Feedback Control Systems

COMPLEX ENGINEERING PROBLEMS

PHASE - I

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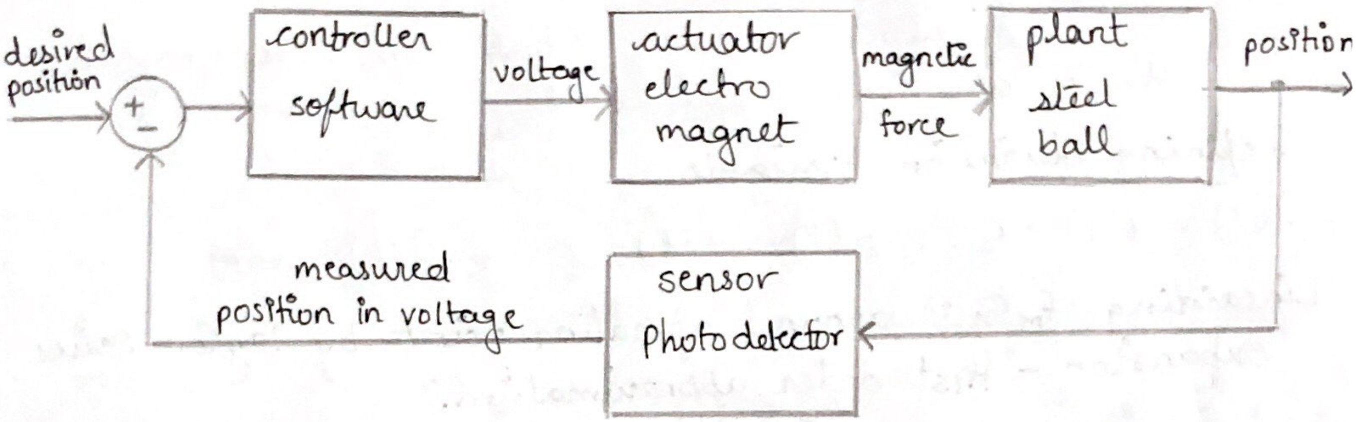
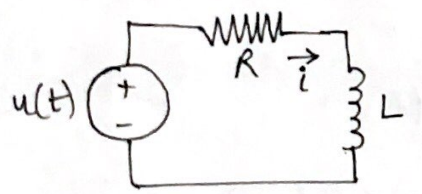
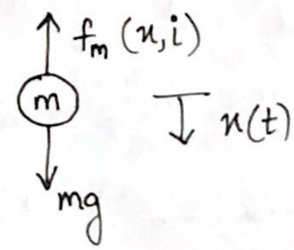
BLOCK DIAGRAM



MODELING (NONLINEAR & LINEAR)





$$u(t) = Ri + L\frac{di}{dt}$$

$$m\frac{d^2x}{dt^2} = mg - f_m$$

Deriving $f_m(x, i)$

$$L(x) = L + \frac{L_0 x_0}{x}$$

$$\delta W = f_m(x, i)\,\partial x$$

$$f_m(x, i) = \frac{\partial W}{\partial x} = \frac{1}{2} i^2 \frac{\partial(L(x))}{\partial x}$$

$$= \frac{1}{2} i^2 \frac{(-L_0 x_0)}{x^2}$$

$$K_c = \frac{-L_0 x_0}{2}$$

$$f_m(x, i) = \frac{K_c i^2}{x^2}$$

Linearizing around operating points

$\frac{di}{dt} = 0$, $\frac{d^2x}{dt^2} = 0$ at equilibrium

$$0 = mg - \frac{K_c i_o^2}{x_o^2} \quad ; \quad u_o - R i_o = 0$$

$$i_o = \sqrt{\frac{mg x_o^2}{K_c}} \qquad u_o = R\sqrt{\frac{mg x_o^2}{K_c}}$$

where i_o is operating current
x_o is operating distance of ball from electromagnet.
u_o is operating input voltage.

Defining deviation variable

$$\tilde{i}(t) = i(t) - i_o \quad , \quad \tilde{x}(t) = x(t) - x_o \quad , \quad \tilde{u}(t) = u(t) - u_o$$

Linearizing $f_m(x,i)$ around operating point by Taylor series expansion - first order approximation.

$$f_m(x, i) \approx f_m(x_o, i_o) + \frac{\partial f(x_o, i_o)}{\partial x}\tilde{x} + \frac{\partial f(x_o, i_o)}{\partial i}\tilde{i}$$

$$\approx \frac{K_c i_o^2}{x_o^2} + \left(-\frac{2K_c i_o^2}{x_o^3}\right)\tilde{x}(t) + \frac{2K_c i_o}{x_o^2}\tilde{i}(t)$$

$$\Rightarrow \frac{m d^2\tilde{x}}{dt^2} = \cancel{mg - \frac{K_c i_o^2}{x_o^2}} + K_x \tilde{x}(t) + K_i \tilde{i}(t)$$

$$K_x = +\frac{2K_c}{x_o^3}\frac{(mg x_o^2)}{K_c} = +\frac{2mg}{x_o} \quad ; \quad K_i = -\frac{2K_c i_o}{i_o^2 K_c}(mg) = -\frac{2mg}{i_o}$$

$$\frac{d^2\tilde{x}}{dt^2} - \frac{K_x \tilde{x}(t)}{m} = \frac{K_i}{m}\tilde{i}(t)$$

$$s^2\tilde{X}(s) - \frac{K_x}{m}\tilde{X}(s) = \frac{K_i}{m}\tilde{I}(s)$$

$$\frac{\tilde{X}(s)}{\tilde{I}(s)} = \frac{K_i/m}{s^2 - K_x/m}$$

Electrical differential equation is already linear
Simply writing in terms of deviation variables

$$\frac{\tilde{u}(t)}{L} - \frac{R\tilde{i}(t)}{L} = \frac{d\tilde{i}(t)}{dt}$$

$$\frac{\tilde{U}(s)}{L} - \frac{R}{L}\tilde{I}(s) = s\tilde{I}(s)$$

$$\frac{\tilde{I}(s)}{\tilde{U}(s)} = \frac{L}{s + R/L}$$

OPEN LOOP TRANSFER FUNCTION

$$\frac{\tilde{X}(s)}{\tilde{U}(s)} = \frac{\tilde{X}(s)}{\tilde{I}(s)} \cdot \frac{\tilde{I}(s)}{\tilde{U}(s)}$$

$$= \frac{K_i/m}{s^2 - K_x/m} \cdot \frac{1/L}{s + R/L}$$

$$= \frac{K_i 1/mL}{s^3 + \frac{R}{L}s^2 + \frac{K_x}{m}s - \frac{K_x R}{Lm}}$$

Design Parameters

$R = 3.5\Omega$, $m = 0.0084\,kg$, $K_c = 68.823\times10^{-6}\,\frac{Nm^2}{A}$, $L = 0.01H$, $g = 9.81\,m/s^2$

$x_0 = 0.01\,m$

$i_0 = \sqrt{\frac{mgx_0^2}{K_c}} = 0.346A$, $V_0 = Ri_0 = 1.211\,V$

$K_x = +\frac{2mg}{x_0} = +16.48$ $\quad K_i = \frac{2mg}{i_0} = -0.476$

$$\frac{\tilde{X}(s)}{\tilde{U}(s)} = \frac{-0.476\left(\frac{1}{0.01}/0.0084\right)}{s^3 + \frac{3.5}{0.01}s^2 \mp \frac{(+16.48)}{0.0084} - \frac{(+16.48)(3.5)}{(0.01)(0.0084)}}$$

$$= \frac{-5670}{s^3 + 350s^2 - 1962s - 686667}$$