Normal Distribution Comparison with Different Standard Deviations

Introduction

The normal distribution is a key statistical concept used to describe how data points are spread around a mean. The spread is determined by the standard deviation (σ). Smaller σ indicates tighter clustering around the mean, while larger σ indicates wider dispersion.

Parameters Given

We are analysing exam scores with:

- Mean (μ) = 55
- $\sigma_{4a} = 4$
- $\sigma_{4\beta} = 10$
- $\sigma_{4c} = 15$

These represent three different scenarios of score distributions.

Case A ($\sigma = 4$)

- Narrow distribution
- Most scores tightly clustered around 55
- Less variability among students

Implication: Predictable performance, but may not capture diversity in ability.

Case B ($\sigma = 10$)

- Moderate spread
- Scores range more widely around the mean
- Balanced variability

Implication: Reflects realistic variation in student performance.

Case C (σ = 15)

- Wide distribution
- Scores spread far from the mean
- High variability

Implication: Greater diversity, but less predictability.

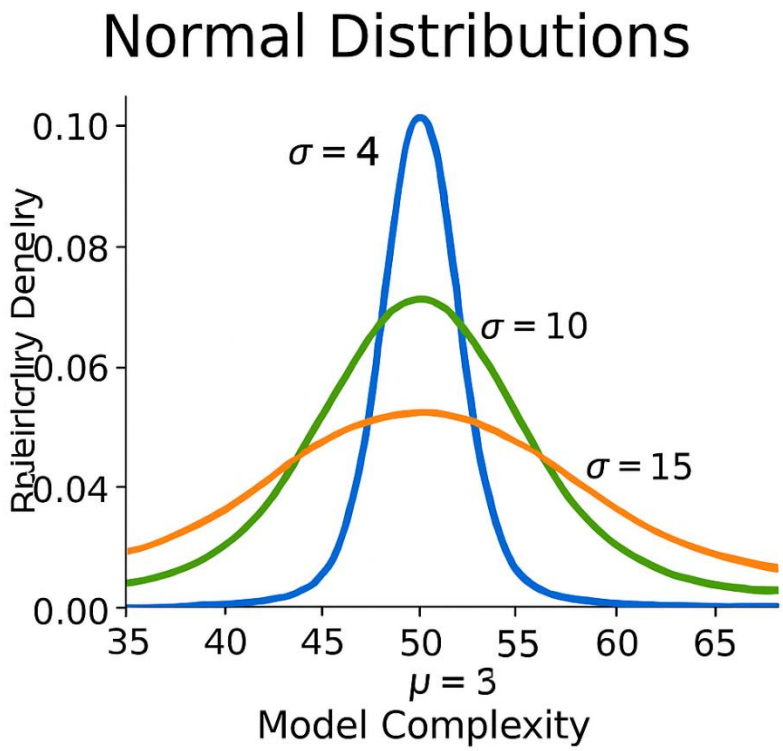
Graphical Representation

Graph: Three normal distribution curves with same mean (55) but different spreads:

- σ = 4 (narrow, tall curve)

- σ = 10 (moderate curve)

- σ = 15 (wide, flat curve)



Empirical Rule Application

For each case:

- σ = 4 → 68% between 51 and 59

- σ = 10 → 68% between 45 and 65

- $\sigma = 15 \rightarrow$ 68% between 40 and 70

Comparison of Cases

- Case A: Too narrow, may underestimate variability
- Case B: Balanced, realistic spread
- Case C: Too wide, may exaggerate variability

Which is Better?

Case B ($\sigma = 10$) is generally better:

- Captures natural variation in scores
- Provides realistic distribution
- Avoids extremes of too narrow or too wide spreads

Conclusion

Among the three scenarios, $\sigma = 10$ offers the most balanced representation of exam scores. It reflects diversity without losing predictability, making it the most suitable choice for modelling student performance using the normal distribution.