Homework #3

#1 Three-phase Definitions : $V_{LL} = 230$ kV

a) $V_{LN} = \frac{1}{\sqrt{3}} V_{LL} = \left(\frac{1}{\sqrt{3}}\right)(230 \text{ kV}) = 132.7906 \approx 132.8$ kV

b) $V_{LL,max} = \sqrt{2}\, V_{LL} = 325.27$ kV

c) $V_{LN,max} = \frac{\sqrt{2}}{\sqrt{3}} V_{LL} = \sqrt{2}\, V_{LN} = 187.7942 \text{ kV} \approx 187.8$ kV

#2 WYE & DELTA connections

$Z_y = 20\angle 30^\circ$, $V_{AN} = 100\angle 0^\circ$, $V_{BN} = 100\angle -120$, $V_{CN} = 100\angle 120^\circ$

This is a balanced three-phase Y- connected system with positive-sequence sources.

a) $I_A = \frac{V_{AN}}{Z_y} = \frac{100\angle 0^\circ}{20\angle 30^\circ} = 5\angle -30^\circ$

$I_B = \frac{V_{BN}}{Z_y} = \frac{100\angle -120^\circ}{20\angle 30^\circ} = 5\angle -150^\circ$

$I_C = \frac{V_{CN}}{Z_y} = \frac{100\angle 120^\circ}{20\angle 30^\circ} = 5\angle 90^\circ$

Line currents are also balanced, since they have equal magnitudes of 5 A & 120° displacement between any two phases

b) $S_A = V_{AN} I_A^* = (100)(5)\angle 0-(-30^\circ) = 500\angle 30^\circ$

$S_Y = S_A + S_B + S_C = 3 S_A$ since we have balanced conditions the complex powers delivered by b & c are identical to a.

$S_y = 3(500\angle 30^\circ)$

$\boxed{S_Y = 1500\angle 30^\circ = 1500(\cos(30^\circ)) + j\,1500(\sin(30^\circ)) = 1299 + j\,750 \text{ VA}}$

c) $I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{\sqrt{3}\, V_{an}\angle 30^\circ}{Z_\Delta} = \frac{(\sqrt{3})(100)\angle 30^\circ}{20\angle 30^\circ} = 8.66\angle 0^\circ$ A

$I_{BC} = \frac{V_{BC}}{Z_\Delta} = \frac{\sqrt{3}\, V_{bn}\angle -120^\circ+30^\circ}{Z_\Delta} = \frac{(\sqrt{3})(100)\angle -90^\circ}{20\angle 30^\circ} = 8.66\angle -120^\circ$ A

$I_{CA} = \frac{V_{CA}}{Z_\Delta} = \frac{\sqrt{3}\, V_{cn}\angle 120^\circ+30^\circ}{Z_\Delta} = \frac{(\sqrt{3})(100)\angle 150^\circ}{20\angle 30^\circ} = 8.66\angle 120^\circ$ A

d) $S_{AB} = V_{AB} I_{AB}^* = \left[(\sqrt{3})(100)\angle 30^\circ\right]\left[8.66\angle 0^\circ\right] = 1499.956\angle 30^\circ$ VA

$S_\Delta = S_{AB} + S_{BC} + S_{CA} = 3 S_{AB} = 3(1499.956\angle 30^\circ) = 4499.868\angle 30^\circ$ VA

$S_\Delta = 3897 + j\,2249.934$ VA

e) $S_{AB} = 1500\angle 30° = 3V_{AB}I_{AB}^{*} = \dfrac{3(V_{AB})(V_{AB}^{*})}{Z_\Delta^{*}}$

$Z_\Delta^{*} = \dfrac{[(\sqrt{3})(100)/30°][\sqrt{3}\cdot 100\angle -30°]\cdot 3}{1500\angle 30°} = 3\cdot 20\angle -30° = 60\angle -30°$

$Z_\Delta = 60\angle 30°$

#3 p.f. = .707 lagging

Power drawn = 240 kW

$V_{LL} = 440$ V

a) $|S| = \dfrac{P}{p.f.} = \dfrac{240\text{ kW}}{.707} = 339.4625$ kVA

$|I| = \dfrac{|S|}{\sqrt{3}|V|} = \dfrac{(339.4625\text{ kVA})}{(\sqrt{3})(440\text{ V})} = 445.4290$ A $= .445$ kA per phase

b) $S_L = P + jQ = 240 + j(|S|\sin(\cos^{-1}(.707))) = 240 + j240$

$S_C = -j60$

$S_L + S_C = 240 + j240 - j60 = 240 + j180 = |S_{tot}|\angle\theta = 300\angle 36.86°$ kW

$|S_{tot}| = \sqrt{(240)^2 + (180)^2} = 300$

$\theta = \tan^{-1}\left(\dfrac{180}{240}\right) = 36.86°$

p.f. $= \cos(36.86°) = .8$ lagging

c) We don't have to know whether it's a delta or wye connection because we have the power it consumes in total. Impedance would have to change.

#4 a) Three-phase motor draws 20 kVA at .707 p.f. lagging from a 220 V source

$$\cos\theta = \frac{P}{|S_m|}$$

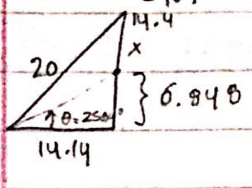
$P = |S_m| \cos(\theta) = (20\text{ kVA})(.707) = 14.14\text{ kW}$

$Q = |S_m| \sin\theta = (20\text{ kVA}) \sin(\cos^{-1}(.707)) = 14.14\text{ kVAR}$

$S_m = 14.14 + j14.14$

$S_{tot} = S_m + S_c = 14.14 + j14.14 - jX$ $\qquad \cos\theta = .90$ lagging

$S_{tot} = 14.14 + j(14.14 - X)$ $\qquad \theta = \cos^{-1}(.90) = 25.84°$



$$\theta = \tan^{-1}\left(\frac{14.14 - X}{14.14}\right)$$

$(14.14)(\tan\theta) = 14.14 - X$

$14.14\tan\theta - 14.14 = -X$

$-7.29 = 6.848 - 14.14 = -X$

$X = 7.29\text{ kVAR}$

b) Before Capacitors are added:

$$|I| = \frac{20\text{ kVA}}{\sqrt{3}\,(220\text{ V})} = 52.4\text{ A}$$

After Capacitors are added:

$S_{tot} = 14.14 + j6.848$

$|S_{tot}| = \sqrt{(14.14)^2 + (6.848)^2} = 15.71\text{ kVA}$

$$|I| = \frac{15.71\text{ kVA}}{\sqrt{3}\,(220\text{ V})} = 41.23\text{ A}$$

5. Power Transfer Problem

$Z = -j5\,\Omega = 5\angle{-90°}$

a) $I = \dfrac{E_1 - E_2}{Z} = \dfrac{(100\cos(0) + j(100)\sin(0)) - (100\cos(30) + j100\sin(30))}{5\angle{-90}}$

$I = \dfrac{100 - 86.6 - j50}{5\angle{-90}} = \dfrac{13.4 - j50}{5\angle{-90}} = \dfrac{51.76\angle{-75}}{5\angle{-90}} = 10.35\angle{15°}$

$S_1 = E_1(-I^*) = -(100)(10.35\angle{-15°}) = -1035\angle{-15°} = -1000 + j267.877$

$S_2 = E_2 I^* = (100)(10.35)\angle{30-15} = 1035\angle{15°} = 1000 + j267.877$

Machine 1 consumes 1000 W, Machine 2 generates 1000 W

b) Machine 1 & 2 are both generating 268 VARs (see above for calculations)

c) $\sum_{\text{generator convention}} = \sum_{\text{Load convention}}$

$Q_1 + Q_2 = -Q_Z$

$268 + 268 = -Q_Z$

$Q_Z = -536$ var

Load absorbs 536 VARs.

d) $I_{New} = \dfrac{51.76\angle{-75°}}{11.18\angle{63.43°}} = 4.629\angle{-138.4°}$

$S_1 = -E_1(I^*_{New}) = (100)(-4.63\angle{+138.4}) = -463\angle{138.4°} = 346.04 - j307.4$

$S_2 = E_2(I^*_{New}) = (100)(4.63\angle{168.4°}) = 463\angle{168.4} = -453.54 + j93.09$

E_1 generates real power while E_2 absorbs it.

e) Machine 1 absorbs 307.4 Vars & machine 2 generates 93.09 vars.

f) $Q_1 + Q_2 = -307.4 + 93.09 = -214.3$ vars ; impedance absorbs 214.3 vars

g) Yes, the imaginary component of the impedance is positive so we know that there is an inductor present & it will consume VARs.

h) Reactive power is <u>not</u> paid for by consumers, so claiming that reactive power is being generated is misleading.