Dynamic Programming

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We are going to see

1. Introduction
2. Properties
3. A problem solved by dp

We are going to see

- Introduction

Dynamic Programming

- Algorithm technique that systematically **records** the answers to sub-problems and **reuses** them those recorded result
- A simple example:
  Calculating the n-th Fibonacci number
  $Fib(n) = Fib(n1) + Fib(n2)$

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Dynamic Programming

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- It breaks down a complicated problem into simpler sub-problems in a recursive manner.
- If optimal solutions can be found recursively for the sub-problems, then it is said to have optimal substructure.

We are going to see

- Properties

Properties of Dynamic Programming

Such problems exhibits following two properties:

- **Optimal Substructure**
- **Overlapping sub-problems**

Optimal Substructure

A problem is said to have optimal substructure if an optimal solution can be constructed from optimal solutions of its sub-problems.
e.g. in Floyd-Warshall algorithm, travelling from node i to j using node k, dist[i][j]=dist[i][k]+dist[k][j]

Overlapping sub-problem

A problem has overlapping sub-problems if finding its solution involves solving the same sub-problem multiple times.
Example: Calculating n-th Fibonacci number $F(n)$

Example of Overlapping sub-problems

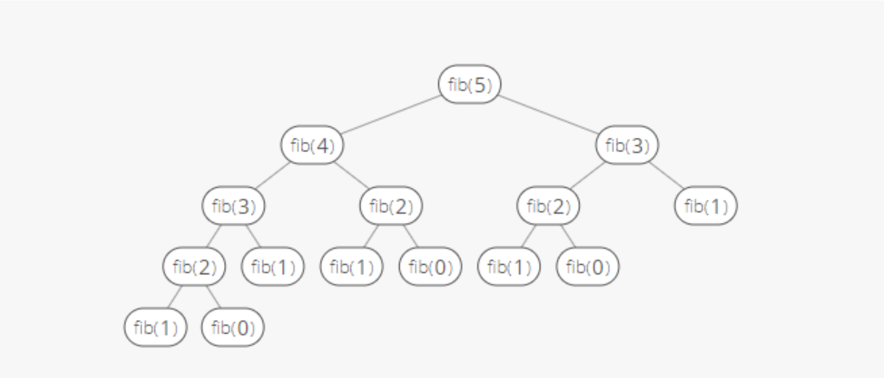


Figure: Overlapping Sub-problems in determination of Fibonacci series

We are going to see

- A problem solved by dp

Binomial Coefficient a.k.a C(n,r)

problem statement:ways to select r objects from n objects regardless of the ordering

Binomial Coefficient a.k.a C(n,r)

Naive approach : calculating $\frac{n!}{r!(nr)!}$

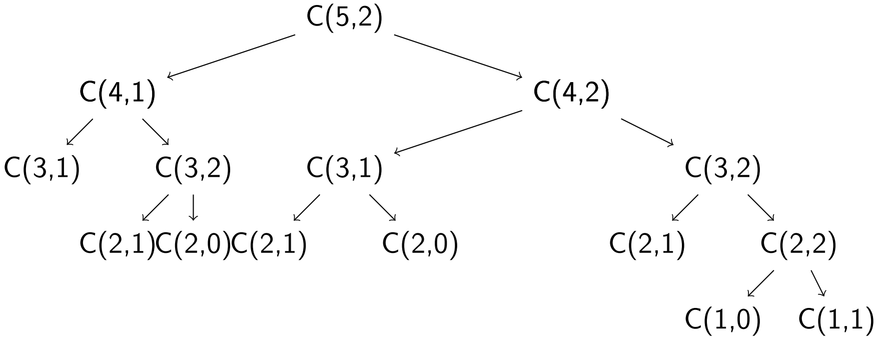
- Problem : overflow will be caused calculating factorials, unsigned long long wouldn't be enough. May be BigInteger would do but not efficient.
- Solution : using dynamic programming.

C(n,r) having dynamic programming properties

Optimal Substructure: C(n,r) can be recursively calculated using the formula,
$C(n,r) = C(n1,r1) + C(n1,r)$
with base cases, $C(n,0) = C(n,n) = 1$ and $C(n,1) = n$

C(n,r) having dynamic programming properties

Overlapping Sub-problems: let n=5, r=2



Algorithm List		
Algorithm name	Time Complexity	Space Complexity
BFS DFS	$O(\lvert V\rvert + \lvert E\rvert)$ $O(\lvert V\rvert + \lvert E\rvert)$	$O(\lvert V\rvert)$ $O(\lvert V\rvert)$
Dijkstra	$O(\lvert V\rvert + \lvert E\rvert log \lvert V\rvert)$	$O(\lvert V\rvert + \lvert E\rvert)$
Bellman Ford	$O(\lvert V\rvert \lvert E\rvert)$	$O(\lvert V\rvert)$
Floyd–Warshall	$O(\lvert V\rvert^3)$	$O(\lvert V\rvert^2)$
Edmonds–Karp	$O(\lvert V\rvert \lvert E\rvert^2)$	$O(\lvert V\rvert + \lvert E\rvert)$