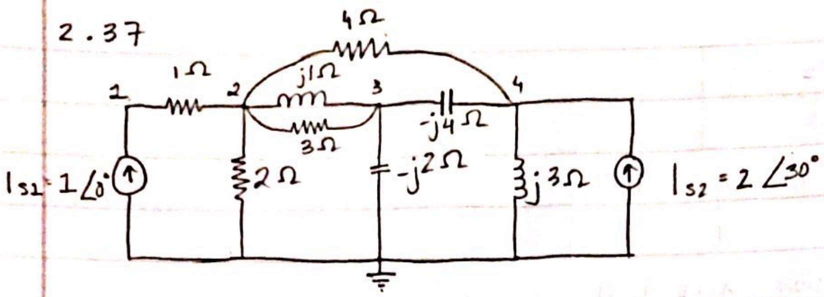
Homework 9

2.37



$Y_{12} = \frac{1}{1\Omega} = 1\ s$

$Y_{2G} = \frac{1}{2\Omega} = .5\ s$

$Y_{23} = \frac{1}{3\Omega} - j1 = .33 - j1\ s$

$Y_{34} = j.25s$

$Y_{24} = \frac{1}{4\Omega} = .25\ s$

$Y_{3G} = +j.5\ s$

$Y_{4G} = -j.33\ s$

$$Y_{BUS} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & (1+.5+.33-j1+.25)-(-j1+.3) & -.25 \\ 0 & -(-j1+.33) & (j.5+j.25-j1+.33) & -(j.25) \\ 0 & -.25 & -(j.25) & (j.25-j.\cancel{33}+.25) \end{bmatrix}$$

$$Y_{BUS} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2.083-j1 & -.33+j1 & -.25 \\ 0 & -.33+j1 & .33-j.25 & -j.25 \\ 0 & -.25 & -j.25 & .25-j.083 \end{bmatrix}$$

6.18:

$f(x) = y$; $y=0$; $f(x) = x^3 + 8x^2 + 2x - 40$; $x(0) = 1$

$$J(i) = \frac{d}{dx}\left(x^3+8x^2+2x-40\right)\Big|_{x=x(i)} 3x^2+16x+2\ \Big|_{x=x(i)} 3x^2(i)+16x(i)+2$$

$$x(i+1) = x(i) + J(i)^{-1}\left[y - f(x(i))\right]$$

$$x(i+1) = x(i) + \left[\frac{-x(i)^3 - 8x(i)^2 - 2x(i) + 40}{3x(i)^2 + 16x(i) + 2}\right]$$

i	x(i)	ε
0	1	.5798
1	2.38	+.208
2	1.97	+.0314
3	1.91	.000397
4	1.9107	

$$x(1) = x(0) + \left[\frac{-x(0)^3 - 8(x(0))^2 - 2(x(0)) + 40}{3x(0)^2 + 16(x(0)) + 2}\right]$$

$$x(1) = 1 + \left[\frac{-1-8-2+40}{3+16+2}\right] = 1 + \frac{29}{21} = 2.38$$

$$x(2) = x(1) + \left[\frac{-x(1)^3 - 8x(1)^2 - 2x(1) + 40}{3x(1)^2 + 16(x(1)) + 2}\right]$$

$$x(2) = 2.38 + \left[\frac{-13.48 - 45.315 - 4.76 + 40}{16.99 + 38.08 + 2}\right] = 2.38 + \left[\frac{-23.55}{57.07}\right] = 1.97$$

$$x(3) = x(2) + \left[\frac{-x(2)^3 - 8x(2)^2 - 2x(2) + 40}{3x(2)^2 + 16x(2) + 2}\right] = 1.97 + \left[\frac{-7.645 - 31.05 - 3.94 + 40}{11.64 + 31.52 + 2}\right] = 1.97 + \left[\frac{-2.635}{45.16}\right] = 1.911$$

#1 p=7 , q=8 | #2 a) True

$$x(4) = x(3) + \left[\frac{-x(3)^3 - 8x(3)^2 - 2x(3) + 40}{3x(3)^2 + 16x(3) + 2}\right]$$

$$x(4) = 1.91 + \left[\frac{-6.967 - 29.18 - 3.82 + 40}{10.94 + 30.56 + 2}\right] = 1.91 + \left[\frac{.03300}{43.5}\right] = 1.9107$$

The solution to the polynomial equation, x is 1.91

6.28

a)

$$Y_{BUS} = \begin{bmatrix} (2-j4+3-j6) & -2+j4 & -3+j6 \\ -2+j4 & 2-j4 & 0 \\ -3+j6 & 0 & 3-j6 \end{bmatrix} = \begin{bmatrix} 5-j10 & -2+j4 & -3+j6 \\ -2+j4 & 2-j4 & 0 \\ -3+j6 & 0 & 3-j6 \end{bmatrix}$$

b) $P_K = V_K \sum_{n=1}^{N} Y_{KN} V_n \cos(\delta_K - \delta_n - \theta_{Kn})$

At Bus 2:

$$P_2 = V_2 \sum_{n=1}^{3} Y_{23} V_n \cos(\delta_2 - \delta_n - \theta_{2n})$$

$$P_2 = V_2 \left[Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \cos(\delta_2 - \delta_2 - \theta_{22}) + Y_{23} V_3 \cos(\delta_2 - \delta_3 - \theta_{23})\right]$$

$$1.5 = (1.1)\left[4.47(1)\cos(\delta_2 - 116.57°) + (4.47)(1.1)\cos(-(-63.43°)) + 0\right]$$

$$1.363 = 4.47\cos(\delta_2 - 116.57°) + 2.199$$

$$-.1869 = \cos(\delta_2 - 116.57°)$$

$$\delta_2 - 116.57° = \pm 100.77°$$

$$\boxed{\delta_2 = 15.8°}$$

c) $P_3 = V_3 \sum_{n=1}^{N} Y_{3n} V_n \cos(\delta_3 - \delta_n - \theta_{3n})$ | $Q_3 = V_3 \sum_{n=1}^{N} Y_{3n} V_n \sin(\delta_3 - \delta_n - \theta_{3n})$

$$P_3 = V_3\left[Y_{31} V_1 \cos(\delta_3 - \delta_1 - \theta_{31}) + Y_{32} V_2 \cos(\delta_3 - \delta_2 - \theta_{32}) + Y_{33} V_3 \cos(\delta_3 - \delta_3 - \theta_{33})\right]$$

$$P_3 = V_3\left[6.708(1)\cos(\delta_3 - 116.57°) + 0 + 6.708(V_3)\cos(63.43°)\right]$$

$$-1.5 = V_3\left[6.71\cos(\delta_3 - 116.57°) + 6.71 V_3(\cos(63.43))\right]$$

$$Q_3 = V_3\left[Y_{31} V_1 \sin(\delta_3 - \delta_1 - \theta_{31}) + Y_{32} V_2 \sin(\delta_3 - \delta_2 - \theta_{32}) + Y_{33} V_3 \sin(\delta_3 - \delta_3 - \theta_{33})\right]$$

$$Q_3 = V_3\left[6.71\sin(\delta_3 - 116.57°) + 0 + 6.71 V_3 \sin(63.43°)\right]$$

#4 $S_{BASE} = 100\text{ MVA}$ $|V_1| = 1.0\text{ p.u.}$ $\theta_1 = 0°$

$$Y_{BUS} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix} \qquad x = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix} \qquad x^{(0)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di} = |V_2||V_1|(10\sin\theta_2) + 1.5 = 0$$

$$Q_i = \sum_{k=1}^{n} |V_i|V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di} = |V_2||V_1|(-10\cos\theta_2) + |V_2|^2(10) + .75 = 0$$

$$P_2(x) = |V_2|(10\sin\theta_2) + 1.50 = 0$$
$$Q_2(x) = |V_2|(-10\cos\theta_2) + |V_2|^2(10) + .75 = 0$$

$$J(x) = \begin{bmatrix} \partial P_2(x)/\partial\theta_2 & \partial P_2(x)/\partial|V_2| \\ \partial Q_2(x)/\partial\theta_2 & \partial Q_2(x)/\partial|V_2| \end{bmatrix}$$

$$J(x) = \begin{bmatrix} |V_2|10\cos\theta_2 & 10\sin\theta_2 \\ |V_2|10\sin\theta_2 & -10\cos\theta_2 + |V_2|20 \end{bmatrix}$$

$$f(x^{(0)}) = \begin{bmatrix} 1.50 \\ .75 \end{bmatrix} \qquad J(x^{(0)}) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$x^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1}\begin{bmatrix} 1.50 \\ .75 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} .1 & 0 \\ 0 & .1 \end{bmatrix}\begin{bmatrix} 1.50 \\ .75 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} .150 \\ .075 \end{bmatrix} = \begin{bmatrix} -.150 \\ .925 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = \frac{1}{100}\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} .1 & 0 \\ 0 & .1 \end{bmatrix}$$

$$f(x^{(1)}) = \begin{bmatrix} (.925)(10)(\sin(-.150) + 1.50 \\ (.925)(-10)(\cos(-.150)) + (.925)^2(10) + .75 \end{bmatrix} = \begin{bmatrix} 1.4757 \\ .05628 \end{bmatrix}$$

$$J(x^{(1)}) = \begin{bmatrix} (.925)(10)(\cos(-.150)) & 10\sin(-.150) \\ (.925)(10)(\sin(-.150)) & -10\cos(-.150) + (.925)(20) \end{bmatrix} = \begin{bmatrix} 9.2499 & -.026179 \\ -.024216 & 8.50034 \end{bmatrix}$$

.01271899

$$J(x^{(1)})^{-1} = .01271833\begin{bmatrix} 8.50034 & .026179 \\ .024216 & 9.2499 \end{bmatrix} = \begin{bmatrix} .1081101 & .0003329 \\ .00030798 & .1176433 \end{bmatrix}$$

$$\begin{bmatrix} .1081140 & .0003324052 \\ .0003079482 & .1176475 \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} -.150 \\ .925 \end{bmatrix} - \begin{bmatrix} .1081101 & .0003329 \\ .00030798 & .1176433 \end{bmatrix}\begin{bmatrix} 1.4757 \\ .05628 \end{bmatrix} = \begin{bmatrix} -.150 \\ .925 \end{bmatrix} - \begin{bmatrix} .1595568 \\ .00707546 \end{bmatrix} = \begin{bmatrix} -.3095568 \\ .9179245 \end{bmatrix}$$

$$f(x^{(2)}) = \begin{bmatrix} 1.450407 \\ -.00325715 \end{bmatrix}$$

$$J(x^{(2)}) = \begin{bmatrix} 9.179111 & -.05402759 \\ -.04959325 & 8.358636 \end{bmatrix}$$

$$J(x^{(2)})^{-1} = .01303404 \begin{bmatrix} 8.358636 & +.05402759 \\ .04959325 & 9.179111 \end{bmatrix} = \begin{bmatrix} .1089469 & .0007041978 \\ .0006464604 & .1196409 \end{bmatrix}$$

$$x^{(3)} = \begin{bmatrix} -.3095568 \\ .9179245 \end{bmatrix} - \begin{bmatrix} .1089468 & .0007041978 \\ .0006464604 & .1196409 \end{bmatrix} \begin{bmatrix} 1.450407 \\ -.00325715 \end{bmatrix}$$

$$x^{(3)} = \begin{bmatrix} -.3095568 \\ .9179245 \end{bmatrix} - \begin{bmatrix} .1580149 \\ .0005478553 \end{bmatrix} = \begin{bmatrix} .1515419 \\ .9173766 \end{bmatrix}$$

$$f(x^{(3)}) = \begin{bmatrix} 1.524264 \\ -.005028539 \end{bmatrix} \qquad J(x^{(3)}) = \begin{bmatrix} 9.173766 & .02644902 \\ .02426371 & 8.347567 \end{bmatrix}$$

$$J(x^{(3)})^{-1} = .01305858 \begin{bmatrix} 8.347567 & -.02644902 \\ -.02426371 & 9.173766 \end{bmatrix} = \begin{bmatrix} .1090074 & -.0003453866 \\ -.0003168496 & .1197964 \end{bmatrix}$$

$$x^{(4)} = \begin{bmatrix} .1515419 \\ .9173766 \end{bmatrix} - \begin{bmatrix} .1090074 & -.0003453866 \\ -.0003168496 & .1197964 \end{bmatrix} \begin{bmatrix} 1.524264 \\ -.005028539 \end{bmatrix}$$

$$x^{(4)} = \begin{bmatrix} .1515419 \\ .9173766 \end{bmatrix} - \begin{bmatrix} .1661578 \\ -.001085363 \end{bmatrix} = \begin{bmatrix} -.01461590 \\ .9184620 \end{bmatrix}$$

#6 $B_{Matrix} = \begin{bmatrix} \cancel{-10} & \cancel{0} \\ \cancel{0} & -10 \end{bmatrix} = -10$

$P = P_{G2} - P_{L2} = 0 - 2.0 = -2.0$

$-2.0 = (-10\,\theta_2)$

$-.02 = \theta_2$ radians

$\theta_2 = -11.459°$

#5 $B = [-10]$ $\qquad [\theta_2]^{(0)} = [0]$ $\qquad f([\theta^2]^{(1)})$

$B^{-1} = [-\frac{1}{10}]$

$[\theta_2]^{(1)} = [0] + [\frac{-1}{10}][2.0]$

$[\theta^2]^{(1)} = [-.2]$

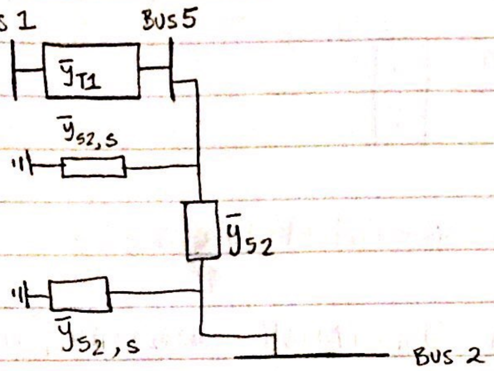
$[\theta^2]^{(2)} = [-.2] + [\frac{-1}{10}][$

Line 3: 50 mi $\left(\frac{1.60934\text{ Km}}{1\text{ mi}}\right)$ = 80 km ; medium Line

Line 2: 100 mi $\left(\frac{1.60934\text{ Km}}{1\text{ mi}}\right)$ = 160.934 km ; medium line

Line 1: 200 mi $\left(\frac{1.60934\text{ Km}}{1\text{ mi}}\right)$ = 321.86 km ; Long line

7. a)



b)

c) Since the load is never changed, the generators are reacting inversely to each other. As the MW on load three are decreased, the MW at Bus one are increased. Real power between Bus 5 & 4 is decreased.

d) Between 520 MW - 540 MW varied at Bus three, the real power losses are minimized to .348 p.u.

e)