Clique Decision Problem: maximal siged clique
(Complete graph)

① Proof that it belongs to NP.

Proof

1. Certificate: let a set S be consisting of nodes in the clique and S is a subset of G.
S is the certificate.

2. Verification:

First step: verifying numbere of nodes in S = K.

Takes O(1) time.

Second step: Verifying whether each vertex has an outdegree of $(k-1)$ takes $O(k^2)$ time.

As total number of complete graph $= \frac{n(n-1)}{2}$.

So, it takes $O(k^2) = O(n^2)$; which is polynomial.
As it's polynomial time verified, it belongs to NP.

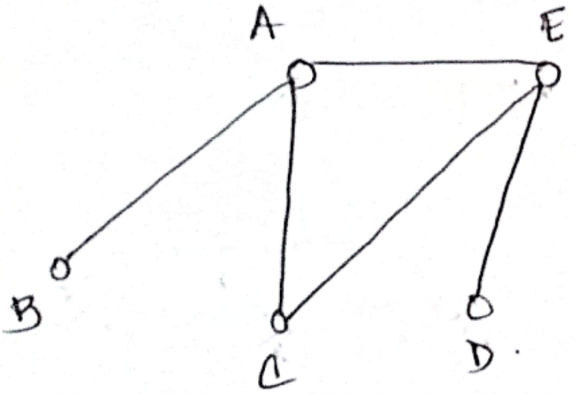
Independent set Problem is a NP-Hard problem.

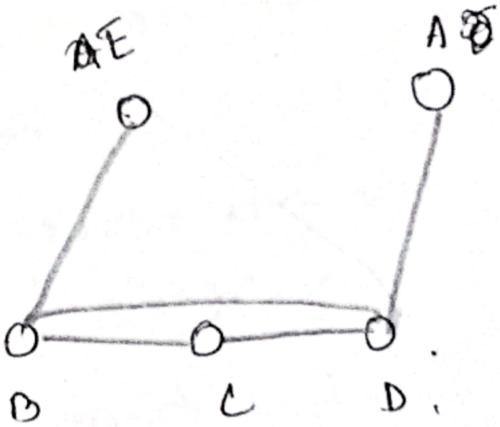
Every instance of Independent set problem, $G(V, E)$ and integer K can be converted to required graph for Clique problem, $G'(V', E'), K'$.

$V' = V$ for all vertices of G.

E' = Complement of E.

G' = complementary graph G'.





An Independent set of K=3 formed by {B, C, D}

A clique of K=3 formed by {B, C, D}

Time required to compute the complementary graph G' requires a traversal over all the vertices and edges. → $O(V+E)$

→ Let us assume G contains K sized clique. So, it implies that there are K vertices in G where each of vertices is connected by an edge with remaining vertices. As these edges are contained in G, they won't be in G'.

∴ These K vertices are not adjacent to each other in G' and form Indepent set of size K.

→ G' has an independent set of vertices of size K'. None of these vertices shares an edge with any other vertices. After complementing edges K' to K to achieve G, these K vertices will share an edge and hence become adjacent to each other.

∴ Graph G will have a clique of size K.

∴ There is a clique of size K in graph G if there is an independent set of size in G'. (complement graph).

∴ Any instance of clique problem can be reduced to an instance of Independent set problem.

∴ Clique problem is NP-Hard.