11.1 a) $\omega_{syn} = 2\pi(f) = 2\pi(60\ Hz) = 377\ rad/s$

$\omega_{msyn} = \frac{2}{P}\omega_{syn} = \frac{2}{4}(377\ rad/s) = 188.5\ rad/s$

b) KE? → $KE = H S_{rated} = (5\ p.u.\cdot s)(500\ MVA) = 2.5 \times 10^9\ J$

c) $P_e(t) = 0\ MW$

$P_{a.p.u}(t) = \frac{P_m(t) - P_e(t)}{S_{rated}} = \frac{500\ MW - 0}{500\ MVA} = 1$

$\frac{2H}{\omega_{syn}}\omega_{p.u}(t)\,\alpha(t) = P_{a\,p.u}(t)$

$\alpha(t) = \frac{P_{ap.u}(t)}{\frac{2H}{\omega_{syn}}(\omega_{p.u}(t))} = \frac{1}{\frac{2(5\ p.u.\cdot s)}{377\ rad/s}(1\ rad/s)} = \frac{1}{.0265\ p.u.\cdot s} = 37.7\ rad/s$

11.2 $\frac{2H S_{rated}}{\omega_{msyn}^2} = J$

$\frac{2(5\ p.u.\cdot s)(500 \times 10^6\ VA)}{(188.5\ rad/s)^2} = \frac{J}{s^2} = 1.407 \times 10^5\ kg \cdot m^2$

11.4 $P_{mp.u.} = .7\ p.u.$

$\omega = \omega_{syn} = 2\pi f\ rad/s = 377\ rad/s$

$\delta = 12°\left(\frac{\pi}{180}\right) = .2094\ rad$

$\omega_{p.u.}(t) = 1$

$P_{ep.u.} = (.7)(.4) = .28$

$H = 5\ p.u.\cdot s$

Per Unit Swing equation:

$2H\frac{\omega_{p.u.}(t)}{\omega_{syn}}\frac{d^2\delta(t)}{dt^2} = P_{mp.u.}(t) - P_{ep.u.}(t)$

$\frac{(2)(5\ p.u.\cdot s)(1)}{377\ rad/s}\frac{d^2\delta(t)}{dt^2} = .7 - .28$

$.02652\ s^2\frac{d^2\delta(t)}{dt^2} = .420$

$\frac{d^2\delta(t)}{dt^2} = \frac{.420}{.02652\ s^2} = 15.834$

$\delta(0) = 12°$

$\frac{d\delta(0)}{dt} = 0$

$\frac{d\delta(t)}{dt} = 15.834t + 0$

$\delta(t) = 7.917t^2 + .2094\ rad$ → $\delta(.05\ sec\ [3\ cycles]) = 7.917(.05)^2 + .2094 = .22919\ rad$

$t = \frac{n}{f}\ sec = \frac{3\ cycles}{60\ Hz} = .05\ sec$

$.22919\ rad\left(\frac{180}{\pi}\right) = 13.13°$

11.6. Number of poles is 16. H is 1.5 p.u.

$P_e = .5$ p.u. $P_m = .5$ p.u.

a) $\omega_{syn} = 2\pi\, 60\ Hz = 377\ rad/s$

$\omega_{msyn} = \frac{2}{P}\omega_{syn} = \left(\frac{2}{16}\right)(377\ rad/s) = 47.12\ \frac{rad}{s}$

b) $\frac{2H}{\omega_{syn}}\,\omega_{p.u.}(t)\,\frac{d^2\delta(t)}{dt^2} = P_{mp.u.}(t) - P_{e.p.u}(t)$

$\frac{2(1.5\ p.u)}{377\ rad/s}\,\omega_{p.u.}(t)\,\frac{d^2\delta(t)}{dt^2} = P_{m\,p.u.}(t) - P_{e\,p.u.}(t)$

c) $\omega = \omega_{syn}$ $\delta(0) = 10^\circ\left(\frac{\pi}{180^\circ}\right) = .1745$ radians

$\frac{d\delta(0)}{dt} = 0$ For $t \geq 0 \rightarrow \left[\frac{2(1.5\ p.u.)}{377\ \frac{rad}{s}}\right]\frac{d^2\delta(t)}{dt^2} = .5$

$t = 3\ cycles = \frac{3\ cycles}{60\ cycles/sec} = .05\ s$

$\frac{d^2\delta(t)}{dt^2} = (.5)\left[\frac{1}{\left[\frac{2(1.5\ p.u.)}{377\ \frac{rad}{s}}\right]}\right]$

$\frac{d^2\delta(t)}{dt^2} = 62.833\ rad$

$\frac{d\delta(t)}{dt} = 62.833t + 0\ rad$

$\delta(t) = 31.41t^2 + .1745\ rad$

$\delta(.05) = 31.41(.05)^2 + .1745$

$\delta(.05) = .078525 + .1745 = .0253\ rad$

$.0253\ rad\left(\frac{180^\circ}{\pi\ rad}\right) = 14.49726^\circ \approx 14.5^\circ$

11.7 $W_{kinetic} = \frac{1}{2} M v^2$

Mass = 80,000 kg

Supply frequency: 60 Hz

Poles: 2

S_{rated}: 100 MVA

Inertia: 3.0 p.u.-s

$KE_{gen} = H S_{rated}$

$KE_{gen} = (3.0 \text{ p.u.-s})(100 \times 10^6 \text{ VA})$

$KE_{gen} = 300 \text{ MJ}$

$KE_{gen} = \frac{1}{2} M v^2$

$\frac{2 KE_{gen}}{M} = v^2$

$v = \sqrt{\frac{2(300 \text{ MJ})}{80{,}000 \text{ kg}}} = 86.6 \text{ m/s}$

$\sqrt{\frac{2 KE_{gen}}{M}} = v$

11.8 a) $X_{13} + X_{23} = .10 + .20 = .30$ j.o

$Y_{13+23} = \frac{1}{.30} = 3.33$

$Y_{12} = \frac{1}{.20} = 5$

$Y_{12} + Y_{13+23} = 5 + 3.33 = 8.33$

$X_{12+13+23} = \frac{1}{8.33} = .12$

$X_{TR} + X_{12+13+23} = .10 + .12 = .22 \text{ p.u.}$

$P_e = \frac{V_T V_{bus}}{X_{TR} + X_{12+13+23}} \sin\delta$

$.8 \text{ pu} = \frac{(1.05 \text{ pu})(1.0 \text{ pu})}{.22 \text{ p.u}} \sin\delta$

$\sin\delta = .168$

$\delta = \sin^{-1}(.168) = 9.649°$

$I = \frac{1.05\angle 9.649° - 1.0\angle 0°}{j.22} = \frac{.035 + j.176}{j.22} = .8 - j.16 = .8158\angle -11.31° \text{ p.u.}$

$S = V_T I^* = (1.05\angle 9.649°)(.8158\angle 11.31° \text{ p.u.}) = .8565\angle 20.95° = .7998 + j.3062 \text{ p.u.}$

$\boxed{Q = .3062 \text{ p.u.}}$

Turns ratio: $n_2/\_ = V_2/\_$ | Referring Impedance

b) Generator Internal voltage

$$E'\angle\delta = V_{BUS} + j(X_d' + X)I$$

$$E'\angle\delta = 1.0\angle 0^\circ + j(.30+.22)(.816\angle -11.29^\circ)$$

$$E'\angle\delta = 1.0\angle 0^\circ + j(.52)(.816\angle -11.29^\circ)$$

$$E'\angle\delta = 1.0\angle 0^\circ + (.52\angle 90^\circ)(.816\angle -11.29^\circ)$$

$$E'\angle\delta = 1.0\angle 0^\circ + (.424\angle 78.71^\circ)$$

$$E'\angle\delta = 1.0 + .083 + j.4158$$

$$E'\angle\delta = 1.083 + j.4158 = 1.16\angle 21.00^\circ \text{ p.u.}$$

c) $$P = \frac{E' V_{BUS} \sin\delta}{X_d' + X_{RT} + X_{12+23+13}} = \frac{(1.16)(1.0)}{.30 + .22}\sin(21.0^\circ)$$

$$P = \frac{1.16}{.52}\sin\delta = 2.23(.358) = .799 \text{ p.u.}$$