HW5

#1 $P = I^2 R$

$65\ kW = (100\ A)^2 R$

$R = 6.5\ \Omega$

$\rho = 1.72 \times 10^{-8}\ \Omega \cdot m$

$L = 70\ km \left(\frac{1000\ m}{1\ km}\right) = 70{,}000\ m$

$A = \frac{\rho L}{R}$

$\frac{\pi D^2}{4} = \frac{\rho L}{R}$

$D^2 = \frac{4 \rho L}{\pi R}$

$D = \left[\frac{4 \rho L}{\pi R}\right]^{1/2} = \left[\frac{4 (1.72 \times 10^{-8}\ \Omega \cdot m)(70{,}000\ m)}{\pi\ (6.5\ \Omega)}\right]^{1/2}$

$\boxed{D = \left[2.358431 \times 10^{-4}\ m^2\right]^{1/2} = 1.5\bar{3}5 \times 10^{-2}\ m \approx 1.54 \times 10^{-2}\ m}$

#2 GMR = .0217 ft.

30 ft - spaced

$L_1 = 2 \times 10^{-7} \ln\left[\frac{D_{eq}}{GMR}\right] = (2 \times 10^{-7}) \ln\left[\frac{30\ ft}{.0217\ ft}\right] \frac{H}{m}\left(1609\ \frac{m}{mi}\right)$

$L_1 = 2.3\bar{2}7142 \times 10^{-3}\ \frac{H}{mi} \approx 2.33 \times 10^{-3}\ H/mi$ (per conductor)

$L = 2 \times L_1 = 2\left(2.33 \times 10^{-3}\ H/mi\right) = 4.66 \times 10^{-3}\ \frac{H}{mi}$ (per circuit)

$X_L = 2\pi f L = 2\pi (60\ Hz)\left(4.66 \times 10^{-3}\ \frac{H}{mi}\right) = 1.7\bar{5}6779 \approx 1.76\ \frac{\Omega}{mi}$ (per circuit)

#3 a) $GMR = \sqrt{r' 2r} = \sqrt{e^{-1/4} \cdot r^2 \cdot 2} = 1.2\bar{4}8039 r \approx 1.25 r$

b) A Geometric mean radius of bundled conductors is the effective conductor size.

c) GMR increases when the distance between the conductors in the bundle increases. As GMR increases, inductance decreases. $GMR = \sqrt{.7788 dr}$.

d) Stranded $\rightarrow \frac{R}{X_L} = \frac{R}{\omega L_s}$

Bundled $\rightarrow \frac{R}{X_L} = \frac{R}{\omega L_B}$

$\frac{R}{\omega L_s} > \frac{R}{\omega L_B}$ because L_s has a smaller GMR than L_B given that the distance between the stranded conductors is less than the difference between the bundled conductors.

#4 $D_{AB} = 6\ m$ Distance between conductors $= 30\ cm \left(\frac{1\ m}{100\ cm}\right) = .3\ m$

$D_{BC} = 6\ m$ radius $= .74\ cm \left(\frac{1 m}{100\ cm}\right) = .0074 m$

$D_{AC} = 12\ m$

$D_{eq} = \sqrt[3]{(6)(6)(12)} = 7.56\ m$

$D_{SL} = \sqrt{e^{-1/4} \cdot r \cdot (.3 m)} = \sqrt{e^{-1/4}(.0074)(.3)} = .0416\ m$

$L = (2\times10^{-7}) \ln\left(\frac{7.56 m}{.0416\ m}\right) = 1.0405 \times 10^{-6}\ H/m \left(\frac{1000\ m}{1\ km}\right) = 1.04\ mH/km$ per phase

$X_L = 2\pi f L = 2\pi(60\ Hz)[1.04\ mH/km] = .392\ \frac{\Omega}{km} \left(\frac{1.6\ km}{1\ mi}\right) = .627\ \frac{\Omega}{mi}$

#5 line length: 25 km

$Z_1 = .19 + j.34 (\Omega/km)$ per phase series impedance

load absorbs 10 MVA @ 33KV

a) ABCD parameters

$V_s = AV_r + BI_R$
$I_s = CV_r + DI_R$ $\rightarrow$ $\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_R \end{bmatrix}$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & .19+j.34 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

b) $V_s = V_r + I_r\left[.389\angle 60.8 \frac{\Omega}{km}(25\ KM)\right] = V_r + I_r(9.73\angle 60.8°\ \Omega)$

$P = |S_1| \cos\theta = 10\,(MVA)(.9) = 9\ MW$ $\theta = \cos^{-1}(.9) = 25.84°$

$P = V_{rms} I_{rm} \cos\theta$

$I_{rms} = \frac{P}{V_{rms}\cos\theta} = \frac{(9\ MW)}{(33KV)(.9)} = 303.03$ Amps

$V_s = 33\angle 0°\ KV + (303.03\angle -25.84°\ A)(9.73\ \angle 60.8°\ \Omega)$

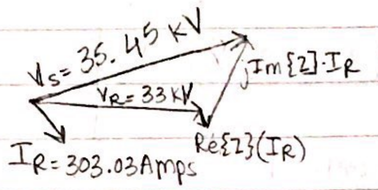
$V_s = 33\angle 0°\ KV + 2.948\angle 34.96°\ KV = 33 + 2.416 + j1.689 = 35.416 + j1.689\ KV$

$\boxed{V_s = 35.45\ \angle 2.73\ \ KV}$

c) From B $\rightarrow$ $P = 9\,MW$ $\quad I_{rms} = 303.03$ Amps $\quad V_{rms} = 33\,kV$

$$V_s = 33\angle 0°\,kV + (303.03\angle 25.84°\,A)(9.73\angle 60.8°\,\Omega)$$
$$V_s = 33\angle 0°\,kV + 2.948\angle 86.64°\,kV$$
$$V_s = 33 + .1727 + j2.9429\,kV$$
$$V_s = 33.1727 + j2.9429\,kV$$
$$V_s = 33.3\angle 5.07°\,kV$$

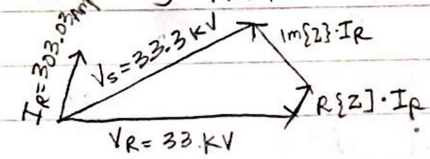
d) Lagging P.F. Load



The lagging load P.F. results in a lower voltage drop for the receiving end.

$$Z = 9.73\cos(60.8) + j\,9.73\sin(60.8) = 4.74 + j8.49$$

Leading P.F. Load



The leading load P.F. results in a higher voltage drop for the receiving end.

#6 500 km, 500 kV, 60 Hz $\quad Z = .03 + j.35\ \Omega/km \quad$ shunt admittance = $j4.4\times10^{-6}$ S/km

a) $$Z_c = \sqrt{\frac{Z}{y}} = \sqrt{\frac{(.35128\angle 85.10°)}{(4.4\times10^{-6}\angle 90°)}} = \sqrt{7.983\times10^{4}\angle -4.9°} = 282.5\angle -2.45°$$

b) $$\gamma l = \sqrt{Zy} = \sqrt{(.35128\angle 85.10°)(4.4\times10^{-6}\angle 90°)}\ [500\,km]$$
$$\gamma l = \sqrt{1.545\times10^{-6}\angle 175.10°} \times 500 = .62149\angle 87.55° = 2.656\times10^{-2} + j\,6.209\times10^{-1}$$ per unit

c) $$e^{\gamma l} = e^{.02656}\,e^{+j.6209} = 1.0269\angle .6209 \text{ radians} = .8352 + j.5974$$
$$e^{-\gamma l} = e^{-.02656}\,e^{-j.6209} = .97378\angle -.6209 \text{ radians} = .7920 - j.5665$$
$$\cosh(\gamma l) = \frac{.8352 + j.5974 + .7920 - j.5665}{2} = \frac{1.6272 + j.0309}{2} = .8136 + j.01545$$
$$\cosh(\gamma l) = .8136 + j.01545 = .8137467\angle 1.087° \approx .8137\angle 1.087°$$
$$\sinh(\gamma l) = \frac{.8352 + j.5974 - .7920 + j.5665}{2} = \frac{.0432 + j1.164}{2} = .0216 + j.5819$$
$$\sinh(\gamma l) = .5823\angle 87.87°$$

$$\boxed{\begin{aligned} &A = D = \cosh(\gamma l) = .8137\angle 1.087^\circ \quad \text{per unit} \\ &B = Z_c \sinh(\gamma l) = (282.5\angle -2.45^\circ)(.5823\angle 87.87^\circ) \\ &B = 164.5\angle 85.42^\circ\ \Omega \\ &C = \frac{\sinh(\gamma l)}{Z_c} = \frac{.5823\angle 87.87^\circ}{282.5\angle -2.45^\circ} = .00206\angle 90.32^\circ\ S \end{aligned}}$$

d) Total series impedance: $Z = zl = (.03 + j.35\ \Omega/km)(500\ km)$

$$Z = 15 + j17.5\ \Omega \rightarrow 23.05\angle 49.39^\circ\ \Omega$$

$$Y = yl = \left(j4.4\times10^{-6}\ \frac{S}{km}\right)(500\ km) = j\,2.2\times10^{-3}\ S \rightarrow 2.2\times10^{-3}\angle 90^\circ\ S$$

$$A = D = 1 + \frac{YZ}{2} = 1 + \left[\frac{(23.05\angle 49.39^\circ\ \Omega)(2.2\times10^{-3}\angle 90^\circ)}{2}\right]$$

$$A = D = 1 + \left[\frac{.05071\angle 139.39^\circ}{2}\right] = 1 + .02535\angle 139.39^\circ$$

$$\boxed{A = D = 1 + (-.019248 + j.01650) = .98075 + j.01650 = .9808\angle .963^\circ \quad \text{per unit}}$$

$$\boxed{B = Z = 23.05\angle 49.39^\circ\ \Omega}$$

$$C = Y\left(1 + \frac{YZ}{4}\right) = (2.2\times10^{-3}\angle 90^\circ\ S)\left[1 + \frac{(2.2\times10^{-3}\angle 90^\circ)(23.05\angle 49.39^\circ)}{4}\right]$$

$$C = (2.2\times10^{-3}\angle 90^\circ\ S)\left[1 + 1.2677\times10^{-2}\angle 139.39\right]$$

$$C = (2.2\times10^{-3}\angle 90^\circ\ S)\left[1 + (-9.6234\times10^{-3} + j\,8.2515\times10^{-3})\right]$$

$$C = (2.2\times10^{-3}\angle 90^\circ S)\left[.99037 + j.0082515\right]$$

$$\boxed{C = (2.2\times10^{-3}\angle 90^\circ\ S)\left[.990411\angle .47736^\circ\right] = .00217\angle 90.47^\circ\ S}$$