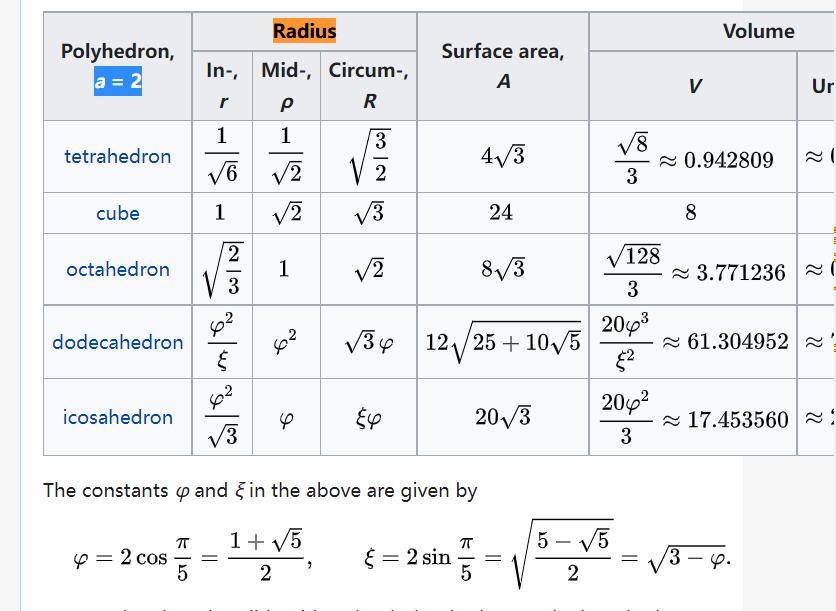
这里有一张图可以很好的说明欧拉降幂是什么



//其实只是想试一下markdown怎么用  
//假装这里有代码

然后下面这个是用 $\LaTeX$公式写的

## 正多面体



4 6 8 12 20

## 组合数

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const ll mod = 1e9 + 7;  
const ll maxn = 3e4 + 5;  
ll inv[maxn], fac[maxn];  
  
ll qpow(ll a, ll b) {  
 ll ans = 1;  
 while (b) {  
 if (b & 1) ans = (ans \* a) % mod;  
 a = (a \* a) % mod;  
 b >>= 1;  
 }  
 return ans;  
}  
  
ll c(ll n, ll m) {  
 if (n < 0 || m < 0 || n < m) return 0;  
 return fac[n] \* inv[n - m] % mod \* inv[m] % mod;  
}  
  
void init() {  
 fac[0] = 1;  
 for (int i = 1; i < maxn; i++) {  
 fac[i] = fac[i - 1] \* i % mod;  
 }  
 inv[maxn - 1] = qpow(fac[maxn - 1], mod - 2);  
 for (ll i = maxn - 2; i >= 0; i--) {  
 inv[i] = (inv[i + 1] \* (i + 1)) % mod;  
 }  
}

## 莫比乌斯反演

莫比乌斯函数

n的所有约数的莫比乌斯的和

反演

构造 使f(n)为目标，F(n)好求

1

求满足 且 gcd(x, y) = k 的xy的对数

#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
  
const int N = 50010;  
  
ll primes[N], mu[N], sum[N], cnt;  
bool st[N];  
  
void init() {  
 mu[1] = 1;  
   
 for(int i = 2; i < N; ++ i) {  
 if(!st[i]) {  
 primes[cnt ++] = i;  
 mu[i] = -1;  
 }  
   
 for(int j = 0; primes[j] \* i < N; ++ j) {  
 st[primes[j] \* i] = 1;  
 if(i % primes[j] == 0) break;  
 mu[primes[j] \* i] = -mu[i];   
 }  
 }  
   
 for(int i = 1; i < N; ++ i) {  
 sum[i] = sum[i - 1] + mu[i];  
 }  
}   
  
ll g(ll n, ll x) {  
 return n / (n / x);  
}  
  
ll f (int a, int b, int k) {  
 a = a / k, b = b / k;  
   
 ll res = 0;  
   
 ll n = min(a, b);  
   
 for(ll l = 1, r; l <= n; l = r + 1) {  
 r = min(n, min(g(a, l), g(b, l)));  
 res += (sum[r] - sum[l - 1]) \* (a / l) \* (b / l);  
 }  
   
 return res;  
}  
  
int main() {  
 ios::sync\_with\_stdio(0); cin.tie(0); cout.tie(0);  
   
 init();  
   
 int T;  
 cin >> T;  
 while(T --) {  
 int a, b, c, d, k;  
 cin >> a >> b >> c >> d >> k;  
 cout << f(b, d, k) - f(a - 1, d, k) - f(b, c - 1, k)   
 + f(a - 1, c - 1, k) << endl;  
 }   
   
 return 0;  
}

2

求

//

两次整数分块

#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
const int N = 50010;  
  
int primes[N], cnt, mu[N], sum[N], h[N];  
bool st[N];  
  
inline int g(int n, int x) {  
 return n / (n / x);  
}  
  
void init() {  
 mu[1] = 1;  
 for(int i = 2; i < N; ++i) {  
 if(!st[i]){  
 primes[cnt++] = i;  
 mu[i] = -1;  
 }  
 for(int j = 0; primes[j] \* i < N; ++j) {  
 st[primes[j] \* i] = 1;  
 if(i % primes[j] == 0) break;  
 mu[primes[j] \* i] = -mu[i];  
 }  
   
   
 }  
   
 for(int i = 1; i < N; ++ i) {  
 sum[i] = sum[i - 1] + mu[i];   
 }  
   
 for(int i = 1; i < N; ++i) {  
 for(int l = 1, r; l <= i; l = r + 1) {  
 r = min(i, g(i, l));  
 h[i] += (r - l + 1) \* (i / l);  
 }  
 }  
}  
  
int main() {  
 //ios::sync\_with\_stdio(0); cin.tie(0); cout.tie(0);   
 init();  
   
 int T;  
 scanf("%d", &T);  
 while(T--) {  
 int n, m;  
 scanf("%d %d", &n, &m);  
 ll res = 0;  
 int k = min(n, m);  
 for(int l = 1, r; l <= k; l = r + 1) {  
 r = min(k, min(g(n, l), g(m, l)));  
 res += (ll)(sum[r] - sum[l - 1]) \* h[n / l] \* h[m / l];  
 }  
 printf("%lld\n", res);  
 }  
   
 return 0;  
}

## 逆元线性递推 inv 阶乘逆元组合数

ll fac[N];// n!  
ll invfac[N]; // n!的inv  
ll invn[N]; //n的inv  
  
inline void init() {  
 fac[0] = fac[1] = invfac[0] = invfac[1] = invn[0] = invn[1] = 1;  
 for (int i = 2; i < N; ++i) {  
 fac[i] = fac[i - 1] \* i % mod;  
 invn[i] = (mod - mod / i) \* invn[mod % i] % mod;  
 invfac[i] = invfac[i - 1] \* invn[i] % mod;  
 }  
}  
  
ll C(ll up, ll down) {  
 if (up > down) return 0;  
 if (up < 0 || down < 0) return 0;  
 ll res = fac[down];  
 res = res \* invfac[down - up] % mod;  
 res = res \* invfac[up] % mod;  
 return res;  
}  
  
//先init

# 杂项

## fread快读

#include <bits/stdc++.h>  
using namespace std;  
  
char next\_char() {  
 static char buf[1 << 20], \*first, \*last;  
 if(first == last) {  
 last = buf + fread(buf, 1, 1 << 20, stdin);  
 first = buf;  
 }  
 return first == last ? EOF : \*first ++;  
}  
  
inline int read(){  
 int x = 0, w = 0; char ch = 0;  
 while(!isdigit(ch)) {w |= ch == '-'; ch = next\_char(); }  
 while(isdigit(ch)) {x = (x << 3) + (x << 1) + (ch ^ 48), ch = next\_char(); }  
 return w ? -x : x;  
}  
  
int main(){  
 freopen("1.txt", "r", stdin); // �������ʱ��һ��Ҫȥ��aaa   
 int T;  
 cin >> T;  
 while(T --){  
 int x = read();  
 cout << x << endl;  
 }  
}

## int128输出

inline void print(\_\_int128 x) {  
 if (x < 0) {  
 putchar('-');  
 x = -x;  
 }  
 if (x > 9)  
 print(x / 10);  
 putchar(x % 10 + '0');  
}

## mt19937

mt19937

#include <random>  
#include <iostream>  
  
int main()  
{  
 std::random\_device rd; //获取随机数种子  
 std::mt19937 gen(rd()); //Standard mersenne\_twister\_engine seeded with rd()  
 std::uniform\_int\_distribution<> dis(0, 9);  
  
 for (int n = 0; n<20; ++n)  
 std::cout << dis(gen) << ' ';  
 std::cout << '\n';  
 system("pause");  
 return 0;  
}  
  
//可能的结果：7 2 2 1 4 1 4 0 4 7 2 1 0 9 1 9 2 3 5 1

**doule ：**std::uniform*real*distribution<> dis(0, 9);

#include <iostream>  
#include <chrono>  
#include <random>  
using namespace std;  
int main()  
{  
 // 随机数种子  
 unsigned seed = std::chrono::system\_clock::now().time\_since\_epoch().count();  
 mt19937 rand\_num(seed); // 大随机数  
 uniform\_int\_distribution<long long> dist(0, 1000000000); // 给定范围  
 cout << dist(rand\_num) << endl;  
 return 0;  
}

**注意：** 代码中的 rand\_num 和 dist 都是自己定义的对象，不是系统的。

## 洗牌算法

#include <random>  
#include <algorithm>  
#include <iterator>  
#include <iostream>  
  
int main()  
{  
 std::vector<int> v = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 };  
  
 std::random\_device rd;  
 std::mt19937 g(rd());  
  
 std::shuffle(v.begin(), v.end(), g);  
  
 std::copy(v.begin(), v.end(), std::ostream\_iterator<int>(std::cout, " "));  
 std::cout << "\n";  
  
 system("pause");  
 return 0;  
}

## 大质数表

https://www.cnblogs.com/ljxtt/p/13514346.html

| 1e17 | 1e18 |
| --- | --- |
| 100000000000000003 | 1000000000000000003 |
| 100000000000000013 | 1000000000000000009 |
| 100000000000000019 | 1000000000000000031 |
| 100000000000000021 | 1000000000000000079 |
| 100000000000000049 | 1000000000000000177 |
| 100000000000000081 | 1000000000000000183 |
| 100000000000000099 | 1000000000000000201 |
| 100000000000000141 | 1000000000000000283 |
| 100000000000000181 | 1000000000000000381 |
| 100000000000000337 | 1000000000000000387 |
| 100000000000000339 | 1000000000000000507 |
| 100000000000000369 | 1000000000000000523 |
| 100000000000000379 | 1000000000000000583 |
| 100000000000000423 | 1000000000000000603 |
| 100000000000000519 | 1000000000000000619 |
| 100000000000000543 | 1000000000000000621 |
| 100000000000000589 | 1000000000000000799 |
| 100000000000000591 | 1000000000000000841 |
| 100000000000000609 | 1000000000000000861 |
| 100000000000000669 | 1000000000000000877 |
| 100000000000000691 | 1000000000000000913 |
| 100000000000000781 | 1000000000000000931 |
| 100000000000000787 | 1000000000000000997 |

## 快读 read

inline int read(){  
 int X=0,w=0;char ch=0;  
 while(!isdigit(ch)){w|=ch=='-';ch=getchar();}  
 while(isdigit(ch))X=(X<<3)+(X<<1)+(ch^48),ch=getchar();  
 return w?-X:X;  
}

## 整体二分

ll bit[N];  
  
void add\_bit(ll k, ll a) {  
 while (k < N) {  
 bit[k] = bit[k] + a;  
 k += k & -k;  
 }  
}  
  
ll query\_bit(ll k) {  
 ll ans = 0;  
 while (k) {  
 ans = ans + bit[k];  
 k -= k & -k;  
 }  
 return ans;  
}  
  
struct node {  
 ll x, y, k, id, type;  
};  
node q[N], q1[N], q2[N];  
ll ans[N], now[N], tot, totx;  
  
void solve(ll l, ll r, ll ql, ll qr) {  
 if (ql > qr) return;  
 if (l == r) {  
 for (ll i = ql; i <= qr; i++) {  
 if (q[i].type == 2) {  
 ans[q[i].id] = l;  
 }  
 }  
 return;  
 }  
 ll mid = (l + r) >> 1;  
 ll cq1 = 0, cq2 = 0;  
 for (ll i = ql; i <= qr; i++) {  
 if (q[i].type == 1) {  
 if (q[i].y <= mid) {  
 add\_bit(q[i].x, q[i].k);  
 q1[++cq1] = q[i];  
 } else {  
 q2[++cq2] = q[i];  
 }  
 } else {  
 ll sum = query\_bit(q[i].y) - query\_bit(q[i].x - 1);  
 if (sum >= q[i].k) {  
 q1[++cq1] = q[i];  
 } else {  
 q2[++cq2] = q[i];  
 q2[cq2].k -= sum;  
 }  
 }  
 }  
 for (ll i = 1; i <= cq1; i++) if (q1[i].type == 1) add\_bit(q1[i].x, -q1[i].k);  
 for (ll i = 1; i <= cq1; i++) q[ql + i - 1] = q1[i];  
 for (ll i = 1; i <= cq2; i++) q[ql + cq1 + i - 1] = q2[i];  
 solve(l, mid, ql, ql + cq1 - 1);  
 solve(mid + 1, r, ql + cq1, qr);  
  
}  
  
void init() {  
 totx = 0;  
 tot = 0;  
 memset(bit, 0, sizeof bit);  
}

## 朝鲜大哥快读

#define FI(n) FastIO::read(n)  
#define FO(n) FastIO::write(n)  
#define Flush FastIO::Fflush()  
//程序末尾写上 Flush;  
  
namespace FastIO {  
 const int SIZE = 1 << 16;  
 char buf[SIZE], obuf[SIZE], str[60];  
 int bi = SIZE, bn = SIZE, opt;  
 double D[] = {0.1, 0.01, 0.001, 0.0001, 0.00001, 0.000001, 0.0000001, 0.00000001, 0.000000001, 0.0000000001};  
  
 int read(char \*s) {  
 while (bn) {  
 for (; bi < bn && buf[bi] <= ' '; bi++);  
 if (bi < bn)  
 break;  
 bn = fread(buf, 1, SIZE, stdin);  
 bi = 0;  
 }  
 int sn = 0;  
 while (bn) {  
 for (; bi < bn && buf[bi] > ' '; bi++)  
 s[sn++] = buf[bi];  
 if (bi < bn)  
 break;  
 bn = fread(buf, 1, SIZE, stdin);  
 bi = 0;  
 }  
 s[sn] = 0;  
 return sn;  
 }  
  
 bool read(int &x) {  
 int n = read(str), bf = 0;  
 if (!n)  
 return 0;  
 int i = 0;  
 if (str[i] == '-')  
 bf = 1, i++;  
 else if (str[i] == '+')  
 i++;  
 for (x = 0; i < n; i++)  
 x = x \* 10 + str[i] - '0';  
 if (bf)  
 x = -x;  
 return 1;  
 }  
  
 bool read(long long &x) {  
 int n = read(str), bf;  
 if (!n)  
 return 0;  
 int i = 0;  
 if (str[i] == '-')  
 bf = -1, i++;  
 else  
 bf = 1;  
 for (x = 0; i < n; i++)  
 x = x \* 10 + str[i] - '0';  
 if (bf < 0)  
 x = -x;  
 return 1;  
 }  
  
 void write(int x) {  
 if (x == 0)  
 obuf[opt++] = '0';  
 else {  
 if (x < 0)  
 obuf[opt++] = '-', x = -x;  
 int sn = 0;  
 while (x)  
 str[sn++] = x % 10 + '0', x /= 10;  
 for (int i = sn - 1; i >= 0; i--)  
 obuf[opt++] = str[i];  
 }  
 if (opt >= (SIZE >> 1)) {  
 fwrite(obuf, 1, opt, stdout);  
 opt = 0;  
 }  
 }  
  
 void write(long long x) {  
 if (x == 0)  
 obuf[opt++] = '0';  
 else {  
 if (x < 0)  
 obuf[opt++] = '-', x = -x;  
 int sn = 0;  
 while (x)  
 str[sn++] = x % 10 + '0', x /= 10;  
 for (int i = sn - 1; i >= 0; i--)  
 obuf[opt++] = str[i];  
 }  
 if (opt >= (SIZE >> 1)) {  
 fwrite(obuf, 1, opt, stdout);  
 opt = 0;  
 }  
 }  
  
 void write(unsigned long long x) {  
 if (x == 0)  
 obuf[opt++] = '0';  
 else {  
 int sn = 0;  
 while (x)  
 str[sn++] = x % 10 + '0', x /= 10;  
 for (int i = sn - 1; i >= 0; i--)  
 obuf[opt++] = str[i];  
 }  
 if (opt >= (SIZE >> 1)) {  
 fwrite(obuf, 1, opt, stdout);  
 opt = 0;  
 }  
 }  
  
 void write(char x) {  
 obuf[opt++] = x;  
 if (opt >= (SIZE >> 1)) {  
 fwrite(obuf, 1, opt, stdout);  
 opt = 0;  
 }  
 }  
  
 void Fflush() {  
 if (opt)  
 fwrite(obuf, 1, opt, stdout);  
 opt = 0;  
 }  
}; // namespace FastIO

## 枚举子集

cin >> n;  
 for (int s = n; s; s = (s - 1) & n) {  
 cout << bitset<8>(s) << endl;  
 }

## 模拟退火

“优化的随机算法”

连续函数找区间最优

// 找一个点，与平面中的n个点的距离和最近

//进行多次模拟退火避免局部最大值

#include <bits/stdc++.h>  
#include <ctime>  
using namespace std;  
  
const int maxn = 110;  
  
int n;  
  
#define x first  
#define y second  
  
typedef pair<double, double> PDD;  
  
PDD q[maxn];   
double ans = 1e8;  
  
double rand(double l, double r) {  
 return (double) rand() / RAND\_MAX \* (r - l) + l;   
}  
  
double getDist(PDD a, PDD b) {  
 double dx = a.x - b.x;  
 double dy = a.y - b.y;  
 return sqrt(dx \* dx + dy \* dy) ;  
}  
  
double calc(PDD p) {  
 double res = 0;  
 for(int i = 0; i < n; ++ i) {  
 res += getDist(q[i], p);  
 }  
 ans = min(ans, res);  
 return res;  
}  
  
double simulate\_anneal() {  
 PDD cur(rand(0, 10000), rand(0, 10000)); // 随机一个起点  
 for(double T = 1e4; T > 1e-4; T = T \* 0.99) { // 初始温度，末态温度，衰减系数，一般调整衰减系数0.999 0.95  
 PDD np(rand(cur.x - T, cur.x + T), rand(cur.y - T, cur.y + T)); // 随机新点  
 double delta = calc(np) - calc(cur);  
 if(exp(-delta / T) > rand(0, 1)) cur = np; //如果新点比现在的点更优，必过去，不然有一定概率过去  
 }  
  
}  
  
int main() {  
 cin >> n;  
 for(int i = 0; i < n; ++ i) {  
 cin >> q[i].x >> q[i].y;   
 }  
  
 while((double) clock() / CLOCKS\_PER\_SEC < 0.8) { // 卡时 // 或for（100）  
 simulate\_anneal();   
 }  
  
 cout << (int)(ans + 0.5) << endl;  
  
 return 0;  
}

// n个点带权费马点 // 平衡点||吊打XXX

//n个二维坐标点，带重物重量，找平衡点

//进行一次模拟退火，但是在局部最大值周围多次跳动（以提高精度

#include <cmath>  
#include <cstdio>  
#include <cstdlib>  
#include <ctime>  
  
const int N = 10005;  
int n, x[N], y[N], w[N];  
double ansx, ansy, dis;  
  
double Rand() { return (double)rand() / RAND\_MAX; }  
double calc(double xx, double yy) {  
 double res = 0;  
 for (int i = 1; i <= n; ++i) {  
 double dx = x[i] - xx, dy = y[i] - yy;  
 res += sqrt(dx \* dx + dy \* dy) \* w[i];  
 }  
 if (res < dis) dis = res, ansx = xx, ansy = yy;  
 return res;  
}  
void simulateAnneal() {  
 double t = 100000;  
 double nowx = ansx, nowy = ansy;  
 while (t > 0.001) {  
 double nxtx = nowx + t \* (Rand() \* 2 - 1);  
 double nxty = nowy + t \* (Rand() \* 2 - 1);  
 double delta = calc(nxtx, nxty) - calc(nowx, nowy);  
 if (exp(-delta / t) > Rand()) nowx = nxtx, nowy = nxty;  
 t \*= 0.97;  
 }  
 for (int i = 1; i <= 1000; ++i) {  
 double nxtx = ansx + t \* (Rand() \* 2 - 1);  
 double nxty = ansy + t \* (Rand() \* 2 - 1);  
 calc(nxtx, nxty);  
 }  
}  
int main() {  
 srand(time(0));  
 scanf("%d", &n);  
 for (int i = 1; i <= n; ++i) {  
 scanf("%d%d%d", &x[i], &y[i], &w[i]);  
 ansx += x[i], ansy += y[i];  
 }  
 ansx /= n, ansy /= n, dis = calc(ansx, ansy);  
 simulateAnneal();  
 printf("%.3lf %.3lf\n", ansx, ansy);  
 return 0;  
}

# 测试时常用的代码

//�ļ����������ʱ�亯���������  
  
#ifdef ONLINE\_JUDGE  
#else  
 freopen("in.txt","r",stdin);  
 //freopen("out.txt","w",stdout);  
#endif  
//���¶���stdin/stdout��in.txt/out.txt��"r"��"w"Ϊֻ����ֻд  
  
  
#include<ctime>  
 clock\_t ST,ED;  
 ST=clock();  
 //��������Եĳ���  
 ED=clock();  
 cout<<ED-ST<<"ms"<<endl;  
  
  
#include<ctime>  
#include<cstdlib>  
 srand(time(0));//��ʼ��  
 rand();//����[0,RAND\_MAX]֮����������(int)��RAND\_MAX��cstdlib�еĺ궨�壬һ��Ϊ0x7fff(32767)

# 线性代数

## 矩阵类模板\_加减乘快速幂

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const ll N = 305;  
const ll mod = 998244353;  
  
//矩阵类模板  
struct Matrix {  
 ll n, m;  
 ll a[N][N];  
  
 void set(ll \_a, ll \_b) {  
 n = \_a, m = \_b;  
 }  
  
 Matrix() {  
 clear();  
 }  
  
 void clear() {  
 n = m = 0;  
 memset(a, 0, sizeof(a));  
 }  
  
 Matrix operator+(const Matrix &b) const {  
 Matrix tmp;  
 tmp.n = n;  
 tmp.m = m;  
 for (ll i = 0; i < n; ++i)  
 for (ll j = 0; j < m; ++j)  
 tmp.a[i][j] = (a[i][j] + b.a[i][j]) % mod;  
 return tmp;  
 }  
  
 Matrix operator-(const Matrix &b) const {  
 Matrix tmp;  
 tmp.n = n;  
 tmp.m = m;  
 for (ll i = 0; i < n; ++i) {  
 for (ll j = 0; j < m; ++j)  
 tmp.a[i][j] = (a[i][j] - b.a[i][j] + mod) % mod;  
 }  
  
 return tmp;  
 }  
  
 Matrix operator\*(const Matrix &b) const {  
 Matrix tmp;  
 tmp.clear();  
 tmp.n = n;  
 tmp.m = b.m;  
 for (ll i = 0; i < n; ++i)  
 for (ll j = 0; j < b.m; ++j)  
 for (ll k = 0; k < m; ++k) {  
 tmp.a[i][j] += a[i][k] \* b.a[k][j];  
 tmp.a[i][j] %= mod;  
 }  
 return tmp;  
 }  
  
 Matrix get(ll x) {//幂运算  
 Matrix E;  
 E.clear();  
 E.set(n, m);  
 for (ll i = 0; i < n; ++i)  
 E.a[i][i] = 1;  
 if (x == 0) return E;  
 else if (x == 1) return \*this;  
 Matrix tmp = get(x / 2);  
 tmp = tmp \* tmp;  
 if (x % 2) tmp = tmp \* (\*this);  
 return tmp;  
 }  
  
 void exgcd(ll \_a, ll \_b, ll &x, ll &y) {  
 if (!\_b)return x = 1, y = 0, void();  
 exgcd(\_b, \_a % \_b, y, x);  
 y -= x \* (\_a / \_b);  
 }  
  
 ll inv(ll p) {  
 ll x, y;  
 exgcd(p, mod, x, y);  
 return (x + mod) % mod;  
 }  
  
 Matrix inv() {  
 Matrix E = \*this;  
 ll is[N], js[N];  
 for (ll k = 0; k < E.n; k++) {  
 is[k] = js[k] = -1;  
 for (ll i = k; i < E.n; i++) // 1  
 for (ll j = k; j < E.n; j++)  
 if (E.a[i][j]) {  
 is[k] = i, js[k] = j;  
 break;  
 }  
 if (is[k] == -1) {  
 E.clear();  
 return E;  
 }  
 for (ll i = 0; i < E.n; i++) // 2  
 swap(E.a[k][i], E.a[is[k]][i]);  
 for (ll i = 0; i < E.n; i++)  
 swap(E.a[i][k], E.a[i][js[k]]);  
 if (!E.a[k][k]) {  
 E.clear();  
 return E;  
 }  
 E.a[k][k] = inv(E.a[k][k]); // 3  
 for (ll j = 0; j < E.n; j++)  
 if (j != k) // 4  
 (E.a[k][j] \*= E.a[k][k]) %= mod;  
 for (ll i = 0; i < E.n; i++)  
 if (i != k) // 5  
 for (ll j = 0; j < E.n; j++)  
 if (j != k)  
 (E.a[i][j] += mod - E.a[i][k] \* E.a[k][j] % mod) %= mod;  
 for (ll i = 0; i < E.n; i++)  
 if (i != k) // 就是这里不同  
 E.a[i][k] = (mod - E.a[i][k] \* E.a[k][k] % mod) % mod;  
 }  
 for (ll k = E.n - 1; k >= 0; k--) { // 6  
 for (ll i = 0; i < E.n; i++)  
 swap(E.a[js[k]][i], E.a[k][i]);  
 for (ll i = 0; i < E.n; i++)  
 swap(E.a[i][is[k]], E.a[i][k]);  
 }  
 return E;  
 }  
};  
//矩阵模板结束

## 矩阵类模板\_稀疏矩阵乘法

struct Matrix{  
 int n,m;  
 int a[maxn][maxn];////  
 void clear(){  
 n=m=0;  
 memset(a,0,sizeof(a));  
 }  
 Matrix operator \* (const Matrix &b) const{  
 Matrix tmp;  
 tmp.clear();  
 tmp.n=n;tmp.m=b.m;  
 for (int k=0;k<m;++k){  
 for (int i=0;i<n;++i){  
 if(a[i][k]==0) continue;  
 for(int j=0;j<b.m;++j){  
 if(b.a[k][j]==0) continue;  
 tmp.a[i][j]+=a[i][k]\*b.a[k][j];  
 tmp.a[i][j]%=mod;  
 }   
 }   
 }  
 return tmp;  
 }  
};  
//稀疏矩阵乘法

## 矩阵行列式

#include <bits/stdc++.h>  
using namespace std;  
typedef long long ll;  
const ll mod = 1e9 + 7;  
struct Matrix {  
 static const ll MAXN = 300;  
 ll a[MAXN][MAXN];  
  
 void init() { memset(a, 0, sizeof(a)); }  
  
 ll det(ll n) {  
 for (int i = 0; i < n; i++)  
 for (int j = 0; j < n; j++) a[i][j] = (a[i][j] + mod) % mod;  
 ll res = 1;  
 for (int i = 0; i < n; i++) {  
 if (!a[i][i]) {  
 bool flag = false;  
 for (int j = i + 1; j < n; j++) {  
 if (a[j][i]) {  
 flag = true;  
 for (int k = i; k < n; k++) {  
 swap(a[i][k], a[j][k]);  
 }  
 res = -res;  
 break;  
 }  
 }  
 if (!flag) return 0;  
 }  
  
 for (int j = i + 1; j < n; j++) {  
 while (a[j][i]) {  
 ll t = a[i][i] / a[j][i];  
 for (int k = i; k < n; k++) {  
 a[i][k] = (a[i][k] - t \* a[j][k]) % mod;  
 swap(a[i][k], a[j][k]);  
 }  
 res = -res;  
 }  
 }  
 res \*= a[i][i];  
 res %= mod;  
 }  
 return (res + mod) % mod;  
 }  
} mat;

## 线性基2

线性基 能表示的线性空间与原向量 能表示的线性空间等价

用高斯消元得到线性基

先输入数组a[] 中

int n, k;  
ll a[N];  
  
void getVec() {  
 k = 0;  
  
 for(int i = 62; i >= 0; -- i) {  
 for(int j = k; j < n; ++ j) {  
 if(a[j] >> i & 1) {  
 swap(a[j], a[k]);  
 break;  
 }  
 }  
 if(!(a[k] >> i & 1)) continue;  
 for(int j = 0; j < n; ++j) {  
 if(j != k && (a[j] >> i & 1)) {  
 a[j] ^= a[k];  
 }  
 }  
 ++k;  
 if(k == n) break;  
 }  
  
}

这里注意最后的线性基是a[]中从0到k-1个，在前的是**高位**

## 线性基模板

//  
  
const int maxbit = 62; //maxbit����̫��  
  
struct L\_B{  
 ll lba[maxbit];  
 L\_B(){  
 memset(lba, 0, sizeof(lba));  
 }  
   
 void Insert(ll val){ //����  
 for(int i = maxbit - 1; i >= 0; -- i) // �Ӹ�λ���λɨ   
 if(val & (1ll << i)){ //   
 if(!lba[i]){  
 lba[i] = val;  
 break;  
 }  
 val ^= lba[i];  
 }  
 }  
};  
//��ԭ���ϵ�ÿ����valתΪ2���ƣ��Ӹ�λ���λɨ�����ڵ�ǰλΪ1�ģ���lba[i]�����ھ���lba[i]=x��������val=val`xor`lba[i]  
//ʹ�ã� ֱ��insert   
// --------------���Ի�ģ��

## 高斯消元

#include <iostream>  
#include <vector>  
using namespace std;  
const double eps = 1e-8;  
void sway(vector<double>& a, vector<double>& b) {  
 vector<double> s;  
 for (int i = 0; i < a.size(); i++) {  
 s.push\_back(a[i]);  
 }  
 a.clear();  
 for (int i = 0; i < b.size(); i++) {  
 a.push\_back(b[i]);  
 }  
 b.clear();  
 for (int i = 0; i < s.size(); i++) {  
 b.push\_back(s[i]);  
 }  
}  
vector<double> gauss\_jordan(const vector<vector<double> >& A,  
 const vector<double>& b) {  
 int n = A.size();  
 vector<vector<double> > B(n, vector<double>(n + 1));  
 for (int i = 0; i < n; i++)  
 for (int j = 0; j < n; j++) B[i][j] = A[i][j];  
 for (int i = 0; i < n; i++) B[i][n] = b[i];  
  
 for (int i = 0; i < n; i++) {  
 int pivot = i;  
 for (int j = i; j < n; j++) {  
 if (abs(B[j][i]) > abs(B[pivot][i])) pivot = j;  
 }  
 swap(B[i], B[pivot]);  
 if (abs(B[i][i]) < eps) return vector<double>();  
 for (int j = i + 1; j <= n; j++) B[i][j] /= B[i][i];  
 for (int j = 0; j < n; j++) {  
 if (i != j) {  
 for (int k = i + 1; k <= n; k++) B[j][k] -= B[j][i] \* B[i][k];  
 }  
 }  
 }  
 vector<double> x(n);  
 for (int i = 0; i < n; i++) x[i] = B[i][n];  
 return x;  
}  
int main() {  
 int n, m;  
 cin >> n >> m;  
 vector<vector<double> > mat(n, vector<double>(m));  
 for (int i = 0; i < n; i++) {  
 for (int j = 0; j < m; j++) {  
 cin >> mat[i][j];  
 }  
 }  
 vector<double> val(n);  
 for (int i = 0; i < n; i++) cin >> val[i];  
 vector<double> ans = gauss\_jordan(mat, val);  
 for (int i = 0; i < ans.size(); i++) cout << ans[i] << ' ';  
}

# 组合数学

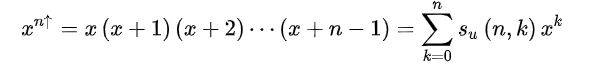
## 斯特林数

[百度百科讲的超好](https://baike.baidu.com/item/%E6%96%AF%E7%89%B9%E6%9E%97%E6%95%B0/4938529?fr=aladdin)

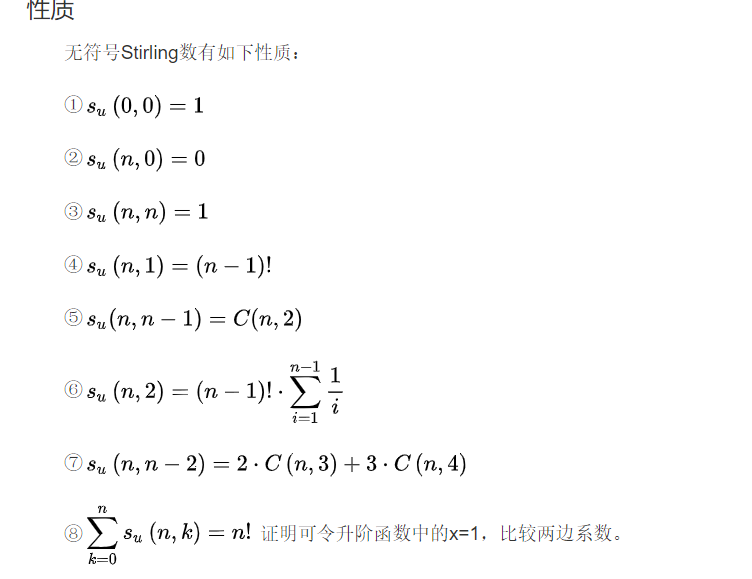
## 第一类斯特林数（无符号第一类）

定义： 表示将n个两两不同的元素，划分为k个非空圆排列的方案数。

递推式

升阶函数

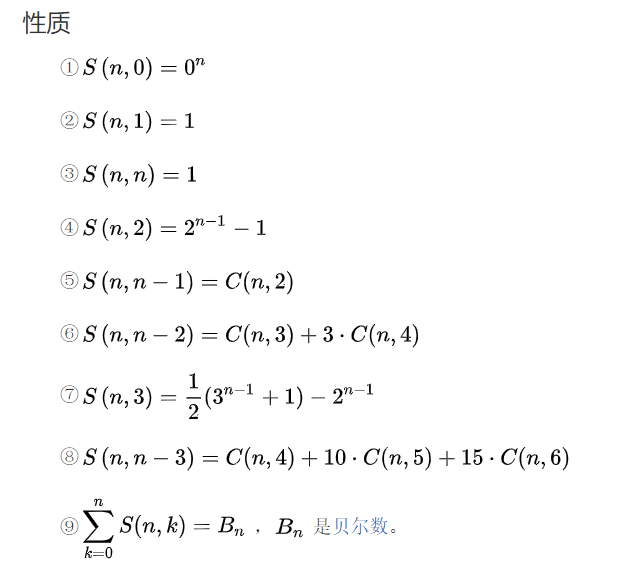
（每一项系数则为无符号第一类斯特林数，求前n项和则为取x=1）

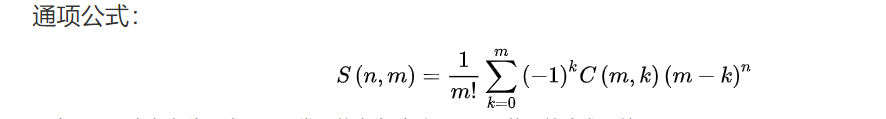


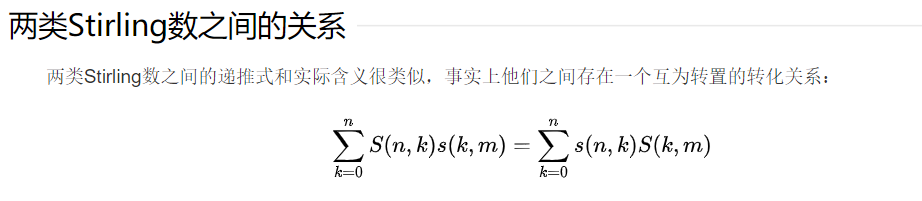
## 第二类斯特林数

定义： 表示将n个两两不同的元素，划分为k个非空子集的方案数。

递推式







# 计算几何

## jls

#define mp make\_pair  
#define fi first  
#define se second  
#define pb push\_back  
typedef double db;  
const db eps=1e-6;  
const db pi=acos(-1);  
int sign(db k){  
 if (k>eps) return 1; else if (k<-eps) return -1; return 0;  
}  
int cmp(db k1,db k2){return sign(k1-k2);}  
int inmid(db k1,db k2,db k3){return sign(k1-k3)\*sign(k2-k3)<=0;}// k3 在 [k1,k2] 内   
struct point{  
 db x,y;  
 point operator + (const point &k1) const{return (point){k1.x+x,k1.y+y};}  
 point operator - (const point &k1) const{return (point){x-k1.x,y-k1.y};}  
 point operator \* (db k1) const{return (point){x\*k1,y\*k1};}  
 point operator / (db k1) const{return (point){x/k1,y/k1};}  
 int operator == (const point &k1) const{return cmp(x,k1.x)==0&&cmp(y,k1.y)==0;}  
 // 逆时针旋转   
 point turn(db k1){return (point){x\*cos(k1)-y\*sin(k1),x\*sin(k1)+y\*cos(k1)};}  
 point turn90(){return (point){-y,x};}  
 bool operator < (const point k1) const{  
 int a=cmp(x,k1.x);  
 if (a==-1) return 1; else if (a==1) return 0; else return cmp(y,k1.y)==-1;  
 }  
 db abs(){return sqrt(x\*x+y\*y);}  
 db abs2(){return x\*x+y\*y;}  
 db dis(point k1){return ((\*this)-k1).abs();}  
 point unit(){db w=abs(); return (point){x/w,y/w};}  
 void scan(){double k1,k2; scanf("%lf%lf",&k1,&k2); x=k1; y=k2;}  
 void print(){printf("%.11lf %.11lf\n",x,y);}  
 db getw(){return atan2(y,x);}   
 point getdel(){if (sign(x)==-1||(sign(x)==0&&sign(y)==-1)) return (\*this)\*(-1); else return (\*this);}  
 int getP() const{return sign(y)==1||(sign(y)==0&&sign(x)==-1);}  
};  
int inmid(point k1,point k2,point k3){return inmid(k1.x,k2.x,k3.x)&&inmid(k1.y,k2.y,k3.y);}  
db cross(point k1,point k2){return k1.x\*k2.y-k1.y\*k2.x;}  
db dot(point k1,point k2){return k1.x\*k2.x+k1.y\*k2.y;}  
db rad(point k1,point k2){return atan2(cross(k1,k2),dot(k1,k2));}  
// -pi -> pi  
int compareangle (point k1,point k2){  
 return k1.getP()<k2.getP()||(k1.getP()==k2.getP()&&sign(cross(k1,k2))>0);  
}  
point proj(point k1,point k2,point q){ // q 到直线 k1,k2 的投影   
 point k=k2-k1; return k1+k\*(dot(q-k1,k)/k.abs2());  
}  
point reflect(point k1,point k2,point q){return proj(k1,k2,q)\*2-q;}  
int clockwise(point k1,point k2,point k3){// k1 k2 k3 逆时针 1 顺时针 -1 否则 0   
 return sign(cross(k2-k1,k3-k1));  
}  
int checkLL(point k1,point k2,point k3,point k4){// 求直线 (L) 线段 (S)k1,k2 和 k3,k4 的交点   
 return cmp(cross(k3-k1,k4-k1),cross(k3-k2,k4-k2))!=0;  
}  
point getLL(point k1,point k2,point k3,point k4){  
 db w1=cross(k1-k3,k4-k3),w2=cross(k4-k3,k2-k3); return (k1\*w2+k2\*w1)/(w1+w2);  
}  
int intersect(db l1,db r1,db l2,db r2){  
 if (l1>r1) swap(l1,r1); if (l2>r2) swap(l2,r2); return cmp(r1,l2)!=-1&&cmp(r2,l1)!=-1;  
}  
int checkSS(point k1,point k2,point k3,point k4){  
 return intersect(k1.x,k2.x,k3.x,k4.x)&&intersect(k1.y,k2.y,k3.y,k4.y)&&  
 sign(cross(k3-k1,k4-k1))\*sign(cross(k3-k2,k4-k2))<=0&&  
 sign(cross(k1-k3,k2-k3))\*sign(cross(k1-k4,k2-k4))<=0;  
}  
db disSP(point k1,point k2,point q){  
 point k3=proj(k1,k2,q);  
 if (inmid(k1,k2,k3)) return q.dis(k3); else return min(q.dis(k1),q.dis(k2));  
}  
db disSS(point k1,point k2,point k3,point k4){  
 if (checkSS(k1,k2,k3,k4)) return 0;  
 else return min(min(disSP(k1,k2,k3),disSP(k1,k2,k4)),min(disSP(k3,k4,k1),disSP(k3,k4,k2)));  
}  
int onS(point k1,point k2,point q){return inmid(k1,k2,q)&&sign(cross(k1-q,k2-k1))==0;}  
struct circle{  
 point o; db r;  
 void scan(){o.scan(); scanf("%lf",&r);}  
 int inside(point k){return cmp(r,o.dis(k));}  
};  
struct line{  
 // p[0]->p[1]  
 point p[2];  
 line(point k1,point k2){p[0]=k1; p[1]=k2;}  
 point& operator [] (int k){return p[k];}  
 int include(point k){return sign(cross(p[1]-p[0],k-p[0]))>0;}  
 point dir(){return p[1]-p[0];}  
 line push(){ // 向外 ( 左手边 ) 平移 eps   
 const db eps = 1e-6;  
 point delta=(p[1]-p[0]).turn90().unit()\*eps;  
 return {p[0]-delta,p[1]-delta};  
 }  
};  
point getLL(line k1,line k2){return getLL(k1[0],k1[1],k2[0],k2[1]);}  
int parallel(line k1,line k2){return sign(cross(k1.dir(),k2.dir()))==0;}  
int sameDir(line k1,line k2){return parallel(k1,k2)&&sign(dot(k1.dir(),k2.dir()))==1;}  
int operator < (line k1,line k2){  
 if (sameDir(k1,k2)) return k2.include(k1[0]);   
 return compareangle(k1.dir(),k2.dir());  
}  
int checkpos(line k1,line k2,line k3){return k3.include(getLL(k1,k2));}  
vector<line> getHL(vector<line> &L){ // 求半平面交 , 半平面是逆时针方向 , 输出按照逆时针  
 sort(L.begin(),L.end()); deque<line> q;  
 for (int i=0;i<(int)L.size();i++){  
 if (i&&sameDir(L[i],L[i-1])) continue;  
 while (q.size()>1&&!checkpos(q[q.size()-2],q[q.size()-1],L[i])) q.pop\_back();  
 while (q.size()>1&&!checkpos(q[1],q[0],L[i])) q.pop\_front();  
 q.push\_back(L[i]);  
 }  
 while (q.size()>2&&!checkpos(q[q.size()-2],q[q.size()-1],q[0])) q.pop\_back();  
 while (q.size()>2&&!checkpos(q[1],q[0],q[q.size()-1])) q.pop\_front();  
 vector<line>ans; for (int i=0;i<q.size();i++) ans.push\_back(q[i]);  
 return ans;  
}  
db closepoint(vector<point>&A,int l,int r){ // 最近点对 , 先要按照 x 坐标排序   
 if (r-l<=5){  
 db ans=1e20;  
 for (int i=l;i<=r;i++) for (int j=i+1;j<=r;j++) ans=min(ans,A[i].dis(A[j]));  
 return ans;  
 }  
 int mid=l+r>>1; db ans=min(closepoint(A,l,mid),closepoint(A,mid+1,r));  
 vector<point>B; for (int i=l;i<=r;i++) if (abs(A[i].x-A[mid].x)<=ans) B.push\_back(A[i]);  
 sort(B.begin(),B.end(),[](point k1,point k2){return k1.y<k2.y;});  
 for (int i=0;i<B.size();i++) for (int j=i+1;j<B.size()&&B[j].y-B[i].y<ans;j++) ans=min(ans,B[i].dis(B[j]));  
 return ans;  
}  
int checkposCC(circle k1,circle k2){// 返回两个圆的公切线数量  
 if (cmp(k1.r,k2.r)==-1) swap(k1,k2);  
 db dis=k1.o.dis(k2.o); int w1=cmp(dis,k1.r+k2.r),w2=cmp(dis,k1.r-k2.r);  
 if (w1>0) return 4; else if (w1==0) return 3; else if (w2>0) return 2;   
 else if (w2==0) return 1; else return 0;  
}  
vector<point> getCL(circle k1,point k2,point k3){ // 沿着 k2->k3 方向给出 , 相切给出两个   
 point k=proj(k2,k3,k1.o); db d=k1.r\*k1.r-(k-k1.o).abs2();  
 if (sign(d)==-1) return {};  
 point del=(k3-k2).unit()\*sqrt(max((db)0.0,d)); return {k-del,k+del};  
}  
vector<point> getCC(circle k1,circle k2){// 沿圆 k1 逆时针给出 , 相切给出两个   
 int pd=checkposCC(k1,k2); if (pd==0||pd==4) return {};  
 db a=(k2.o-k1.o).abs2(),cosA=(k1.r\*k1.r+a-k2.r\*k2.r)/(2\*k1.r\*sqrt(max(a,(db)0.0)));  
 db b=k1.r\*cosA,c=sqrt(max((db)0.0,k1.r\*k1.r-b\*b));  
 point k=(k2.o-k1.o).unit(),m=k1.o+k\*b,del=k.turn90()\*c;  
 return {m-del,m+del};  
}   
vector<point> TangentCP(circle k1,point k2){// 沿圆 k1 逆时针给出   
 db a=(k2-k1.o).abs(),b=k1.r\*k1.r/a,c=sqrt(max((db)0.0,k1.r\*k1.r-b\*b));  
 point k=(k2-k1.o).unit(),m=k1.o+k\*b,del=k.turn90()\*c;  
 return {m-del,m+del};  
}   
vector<line> TangentoutCC(circle k1,circle k2){  
 int pd=checkposCC(k1,k2); if (pd==0) return {};   
 if (pd==1){point k=getCC(k1,k2)[0]; return {(line){k,k}};}  
 if (cmp(k1.r,k2.r)==0){  
 point del=(k2.o-k1.o).unit().turn90().getdel();  
 return {(line){k1.o-del\*k1.r,k2.o-del\*k2.r},(line){k1.o+del\*k1.r,k2.o+del\*k2.r}};  
 } else {  
 point p=(k2.o\*k1.r-k1.o\*k2.r)/(k1.r-k2.r);  
 vector<point>A=TangentCP(k1,p),B=TangentCP(k2,p);  
 vector<line>ans; for (int i=0;i<A.size();i++) ans.push\_back((line){A[i],B[i]});   
 return ans;  
 }  
}  
vector<line> TangentinCC(circle k1,circle k2){  
 int pd=checkposCC(k1,k2); if (pd<=2) return {};  
 if (pd==3){point k=getCC(k1,k2)[0]; return {(line){k,k}};}   
 point p=(k2.o\*k1.r+k1.o\*k2.r)/(k1.r+k2.r);  
 vector<point>A=TangentCP(k1,p),B=TangentCP(k2,p);  
 vector<line>ans; for (int i=0;i<A.size();i++) ans.push\_back((line){A[i],B[i]});   
 return ans;  
}  
vector<line> TangentCC(circle k1,circle k2){  
 int flag=0; if (k1.r<k2.r) swap(k1,k2),flag=1;  
 vector<line>A=TangentoutCC(k1,k2),B=TangentinCC(k1,k2);  
 for (line k:B) A.push\_back(k);   
 if (flag) for (line &k:A) swap(k[0],k[1]);  
 return A;  
}  
db getarea(circle k1,point k2,point k3){  
 // 圆 k1 与三角形 k2 k3 k1.o 的有向面积交  
 point k=k1.o; k1.o=k1.o-k; k2=k2-k; k3=k3-k;  
 int pd1=k1.inside(k2),pd2=k1.inside(k3);   
 vector<point>A=getCL(k1,k2,k3);  
 if (pd1>=0){  
 if (pd2>=0) return cross(k2,k3)/2;  
 return k1.r\*k1.r\*rad(A[1],k3)/2+cross(k2,A[1])/2;  
 } else if (pd2>=0){   
 return k1.r\*k1.r\*rad(k2,A[0])/2+cross(A[0],k3)/2;  
 }else {  
 int pd=cmp(k1.r,disSP(k2,k3,k1.o));  
 if (pd<=0) return k1.r\*k1.r\*rad(k2,k3)/2;  
 return cross(A[0],A[1])/2+k1.r\*k1.r\*(rad(k2,A[0])+rad(A[1],k3))/2;  
 }  
}  
circle getcircle(point k1,point k2,point k3){  
 db a1=k2.x-k1.x,b1=k2.y-k1.y,c1=(a1\*a1+b1\*b1)/2;  
 db a2=k3.x-k1.x,b2=k3.y-k1.y,c2=(a2\*a2+b2\*b2)/2;  
 db d=a1\*b2-a2\*b1;  
 point o=(point){k1.x+(c1\*b2-c2\*b1)/d,k1.y+(a1\*c2-a2\*c1)/d};  
 return (circle){o,k1.dis(o)};  
}  
circle getScircle(vector<point> A){  
 random\_shuffle(A.begin(),A.end());  
 circle ans=(circle){A[0],0};  
 for (int i=1;i<A.size();i++)  
 if (ans.inside(A[i])==-1){  
 ans=(circle){A[i],0};  
 for (int j=0;j<i;j++)  
 if (ans.inside(A[j])==-1){  
 ans.o=(A[i]+A[j])/2; ans.r=ans.o.dis(A[i]);  
 for (int k=0;k<j;k++)  
 if (ans.inside(A[k])==-1)  
 ans=getcircle(A[i],A[j],A[k]);  
 }  
 }  
 return ans;  
}  
db area(vector<point> A){ // 多边形用 vector<point> 表示 , 逆时针   
 db ans=0;  
 for (int i=0;i<A.size();i++) ans+=cross(A[i],A[(i+1)%A.size()]);  
 return ans/2;  
}  
int checkconvex(vector<point>A){  
 int n=A.size(); A.push\_back(A[0]); A.push\_back(A[1]);  
 for (int i=0;i<n;i++) if (sign(cross(A[i+1]-A[i],A[i+2]-A[i]))==-1) return 0;  
 return 1;  
}  
int contain(vector<point>A,point q){ // 2 内部 1 边界 0 外部  
 int pd=0; A.push\_back(A[0]);  
 for (int i=1;i<A.size();i++){  
 point u=A[i-1],v=A[i];  
 if (onS(u,v,q)) return 1; if (cmp(u.y,v.y)>0) swap(u,v);  
 if (cmp(u.y,q.y)>=0||cmp(v.y,q.y)<0) continue;  
 if (sign(cross(u-v,q-v))<0) pd^=1;  
 }  
 return pd<<1;  
}  
vector<point> ConvexHull(vector<point>A,int flag=1){ // flag=0 不严格 flag=1 严格   
 int n=A.size(); vector<point>ans(n\*2);   
 sort(A.begin(),A.end()); int now=-1;  
 for (int i=0;i<A.size();i++){  
 while (now>0&&sign(cross(ans[now]-ans[now-1],A[i]-ans[now-1]))<flag) now--;  
 ans[++now]=A[i];  
 } int pre=now;  
 for (int i=n-2;i>=0;i--){  
 while (now>pre&&sign(cross(ans[now]-ans[now-1],A[i]-ans[now-1]))<flag) now--;  
 ans[++now]=A[i];  
 } ans.resize(now); return ans;  
}  
db convexDiameter(vector<point>A){  
 int now=0,n=A.size(); db ans=0;  
 for (int i=0;i<A.size();i++){  
 now=max(now,i);  
 while (1){  
 db k1=A[i].dis(A[now%n]),k2=A[i].dis(A[(now+1)%n]);  
 ans=max(ans,max(k1,k2)); if (k2>k1) now++; else break;  
 }  
 }  
 return ans;  
}  
int rotating\_calipers()  //卡壳    
{    
    int i , q=1;    
    int ans = 0;    
    stack[top]=0;    
    for(i = 0 ; i < top ; i++)    
    {    
        while( xmult( p[stack[i+1]] , p[stack[q+1]] , p[stack[i]] ) >   
 xmult( p[stack[i+1]] , p[stack[q]] , p[stack[i]] ) )    
            q = (q+1)%(top);    
        ans = max(ans , max( dis(p[stack[i]] , p[stack[q]]) ,   
 dis(p[stack[i+1]] , p[stack[q+1]])));    
    }    
    return ans;    
}    
vector<point> convexcut(vector<point>A,point k1,point k2){  
 // 保留 k1,k2,p 逆时针的所有点  
 int n=A.size(); A.push\_back(A[0]); vector<point>ans;  
 for (int i=0;i<n;i++){  
 int w1=clockwise(k1,k2,A[i]),w2=clockwise(k1,k2,A[i+1]);  
 if (w1>=0) ans.push\_back(A[i]);  
 if (w1\*w2<0) ans.push\_back(getLL(k1,k2,A[i],A[i+1]));  
 }  
 return ans;  
}  
int checkPoS(vector<point>A,point k1,point k2){  
 // 多边形 A 和直线 ( 线段 )k1->k2 严格相交 , 注释部分为线段  
 struct ins{  
 point m,u,v;  
 int operator < (const ins& k) const {return m<k.m;}  
 }; vector<ins>B;  
 //if (contain(A,k1)==2||contain(A,k2)==2) return 1;  
 vector<point>poly=A; A.push\_back(A[0]);   
 for (int i=1;i<A.size();i++) if (checkLL(A[i-1],A[i],k1,k2)){  
 point m=getLL(A[i-1],A[i],k1,k2);   
 if (inmid(A[i-1],A[i],m)/\*&&inmid(k1,k2,m)\*/) B.push\_back((ins){m,A[i-1],A[i]});  
 }  
 if (B.size()==0) return 0; sort(B.begin(),B.end());   
 int now=1; while (now<B.size()&&B[now].m==B[0].m) now++;   
 if (now==B.size()) return 0;  
 int flag=contain(poly,(B[0].m+B[now].m)/2);  
 if (flag==2) return 1;  
 point d=B[now].m-B[0].m;  
 for (int i=now;i<B.size();i++){  
 if (!(B[i].m==B[i-1].m)&&flag==2) return 1;  
 int tag=sign(cross(B[i].v-B[i].u,B[i].m+d-B[i].u));  
 if (B[i].m==B[i].u||B[i].m==B[i].v) flag+=tag; else flag+=tag\*2;  
 }  
 //return 0;  
 return flag==2;  
}  
int checkinp(point r,point l,point m){  
 if (compareangle(l,r)){return compareangle(l,m)&&compareangle(m,r);}  
 return compareangle(l,m)||compareangle(m,r);  
}  
int checkPosFast(vector<point>A,point k1,point k2){ // 快速检查线段是否和多边形严格相交  
 if (contain(A,k1)==2||contain(A,k2)==2) return 1; if (k1==k2) return 0;  
 A.push\_back(A[0]); A.push\_back(A[1]);  
 for (int i=1;i+1<A.size();i++)  
 if (checkLL(A[i-1],A[i],k1,k2)){  
 point now=getLL(A[i-1],A[i],k1,k2);  
 if (inmid(A[i-1],A[i],now)==0||inmid(k1,k2,now)==0) continue;  
 if (now==A[i]){  
 if (A[i]==k2) continue;  
 point pre=A[i-1],ne=A[i+1];  
 if (checkinp(pre-now,ne-now,k2-now)) return 1;  
 } else if (now==k1){  
 if (k1==A[i-1]||k1==A[i]) continue;  
 if (checkinp(A[i-1]-k1,A[i]-k1,k2-k1)) return 1;  
 } else if (now==k2||now==A[i-1]) continue;  
 else return 1;  
 }  
 return 0;  
}  
// 拆分凸包成上下凸壳 凸包尽量都随机旋转一个角度来避免出现相同横坐标   
// 尽量特判只有一个点的情况 凸包逆时针  
void getUDP(vector<point>A,vector<point>&U,vector<point>&D){  
 db l=1e100,r=-1e100;  
 for (int i=0;i<A.size();i++) l=min(l,A[i].x),r=max(r,A[i].x);  
 int wherel,wherer;  
 for (int i=0;i<A.size();i++) if (cmp(A[i].x,l)==0) wherel=i;  
 for (int i=A.size();i;i--) if (cmp(A[i-1].x,r)==0) wherer=i-1;  
 U.clear(); D.clear(); int now=wherel;  
 while (1){D.push\_back(A[now]); if (now==wherer) break; now++; if (now>=A.size()) now=0;}  
 now=wherel;  
 while (1){U.push\_back(A[now]); if (now==wherer) break; now--; if (now<0) now=A.size()-1;}  
}  
// 需要保证凸包点数大于等于 3,2 内部 ,1 边界 ,0 外部  
int containCoP(const vector<point>&U,const vector<point>&D,point k){  
 db lx=U[0].x,rx=U[U.size()-1].x;  
 if (k==U[0]||k==U[U.size()-1]) return 1;  
 if (cmp(k.x,lx)==-1||cmp(k.x,rx)==1) return 0;  
 int where1=lower\_bound(U.begin(),U.end(),(point){k.x,-1e100})-U.begin();  
 int where2=lower\_bound(D.begin(),D.end(),(point){k.x,-1e100})-D.begin();  
 int w1=clockwise(U[where1-1],U[where1],k),w2=clockwise(D[where2-1],D[where2],k);  
 if (w1==1||w2==-1) return 0; else if (w1==0||w2==0) return 1; return 2;  
}  
// d 是方向 , 输出上方切点和下方切点  
pair<point,point> getTangentCow(const vector<point> &U,const vector<point> &D,point d){  
 if (sign(d.x)<0||(sign(d.x)==0&&sign(d.y)<0)) d=d\*(-1);  
 point whereU,whereD;  
 if (sign(d.x)==0) return mp(U[0],U[U.size()-1]);  
 int l=0,r=U.size()-1,ans=0;  
 while (l<r){int mid=l+r>>1; if (sign(cross(U[mid+1]-U[mid],d))<=0) l=mid+1,ans=mid+1; else r=mid;}  
 whereU=U[ans]; l=0,r=D.size()-1,ans=0;  
 while (l<r){int mid=l+r>>1; if (sign(cross(D[mid+1]-D[mid],d))>=0) l=mid+1,ans=mid+1; else r=mid;}  
 whereD=D[ans]; return mp(whereU,whereD);  
}  
// 先检查 contain, 逆时针给出  
pair<point,point> getTangentCoP(const vector<point>&U,const vector<point>&D,point k){  
 db lx=U[0].x,rx=U[U.size()-1].x;  
 if (k.x<lx){  
 int l=0,r=U.size()-1,ans=U.size()-1;  
 while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid+1])==1) l=mid+1; else ans=mid,r=mid;}  
 point w1=U[ans]; l=0,r=D.size()-1,ans=D.size()-1;  
 while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid+1])==-1) l=mid+1; else ans=mid,r=mid;}  
 point w2=D[ans]; return mp(w1,w2);  
 } else if (k.x>rx){  
 int l=1,r=U.size(),ans=0;  
 while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid-1])==-1) r=mid; else ans=mid,l=mid+1;}  
 point w1=U[ans]; l=1,r=D.size(),ans=0;  
 while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid-1])==1) r=mid; else ans=mid,l=mid+1;}  
 point w2=D[ans]; return mp(w2,w1);  
 } else {  
 int where1=lower\_bound(U.begin(),U.end(),(point){k.x,-1e100})-U.begin();  
 int where2=lower\_bound(D.begin(),D.end(),(point){k.x,-1e100})-D.begin();  
 if ((k.x==lx&&k.y>U[0].y)||(where1&&clockwise(U[where1-1],U[where1],k)==1)){  
 int l=1,r=where1+1,ans=0;  
 while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid-1])==1) ans=mid,l=mid+1; else r=mid;}  
 point w1=U[ans]; l=where1,r=U.size()-1,ans=U.size()-1;  
 while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid+1])==1) l=mid+1; else ans=mid,r=mid;}  
 point w2=U[ans]; return mp(w2,w1);  
 } else {  
 int l=1,r=where2+1,ans=0;  
 while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid-1])==-1) ans=mid,l=mid+1; else r=mid;}  
 point w1=D[ans]; l=where2,r=D.size()-1,ans=D.size()-1;  
 while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid+1])==-1) l=mid+1; else ans=mid,r=mid;}  
 point w2=D[ans]; return mp(w1,w2);  
 }  
 }  
}  
struct P3{  
 db x,y,z;  
 P3 operator + (P3 k1){return (P3){x+k1.x,y+k1.y,z+k1.z};}  
 P3 operator - (P3 k1){return (P3){x-k1.x,y-k1.y,z-k1.z};}  
 P3 operator \* (db k1){return (P3){x\*k1,y\*k1,z\*k1};}  
 P3 operator / (db k1){return (P3){x/k1,y/k1,z/k1};}  
 db abs2(){return x\*x+y\*y+z\*z;}  
 db abs(){return sqrt(x\*x+y\*y+z\*z);}  
 P3 unit(){return (\*this)/abs();}  
 int operator < (const P3 k1) const{  
 if (cmp(x,k1.x)!=0) return x<k1.x;  
 if (cmp(y,k1.y)!=0) return y<k1.y;  
 return cmp(z,k1.z)==-1;  
 }  
 int operator == (const P3 k1){  
 return cmp(x,k1.x)==0&&cmp(y,k1.y)==0&&cmp(z,k1.z)==0;  
 }  
 void scan(){  
 double k1,k2,k3; scanf("%lf%lf%lf",&k1,&k2,&k3);  
 x=k1; y=k2; z=k3;  
 }  
};  
P3 cross(P3 k1,P3 k2){return (P3){k1.y\*k2.z-k1.z\*k2.y,k1.z\*k2.x-k1.x\*k2.z,k1.x\*k2.y-k1.y\*k2.x};}  
db dot(P3 k1,P3 k2){return k1.x\*k2.x+k1.y\*k2.y+k1.z\*k2.z;}  
//p=(3,4,5),l=(13,19,21),theta=85 ans=(2.83,4.62,1.77)  
P3 turn3D(db k1,P3 l,P3 p){  
 l=l.unit(); P3 ans; db c=cos(k1),s=sin(k1);  
 ans.x=p.x\*(l.x\*l.x\*(1-c)+c)+p.y\*(l.x\*l.y\*(1-c)-l.z\*s)+p.z\*(l.x\*l.z\*(1-c)+l.y\*s);  
 ans.y=p.x\*(l.x\*l.y\*(1-c)+l.z\*s)+p.y\*(l.y\*l.y\*(1-c)+c)+p.z\*(l.y\*l.z\*(1-c)-l.x\*s);  
 ans.z=p.x\*(l.x\*l.z\*(1-c)-l.y\*s)+p.y\*(l.y\*l.z\*(1-c)+l.x\*s)+p.z\*(l.z\*l.z\*(1-c)+c);  
 return ans;  
}  
typedef vector<P3> VP;  
typedef vector<VP> VVP;  
db Acos(db x){return acos(max(-(db)1,min(x,(db)1)));}  
// 球面距离 , 圆心原点 , 半径 1  
db Odist(P3 a,P3 b){db r=Acos(dot(a,b)); return r;}  
db r; P3 rnd;  
vector<db> solve(db a,db b,db c){  
 db r=sqrt(a\*a+b\*b),th=atan2(b,a);  
 if (cmp(c,-r)==-1) return {0};  
 else if (cmp(r,c)<=0) return {1};  
 else {  
 db tr=pi-Acos(c/r); return {th+pi-tr,th+pi+tr};  
 }  
}  
vector<db> jiao(P3 a,P3 b){  
 // dot(rd+x\*cos(t)+y\*sin(t),b) >= cos(r)  
 if (cmp(Odist(a,b),2\*r)>0) return {0};  
 P3 rd=a\*cos(r),z=a.unit(),y=cross(z,rnd).unit(),x=cross(y,z).unit();  
 vector<db> ret = solve(-(dot(x,b)\*sin(r)),-(dot(y,b)\*sin(r)),-(cos(r)-dot(rd,b)));   
 return ret;  
}  
db norm(db x,db l=0,db r=2\*pi){ // change x into [l,r)  
 while (cmp(x,l)==-1) x+=(r-l); while (cmp(x,r)>=0) x-=(r-l);  
 return x;  
}  
db disLP(P3 k1,P3 k2,P3 q){  
 return (cross(k2-k1,q-k1)).abs()/(k2-k1).abs();  
}  
db disLL(P3 k1,P3 k2,P3 k3,P3 k4){  
 P3 dir=cross(k2-k1,k4-k3); if (sign(dir.abs())==0) return disLP(k1,k2,k3);  
 return fabs(dot(dir.unit(),k1-k2));  
}  
VP getFL(P3 p,P3 dir,P3 k1,P3 k2){  
 db a=dot(k2-p,dir),b=dot(k1-p,dir),d=a-b;  
 if (sign(fabs(d))==0) return {};  
 return {(k1\*a-k2\*b)/d};  
}  
VP getFF(P3 p1,P3 dir1,P3 p2,P3 dir2){// 返回一条线  
 P3 e=cross(dir1,dir2),v=cross(dir1,e);  
 db d=dot(dir2,v); if (sign(abs(d))==0) return {};  
 P3 q=p1+v\*dot(dir2,p2-p1)/d; return {q,q+e};  
}  
// 3D Covex Hull Template  
db getV(P3 k1,P3 k2,P3 k3,P3 k4){ // get the Volume  
 return dot(cross(k2-k1,k3-k1),k4-k1);  
}  
db rand\_db(){return 1.0\*rand()/RAND\_MAX;}  
VP convexHull2D(VP A,P3 dir){  
 P3 x={(db)rand(),(db)rand(),(db)rand()}; x=x.unit();  
 x=cross(x,dir).unit(); P3 y=cross(x,dir).unit();  
 P3 vec=dir.unit()\*dot(A[0],dir);  
 vector<point>B;  
 for (int i=0;i<A.size();i++) B.push\_back((point){dot(A[i],x),dot(A[i],y)});  
 B=ConvexHull(B); A.clear();  
 for (int i=0;i<B.size();i++) A.push\_back(x\*B[i].x+y\*B[i].y+vec);  
 return A;  
}  
namespace CH3{  
 VVP ret; set<pair<int,int> >e;  
 int n; VP p,q;  
 void wrap(int a,int b){  
 if (e.find({a,b})==e.end()){  
 int c=-1;  
 for (int i=0;i<n;i++) if (i!=a&&i!=b){  
 if (c==-1||sign(getV(q[c],q[a],q[b],q[i]))>0) c=i;  
 }  
 if (c!=-1){  
 ret.push\_back({p[a],p[b],p[c]});  
 e.insert({a,b}); e.insert({b,c}); e.insert({c,a});  
 wrap(c,b); wrap(a,c);  
 }  
 }  
 }  
 VVP ConvexHull3D(VP \_p){  
 p=q=\_p; n=p.size();  
 ret.clear(); e.clear();  
 for (auto &i:q) i=i+(P3){rand\_db()\*1e-4,rand\_db()\*1e-4,rand\_db()\*1e-4};  
 for (int i=1;i<n;i++) if (q[i].x<q[0].x) swap(p[0],p[i]),swap(q[0],q[i]);  
 for (int i=2;i<n;i++) if ((q[i].x-q[0].x)\*(q[1].y-q[0].y)>(q[i].y-q[0].y)\*(q[1].x-q[0].x)) swap(q[1],q[i]),swap(p[1],p[i]);  
 wrap(0,1);  
 return ret;  
 }  
}  
VVP reduceCH(VVP A){  
 VVP ret; map<P3,VP> M;  
 for (VP nowF:A){  
 P3 dir=cross(nowF[1]-nowF[0],nowF[2]-nowF[0]).unit();  
 for (P3 k1:nowF) M[dir].pb(k1);  
 }  
 for (pair<P3,VP> nowF:M) ret.pb(convexHull2D(nowF.se,nowF.fi));  
 return ret;  
}  
// 把一个面变成 ( 点 , 法向量 ) 的形式  
pair<P3,P3> getF(VP F){  
 return mp(F[0],cross(F[1]-F[0],F[2]-F[0]).unit());  
}  
// 3D Cut 保留 dot(dir,x-p)>=0 的部分  
VVP ConvexCut3D(VVP A,P3 p,P3 dir){  
 VVP ret; VP sec;  
 for (VP nowF: A){  
 int n=nowF.size(); VP ans; int dif=0;  
 for (int i=0;i<n;i++){  
 int d1=sign(dot(dir,nowF[i]-p));  
 int d2=sign(dot(dir,nowF[(i+1)%n]-p));  
 if (d1>=0) ans.pb(nowF[i]);  
 if (d1\*d2<0){  
 P3 q=getFL(p,dir,nowF[i],nowF[(i+1)%n])[0];  
 ans.push\_back(q); sec.push\_back(q);  
 }  
 if (d1==0) sec.push\_back(nowF[i]); else dif=1;  
 dif|=(sign(dot(dir,cross(nowF[(i+1)%n]-nowF[i],nowF[(i+1)%n]-nowF[i])))==-1);  
 }  
 if (ans.size()>0&&dif) ret.push\_back(ans);  
 }  
 if (sec.size()>0) ret.push\_back(convexHull2D(sec,dir));  
 return ret;  
}  
db vol(VVP A){  
 if (A.size()==0) return 0; P3 p=A[0][0]; db ans=0;  
 for (VP nowF:A)  
 for (int i=2;i<nowF.size();i++)  
 ans+=abs(getV(p,nowF[0],nowF[i-1],nowF[i]));  
 return ans/6;  
}  
VVP init(db INF) {  
 VVP pss(6,VP(4));  
 pss[0][0] = pss[1][0] = pss[2][0] = {-INF, -INF, -INF};  
 pss[0][3] = pss[1][1] = pss[5][2] = {-INF, -INF, INF};  
 pss[0][1] = pss[2][3] = pss[4][2] = {-INF, INF, -INF};  
 pss[0][2] = pss[5][3] = pss[4][1] = {-INF, INF, INF};  
 pss[1][3] = pss[2][1] = pss[3][2] = {INF, -INF, -INF};  
 pss[1][2] = pss[5][1] = pss[3][3] = {INF, -INF, INF};  
 pss[2][2] = pss[4][3] = pss[3][1] = {INF, INF, -INF};  
 pss[5][0] = pss[4][0] = pss[3][0] = {INF, INF, INF};  
 return pss;  
}

## zyx的计算几何

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const int N = 1e6 + 10;  
  
const double eps = 1e-9;  
const double PI = acos(-1.0);  
const double dinf = 1e99;  
const ll inf = 0x3f3f3f3f3f3f3f3f;  
struct Line;  
  
struct Point {  
 double x, y;  
  
 Point() { x = y = 0; }  
  
 Point(const Line &a);  
  
 Point(const double &a, const double &b) : x(a), y(b) {}  
  
 Point operator+(const Point &a) const {  
 return {x + a.x, y + a.y};  
 }  
  
 Point operator-(const Point &a) const {  
 return {x - a.x, y - a.y};  
 }  
  
 Point operator\*(const double &a) const {  
 return {x \* a, y \* a};  
 }  
  
 Point operator/(const double &d) const {  
 return {x / d, y / d};  
 }  
  
 bool operator==(const Point &a) const {  
 return abs(x - a.x) + abs(y - a.y) < eps;  
 }  
  
 // 标准化，转化为膜长为1  
 void standardize() {  
 \*this = \*this / sqrt(x \* x + y \* y);  
 }  
};  
  
  
double norm(const Point &p) { return p.x \* p.x + p.y \* p.y; }  
  
//逆时针转90度  
Point orth(const Point &a) { return Point(-a.y, a.x); }  
  
//两点间距离  
double dist(const Point &a, const Point &b) {  
 return sqrt((a.x - b.x) \* (a.x - b.x) + (a.y - b.y) \* (a.y - b.y));  
}  
  
//两点间距离的平方  
double dist2(const Point &a, const Point &b) {  
 return (a.x - b.x) \* (a.x - b.x) + (a.y - b.y) \* (a.y - b.y);  
}  
  
struct Line {  
 Point s, t;  
  
 Line() {}  
  
 Line(const Point &a, const Point &b) : s(a), t(b) {}  
  
};  
  
  
struct Circle {  
 Point o;  
 double r;  
  
 Circle() {}  
  
 Circle(Point P, double R = 0) { o = P, r = R; }  
};  
  
//向量的膜长  
double length(const Point &p) {  
 return sqrt(p.x \* p.x + p.y \* p.y);  
}  
  
//线段的长度  
double length(const Line &l) {  
 Point p(l);  
 return length(p);  
}  
  
Point::Point(const Line &a) { \*this = a.t - a.s; }  
  
istream &operator>>(istream &in, Point &a) {  
 in >> a.x >> a.y;  
 return in;  
}  
  
ostream &operator<<(ostream &out, Point &a) {  
 out << fixed << setprecision(10) << a.x << ' ' << a.y;  
 return out;  
}  
  
//点积  
double dot(const Point &a, const Point &b) { return a.x \* b.x + a.y \* b.y; }  
  
//叉积  
double det(const Point &a, const Point &b) { return a.x \* b.y - a.y \* b.x; }  
  
//正负判断  
int sgn(const double &x) { return fabs(x) < eps ? 0 : (x > 0 ? 1 : -1); }  
  
//平方  
double sqr(const double &x) { return x \* x; }  
  
//将向量a逆时针旋转ang（弧度制）  
Point rotate(const Point &a, const double &ang) {  
 double x = cos(ang) \* a.x - sin(ang) \* a.y;  
 double y = sin(ang) \* a.x + cos(ang) \* a.y;  
 return {x, y};  
}  
  
//点p在线段seg上，<=0则包含端点  
bool sp\_on(const Line &seg, const Point &p) {  
 Point a = seg.s, b = seg.t;  
 return !sgn(det(p - a, b - a)) && sgn(dot(p - a, p - b)) <= 0;  
}  
  
//点p在直线line上  
bool lp\_on(const Line &line, const Point &p) {  
 Point a = line.s, b = line.t;  
 return !sgn(det(p - a, b - a));  
}  
  
//凸包，下标从0开始，<=0则凸包中不包含共线点  
int andrew(Point \*point, Point \*convex, int n) {  
 sort(point, point + n, [](Point a, Point b) {  
 if (a.x != b.x) return a.x < b.x;  
 return a.y < b.y;  
 });  
 int top = 0;  
 for (int i = 0; i < n; i++) {  
 while ((top > 1) && det(convex[top - 1] - convex[top - 2], point[i] - convex[top - 1]) <= 0)  
 top--;  
 convex[top++] = point[i];  
 }  
 int tmp = top;  
 for (int i = n - 2; i >= 0; i--) {  
 while ((top > tmp) && det(convex[top - 1] - convex[top - 2], point[i] - convex[top - 1]) <= 0)  
 top--;  
 convex[top++] = point[i];  
 }  
 if (n > 1) top--;  
 return top;  
}  
  
//斜率  
double slope(const Point &a, const Point &b) { return (a.y - b.y) / (a.x - b.x); }  
  
//斜率  
double slope(const Line &a) { return slope(a.s, a.t); }  
  
//两直线的焦点  
Point ll\_intersection(const Line &a, const Line &b) {  
 double s1 = det(Point(a), b.s - a.s), s2 = det(Point(a), b.t - a.s);  
 if (sgn(s1) == 0 && sgn(s2) == 0) return a.s;  
 return (b.s \* s2 - b.t \* s1) / (s2 - s1);  
}  
  
//两线段交点p，返回0为无交点，2为交点为端点，1为相交  
int ss\_cross(const Line &a, const Line &b, Point &p) {  
 int d1 = sgn(det(a.t - a.s, b.s - a.s));  
 int d2 = sgn(det(a.t - a.s, b.t - a.s));  
 int d3 = sgn(det(b.t - b.s, a.s - b.s));  
 int d4 = sgn(det(b.t - b.s, a.t - b.s));  
 if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) {  
 p = ll\_intersection(a, b);  
 return 1;  
 }  
 if (!d1 && sp\_on(a, b.s)) {  
 p = b.s;  
 return 2;  
 }  
 if (!d2 && sp\_on(a, b.t)) {  
 p = b.t;  
 return 2;  
 }  
 if (!d3 && sp\_on(b, a.s)) {  
 p = a.s;  
 return 2;  
 }  
 if (!d4 && sp\_on(b, a.t)) {  
 p = a.t;  
 return 2;  
 }  
 return 0;  
}  
  
//两向量直接的相对位置关系，含义见英文注释  
int ccw(const Point &a, Point b, Point c) {  
 b = b - a, c = c - a;  
 if (sgn(det(b, c)) > 0) return +1; // "COUNTER\_CLOCKWISE"  
 if (sgn(det(b, c)) < 0) return -1; // "CLOCKWISE"  
 if (sgn(dot(b, c)) < 0) return +2; // "ONLINE\_BACK"  
 if (sgn(norm(b) - norm(c)) < 0) return -2; // "ONLINE\_FRONT"  
 return 0; // "ON\_SEGMENT"  
}  
  
  
//点p在线l上的投影位置  
Point project(const Line &l, const Point &p) {  
 Point base(l);  
 double r = dot(base, p - l.s) / sqr(length(base));  
 return l.s + (base \* r);  
}  
  
//线段l和点p的距离  
double sp\_dist(const Line &l, const Point &p) {  
 if (l.s == l.t) return dist(l.s, p);  
 Point x = p - l.s, y = p - l.t, z = l.t - l.s;  
 if (sgn(dot(x, z)) < 0)return length(x);//P距离A更近  
 if (sgn(dot(y, z)) > 0)return length(y);//P距离B更近  
 return abs(det(x, z) / length(z));//面积除以底边长  
}  
  
//直线l和点p的距离  
double lp\_dist(const Line &l, const Point &p) {  
 Point x = p - l.s, y = p - l.t, z = l.t - l.s;  
 return abs(det(x, z) / length(z));//面积除以底边长  
}  
  
//圆c和直线l的交点，返回值为交点的数量，ans为交点位置  
int cl\_cross(const Circle &c, const Line &l, pair<Point, Point> &ans) {  
 Point a = c.o;  
 double r = c.r;  
 Point pr = project(l, a);  
 double dis = dist(pr, a);  
 double tmp = r \* r - dis \* dis;  
 if (sgn(tmp) == 1) {  
 double base = sqrt(max(0.0, r \* r - dis \* dis));  
 Point e(l);  
 e.standardize();  
 e = e \* base;  
 ans = make\_pair(pr + e, pr - e);  
 return 2;  
 } else if (sgn(tmp) == 0) {  
 ans = make\_pair(pr, pr);  
 return 1;  
 } else return 0;  
}  
  
//圆c和线段l交点个数，下面cs\_cross用到  
int intersectCS(Circle c, Line l) {  
 if (sgn(norm(project(l, c.o) - c.o) - c.r \* c.r) > 0) return 0;  
 double d1 = length(c.o - l.s), d2 = length(c.o - l.t);  
 if (sgn(d1 - c.r) <= 0 && sgn(d2 - c.r) <= 0) return 0;  
 if ((sgn(d1 - c.r) < 0 && sgn(d2 - c.r) > 0) || (sgn(d1 - c.r) > 0 && sgn(d2 - c.r) < 0)) return 1;  
 Point h = project(l, c.o);  
 if (dot(l.s - h, l.t - h) < 0) return 2;  
 return 0;  
}  
  
//圆和线段交点，返回交点数量  
int cs\_cross(Circle c, Line s, pair<Point, Point> &ans) {  
 Line l(s);  
 int num = cl\_cross(c, l, ans);  
 int res = intersectCS(c, s);  
 if (res == 2) return 2;  
 if (num > 1) {  
 if (dot(l.s - ans.first, l.t - ans.first) > 0) swap(ans.first, ans.second);  
 ans.second = ans.first;  
 }  
 return res;  
}  
  
//两圆交点，位置关系见注释  
int cc\_cross(const Circle &cir1, const Circle &cir2, pair<Point, Point> &ans) {  
 const Point &c1 = cir1.o, &c2 = cir2.o;  
 const double &r1 = cir1.r, &r2 = cir2.r;  
 double x1 = c1.x, x2 = c2.x, y1 = c1.y, y2 = c2.y;  
 double d = length(c1 - c2);  
 if (sgn(fabs(r1 - r2) - d) > 0) return 0; //内含  
 if (sgn(r1 + r2 - d) < 0) return 4; //相离  
 double a = r1 \* (x1 - x2) \* 2, b = r1 \* (y1 - y2) \* 2, c = r2 \* r2 - r1 \* r1 - d \* d;  
 double p = a \* a + b \* b, q = -a \* c \* 2, r = c \* c - b \* b;  
  
 double cosa, sina, cosb, sinb;  
 //One Intersection  
 if (sgn(d - (r1 + r2)) == 0 || sgn(d - fabs(r1 - r2)) == 0) {  
 cosa = -q / p / 2;  
 sina = sqrt(1 - sqr(cosa));  
 Point p0(x1 + r1 \* cosa, y1 + r1 \* sina);  
 if (sgn(dist(p0, c2) - r2)) p0.y = y1 - r1 \* sina;  
 ans = pair<Point, Point>(p0, p0);  
 if (sgn(r1 + r2 - d) == 0) return 3; //外切  
 else return 1; //内切  
 }  
 //Two Intersections  
 double delta = sqrt(q \* q - p \* r \* 4);  
 cosa = (delta - q) / p / 2;  
 cosb = (-delta - q) / p / 2;  
 sina = sqrt(1 - sqr(cosa));  
 sinb = sqrt(1 - sqr(cosb));  
 Point p1(x1 + r1 \* cosa, y1 + r1 \* sina);  
 Point p2(x1 + r1 \* cosb, y1 + r1 \* sinb);  
 if (sgn(dist(p1, c2) - r2)) p1.y = y1 - r1 \* sina;  
 if (sgn(dist(p2, c2) - r2)) p2.y = y1 - r1 \* sinb;  
 if (p1 == p2) p1.y = y1 - r1 \* sina;  
 ans = pair<Point, Point>(p1, p2);  
 return 2; // 相交  
}  
  
//点p关于直线l的对称点  
Point lp\_sym(const Line &l, const Point &p) {  
 return p + (project(l, p) - p) \* 2;  
}  
  
//返回两向量的夹角  
double alpha(const Point &t1, const Point &t2) {  
 double theta;  
 theta = atan2((double) t2.y, (double) t2.x) - atan2((double) t1.y, (double) t1.x);  
 if (sgn(theta) < 0)  
 theta += 2.0 \* PI;  
 return theta;  
}  
  
//【射线法】判断点A是否在任意多边形Poly以内，下标从1开始（为保险起见，可以在判断前将所有点随机旋转一个角度防止被卡）  
int pip(const Point \*P, const int &n, const Point &a) {  
 int cnt = 0;  
 double tmp;  
 for (int i = 1; i <= n; ++i) {  
 int j = i < n ? i + 1 : 1;  
 if (sp\_on(Line(P[i], P[j]), a))return 2;//点在多边形上  
 if (a.y >= min(P[i].y, P[j].y) && a.y < max(P[i].y, P[j].y))//纵坐标在该线段两端点之间  
 tmp = P[i].x + (a.y - P[i].y) / (P[j].y - P[i].y) \* (P[j].x - P[i].x), cnt += sgn(tmp - a.x) > 0;//交点在A右方  
 }  
 return cnt & 1;//穿过奇数次则在多边形以内  
}  
  
//判断AL是否在AR右边  
bool pip\_convex\_jud(const Point &a, const Point &L, const Point &R) {  
 return sgn(det(L - a, R - a)) > 0;//必须严格以内  
}  
  
//【二分法】判断点A是否在凸多边形Poly以内，下标从0开始  
bool pip\_convex(const Point \*P, const int &n, const Point &a) {  
 //点按逆时针给出  
 if (pip\_convex\_jud(P[0], a, P[1]) || pip\_convex\_jud(P[0], P[n - 1], a)) return 0;//在P[0\_1]或P[0\_n-1]外  
 if (sp\_on(Line(P[0], P[1]), a) || sp\_on(Line(P[0], P[n - 1]), a)) return 2;//在P[0\_1]或P[0\_n-1]上  
 int l = 1, r = n - 2;  
 while (l < r) {//二分找到一个位置pos使得P[0]\_A在P[0\_pos],P[0\_(pos+1)]之间  
 int mid = (l + r + 1) >> 1;  
 if (pip\_convex\_jud(P[0], P[mid], a))l = mid;  
 else r = mid - 1;  
 }  
 if (pip\_convex\_jud(P[l], a, P[l + 1]))return 0;//在P[pos\_(pos+1)]外  
 if (sp\_on(Line(P[l], P[l + 1]), a))return 2;//在P[pos\_(pos+1)]上  
 return 1;  
}  
// 多边形是否包含线段  
// 因此我们可以先求出所有和线段相交的多边形的顶点，然后按照X-Y坐标排序(X坐标小的排在前面，对于X坐标相同的点，Y坐标小的排在前面，  
// 这种排序准则也是为了保证水平和垂直情况的判断正确)，这样相邻的两个点就是在线段上相邻的两交点，如果任意相邻两点的中点也在多边形内，  
// 则该线段一定在多边形内。  
  
//【判断多边形A与多边形B是否相离】  
int pp\_judge(Point \*A, int n, Point \*B, int m) {  
 for (int i1 = 1; i1 <= n; ++i1) {  
 int j1 = i1 < n ? i1 + 1 : 1;  
 for (int i2 = 1; i2 <= m; ++i2) {  
 int j2 = i2 < m ? i2 + 1 : 1;  
 Point tmp;  
 if (ss\_cross(Line(A[i1], A[j1]), Line(B[i2], B[j2]), tmp)) return 0;//两线段相交  
 if (pip(B, m, A[i1]) || pip(A, n, B[i2]))return 0;//点包含在内  
 }  
 }  
 return 1;  
}  
  
//【任意多边形P的面积】,下标从0开始  
double area(Point \*P, int n) {  
 double S = 0;  
 for (int i = 0; i < n; i++) S += det(P[i], P[(i + 1) % n]);  
 return S \* 0.5;  
}  
  
//多边形和圆的面积交 ，下表从0开始  
double pc\_area(Point \*p, int n, const Circle &c) {  
 if (n < 3) return 0;  
 function<double(Circle, Point, Point)> dfs = [&](Circle c, Point a, Point b) {  
 Point va = c.o - a, vb = c.o - b;  
 double f = det(va, vb), res = 0;  
 if (sgn(f) == 0) return res;  
 if (sgn(max(length(va), length(vb)) - c.r) <= 0) return f;  
 Point d(dot(va, vb), det(va, vb));  
 if (sgn(sp\_dist(Line(a, b), c.o) - c.r) >= 0) return c.r \* c.r \* atan2(d.y, d.x);  
 pair<Point, Point> u;  
 int cnt = cs\_cross(c, Line(a, b), u);  
 if (cnt == 0) return res;  
 if (cnt > 1 && sgn(dot(u.second - u.first, a - u.first)) > 0) swap(u.first, u.second);  
 res += dfs(c, a, u.first);  
 if (cnt == 2) res += dfs(c, u.first, u.second) + dfs(c, u.second, b);  
 else if (cnt == 1) res += dfs(c, u.first, b);  
 return res;  
 };  
 double res = 0;  
 for (int i = 0; i < n; i++) {  
 res += dfs(c, p[i], p[(i + 1) % n]);  
 }  
 return res \* 0.5;  
}  
  
Line Q[N];  
  
//【半平面交】  
int judge(Line L, Point a) { return sgn(det(a - L.s, L.t - L.s)) > 0; }//判断点a是否在直线L的右边  
int halfcut(Line \*L, int n, Point \*P) {  
 sort(L, L + n, [](const Line &a, const Line &b) {  
 double d = atan2((a.t - a.s).y, (a.t - a.s).x) - atan2((b.t - b.s).y, (b.t - b.s).x);  
 return sgn(d) ? sgn(d) < 0 : judge(a, b.s);  
 });  
  
 int m = n;  
 n = 0;  
 for (int i = 0; i < m; ++i)  
 if (i == 0 || sgn(atan2(Point(L[i]).y, Point(L[i]).x) - atan2(Point(L[i - 1]).y, Point(L[i - 1]).x)))  
 L[n++] = L[i];  
 int h = 1, t = 0;  
 for (int i = 0; i < n; ++i) {  
 while (h < t && judge(L[i], ll\_intersection(Q[t], Q[t - 1]))) --t;//当队尾两个直线交点不是在直线L[i]上或者左边时就出队  
 while (h < t && judge(L[i], ll\_intersection(Q[h], Q[h + 1]))) ++h;//当队头两个直线交点不是在直线L[i]上或者左边时就出队  
 Q[++t] = L[i];  
  
 }  
 while (h < t && judge(Q[h], ll\_intersection(Q[t], Q[t - 1]))) --t;  
 while (h < t && judge(Q[t], ll\_intersection(Q[h], Q[h + 1]))) ++h;  
 n = 0;  
 for (int i = h; i <= t; ++i) {  
 P[n++] = ll\_intersection(Q[i], Q[i < t ? i + 1 : h]);  
 }  
 return n;  
}  
  
Point V1[N], V2[N];  
  
//【闵可夫斯基和】求两个凸包{P1},{P2}的向量集合{V}={P1+P2}构成的凸包  
int mincowski(Point \*P1, int n, Point \*P2, int m, Point \*V) {  
 for (int i = 0; i < n; ++i) V1[i] = P1[(i + 1) % n] - P1[i];  
 for (int i = 0; i < m; ++i) V2[i] = P2[(i + 1) % m] - P2[i];  
 int t = 0, i = 0, j = 0;  
 V[t++] = P1[0] + P2[0];  
 while (i < n && j < m) V[t] = V[t - 1] + (sgn(det(V1[i], V2[j])) > 0 ? V1[i++] : V2[j++]), t++;  
 while (i < n) V[t] = V[t - 1] + V1[i++], t++;  
 while (j < m) V[t] = V[t - 1] + V2[j++], t++;  
 return t;  
}  
  
//【三点确定一圆】向量垂心法  
Circle external\_circle(const Point &A, const Point &B, const Point &C) {  
 Point P1 = (A + B) \* 0.5, P2 = (A + C) \* 0.5;  
 Line R1 = Line(P1, P1 + orth(B - A));  
 Line R2 = Line(P2, P2 + orth(C - A));  
 Circle O;  
 O.o = ll\_intersection(R1, R2);  
 O.r = length(A - O.o);  
 return O;  
}  
  
//三角形内接圆  
Circle internal\_circle(const Point &A, const Point &B, const Point &C) {  
 double a = dist(B, C), b = dist(A, C), c = dist(A, B);  
 double s = (a + b + c) / 2;  
 double S = sqrt(max(0.0, s \* (s - a) \* (s - b) \* (s - c)));  
 double r = S / s;  
  
 return Circle((A \* a + B \* b + C \* c) / (a + b + c), r);  
}  
  
//动态凸包  
struct ConvexHull {  
  
 int op;  
  
 struct cmp {  
 bool operator()(const Point &a, const Point &b) const {  
 return sgn(a.x - b.x) < 0 || sgn(a.x - b.x) == 0 && sgn(a.y - b.y) < 0;  
 }  
 };  
  
 set<Point, cmp> s;  
  
 ConvexHull(int o) {  
 op = o;  
 s.clear();  
 }  
  
 inline int PIP(Point P) {  
 set<Point>::iterator it = s.lower\_bound(Point(P.x, -dinf));//找到第一个横坐标大于P的点  
 if (it == s.end())return 0;  
 if (sgn(it->x - P.x) == 0) return sgn((P.y - it->y) \* op) <= 0;//比较纵坐标大小  
 if (it == s.begin())return 0;  
 set<Point>::iterator j = it, k = it;  
 --j;  
 return sgn(det(P - \*j, \*k - \*j) \* op) >= 0;//看叉姬1  
 }  
  
 inline int judge(set<Point>::iterator it) {  
 set<Point>::iterator j = it, k = it;  
 if (j == s.begin())return 0;  
 --j;  
 if (++k == s.end())return 0;  
 return sgn(det(\*it - \*j, \*k - \*j) \* op) >= 0;//看叉姬  
 }  
  
 inline void insert(Point P) {  
 if (PIP(P))return;//如果点P已经在凸壳上或凸包里就不插入了  
 set<Point>::iterator tmp = s.lower\_bound(Point(P.x, -dinf));  
 if (tmp != s.end() && sgn(tmp->x - P.x) == 0)s.erase(tmp);//特判横坐标相等的点要去掉  
 s.insert(P);  
 set<Point>::iterator it = s.find(P), p = it;  
 if (p != s.begin()) {  
 --p;  
 while (judge(p)) {  
 set<Point>::iterator temp = p--;  
 s.erase(temp);  
 }  
 }  
 if ((p = ++it) != s.end()) {  
 while (judge(p)) {  
 set<Point>::iterator temp = p++;  
 s.erase(temp);  
 }  
 }  
 }  
} up(1), down(-1);  
  
int PIC(Circle C, Point a) { return sgn(length(a - C.o) - C.r) <= 0; }//判断点A是否在圆C内  
void Random(Point \*P, int n) { for (int i = 0; i < n; ++i)swap(P[i], P[(rand() + 1) % n]); }//随机一个排列  
//【求点集P的最小覆盖圆】 O(n)  
Circle min\_circle(Point \*P, int n) {  
// random\_shuffle(P,P+n);  
 Random(P, n);  
 Circle C = Circle(P[0], 0);  
 for (int i = 1; i < n; ++i)  
 if (!PIC(C, P[i])) {  
 C = Circle(P[i], 0);  
 for (int j = 0; j < i; ++j)  
 if (!PIC(C, P[j])) {  
 C.o = (P[i] + P[j]) \* 0.5, C.r = length(P[j] - C.o);  
 for (int k = 0; k < j; ++k) if (!PIC(C, P[k])) C = external\_circle(P[i], P[j], P[k]);  
 }  
 }  
 return C;  
}  
  
  
int temp[N];  
  
//最近点对  
double closest\_point(Point \*p, int n) {  
 function<double(int, int)> merge = [&](int l, int r) {  
 double d = dinf;  
 if (l == r) return d;  
 if (l + 1 == r) return dist(p[l], p[r]);  
 int mid = (l + r) >> 1;  
 double d1 = merge(l, mid);  
 double d2 = merge(mid + 1, r);  
 d = min(d1, d2);  
 int i, j, k = 0;  
 for (i = l; i <= r; i++) {  
 if (sgn(abs(p[mid].x - p[i].x) - d) <= 0)  
 temp[k++] = i;  
  
 }  
 sort(temp, temp + k, [&](const int &a, const int &b) {  
 return sgn(p[a].y - p[b].y) < 0;  
 });  
 for (i = 0; i < k; i++) {  
 for (j = i + 1; j < k && sgn(p[temp[j]].y - p[temp[i]].y - d) <= 0; j++) {  
 double d3 = dist(p[temp[i]], p[temp[j]]);  
 d = min(d, d3);  
 }  
 }  
 return d;  
 };  
 sort(p, p + n, [&](const Point &a, const Point &b) {  
 if (sgn(a.x - b.x) == 0) return sgn(a.y - b.y) < 0;  
 else return sgn(a.x - b.x) < 0;  
 });  
 return merge(0, n - 1);  
}  
  
//圆和点的切线  
int tangent(const Circle &c1, const Point &p2, pair<Point, Point> &ans) {  
 Point tmp = c1.o - p2;  
 int sta;  
 if (sgn(norm(tmp) - c1.r \* c1.r) < 0) return 0;  
 else if (sgn(norm(tmp) - c1.r \* c1.r) == 0) sta = 1;  
 else sta = 2;  
 Circle c2 = Circle(p2, sqrt(max(0.0, norm(tmp) - c1.r \* c1.r)));  
 cc\_cross(c1, c2, ans);  
 return sta;  
}  
  
//圆和圆的切线  
int tangent(Circle c1, Circle c2, vector<Line> &ans) {  
 ans.clear();  
 if (sgn(c1.r - c2.r) < 0) swap(c1, c2);  
 double g = norm(c1.o - c2.o);  
 if (sgn(g) == 0) return 0;  
 Point u = (c2.o - c1.o) / sqrt(g);  
 Point v = orth(u);  
 for (int s = 1; s >= -1; s -= 2) {  
 double h = (c1.r + s \* c2.r) / sqrt(g);  
 if (sgn(1 - h \* h) == 0) {  
 ans.push\_back(Line(c1.o + u \* c1.r, c1.o + (u + v) \* c1.r));  
 } else if (sgn(1 - h \* h) >= 0) {  
 Point uu = u \* h, vv = v \* sqrt(1 - h \* h);  
 ans.push\_back(Line(c1.o + (uu + vv) \* c1.r, c2.o - (uu + vv) \* c2.r \* s));  
 ans.push\_back(Line(c1.o + (uu - vv) \* c1.r, c2.o - (uu - vv) \* c2.r \* s));  
 }  
 }  
  
 return ans.size();  
}  
  
//两圆面积交  
double areaofCC(Circle c1, Circle c2) {  
 if (c1.r > c2.r) swap(c1, c2);  
 double nor = norm(c1.o - c2.o);  
 double dist = sqrt(max(0.0, nor));  
  
 if (sgn(c1.r + c2.r - dist) <= 0) return 0;  
  
 if (sgn(dist + c1.r - c2.r) <= 0) return c1.r \* c1.r \* PI;  
  
 double val;  
 val = (nor + c1.r \* c1.r - c2.r \* c2.r) / (2 \* c1.r \* dist);  
 val = max(val, -1.0), val = min(val, 1.0);  
 double theta1 = acos(val);  
 val = (nor + c2.r \* c2.r - c1.r \* c1.r) / (2 \* c2.r \* dist);  
 val = max(val, -1.0), val = min(val, 1.0);  
 double theta2 = acos(val);  
 return (theta1 - sin(theta1 + theta1) \* 0.5) \* c1.r \* c1.r + (theta2 - sin(theta2 + theta2) \* 0.5) \* c2.r \* c2.r;  
}  
  
//https://onlinejudge.u-aizu.ac.jp/courses/library/4/CGL/all/CGL\_4\_C  
//把凸包切一刀  
int convexCut(Point \*p, Point \*ans, int n, Line l) {  
 int top = 0;  
 for (int i = 0; i < n; i++) {  
 Point a = p[i], b = p[(i + 1) % n];  
 if (ccw(l.s, l.t, a) != -1) ans[top++] = a;  
 if (ccw(l.s, l.t, a) \* ccw(l.s, l.t, b) < 0)  
 ans[top++] = ll\_intersection(Line(a, b), l);  
 }  
 return top;  
}  
  
//两球体积交  
double SphereCross(double d, double r1, double r2) {  
 if (r1 < r2) swap(r1, r2);  
 if (sgn(d - r1 - r2) >= 0) return 0;  
 if (sgn(d + r2 - r1) <= 0) return 4.0 / 3 \* PI \* r2 \* r2 \* r2;  
 double co = (r1 \* r1 + d \* d - r2 \* r2) / (2.0 \* d \* r1);  
 double h = r1 \* (1 - co);  
 double ans = (1.0 / 3) \* PI \* (3.0 \* r1 - h) \* h \* h;  
 co = (r2 \* r2 + d \* d - r1 \* r1) / (2.0 \* d \* r2);  
 h = r2 \* (1 - co);  
 ans += (1.0 / 3) \* PI \* (3.0 \* r2 - h) \* h \* h;  
 return ans;  
}

## 几何一些定理（或知识点？

## 多面体欧拉定理

多面体欧拉定理是指对于简单多面体，其各维对象数总满足一定的数学关系，在三维空间中多面体欧拉定理可表示为：  
“顶点数-棱长数+表面数=2”。  
简单多面体即表面经过连续变形可以变为球面的多面体。

## 单纯形体积

空间下标准单纯形与原点围成的体积为

## 球缺体积公式

球缺体积V=(π/3)(3R-h)\*h² 或写成V=πh²(R-h/3)，（R是球的半径,h是球缺的高).如果已知球缺高h，底面半径r,则V=[πh(3r²+h²)]/6

## 海伦公式

## 球体积交和并

#include<bits/stdc++.h>  
#define fi first  
#define sf scanf  
#define se second  
#define pf printf  
#define pb push\_back  
#define mp make\_pair  
#define sz(x) ((int)(x).size())  
#define all(x) (x).begin(),(x).end()  
#define mem(x,y) memset((x),(y),sizeof(x))  
#define fup(i,x,y) for(int i=(x);i<=(y);++i)  
#define fdn(i,x,y) for(int i=(x);i>=(y);--i)  
typedef long long ll;  
typedef long double ld;  
typedef unsigned long long ull;  
typedef std::pair<int,int> pii;  
using namespace std;  
   
const ld pi=acos(-1);  
   
ld pow2(ld x){return x\*x;}  
   
ld pow3(ld x){return x\*x\*x;}  
   
ld dis(ld x1,ld y1,ld z1,ld x2,ld y2,ld z2)  
{  
 return pow2(x1-x2)+pow2(y1-y2)+pow2(z1-z2);  
}  
   
ld cos(ld a,ld b,ld c){return (b\*b+c\*c-a\*a)/(2\*b\*c);}  
   
ld cap(ld r,ld h){return pi\*(r\*3-h)\*h\*h/3;} // 球缺体积公式，h为球缺的高  
   
//2球体积交  
ld sphere\_intersect(ld x1,ld y1,ld z1,ld r1,ld x2,ld y2,ld z2,ld r2)  
{  
 ld d=dis(x1,y1,z1,x2,y2,z2);  
 //相离  
 if(d>=pow2(r1+r2))return 0;  
 //包含  
 if(d<=pow2(r1-r2))return pow3(min(r1,r2))\*4\*pi/3;  
 //相交  
 ld h1=r1-r1\*cos(r2,r1,sqrt(d)),h2=r2-r2\*cos(r1,r2,sqrt(d));  
 return cap(r1,h1)+cap(r2,h2);  
}  
   
//2球体积并  
ld sphere\_union(ld x1,ld y1,ld z1,ld r1,ld x2,ld y2,ld z2,ld r2)  
{  
 ld d=dis(x1,y1,z1,x2,y2,z2);  
 //相离  
 if(d>=pow2(r1+r2))return (pow3(r1)+pow3(r2))\*4\*pi/3;  
 //包含  
 if(d<=pow2(r1-r2))return pow3(max(r1,r2))\*4\*pi/3;  
 //相交  
 ld h1=r1+r1\*cos(r2,r1,sqrt(d)),h2=r2+r2\*cos(r1,r2,sqrt(d));  
 return cap(r1,h1)+cap(r2,h2);  
}  
   
int main()  
{  
 double x1,y1,z1,r1,x2,y2,z2,r2;  
 sf("%lf%lf%lf%lf%lf%lf%lf%lf",&x1,&y1,&z1,&r1,&x2,&y2,&z2,&r2);  
 pf("%.12Lf\n",sphere\_union(x1,y1,z1,r1,x2,y2,z2,r2));  
 return 0;  
}

## 自适应辛普森

double f(double x) {  
}  
  
double simpson(double l, double r) {  
 double mid = (l + r) / 2;  
 return (r - l) \* (f(l) + 4 \* f(mid) + f(r)) / 6; // 辛普森公式  
}  
  
double asr(double l, double r, double EPS, double ans) {  
 double mid = (l + r) / 2;  
 double fl = simpson(l, mid), fr = simpson(mid, r);  
 if (abs(fl + fr - ans) <= 15 \* EPS)  
 return fl + fr + (fl + fr - ans) / 15; // 足够相似的话就直接返回  
 return asr(l, mid, EPS / 2, fl) +  
 asr(mid, r, EPS / 2, fr); // 否则分割成两段递归求解  
}

## 计算几何全家桶

#include <bits/stdc++.h>  
  
using namespace std;  
typedef long long ll;  
const ll N = 1 << 20;  
const ll mod = 1e9 + 7;  
const double dinf = 1e99;  
const int inf = 0x3f3f3f3f;  
const ll linf = 0x3f3f3f3f3f3f3f3f;  
  
const double eps = 1e-9;  
const double PI = acos(-1.0);  
  
struct Line;  
  
struct Point {  
 double x, y;  
  
 Point() { x = y = 0; }  
  
 Point(const Line &a);  
  
 Point(const double &a, const double &b) : x(a), y(b) {}  
  
 Point operator+(const Point &a) const {  
 return {x + a.x, y + a.y};  
 }  
  
 Point operator-(const Point &a) const {  
 return {x - a.x, y - a.y};  
 }  
  
 Point operator\*(const double &a) const {  
 return {x \* a, y \* a};  
 }  
  
 Point operator/(const double &d) const {  
 return {x / d, y / d};  
 }  
  
 bool operator==(const Point &a) const {  
 return abs(x - a.x) + abs(y - a.y) < eps;  
 }  
  
 void standardize() {  
 \*this = \*this / sqrt(x \* x + y \* y);  
 }  
};  
  
Point normal(const Point &a) { return Point(-a.y, a.x); }  
  
double dist(const Point &a, const Point &b) {  
 return sqrt((a.x - b.x) \* (a.x - b.x) + (a.y - b.y) \* (a.y - b.y));  
}  
  
double dist2(const Point &a, const Point &b) {  
 return (a.x - b.x) \* (a.x - b.x) + (a.y - b.y) \* (a.y - b.y);  
}  
  
struct Line {  
 Point s, t;  
  
 Line() {}  
  
 Line(const Point &a, const Point &b) : s(a), t(b) {}  
  
};  
  
struct circle {  
 Point o;  
 double r;  
  
 circle() {}  
  
 circle(Point P, double R = 0) { o = P, r = R; }  
};  
  
double length(const Point &p) {  
 return sqrt(p.x \* p.x + p.y \* p.y);  
}  
  
double length(const Line &l) {  
 Point p(l);  
 return length(p);  
}  
  
Point::Point(const Line &a) { \*this = a.t - a.s; }  
  
istream &operator>>(istream &in, Point &a) {  
 in >> a.x >> a.y;  
 return in;  
}  
  
double dot(const Point &a, const Point &b) {  
 return a.x \* b.x + a.y \* b.y;  
}  
  
double det(const Point &a, const Point &b) {  
 return a.x \* b.y - a.y \* b.x;  
}  
  
int sgn(const double &x) { return fabs(x) < eps ? 0 : (x > 0 ? 1 : -1); }  
  
double sqr(const double &x) { return x \* x; }  
  
Point rotate(const Point &a, const double &ang) {  
 double x = cos(ang) \* a.x - sin(ang) \* a.y;  
 double y = sin(ang) \* a.x + cos(ang) \* a.y;  
 return {x, y};  
}  
  
//点在线段上 <=0 包含端点  
bool sp\_on(const Line &seg, const Point &p) {  
 Point a = seg.s, b = seg.t;  
 return !sgn(det(p - a, b - a)) && sgn(dot(p - a, p - b)) <= 0;  
}  
  
bool lp\_on(const Line &line, const Point &p) {  
 Point a = line.s, b = line.t;  
 return !sgn(det(p - a, b - a));  
}  
  
//等于不包含共线  
int andrew(Point \*point, Point \*convex, int n) {  
 sort(point, point + n, [](Point a, Point b) {  
 if (a.x != b.x) return a.x < b.x;  
 return a.y < b.y;  
 });  
 int top = 0;  
 for (int i = 0; i < n; i++) {  
 while ((top > 1) && det(convex[top - 1] - convex[top - 2], point[i] - convex[top - 1]) <= 0)  
 top--;  
 convex[top++] = point[i];  
 }  
 int tmp = top;  
 for (int i = n - 2; i >= 0; i--) {  
 while ((top > tmp) && det(convex[top - 1] - convex[top - 2], point[i] - convex[top - 1]) <= 0)  
 top--;  
 convex[top++] = point[i];  
 }  
 if (n > 1) top--;  
 return top;  
}  
  
double slope(const Point &a, const Point &b) {  
 return (a.y - b.y) / (a.x - b.x);  
}  
  
double slope(const Line &a) {  
 return slope(a.s, a.t);  
}  
  
Point ll\_intersection(const Line &a, const Line &b) {  
 double s1 = det(Point(a), b.s - a.s), s2 = det(Point(a), b.t - a.s);  
 return (b.s \* s2 - b.t \* s1) / (s2 - s1);  
}  
  
int ss\_cross(const Line &a, const Line &b, Point &p) {  
 int d1 = sgn(det(a.t - a.s, b.s - a.s));  
 int d2 = sgn(det(a.t - a.s, b.t - a.s));  
 int d3 = sgn(det(b.t - b.s, a.s - b.s));  
 int d4 = sgn(det(b.t - b.s, a.t - b.s));  
 if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) {  
 p = ll\_intersection(a, b);  
 return 1;  
 }  
 if (!d1 && sp\_on(a, b.s)) {  
 p = b.s;  
 return 2;  
 }  
 if (!d2 && sp\_on(a, b.t)) {  
 p = b.t;  
 return 2;  
 }  
 if (!d3 && sp\_on(b, a.s)) {  
 p = a.s;  
 return 2;  
 }  
 if (!d4 && sp\_on(b, a.t)) {  
 p = a.t;  
 return 2;  
 }  
 return 0;  
}  
  
Point project(const Line &l, const Point &p) {  
 Point base(l);  
 double r = dot(base, p - l.s) / sqr(length(base));  
 return l.s + (base \* r);  
}  
  
double sp\_dist(const Line &l, const Point &p) {  
 if (l.s == l.t) return dist(l.s, p);  
 Point x = p - l.s, y = p - l.t, z = l.t - l.s;  
 if (sgn(dot(x, z)) < 0)return length(x);//P距离A更近  
 if (sgn(dot(y, z)) > 0)return length(y);//P距离B更近  
 return abs(det(x, z) / length(z));//面积除以底边长  
}  
  
double lp\_dist(const Line &l, const Point &p) {  
 Point x = p - l.s, y = p - l.t, z = l.t - l.s;  
 return abs(det(x, z) / length(z));//面积除以底边长  
}  
  
int lc\_cross(const Line &l, const Point &a, const double &r, pair<Point, Point> &ans) {  
 int num = 0;  
 Point pr = project(l, a);  
 double dis = dist(pr, a);  
 double tmp = r \* r - dis \* dis;  
 if (sgn(tmp) == 1) num = 2;  
 else if (sgn(tmp) == 0) num = 1;  
 else return 0;  
 double base = sqrt(r \* r - dis \* dis);  
 Point e(l);  
 e.standardize();  
 e = e \* base;  
 ans = make\_pair(pr + e, pr - e);  
 return num;  
}  
  
int cc\_cross(const Point &c1, const double &r1, const Point &c2, const double &r2, pair<Point, Point> &ans) {  
 double x1 = c1.x, x2 = c2.x, y1 = c1.y, y2 = c2.y;  
 double d = length(c1 - c2);  
 if (sgn(fabs(r1 - r2) - d) > 0) return -1; //内含  
 if (sgn(r1 + r2 - d) < 0) return 0; //相离  
 double a = r1 \* (x1 - x2) \* 2, b = r1 \* (y1 - y2) \* 2, c = r2 \* r2 - r1 \* r1 - d \* d;  
 double p = a \* a + b \* b, q = -a \* c \* 2, r = c \* c - b \* b;  
  
 double cosa, sina, cosb, sinb;  
 //One Intersection  
 if (sgn(d - (r1 + r2)) == 0 || sgn(d - fabs(r1 - r2)) == 0) {  
 cosa = -q / p / 2;  
 sina = sqrt(1 - sqr(cosa));  
 Point p0(x1 + r1 \* cosa, y1 + r1 \* sina);  
 if (sgn(dist(p0, c2) - r2)) p0.y = y1 - r1 \* sina;  
 ans = pair<Point, Point>(p0, p0);  
 return 1;  
 }  
 //Two Intersections  
 double delta = sqrt(q \* q - p \* r \* 4);  
 cosa = (delta - q) / p / 2;  
 cosb = (-delta - q) / p / 2;  
 sina = sqrt(1 - sqr(cosa));  
 sinb = sqrt(1 - sqr(cosb));  
 Point p1(x1 + r1 \* cosa, y1 + r1 \* sina);  
 Point p2(x1 + r1 \* cosb, y1 + r1 \* sinb);  
 if (sgn(dist(p1, c2) - r2)) p1.y = y1 - r1 \* sina;  
 if (sgn(dist(p2, c2) - r2)) p2.y = y1 - r1 \* sinb;  
 if (p1 == p2) p1.y = y1 - r1 \* sina;  
 ans = pair<Point, Point>(p1, p2);  
 return 2;  
}  
  
Point lp\_sym(const Line &l, const Point &p) {  
 return p + (project(l, p) - p) \* 2;  
}  
  
double alpha(const Point &t1, const Point &t2) {  
 double theta;  
 theta = atan2((double) t2.y, (double) t2.x) - atan2((double) t1.y, (double) t1.x);  
 if (sgn(theta) < 0)  
 theta += 2.0 \* PI;  
 return theta;  
}  
  
int pip(const Point \*P, const int &n, const Point &a) {//【射线法】判断点A是否在任意多边形Poly以内  
 int cnt = 0;  
 int tmp;  
 for (int i = 1; i <= n; ++i) {  
 int j = i < n ? i + 1 : 1;  
 if (sp\_on(Line(P[i], P[j]), a))return 2;//点在多边形上  
 if (a.y >= min(P[i].y, P[j].y) && a.y < max(P[i].y, P[j].y))//纵坐标在该线段两端点之间  
 tmp = P[i].x + (a.y - P[i].y) / (P[j].y - P[i].y) \* (P[j].x - P[i].x), cnt += sgn(tmp - a.x) > 0;//交点在A右方  
 }  
 return cnt & 1;//穿过奇数次则在多边形以内  
}  
  
bool pip\_convex\_jud(const Point &a, const Point &L, const Point &R) {//判断AL是否在AR右边  
 return sgn(det(L - a, R - a)) > 0;//必须严格以内  
}  
  
bool pip\_convex(const Point \*P, const int &n, const Point &a) {//【二分法】判断点A是否在凸多边形Poly以内  
 //点按逆时针给出  
 if (pip\_convex\_jud(P[0], a, P[1]) || pip\_convex\_jud(P[0], P[n - 1], a)) return 0;//在P[0\_1]或P[0\_n-1]外  
 if (sp\_on(Line(P[0], P[1]), a) || sp\_on(Line(P[0], P[n - 1]), a)) return 2;//在P[0\_1]或P[0\_n-1]上  
 int l = 1, r = n - 2;  
 while (l < r) {//二分找到一个位置pos使得P[0]\_A在P[0\_pos],P[0\_(pos+1)]之间  
 int mid = (l + r + 1) >> 1;  
 if (pip\_convex\_jud(P[0], P[mid], a))l = mid;  
 else r = mid - 1;  
 }  
 if (pip\_convex\_jud(P[l], a, P[l + 1]))return 0;//在P[pos\_(pos+1)]外  
 if (sp\_on(Line(P[l], P[l + 1]), a))return 2;//在P[pos\_(pos+1)]上  
 return 1;  
}  
// 多边形是否包含线段  
// 因此我们可以先求出所有和线段相交的多边形的顶点，然后按照X-Y坐标排序(X坐标小的排在前面，对于X坐标相同的点，Y坐标小的排在前面，  
// 这种排序准则也是为了保证水平和垂直情况的判断正确)，这样相邻的两个点就是在线段上相邻的两交点，如果任意相邻两点的中点也在多边形内，  
// 则该线段一定在多边形内。  
  
int pp\_judge(Point \*A, int n, Point \*B, int m) {//【判断多边形A与多边形B是否相离】  
 for (int i1 = 1; i1 <= n; ++i1) {  
 int j1 = i1 < n ? i1 + 1 : 1;  
 for (int i2 = 1; i2 <= m; ++i2) {  
 int j2 = i2 < m ? i2 + 1 : 1;  
 Point tmp;  
 if (ss\_cross(Line(A[i1], A[j1]), Line(B[i2], B[j2]), tmp)) return 0;//两线段相交  
 if (pip(B, m, A[i1]) || pip(A, n, B[i2]))return 0;//点包含在内  
 }  
 }  
 return 1;  
}  
  
double area(Point \*P, int n) {//【任意多边形P的面积】  
 double S = 0;  
 for (int i = 1; i <= n; i++) S += det(P[i], P[i < n ? i + 1 : 1]);  
 return S / 2.0;  
}  
  
Line Q[N];  
  
int judge(Line L, Point a) { return sgn(det(a - L.s, L.t - L.s)) > 0; }//判断点a是否在直线L的右边  
int halfcut(Line \*L, int n, Point \*P) {//【半平面交】  
 sort(L, L + n, [](const Line &a, const Line &b) {  
 double d = atan2((a.t - a.s).y, (a.t - a.s).x) - atan2((b.t - b.s).y, (b.t - b.s).x);  
 return sgn(d) ? sgn(d) < 0 : judge(a, b.s);  
 });  
  
 int m = n;  
 n = 0;  
 for (int i = 0; i < m; ++i)  
 if (i == 0 || sgn(atan2(Point(L[i]).y, Point(L[i]).x) - atan2(Point(L[i - 1]).y, Point(L[i - 1]).x)))  
 L[n++] = L[i];  
 int h = 1, t = 0;  
 for (int i = 0; i < n; ++i) {  
 while (h < t && judge(L[i], ll\_intersection(Q[t], Q[t - 1]))) --t;//当队尾两个直线交点不是在直线L[i]上或者左边时就出队  
 while (h < t && judge(L[i], ll\_intersection(Q[h], Q[h + 1]))) ++h;//当队头两个直线交点不是在直线L[i]上或者左边时就出队  
 Q[++t] = L[i];  
  
 }  
 while (h < t && judge(Q[h], ll\_intersection(Q[t], Q[t - 1]))) --t;  
 while (h < t && judge(Q[t], ll\_intersection(Q[h], Q[h + 1]))) ++h;  
 n = 0;  
 for (int i = h; i <= t; ++i) {  
 P[n++] = ll\_intersection(Q[i], Q[i < t ? i + 1 : h]);  
 }  
 return n;  
}  
  
Point V1[N], V2[N];  
  
int mincowski(Point \*P1, int n, Point \*P2, int m, Point \*V) {//【闵可夫斯基和】求两个凸包{P1},{P2}的向量集合{V}={P1+P2}构成的凸包  
 for (int i = 0; i < n; ++i) V1[i] = P1[(i + 1) % n] - P1[i];  
 for (int i = 0; i < m; ++i) V2[i] = P2[(i + 1) % m] - P2[i];  
 int t = 0, i = 0, j = 0;  
 V[t++] = P1[0] + P2[0];  
 while (i < n && j < m) V[t] = V[t - 1] + (sgn(det(V1[i], V2[j])) > 0 ? V1[i++] : V2[j++]), t++;  
 while (i < n) V[t] = V[t - 1] + V1[i++], t++;  
 while (j < m) V[t] = V[t - 1] + V2[j++], t++;  
 return t;  
}  
  
circle getcircle(const Point &A, const Point &B, const Point &C) {//【三点确定一圆】向量垂心法  
 Point P1 = (A + B) \* 0.5, P2 = (A + C) \* 0.5;  
 Line R1 = Line(P1, P1 + normal(B - A));  
 Line R2 = Line(P2, P2 + normal(C - A));  
 circle O;  
 O.o = ll\_intersection(R1, R2);  
 O.r = length(A - O.o);  
 return O;  
}  
  
struct ConvexHull {  
  
 int op;  
  
 struct cmp {  
 bool operator()(const Point &a, const Point &b) const {  
 return sgn(a.x - b.x) < 0 || sgn(a.x - b.x) == 0 && sgn(a.y - b.y) < 0;  
 }  
 };  
  
 set<Point, cmp> s;  
  
 ConvexHull(int o) {  
 op = o;  
 s.clear();  
 }  
  
 inline int PIP(Point P) {  
 set<Point>::iterator it = s.lower\_bound(Point(P.x, -dinf));//找到第一个横坐标大于P的点  
 if (it == s.end())return 0;  
 if (sgn(it->x - P.x) == 0) return sgn((P.y - it->y) \* op) <= 0;//比较纵坐标大小  
 if (it == s.begin())return 0;  
 set<Point>::iterator j = it, k = it;  
 --j;  
 return sgn(det(P - \*j, \*k - \*j) \* op) >= 0;//看叉姬1  
 }  
  
 inline int judge(set<Point>::iterator it) {  
 set<Point>::iterator j = it, k = it;  
 if (j == s.begin())return 0;  
 --j;  
 if (++k == s.end())return 0;  
 return sgn(det(\*it - \*j, \*k - \*j) \* op) >= 0;//看叉姬  
 }  
  
 inline void insert(Point P) {  
 if (PIP(P))return;//如果点P已经在凸壳上或凸包里就不插入了  
 set<Point>::iterator tmp = s.lower\_bound(Point(P.x, -inf));  
 if (tmp != s.end() && sgn(tmp->x - P.x) == 0)s.erase(tmp);//特判横坐标相等的点要去掉  
 s.insert(P);  
 set<Point>::iterator it = s.find(P), p = it;  
 if (p != s.begin()) {  
 --p;  
 while (judge(p)) {  
 set<Point>::iterator temp = p--;  
 s.erase(temp);  
 }  
 }  
 if ((p = ++it) != s.end()) {  
 while (judge(p)) {  
 set<Point>::iterator temp = p++;  
 s.erase(temp);  
 }  
 }  
 }  
} up(1), down(-1);  
  
int PIC(circle C, Point a) { return sgn(length(a - C.o) - C.r) <= 0; }//判断点A是否在圆C内  
void Random(Point \*P, int n) { for (int i = 0; i < n; ++i)swap(P[i], P[(rand() + 1) % n]); }//随机一个排列  
circle min\_circle(Point \*P, int n) {//【求点集P的最小覆盖圆】 O(n)  
// random\_shuffle(P,P+n);  
 Random(P, n);  
 circle C = circle(P[0], 0);  
 for (int i = 1; i < n; ++i)  
 if (!PIC(C, P[i])) {  
 C = circle(P[i], 0);  
 for (int j = 0; j < i; ++j)  
 if (!PIC(C, P[j])) {  
 C.o = (P[i] + P[j]) \* 0.5, C.r = length(P[j] - C.o);  
 for (int k = 0; k < j; ++k) if (!PIC(C, P[k])) C = getcircle(P[i], P[j], P[k]);  
 }  
 }  
 return C;  
}

# 高精度

## 高精度GCD

#include <bits/stdc++.h>  
using namespace std;  
string add(string a, string b) {  
 const int L = 1e5;  
 string ans;  
 int na[L] = {0}, nb[L] = {0};  
 int la = a.size(), lb = b.size();  
 for (int i = 0; i < la; i++) na[la - 1 - i] = a[i] - '0';  
 for (int i = 0; i < lb; i++) nb[lb - 1 - i] = b[i] - '0';  
 int lmax = la > lb ? la : lb;  
 for (int i = 0; i < lmax; i++)  
 na[i] += nb[i], na[i + 1] += na[i] / 10, na[i] %= 10;  
 if (na[lmax]) lmax++;  
 for (int i = lmax - 1; i >= 0; i--) ans += na[i] + '0';  
 return ans;  
}  
string mul(string a, string b) {  
 const int L = 1e5;  
 string s;  
 int na[L], nb[L], nc[L],  
 La = a.size(), Lb = b.size(); // na存储被乘数，nb存储乘数，nc存储积  
 fill(na, na + L, 0);  
 fill(nb, nb + L, 0);  
 fill(nc, nc + L, 0); //将na,nb,nc都置为0  
 for (int i = La - 1; i >= 0; i--)  
 na[La - i] =  
 a[i] - '0'; //将字符串表示的大整形数转成i整形数组表示的大整形数  
 for (int i = Lb - 1; i >= 0; i--) nb[Lb - i] = b[i] - '0';  
 for (int i = 1; i <= La; i++)  
 for (int j = 1; j <= Lb; j++)  
 nc[i + j - 1] +=  
 na[i] \*  
 nb[j]; // a的第i位乘以b的第j位为积的第i+j-1位（先不考虑进位）  
 for (int i = 1; i <= La + Lb; i++)  
 nc[i + 1] += nc[i] / 10, nc[i] %= 10; //统一处理进位  
 if (nc[La + Lb]) s += nc[La + Lb] + '0'; //判断第i+j位上的数字是不是0  
 for (int i = La + Lb - 1; i >= 1; i--)  
 s += nc[i] + '0'; //将整形数组转成字符串  
 return s;  
}  
int sub(int \*a, int \*b, int La, int Lb) {  
 if (La < Lb) return -1; //如果a小于b，则返回-1  
 if (La == Lb) {  
 for (int i = La - 1; i >= 0; i--)  
 if (a[i] > b[i])  
 break;  
 else if (a[i] < b[i])  
 return -1; //如果a小于b，则返回-1  
 }  
 for (int i = 0; i < La; i++) //高精度减法  
 {  
 a[i] -= b[i];  
 if (a[i] < 0) a[i] += 10, a[i + 1]--;  
 }  
 for (int i = La - 1; i >= 0; i--)  
 if (a[i]) return i + 1; //返回差的位数  
 return 0; //返回差的位数  
}  
string div(string n1, string n2,  
 int nn) // n1,n2是字符串表示的被除数，除数,nn是选择返回商还是余数  
{  
 const int L = 1e5;  
 string s, v; // s存商,v存余数  
 int a[L], b[L], r[L],  
 La = n1.size(), Lb = n2.size(), i,  
 tp = La; // a，b是整形数组表示被除数，除数，tp保存被除数的长度  
 fill(a, a + L, 0);  
 fill(b, b + L, 0);  
 fill(r, r + L, 0); //数组元素都置为0  
 for (i = La - 1; i >= 0; i--) a[La - 1 - i] = n1[i] - '0';  
 for (i = Lb - 1; i >= 0; i--) b[Lb - 1 - i] = n2[i] - '0';  
 if (La < Lb || (La == Lb && n1 < n2)) {  
 // cout<<0<<endl;  
 return n1;  
 } //如果a<b,则商为0，余数为被除数  
 int t = La - Lb; //除被数和除数的位数之差  
 for (int i = La - 1; i >= 0; i--) //将除数扩大10^t倍  
 if (i >= t)  
 b[i] = b[i - t];  
 else  
 b[i] = 0;  
 Lb = La;  
 for (int j = 0; j <= t; j++) {  
 int temp;  
 while ((temp = sub(a, b + j, La, Lb - j)) >=  
 0) //如果被除数比除数大继续减  
 {  
 La = temp;  
 r[t - j]++;  
 }  
 }  
 for (i = 0; i < L - 10; i++)  
 r[i + 1] += r[i] / 10, r[i] %= 10; //统一处理进位  
 while (!r[i]) i--; //将整形数组表示的商转化成字符串表示的  
 while (i >= 0) s += r[i--] + '0';  
 // cout<<s<<endl;  
 i = tp;  
 while (!a[i]) i--; //将整形数组表示的余数转化成字符串表示的</span>  
 while (i >= 0) v += a[i--] + '0';  
 if (v.empty()) v = "0";  
 // cout<<v<<endl;  
 if (nn == 1) return s;  
 if (nn == 2) return v;  
}  
bool judge(string s) //判断s是否为全0串  
{  
 for (int i = 0; i < s.size(); i++)  
 if (s[i] != '0') return false;  
 return true;  
}  
string gcd(string a, string b) //求最大公约数  
{  
 string t;  
 while (!judge(b)) //如果余数不为0，继续除  
 {  
 t = a; //保存被除数的值  
 a = b; //用除数替换被除数  
 b = div(t, b, 2); //用余数替换除数  
 }  
 return a;  
}  
  
//o(无法估计)

## 高精度乘法（FFT）

#include <bits/stdc++.h>  
using namespace std;  
#define L(x) (1 << (x))  
const double PI = acos(-1.0);  
const int Maxn = 133015;  
double ax[Maxn], ay[Maxn], bx[Maxn], by[Maxn];  
char sa[Maxn / 2], sb[Maxn / 2];  
int sum[Maxn];  
int x1[Maxn], x2[Maxn];  
int revv(int x, int bits) {  
 int ret = 0;  
 for (int i = 0; i < bits; i++) {  
 ret <<= 1;  
 ret |= x & 1;  
 x >>= 1;  
 }  
 return ret;  
}  
void fft(double\* a, double\* b, int n, bool rev) {  
 int bits = 0;  
 while (1 << bits < n) ++bits;  
 for (int i = 0; i < n; i++) {  
 int j = revv(i, bits);  
 if (i < j) swap(a[i], a[j]), swap(b[i], b[j]);  
 }  
 for (int len = 2; len <= n; len <<= 1) {  
 int half = len >> 1;  
 double wmx = cos(2 \* PI / len), wmy = sin(2 \* PI / len);  
 if (rev) wmy = -wmy;  
 for (int i = 0; i < n; i += len) {  
 double wx = 1, wy = 0;  
 for (int j = 0; j < half; j++) {  
 double cx = a[i + j], cy = b[i + j];  
 double dx = a[i + j + half], dy = b[i + j + half];  
 double ex = dx \* wx - dy \* wy, ey = dx \* wy + dy \* wx;  
 a[i + j] = cx + ex, b[i + j] = cy + ey;  
 a[i + j + half] = cx - ex, b[i + j + half] = cy - ey;  
 double wnx = wx \* wmx - wy \* wmy, wny = wx \* wmy + wy \* wmx;  
 wx = wnx, wy = wny;  
 }  
 }  
 }  
 if (rev) {  
 for (int i = 0; i < n; i++) a[i] /= n, b[i] /= n;  
 }  
}  
int solve(int a[], int na, int b[], int nb, int ans[]) {  
 int len = max(na, nb), ln;  
 for (ln = 0; L(ln) < len; ++ln)  
 ;  
 len = L(++ln);  
 for (int i = 0; i < len; ++i) {  
 if (i >= na)  
 ax[i] = 0, ay[i] = 0;  
 else  
 ax[i] = a[i], ay[i] = 0;  
 }  
 fft(ax, ay, len, 0);  
 for (int i = 0; i < len; ++i) {  
 if (i >= nb)  
 bx[i] = 0, by[i] = 0;  
 else  
 bx[i] = b[i], by[i] = 0;  
 }  
 fft(bx, by, len, 0);  
 for (int i = 0; i < len; ++i) {  
 double cx = ax[i] \* bx[i] - ay[i] \* by[i];  
 double cy = ax[i] \* by[i] + ay[i] \* bx[i];  
 ax[i] = cx, ay[i] = cy;  
 }  
 fft(ax, ay, len, 1);  
 for (int i = 0; i < len; ++i) ans[i] = (int)(ax[i] + 0.5);  
 return len;  
}  
string mul(string sa, string sb) {  
 int l1, l2, l;  
 int i;  
 string ans;  
 memset(sum, 0, sizeof(sum));  
 l1 = sa.size();  
 l2 = sb.size();  
 for (i = 0; i < l1; i++) x1[i] = sa[l1 - i - 1] - '0';  
 for (i = 0; i < l2; i++) x2[i] = sb[l2 - i - 1] - '0';  
 l = solve(x1, l1, x2, l2, sum);  
 for (i = 0; i < l || sum[i] >= 10; i++) // 进位  
 {  
 sum[i + 1] += sum[i] / 10;  
 sum[i] %= 10;  
 }  
 l = i;  
 while (sum[l] <= 0 && l > 0) l--; // 检索最高位  
 for (i = l; i >= 0; i--) ans += sum[i] + '0'; // 倒序输出  
 return ans;  
}  
int main() {  
 cin.sync\_with\_stdio(false);  
 string a, b;  
 while (cin >> a >> b) cout << mul(a, b) << endl;  
 return 0;  
}  
  
//o(nlogn)

## 高精度乘法（乘单精）

#include <bits/stdc++.h>  
using namespace std;  
string mul(string a, int b) //高精度a乘单精度b  
{  
 const int L = 100005;  
 int na[L];  
 string ans;  
 int La = a.size();  
 fill(na, na + L, 0);  
 for (int i = La - 1; i >= 0; i--) na[La - i - 1] = a[i] - '0';  
 int w = 0;  
 for (int i = 0; i < La; i++)  
 na[i] = na[i] \* b + w, w = na[i] / 10, na[i] = na[i] % 10;  
 while (w) na[La++] = w % 10, w /= 10;  
 La--;  
 while (La >= 0) ans += na[La--] + '0';  
 return ans;  
}  
  
//o(n)

## 高精度乘法（朴素）

#include <bits/stdc++.h>  
using namespace std;  
string mul(string a, string b) //高精度乘法a,b,均为非负整数  
{  
 const int L = 1e5;  
 string s;  
 int na[L], nb[L], nc[L],  
 La = a.size(), Lb = b.size(); // na存储被乘数，nb存储乘数，nc存储积  
 fill(na, na + L, 0);  
 fill(nb, nb + L, 0);  
 fill(nc, nc + L, 0); //将na,nb,nc都置为0  
 for (int i = La - 1; i >= 0; i--)  
 na[La - i] =  
 a[i] - '0'; //将字符串表示的大整形数转成i整形数组表示的大整形数  
 for (int i = Lb - 1; i >= 0; i--) nb[Lb - i] = b[i] - '0';  
 for (int i = 1; i <= La; i++)  
 for (int j = 1; j <= Lb; j++)  
 nc[i + j - 1] +=  
 na[i] \*  
 nb[j]; // a的第i位乘以b的第j位为积的第i+j-1位（先不考虑进位）  
 for (int i = 1; i <= La + Lb; i++)  
 nc[i + 1] += nc[i] / 10, nc[i] %= 10; //统一处理进位  
 if (nc[La + Lb]) s += nc[La + Lb] + '0'; //判断第i+j位上的数字是不是0  
 for (int i = La + Lb - 1; i >= 1; i--)  
 s += nc[i] + '0'; //将整形数组转成字符串  
 return s;  
}  
  
//o(n^2)

## 高精度减法

#include <bits/stdc++.h>  
using namespace std;  
string sub(string a, string b) //只限大的非负整数减小的非负整数  
{  
 const int L = 1e5;  
 string ans;  
 int na[L] = {0}, nb[L] = {0};  
 int la = a.size(), lb = b.size();  
 for (int i = 0; i < la; i++) na[la - 1 - i] = a[i] - '0';  
 for (int i = 0; i < lb; i++) nb[lb - 1 - i] = b[i] - '0';  
 int lmax = la > lb ? la : lb;  
 for (int i = 0; i < lmax; i++) {  
 na[i] -= nb[i];  
 if (na[i] < 0) na[i] += 10, na[i + 1]--;  
 }  
 while (!na[--lmax] && lmax > 0)  
 ;  
 lmax++;  
 for (int i = lmax - 1; i >= 0; i--) ans += na[i] + '0';  
 return ans;  
}  
  
//o(n)

## 高精度加法

#include <bits/stdc++.h>  
using namespace std;  
string add(string a, string b) //只限两个非负整数相加  
{  
 const int L = 1e5;  
 string ans;  
 int na[L] = {0}, nb[L] = {0};  
 int la = a.size(), lb = b.size();  
 for (int i = 0; i < la; i++) na[la - 1 - i] = a[i] - '0';  
 for (int i = 0; i < lb; i++) nb[lb - 1 - i] = b[i] - '0';  
 int lmax = la > lb ? la : lb;  
 for (int i = 0; i < lmax; i++)  
 na[i] += nb[i], na[i + 1] += na[i] / 10, na[i] %= 10;  
 if (na[lmax]) lmax++;  
 for (int i = lmax - 1; i >= 0; i--) ans += na[i] + '0';  
 return ans;  
}  
  
//o(n)

## 高精度取模（对单精）

#include <bits/stdc++.h>  
using namespace std;  
int mod(string a,int b)//高精度a除以单精度b  
{  
 int d=0;  
 for(int i=0;i<a.size();i++) d=(d\*10+(a[i]-'0'))%b;//求出余数  
 return d;  
}  
  
//o(n)

## 高精度幂

#include <bits/stdc++.h>  
#define L(x) (1 << (x))  
using namespace std;  
const double PI = acos(-1.0);  
const int Maxn = 133015;  
double ax[Maxn], ay[Maxn], bx[Maxn], by[Maxn];  
char sa[Maxn / 2], sb[Maxn / 2];  
int sum[Maxn];  
int x1[Maxn], x2[Maxn];  
int revv(int x, int bits) {  
 int ret = 0;  
 for (int i = 0; i < bits; i++) {  
 ret <<= 1;  
 ret |= x & 1;  
 x >>= 1;  
 }  
 return ret;  
}  
void fft(double\* a, double\* b, int n, bool rev) {  
 int bits = 0;  
 while (1 << bits < n) ++bits;  
 for (int i = 0; i < n; i++) {  
 int j = revv(i, bits);  
 if (i < j) swap(a[i], a[j]), swap(b[i], b[j]);  
 }  
 for (int len = 2; len <= n; len <<= 1) {  
 int half = len >> 1;  
 double wmx = cos(2 \* PI / len), wmy = sin(2 \* PI / len);  
 if (rev) wmy = -wmy;  
 for (int i = 0; i < n; i += len) {  
 double wx = 1, wy = 0;  
 for (int j = 0; j < half; j++) {  
 double cx = a[i + j], cy = b[i + j];  
 double dx = a[i + j + half], dy = b[i + j + half];  
 double ex = dx \* wx - dy \* wy, ey = dx \* wy + dy \* wx;  
 a[i + j] = cx + ex, b[i + j] = cy + ey;  
 a[i + j + half] = cx - ex, b[i + j + half] = cy - ey;  
 double wnx = wx \* wmx - wy \* wmy, wny = wx \* wmy + wy \* wmx;  
 wx = wnx, wy = wny;  
 }  
 }  
 }  
 if (rev) {  
 for (int i = 0; i < n; i++) a[i] /= n, b[i] /= n;  
 }  
}  
int solve(int a[], int na, int b[], int nb, int ans[]) {  
 int len = max(na, nb), ln;  
 for (ln = 0; L(ln) < len; ++ln)  
 ;  
 len = L(++ln);  
 for (int i = 0; i < len; ++i) {  
 if (i >= na)  
 ax[i] = 0, ay[i] = 0;  
 else  
 ax[i] = a[i], ay[i] = 0;  
 }  
 fft(ax, ay, len, 0);  
 for (int i = 0; i < len; ++i) {  
 if (i >= nb)  
 bx[i] = 0, by[i] = 0;  
 else  
 bx[i] = b[i], by[i] = 0;  
 }  
 fft(bx, by, len, 0);  
 for (int i = 0; i < len; ++i) {  
 double cx = ax[i] \* bx[i] - ay[i] \* by[i];  
 double cy = ax[i] \* by[i] + ay[i] \* bx[i];  
 ax[i] = cx, ay[i] = cy;  
 }  
 fft(ax, ay, len, 1);  
 for (int i = 0; i < len; ++i) ans[i] = (int)(ax[i] + 0.5);  
 return len;  
}  
string mul(string sa, string sb) {  
 int l1, l2, l;  
 int i;  
 string ans;  
 memset(sum, 0, sizeof(sum));  
 l1 = sa.size();  
 l2 = sb.size();  
 for (i = 0; i < l1; i++) x1[i] = sa[l1 - i - 1] - '0';  
 for (i = 0; i < l2; i++) x2[i] = sb[l2 - i - 1] - '0';  
 l = solve(x1, l1, x2, l2, sum);  
 for (i = 0; i < l || sum[i] >= 10; i++) // 进位  
 {  
 sum[i + 1] += sum[i] / 10;  
 sum[i] %= 10;  
 }  
 l = i;  
 while (sum[l] <= 0 && l > 0) l--; // 检索最高位  
 for (i = l; i >= 0; i--) ans += sum[i] + '0'; // 倒序输出  
 return ans;  
}  
string Pow(string a, int n) {  
 if (n == 1) return a;  
 if (n & 1) return mul(Pow(a, n - 1), a);  
 string ans = Pow(a, n / 2);  
 return mul(ans, ans);  
}  
  
//o(nlognlogm)

## 高精度平方根

#include <bits/stdc++.h>  
using namespace std;  
const int L = 2015;  
string add(string a, string b) //只限两个非负整数相加  
{  
 string ans;  
 int na[L] = {0}, nb[L] = {0};  
 int la = a.size(), lb = b.size();  
 for (int i = 0; i < la; i++) na[la - 1 - i] = a[i] - '0';  
 for (int i = 0; i < lb; i++) nb[lb - 1 - i] = b[i] - '0';  
 int lmax = la > lb ? la : lb;  
 for (int i = 0; i < lmax; i++)  
 na[i] += nb[i], na[i + 1] += na[i] / 10, na[i] %= 10;  
 if (na[lmax]) lmax++;  
 for (int i = lmax - 1; i >= 0; i--) ans += na[i] + '0';  
 return ans;  
}  
string sub(string a, string b) //只限大的非负整数减小的非负整数  
{  
 string ans;  
 int na[L] = {0}, nb[L] = {0};  
 int la = a.size(), lb = b.size();  
 for (int i = 0; i < la; i++) na[la - 1 - i] = a[i] - '0';  
 for (int i = 0; i < lb; i++) nb[lb - 1 - i] = b[i] - '0';  
 int lmax = la > lb ? la : lb;  
 for (int i = 0; i < lmax; i++) {  
 na[i] -= nb[i];  
 if (na[i] < 0) na[i] += 10, na[i + 1]--;  
 }  
 while (!na[--lmax] && lmax > 0)  
 ;  
 lmax++;  
 for (int i = lmax - 1; i >= 0; i--) ans += na[i] + '0';  
 return ans;  
}  
string mul(string a, string b) //高精度乘法a,b,均为非负整数  
{  
 string s;  
 int na[L], nb[L], nc[L],  
 La = a.size(), Lb = b.size(); // na存储被乘数，nb存储乘数，nc存储积  
 fill(na, na + L, 0);  
 fill(nb, nb + L, 0);  
 fill(nc, nc + L, 0); //将na,nb,nc都置为0  
 for (int i = La - 1; i >= 0; i--)  
 na[La - i] =  
 a[i] - '0'; //将字符串表示的大整形数转成i整形数组表示的大整形数  
 for (int i = Lb - 1; i >= 0; i--) nb[Lb - i] = b[i] - '0';  
 for (int i = 1; i <= La; i++)  
 for (int j = 1; j <= Lb; j++)  
 nc[i + j - 1] +=  
 na[i] \*  
 nb[j]; // a的第i位乘以b的第j位为积的第i+j-1位（先不考虑进位）  
 for (int i = 1; i <= La + Lb; i++)  
 nc[i + 1] += nc[i] / 10, nc[i] %= 10; //统一处理进位  
 if (nc[La + Lb]) s += nc[La + Lb] + '0'; //判断第i+j位上的数字是不是0  
 for (int i = La + Lb - 1; i >= 1; i--)  
 s += nc[i] + '0'; //将整形数组转成字符串  
 return s;  
}  
int sub(int \*a, int \*b, int La, int Lb) {  
 if (La < Lb) return -1; //如果a小于b，则返回-1  
 if (La == Lb) {  
 for (int i = La - 1; i >= 0; i--)  
 if (a[i] > b[i])  
 break;  
 else if (a[i] < b[i])  
 return -1; //如果a小于b，则返回-1  
 }  
 for (int i = 0; i < La; i++) //高精度减法  
 {  
 a[i] -= b[i];  
 if (a[i] < 0) a[i] += 10, a[i + 1]--;  
 }  
 for (int i = La - 1; i >= 0; i--)  
 if (a[i]) return i + 1; //返回差的位数  
 return 0; //返回差的位数  
}  
string div(string n1, string n2,  
 int nn) // n1,n2是字符串表示的被除数，除数,nn是选择返回商还是余数  
{  
 string s, v; // s存商,v存余数  
 int a[L], b[L], r[L],  
 La = n1.size(), Lb = n2.size(), i,  
 tp = La; // a，b是整形数组表示被除数，除数，tp保存被除数的长度  
 fill(a, a + L, 0);  
 fill(b, b + L, 0);  
 fill(r, r + L, 0); //数组元素都置为0  
 for (i = La - 1; i >= 0; i--) a[La - 1 - i] = n1[i] - '0';  
 for (i = Lb - 1; i >= 0; i--) b[Lb - 1 - i] = n2[i] - '0';  
 if (La < Lb || (La == Lb && n1 < n2)) {  
 // cout<<0<<endl;  
 return n1;  
 } //如果a<b,则商为0，余数为被除数  
 int t = La - Lb; //除被数和除数的位数之差  
 for (int i = La - 1; i >= 0; i--) //将除数扩大10^t倍  
 if (i >= t)  
 b[i] = b[i - t];  
 else  
 b[i] = 0;  
 Lb = La;  
 for (int j = 0; j <= t; j++) {  
 int temp;  
 while ((temp = sub(a, b + j, La, Lb - j)) >=  
 0) //如果被除数比除数大继续减  
 {  
 La = temp;  
 r[t - j]++;  
 }  
 }  
 for (i = 0; i < L - 10; i++)  
 r[i + 1] += r[i] / 10, r[i] %= 10; //统一处理进位  
 while (!r[i]) i--; //将整形数组表示的商转化成字符串表示的  
 while (i >= 0) s += r[i--] + '0';  
 // cout<<s<<endl;  
 i = tp;  
 while (!a[i]) i--; //将整形数组表示的余数转化成字符串表示的</span>  
 while (i >= 0) v += a[i--] + '0';  
 if (v.empty()) v = "0";  
 // cout<<v<<endl;  
 if (nn == 1) return s;  
 if (nn == 2) return v;  
}  
bool cmp(string a, string b) {  
 if (a.size() < b.size()) return 1; // a小于等于b返回真  
 if (a.size() == b.size() && a <= b) return 1;  
 return 0;  
}  
string DeletePreZero(string s) {  
 int i;  
 for (i = 0; i < s.size(); i++)  
 if (s[i] != '0') break;  
 return s.substr(i);  
}  
  
string BigInterSqrt(string n) {  
 n = DeletePreZero(n);  
 string l = "1", r = n, mid, ans;  
 while (cmp(l, r)) {  
 mid = div(add(l, r), "2", 1);  
 if (cmp(mul(mid, mid), n))  
 ans = mid, l = add(mid, "1");  
 else  
 r = sub(mid, "1");  
 }  
 return ans;  
}  
  
// o(n^3)

## 高精度进制转换

#include <bits/stdc++.h>  
using namespace std;  
//将字符串表示的10进制大整数转换为m进制的大整数  
//并返回m进制大整数的字符串  
bool judge(string s) //判断串是否为全零串  
{  
 for (int i = 0; i < s.size(); i++)  
 if (s[i] != '0') return 1;  
 return 0;  
}  
string solve(  
 string s, int n,  
 int m) // n进制转m进制只限0-9进制，若涉及带字母的进制，稍作修改即可  
{  
 string r, ans;  
 int d = 0;  
 if (!judge(s)) return "0"; //特判  
 while (judge(s)) //被除数不为0则继续  
 {  
 for (int i = 0; i < s.size(); i++) {  
 r += (d \* n + s[i] - '0') / m + '0'; //求出商  
 d = (d \* n + (s[i] - '0')) % m; //求出余数  
 }  
 s = r; //把商赋给下一次的被除数  
 r = ""; //把商清空  
 ans += d + '0'; //加上进制转换后数字  
 d = 0; //清空余数  
 }  
 reverse(ans.begin(), ans.end()); //倒置下  
 return ans;  
}  
  
//o(n^2)

## 高精度阶乘

#include <bits/stdc++.h>  
using namespace std;  
string fac(int n) {  
 const int L = 100005;  
 int a[L];  
 string ans;  
 if (n == 0) return "1";  
 fill(a, a + L, 0);  
 int s = 0, m = n;  
 while (m) a[++s] = m % 10, m /= 10;  
 for (int i = n - 1; i >= 2; i--) {  
 int w = 0;  
 for (int j = 1; j <= s; j++)  
 a[j] = a[j] \* i + w, w = a[j] / 10, a[j] = a[j] % 10;  
 while (w) a[++s] = w % 10, w /= 10;  
 }  
 while (!a[s]) s--;  
 while (s >= 1) ans += a[s--] + '0';  
 return ans;  
}  
  
//o(n^2)

## 高精度除法（除单精）

#include <bits/stdc++.h>  
using namespace std;  
string div(string a, int b) //高精度a除以单精度b  
{  
 string r, ans;  
 int d = 0;  
 if (a == "0") return a; //特判  
 for (int i = 0; i < a.size(); i++) {  
 r += (d \* 10 + a[i] - '0') / b + '0'; //求出商  
 d = (d \* 10 + (a[i] - '0')) % b; //求出余数  
 }  
 int p = 0;  
 for (int i = 0; i < r.size(); i++)  
 if (r[i] != '0') {  
 p = i;  
 break;  
 }  
 return r.substr(p);  
}  
  
//o(n)

## 高精度除法（除高精）

#include <bits/stdc++.h>  
using namespace std;  
int sub(int \*a, int \*b, int La, int Lb) {  
 if (La < Lb) return -1; //如果a小于b，则返回-1  
 if (La == Lb) {  
 for (int i = La - 1; i >= 0; i--)  
 if (a[i] > b[i])  
 break;  
 else if (a[i] < b[i])  
 return -1; //如果a小于b，则返回-1  
 }  
 for (int i = 0; i < La; i++) //高精度减法  
 {  
 a[i] -= b[i];  
 if (a[i] < 0) a[i] += 10, a[i + 1]--;  
 }  
 for (int i = La - 1; i >= 0; i--)  
 if (a[i]) return i + 1; //返回差的位数  
 return 0; //返回差的位数  
}  
string div(string n1, string n2, int nn)  
// n1,n2是字符串表示的被除数，除数,nn是选择返回商还是余数  
{  
 const int L = 1e5;  
 string s, v; // s存商,v存余数  
 int a[L], b[L], r[L], La = n1.size(), Lb = n2.size(), i, tp = La;  
 // a，b是整形数组表示被除数，除数，tp保存被除数的长度  
 fill(a, a + L, 0);  
 fill(b, b + L, 0);  
 fill(r, r + L, 0); //数组元素都置为0  
 for (i = La - 1; i >= 0; i--) a[La - 1 - i] = n1[i] - '0';  
 for (i = Lb - 1; i >= 0; i--) b[Lb - 1 - i] = n2[i] - '0';  
 if (La < Lb || (La == Lb && n1 < n2)) {  
 // cout<<0<<endl;  
 return n1;  
 } //如果a<b,则商为0，余数为被除数  
 int t = La - Lb; //除被数和除数的位数之差  
 for (int i = La - 1; i >= 0; i--) //将除数扩大10^t倍  
 if (i >= t)  
 b[i] = b[i - t];  
 else  
 b[i] = 0;  
 Lb = La;  
 for (int j = 0; j <= t; j++) {  
 int temp;  
 while ((temp = sub(a, b + j, La, Lb - j)) >=  
 0) //如果被除数比除数大继续减  
 {  
 La = temp;  
 r[t - j]++;  
 }  
 }  
 for (i = 0; i < L - 10; i++)  
 r[i + 1] += r[i] / 10, r[i] %= 10; //统一处理进位  
 while (!r[i]) i--; //将整形数组表示的商转化成字符串表示的  
 while (i >= 0) s += r[i--] + '0';  
 // cout<<s<<endl;  
 i = tp;  
 while (!a[i]) i--; //将整形数组表示的余数转化成字符串表示的</span>  
 while (i >= 0) v += a[i--] + '0';  
 if (v.empty()) v = "0";  
 // cout<<v<<endl;  
 if (nn == 1) return s; //返回商  
 if (nn == 2) return v; //返回余数  
}  
  
//o(n^2)

## 龟速乘快速幂（快速幂爆longlong

#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
  
ll qmul(ll a, ll b, ll p) {  
 ll res = 0;  
 while(b) {  
 if(b & 1) res = (res + a) % p;  
 a = (a + a) % p;  
 b >>= 1;   
 }  
 return res;  
}  
  
ll qpow(ll x, ll n, ll p) {  
 ll res = 1;  
 while(n) {  
 if(n & 1) res = qmul(res, x, p);  
 x = qmul(x, x, p);  
 n >>= 1;  
 }  
 return res % p; // 1 0 1  
}  
  
int main() {  
 ll b, p, k;  
 cin >> b >> p >> k;  
 ll ans = qpow(b, p, k);  
 printf("%lld^%lld mod %lld=%lld", b, p, k, ans);  
   
 return 0;  
}