

Module: 6SENG001W Reasoning about Programs
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Tutorial Exercises: 5
Subject: Evaluate Relation expressions using **Atelier B** & **ProB**
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1 Introduction

Using the B tools load & type check the following B relation definitions B machine called `Relations` into **Atelier B** & then animate/evaluate the expressions given below using **ProB**'s "*Eval terminal*" or add them to the `Relations` Machine directly as `ASSERTIONS` & use **ProB**'s "*Eval Assertions terminal*" to check if they are true or false.

2 Evaluate the following expressions

Load the `Relations.mch` machine into **Atelier B** & then **ProB** & evaluate the following expressions.

2.1 Value Expressions

1. $AAxXX$
2. *favourite*
3. *speaks*
4. *alphabet*
5. `card(favourite)`
6. `card(AAxXX)`
7. `card({ (1,2), (3,4) })`

8. $\text{card}(\textit{speaks})$
9. $\text{card}(\textit{alphabet})$

2.2 Predicate Expressions

1. $(\textit{Paul}, \textit{purple}) \in \textit{favourite}$
2. $(\textit{Paul}, \textit{pink}) \in \textit{favourite}$
3. $(\textit{Paul}, \textit{blue}) \notin \textit{favourite}$
4. $(\textit{Wales}, \textit{French}) \in \textit{speaks}$
5. $(\textit{Canda}, \textit{Welsh}) \notin \textit{speaks}$

2.3 Cartesian Products

1. $XX \times AA$
2. $AA \times AA$
3. $XX \times XX$
4. $AA \times \textit{COLOUR}$
5. $\text{prj}_1(\textit{COUNTRY}, \textit{LANGUAGE})(\textit{Wales}, \textit{Welsh})$
6. $\text{prj}_2(\textit{COUNTRY}, \textit{LANGUAGE})(\textit{Wales}, \textit{Welsh})$

2.4 Relational Domain & Range

1. $\text{dom}(\textit{favourite})$
2. $\text{ran}(\textit{favourite})$
3. $\text{dom}(\textit{speaks})$
4. $\text{ran}(\textit{speaks})$
5. $\text{dom}(AAxXX)$
6. $\text{ran}(AAxXX)$

7. $\text{dom}(R1)$
8. $\text{ran}(R1)$
9. $\text{dom}(R2)$
10. $\text{ran}(R2)$
11. $\text{dom}(R3)$
12. $\text{ran}(R3)$
13. $\text{dom}(RR)$
14. $\text{ran}(RR)$
15. $\text{dom}(QQ)$
16. $\text{ran}(QQ)$
17. $\text{dom}(RR) \cap \text{dom}(QQ)$

2.5 Relational Image Operator

1. $\text{favourite} [\{ \text{Paul}, \text{Sue} \}]$
2. $\text{speaks} [\{ \text{Canada}, \text{Wales} \}]$
3. $R1 [\{ aa, bb, cc \}]$
4. $R2 [\{ cc, dd \}]$
5. $\text{alphabet} [\{ aa, bb, cc \}]$
6. $QQ [\text{dom}(RR)]$

2.6 Relational Restriction Operators

1. $\{ \text{Jim}, \text{Ian} \} \triangleleft \text{favourite}$
2. $\text{favourite} \triangleright \{ \text{blue}, \text{red} \}$
3. $\{ \text{Wales}, \text{Scotland}, \text{England}, \text{NIreland} \} \triangleleft \text{speaks}$
4. $\text{speaks} \triangleright \{ \text{French} \}$

5. $\{ Wales, Scotland, England, NIreland \} \Leftarrow speaks$
6. $speaks \triangleright \{ English \}$
7. $\{ 2, 3 \} \triangleleft RR$
8. $RR \triangleright \{ 3, 5 \}$
9. $\{ 2, 3 \} \Leftarrow RR$
10. $RR \triangleright \{ 3, 5 \}$

2.7 Relational Composition

Using the *alphabet* relation calculate the following compositions:

1. $alphabet ; alphabet$
2. $alphabet ; alphabet$
3. $alphabet ; alphabet ; alphabet$
4. $alphabet^5$
5. $alphabet^9$
6. $alphabet^{10}$
7. $RR ; QQ$
8. $QQ ; RR$
9. $RR ; RR$
10. $QQ ; QQ$

2.8 Relational Overriding

Try evaluating the following “override” expressions using both the \Leftarrow operator & using its definitions:

$$RR \Leftarrow QQ = (\text{dom}(QQ) \Leftarrow RR) \cup QQ$$

2.8.1 Overriding Expression

1. $R1 \Leftarrow \{ aa \mapsto 10 \}$
2. $R1 \Leftarrow \{ bb \mapsto 9 \}$
3. $R2 \Leftarrow \{ dd \mapsto 2, dd \mapsto 10 \}$
4. $R2 \Leftarrow \{ aa \mapsto 9, bb \mapsto 10 \}$
5. $R3 \Leftarrow \{ gg \mapsto 9, hh \mapsto 6, hh \mapsto 10, zz \mapsto 99 \}$

2.8.2 Using Overriding

Using the two relations R & Q , work out the new relation given by R overriding Q ($Q \Leftarrow R$) then compare this with $R \Leftarrow Q$ given in the lecture notes.

$$RR \Leftarrow QQ = \{ (0, 1), (1, 2), (2, 3), (3, 3), (4, 5), (4, 6), (5, 5), (6, 7) \}$$

Finally, compare these two with the relation you get by just unioning the two relations: $RR \cup QQ$.

Show how the following people's choice of their *favourite colour(s)* can be modifying, i.e. the *favourite* relation is modified using the overriding operator \Leftarrow .

1. *Paul's favourite colour is now blue.*
2. *Sue's favourite colours are now pink & purple.*
3. *Ian's favourite colours are now green & yellow.*

3 Analyse the Hotel Rooms B Specification

Download the `HotelRooms.mch B` specification used in Relations Lecture.

Type check it using **Atelier B**.

Animate it in **ProB** & execute a sample of the operations so that several rooms have guests.

Then use **ProB's Eval terminal** to evaluate the relation expressions used in the specification. For example:

1. In **INITIALISATION**, first:

```
ROOM
then
ROOM * { empty }
```

2. In operation **guestsCheckIn**:

```
{ rm2 } * { Ian, Sue, Tom }

First:
guests
then
guests <+ { rm2 } * { Ian, Sue, Tom }
```

3. In operation **guestsCheckOut**:

```
guests <+ { rm2 } -> empty }
```

4. In operation **roomOccupants**:

```
ran( { rm1 } <| guests )
guests[ { rm1 } ]
guests[ { rm2 } ]
guests[ { rm3 } ]
guests[ { rm4 } ]
guests[ { rm5 } ]
```

5. In operation **hasGuestCheckedIn**:

```
dom(guests)
ran(guests)
```

6. In operation **guestsSwapRoom** pick two rooms that have guests in them.

Assuming that **rm1** & **rm3** have guests then:

```
guests[ { rm1 } ]
guests[ { rm3 } ]
{ rm1 } * guests[ { rm3 } ]
{ rm3 } * guests[ { rm1 } ]
{ rm1 } * guests[ { rm3 } ] \ / { rm3 } * guests[ { rm1 } ]
guests
guests <+ ( { rm1 } * guests[ { rm3 } ] \ / { rm3 } * guests[ { rm1 } ]
```