Planar Graph

Ayesha Binte Mostofa
1805062
Anup Bhowmik
1805082

Department of Computer Science and Engineering
Bangladesh University of Engineering and Technology

August 31, 2022

We are going to see

Motivation

- PCB (Printed Circuit Board) is designed by planar graph.

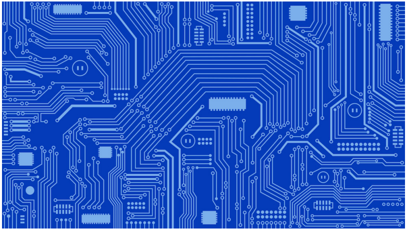


Figure: PCB Circuit

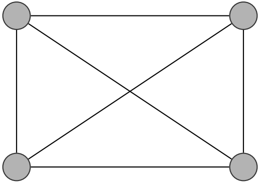
Planar Graph

Definition

A graph is called **planar** whether it can be drawn in the plane in such a way that no two edges cross.

Planar Graph

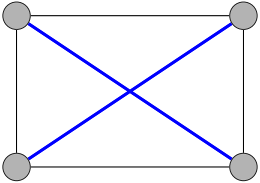
A clique on 4 nodes



- Let's see whether it's a planar graph or not
- It has 6 edges, two of which crossing each other
- So,it is not a Planar Graph

Planar Graph

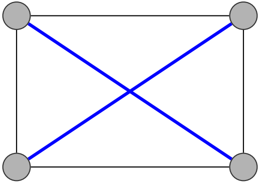
A clique on 4 nodes



- Let's see whether it's a planar graph or not
- It has 6 edges, two of which crossing each other
- So,it is not a Planar Graph

Planar Graph

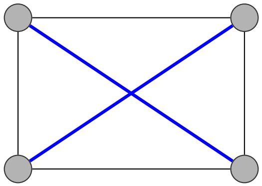
A clique on 4 nodes



- Let's see whether it's a planar graph or not
- It has 6 edges, two of which crossing each other
- So,it is not a Planar Graph

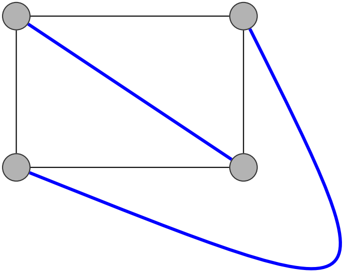
Planar Graph

A clique on 4 nodes



- Let's see whether it's a planar graph or not
- It has 6 edges, two of which crossing each other
- So,it is not a Planar Graph

Planar Graph



Another clique on 4 nodes with six edges
It's a planar Graph because no edges are crossing each other.

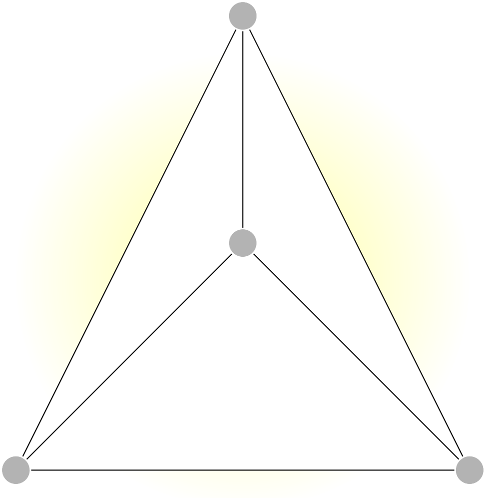
Faces on a Planar Graph

Definition

Consider a planar graph $G = (V, E)$ .A face is defined to be an area of the plane that is bounded by edges. A planar graph divides the planes into one or more **faces**. One of these faces always will be *infinite*.

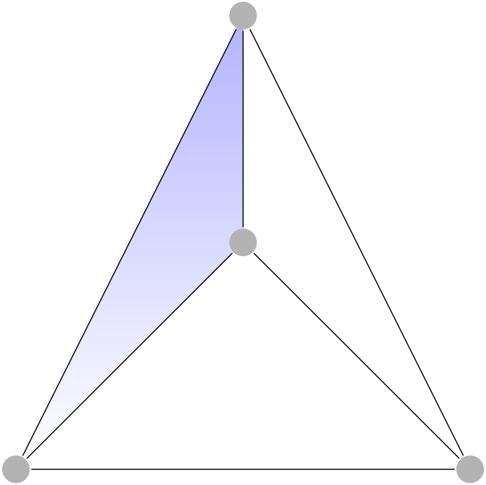
Finding Faces of a Planar Graph

Infinite Face



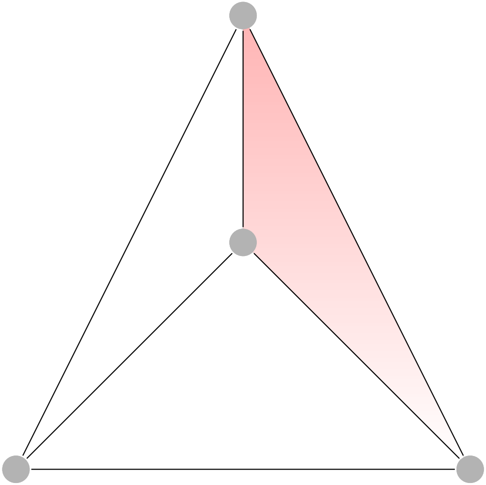
Finding Faces of a Planar Graph

Finite Face



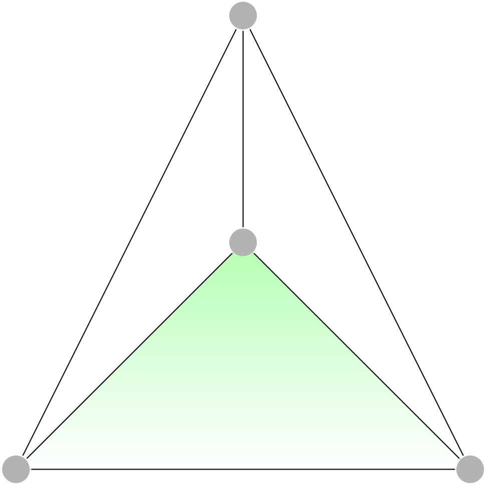
Finding Faces of a Planar Graph

Finite Face



Finding Faces of a Planar Graph

Finite Face



Euler's Theorem on Planar Graph

- Let G be a connected planar graph (drawn without crossing edges).
- Define
  V = number of vertices
  E = number of edges
  F = number of faces, including the "infinite" face
- Then $V - E + F = 2$.

Euler's Theorem on Planar Graph

- Let G be a connected planar graph (drawn without crossing edges).
- Define
  V = number of vertices
  E = number of edges
  F = number of faces, including the "infinite" face
- Then $V - E + F = 2$.

Euler's Theorem on Planar Graph

- Let G be a connected planar graph (drawn without crossing edges).
- Define
  V = number of vertices
  E = number of edges
  F = number of faces, including the "infinite" face
- Then $V - E + F = 2$.

Proof

PROOF IDEA

Proof by induction by the number of cycles

BASE CASE

- G has no cycles
- Since G is connected, it must be a tree. So, $e = v - 1$ and $f = 1$.

$$\begin{aligned} v - e + f &= v - (v - 1) + 1 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

Proof

PROOF IDEA

Proof by induction by the number of cycles

BASE CASE

- G has no cycles
- Since G is connected, it must be a tree. So, $e = v - 1$ and $f = 1$.

$$
\begin{aligned}
v - e + f &= v - (v - 1) + 1 \\
&= 1 + 1 \\
&= 2
\end{aligned}
$$

Proof – Continued

INDUCTION

- Let G has at least one cycle containing edge e
- let $G' = G - e$

G

v' = number of vertices
e' = number of edges
f' = number of faces

- Now, in $G', f' = f - 1$ and $v' = v$ and $e' = e - 1$
- **By induction hypothesis:**

$$v' - e' + f' = 2$$

$$v - (e - 1) + (f - 1) = 2$$

$$v - e + f = 2$$

Proof – Continued

INDUCTION

- Let G has at least one cycle containing edge e
- let $G' = G - e$

G

v' = number of vertices
e' = number of edges
f' = number of faces

- Now, in $G', f' = f - 1$ and $v' = v$ and $e' = e - 1$
- **By induction hypothesis:**

$$v' - e' + f' = 2$$

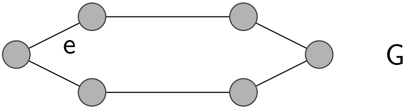
$$v - (e - 1) + (f - 1) = 2$$

$$v - e + f = 2$$

Proof – Continued

INDUCTION

- Let G has at least one cycle containing edge e
- let $G' = G - e$



v' = number of vertices
e' = number of edges
f' = number of faces

- Now, in G', $f' = f - 1$ and $v' = v$ and $e' = e - 1$
- **By induction hypothesis:**

$$v' - e' + f' = 2$$

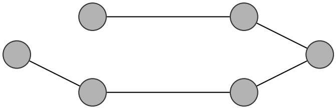
$$v - (e - 1) + (f - 1) = 2$$

$$v - e + f = 2$$

Proof – Continued

INDUCTION

- Let G has at least one cycle containing edge e
- let $G' = G - e$



G'

v' = number of vertices
e' = number of edges
f' = number of faces

- Now, in $G', f' = f - 1$ and $v' = v$ and $e' = e - 1$
- **By induction hypothesis:**

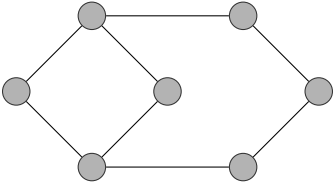
$$v' - e' + f' = 2$$

$$v - (e - 1) + (f - 1) = 2$$

$$v - e + f = 2$$

Question

Can you redraw this graph to alter the number of faces?



Corollary of Euler's Theorem

An interesting fact

No matter how we redraw a planar graph it will always have the same number of faces

Proof

$f = 2 - v + e$

Acknowledgement

- https://mathweb.ucsd.edu/~gptesler/154/slides/154_planar_20-handout.pdf
- https://www.slideserve.com/thor/planar-graphs
- https://en.wikipedia.org/wiki/Planar_graph

Further Reads

`https://ieeexplore.ieee.org/document/5206863`

A very interesting application of planar graph is in ComputerVision
Efficient planar graph cuts with applications in Computer Vision
by Frank R. Schmidt et al.