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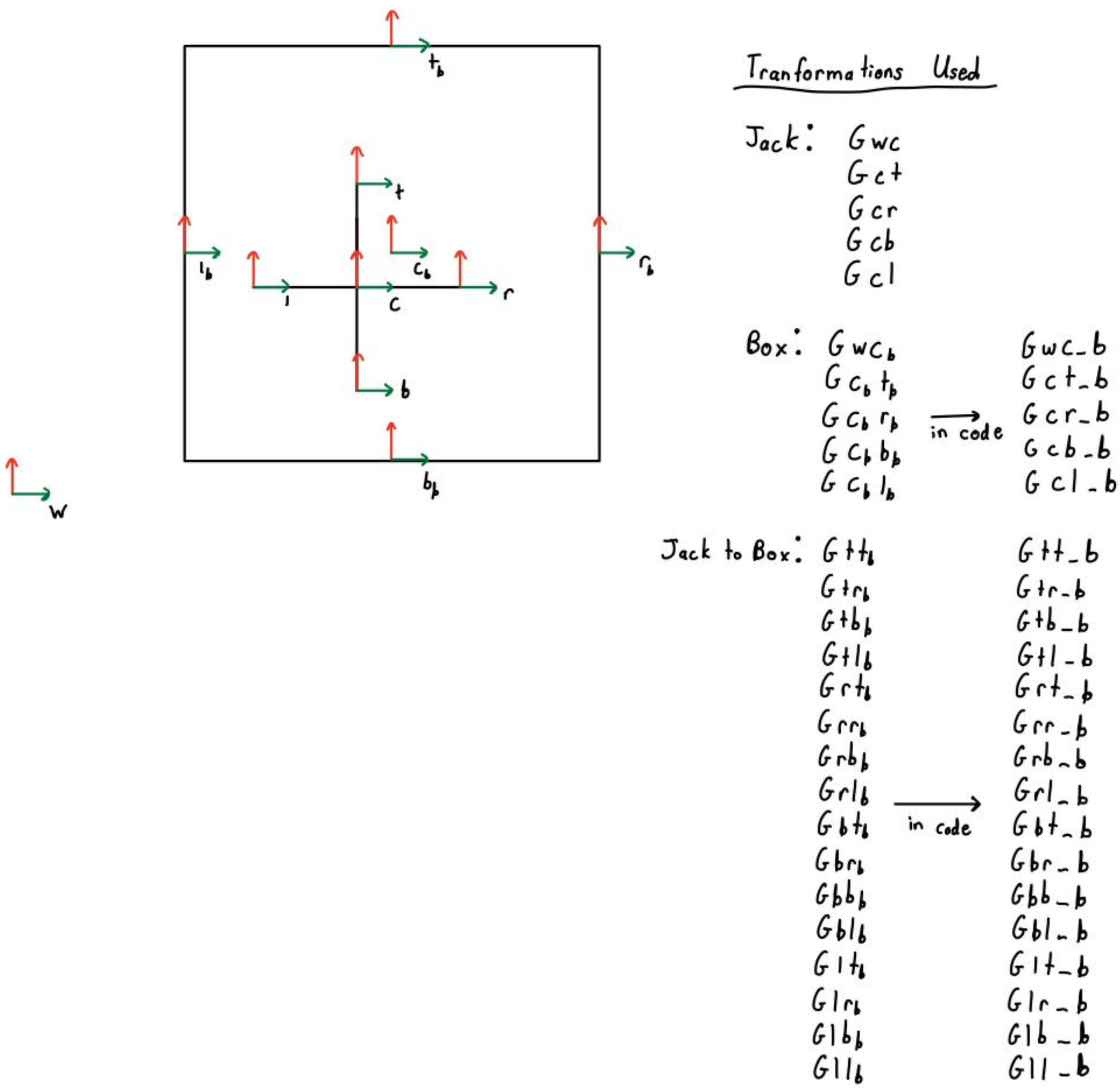
ME 314
Final Project: Jack and Box

Project Description

I pursued the default final project.

Transformations Diagram:



Calculations:

Euler-Lagrange Equations - To calculate the EL equations, I started with potential and kinetic energy. I used the equation $KE = \frac{1}{2}V_b^T * I * V_b$ where V_b is the body velocity and I is the combined mass and inertia matrix. I used the transformations Gwc and Gwc_b in the V_b

equations. I set the potential energy to 0 under the assumption that gravity is acting in the z direction (into the screen). By subtracting the potential energy from the kinetic energy, I calculated the Lagrangian. I then differentiated the Lagrangian to find $\frac{\delta L}{\delta q}$ and $\frac{d}{dt}(\frac{\delta L}{\delta(\frac{dq}{dt})})$. Finally, I formed the EL equations using the following formula, $\frac{d}{dt}(\frac{\delta L}{\delta(\frac{dq}{dt})}) - \frac{\delta L}{\delta q} = F$ where F is a matrix of external forces. The external forces can be set however the user desires.

Constraint Equations - There are 16 constraints because each side of the jack can hit any of the four walls of the box. I calculated the constraints by taking the inverse of the "Jack to Box" transforms listed in the included diagram, and then I set the x or y coordinate of the transformation matrix equal to 0. For the transformations between the jack and the left and right walls of the box, I set the x coordinate equal to zero. For the transformations between the jack and the top and bottom walls of the box, I set the y coordinate equal to zero.

Impact Update Laws - To calculate the impact update, I had to solve for the time derivative of the configuration variables after impact. The configuration variables before and after impact are equivalent. I made two substitutions dictionaries to do this (one with configuration variables before impact and one with configuration variables after impact). I used the substitution dictionaries in the impact update equations which are $\frac{\delta L}{\delta(\frac{dq}{dt})}^{+} - \frac{\delta L}{\delta(\frac{dq}{dt})}^{-} = \lambda\frac{\delta\phi}{\delta q}$ and $H^{+} - H^{-} = 0$ where H is the Hamiltonian and ϕ is the impact constraint. Each constraint requires a different impact update equation. I used the following equation to calculate the Hamiltonian $H = \frac{\delta L}{\delta(\frac{dq}{dt})} * (\frac{dq}{dt}) - L$, Because there are too many variables to solve these equations symbolically in python, I solved the equations when an impact is detected during simulation. When an impact is detected, I substitute current values of the configuration variables and their time derivative into my impact update equations reducing the computational intensity of solving the equations. During each step of the simulation, I check to see if any of the impact constraints have been met. If the code detects impact, the impact equations are solved. If the code detects no impact, the EL equations are solved.

Testing:
I heavily tested my code to ensure that it is functioning properly. Since I modeled gravity in the z direction, it was easy to see how changing the initial condition affected my simulation. I tested each initial condition individually by setting all other initial conditions and external forces to 0 and animating my solution to determine whether or not my simulation is realistic. I then started to combine various initial conditions for testing. After testing initial conditions, I tested external forces. I used the same process to test external forces as I used to test initial conditions. Once I was confident the external forces were accurate, I started to combine external forces and initial conditions. I used the animation to check whether or not the simulation is realistic.
The screencast I submitted uses an initial box angular velocity of 1rad/s, and applies external forces to the box. The external forces are $Fx = 300sin(t)$ and $Fy = 300cos(t)$. If you neglect energy transfer between the jack and the box, these initial conditions and forces would cause

the box to rotate about its center and follow a repeating semi-circle trajectory moving in the positive x direction. My animation shows the box performing as predicted with slight variations due to energy transfer between the box and the jack. The trajectory of the jack within the box looks realistic given the initial conditions and constants. The mass of the jack is 4kg (a 1kg point mass at each end of the jack) and the mass of each wall of the box is 25kg.