differential_flatness

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[1]:
```python
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import sympy as sym
from sympy.abc import t

#Functions of t
th = sym.Function(r'\theta')(t)
thd = sym.Function(r'\theta_d')(t)
v = sym.Function(r'v')(t)
x = sym.Function(r'x')(t)
y = sym.Function(r'y')(t)
omega = sym.Function(r'\omega')(t)
xd = sym.Function(r'x_d')(t)
yd = sym.Function(r'y_d')(t)
vd = sym.Function(r'v_d')(t)
wd = sym.Function(r'\omega_d')(t)

#symbols
H,W,T = sym.symbols('H W T')

#Take derivates
xdot = x.diff(t)
ydot = y.diff(t)
xddot = xdot.diff(t)
yddot = ydot.diff(t)
thdot = th.diff(t)

#solve for v and w
#component velocity equations
xdoteq = sym.Eq(xdot,v*sym.cos(th))
ydoteq = sym.Eq(ydot,v*sym.sin(th))
#solve for velocity
vsol = sym.solve(xdoteq,[v])
#sub in vsol for v
ydoteq_sub = sym.simplify(ydoteq.subs({v:vsol[0]}))
#solve for th in terms of xdot and ydot
thsol = sym.solve(ydoteq_sub,[th])
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#calculate omega
th_dot = sym.simplify(thsol[0].diff(t))
#velocity equation
v_sol = sym.Eq(v,sym.sqrt(xdot**2+ydot**2))

#Display v and w
display(sym.Eq(omega,th_dot))
display(v_sol)
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$$\omega(t) = \frac{\frac{d}{dt}x(t)\frac{d^2}{dt^2}y(t) - \frac{d^2}{dt^2}x(t)\frac{d}{dt}y(t)}{\left(\frac{d}{dt}x(t)\right)^2 + \left(\frac{d}{dt}y(t)\right)^2}$$

$$v(t) = \sqrt{\left(\frac{d}{dt}x(t)\right)^2 + \left(\frac{d}{dt}y(t)\right)^2}$$

[9]:
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#Trajectory Equations
x_d = W/2*sym.sin((2*sym.pi*t)/T)
y_d = H/2*sym.sin((4*sym.pi*t)/T)

#Differentiate Trajectories
x_d_dot = x_d.diff(t)
y_d_dot = y_d.diff(t)
x_d_ddot = x_d_dot.diff(t)
y_d_ddot = y_d_dot.diff(t)

#Plug trajectories into v
v_traj = sym.simplify(sym.sqrt(xdot**2+ydot**2).subs({xdot:x_d_dot,ydot:
↪y_d_dot}))
v_d = sym.Eq(vd,v_traj)

#Plug trajectories into w
w_traj = sym.simplify(th_dot.subs({xdot:x_d_dot,ydot:y_d_dot,xddot:
↪x_d_ddot,yddot:y_d_ddot}))
w_d = sym.Eq(wd,w_traj)

#Find theta desired
th_traj = sym.simplify(thsol[0].subs({ydot:y_d_dot,xdot:x_d_dot}))
th_d = sym.Eq(thd,th_traj)

#display important equations
display(sym.Eq(xd,x_d))
display(sym.Eq(yd,y_d))
display(sym.Eq(xd.diff(t),x_d_dot))
display(sym.Eq(yd.diff(t),y_d_dot))
display(sym.Eq(xd.diff(t).diff(t),x_d_ddot))
display(sym.Eq(yd.diff(t).diff(t),y_d_ddot))
display(v_d)
```

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display(w_d)
display(th_d)
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$$\mathrm{x_d}(t) = \frac{W \sin\left(\frac{2\pi t}{T}\right)}{2}$$

$$\mathrm{y_d}(t) = \frac{H \sin\left(\frac{4\pi t}{T}\right)}{2}$$

$$\frac{d}{dt} \mathrm{x_d}(t) = \frac{\pi W \cos\left(\frac{2\pi t}{T}\right)}{T}$$

$$\frac{d}{dt} \mathrm{y_d}(t) = \frac{2\pi H \cos\left(\frac{4\pi t}{T}\right)}{T}$$

$$\frac{d^2}{dt^2} \mathrm{x_d}(t) = -\frac{2\pi^2 W \sin\left(\frac{2\pi t}{T}\right)}{T^2}$$

$$\frac{d^2}{dt^2} \mathrm{y_d}(t) = -\frac{8\pi^2 H \sin\left(\frac{4\pi t}{T}\right)}{T^2}$$

$$\mathrm{v_d}(t) = \pi \sqrt{\frac{4H^2 \cos^2\left(\frac{4\pi t}{T}\right) + W^2 \cos^2\left(\frac{2\pi t}{T}\right)}{T^2}}$$

$$\omega_d(t) = \frac{4\pi H W \left(\sin\left(\frac{2\pi t}{T}\right) \cos\left(\frac{4\pi t}{T}\right) - 2 \sin\left(\frac{4\pi t}{T}\right) \cos\left(\frac{2\pi t}{T}\right)\right)}{T \left(4H^2 \cos^2\left(\frac{4\pi t}{T}\right) + W^2 \cos^2\left(\frac{2\pi t}{T}\right)\right)}$$

$$\theta_d(t) = \operatorname{atan}\left(\frac{2H \cos\left(\frac{4\pi t}{T}\right)}{W \cos\left(\frac{2\pi t}{T}\right)}\right)$$