
Thresholding
ONWARD


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Thresholding

In digital image processing, **thresholding** is the simplest method of segmenting images. From a grayscale image, thresholding can be used to create binary images [1].

A **grayscale** image is one in which the value of each pixel is a single sample representing only an *amount* of light; that is, it carries only intensity information.

Grayscale images can be the result of measuring the intensity of light at each pixel according to a particular weighted combination of frequencies (or wavelengths), and in such cases they are monochromatic proper when only a single frequency (in practice, a narrow band of frequencies) is captured.

[1] Shapiro, Linda G. & Stockman, George C. (2002). "Computer Vision". Prentice Hall. ISBN 0-13-030796-3

Thresholding

In physics, **monochromatic radiation** is electromagnetic radiation with a single constant frequency. When that frequency is part of the visible spectrum the term **monochromatic light** is often used. Monochromatic light is perceived by the human eye as a spectral colour.

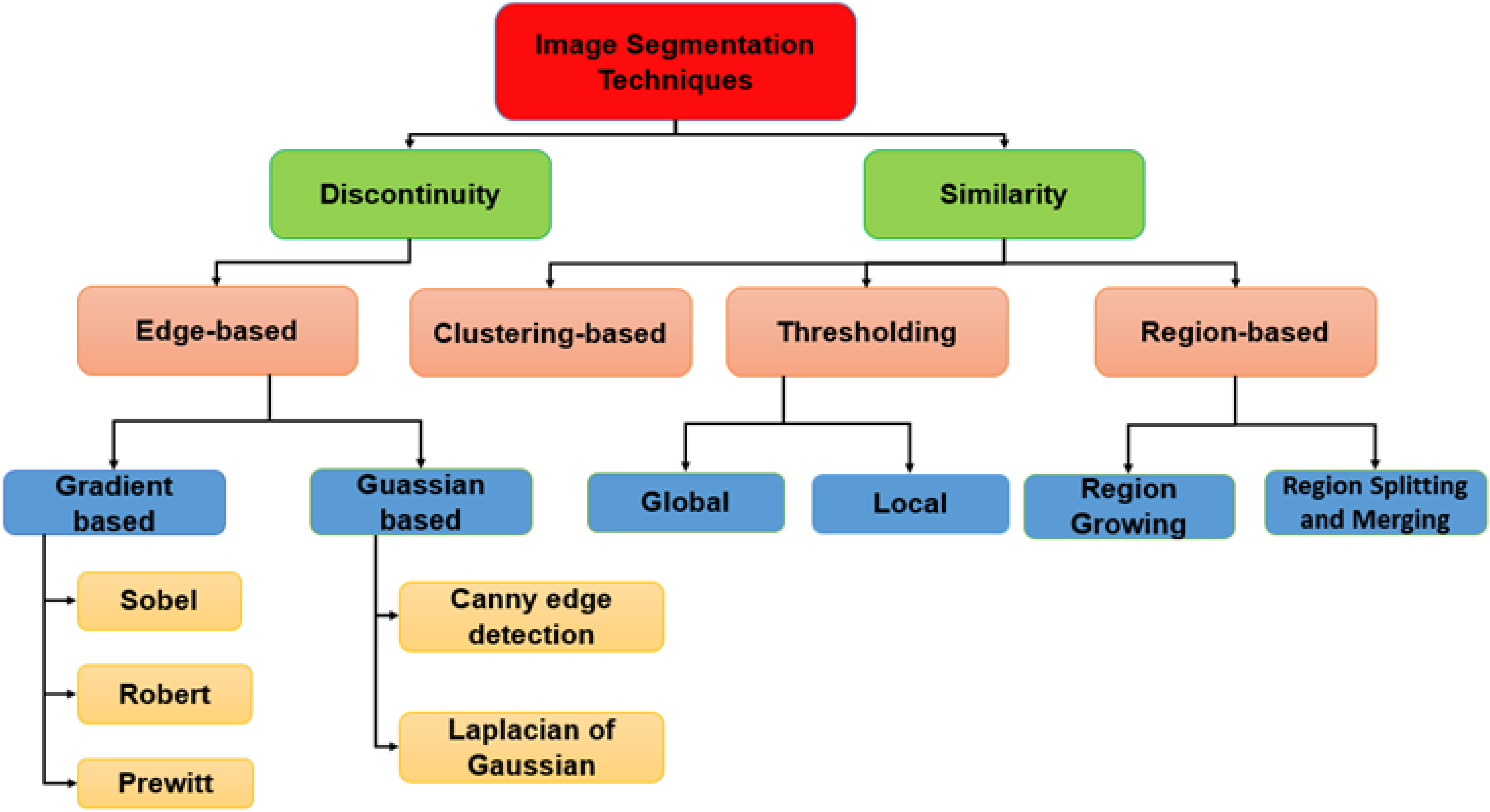
Thresholding Contd..

A **binary image** is one that consists of pixels that can have one of exactly two colors, usually black and white. Binary images are also called *bi-level* or *two-level*, Pixelart made of two colours is often referred to as *1-Bit* or *1bit.*

In digital image processing and computer vision, **image segmentation** is the process of partitioning a digital image into multiple **image segments**, also known as **image regions** or **image objects** (sets of pixels).

The goal of segmentation is to simplify and/or change the representation of an image into something that is more meaningful and easier to analyze.

[1] Shapiro, Linda G. & Stockman, George C. (2002). "Computer Vision". Prentice Hall. ISBN 0-13-030796-3



https://pooja-bagad18.medium.com/classification-of-image-segmentation-techniques-cc6a031b75fc

Type of thresholding

- Global thresholding – ***T*** is constant.
- Local or regional / Adaptive thresholding.

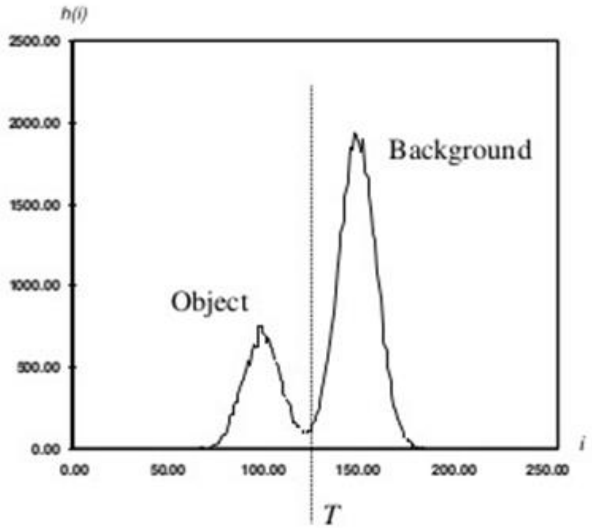
In most methods, the same threshold is applied to all pixels of an image. However, in some cases, it can be advantageous to apply a different threshold to different parts of the image, based on the local value of the pixels.

They are particularly adapted to cases where images have inhomogeneous lighting.

A neighborhood is defined and a threshold is computed for each pixel and its neighborhood.

Global thresholding

- This method is used when the object are easily differentiated from each other, so we can use a single value as threshold for the entire image.



- Threshold value should not be too high or too low, it must be optimal.

n: Original image n1: Threshold too low n2: Threshold too high

Local or regional / Adaptive thresholding –

Local thresholding can be defined as:

$$g(x, y) = \begin{cases} 0, & \text{if } I(x,y) <= T(x,y) \\ 1, & \text{otherwise} \end{cases}$$

Where, **g(x, y) - binary image I(x, y), intensity of the each pixel T(x, y)-threshold value.**

This method decides multiple threshold values for every pixel in the image on the basis of attributes (range, variance or surface-fitting parameters) of adjacent pixels.
As we can set multiple threshold values in local thresholding, this method works well on high grayscale contrast images where global thresholding method will not work effectively.

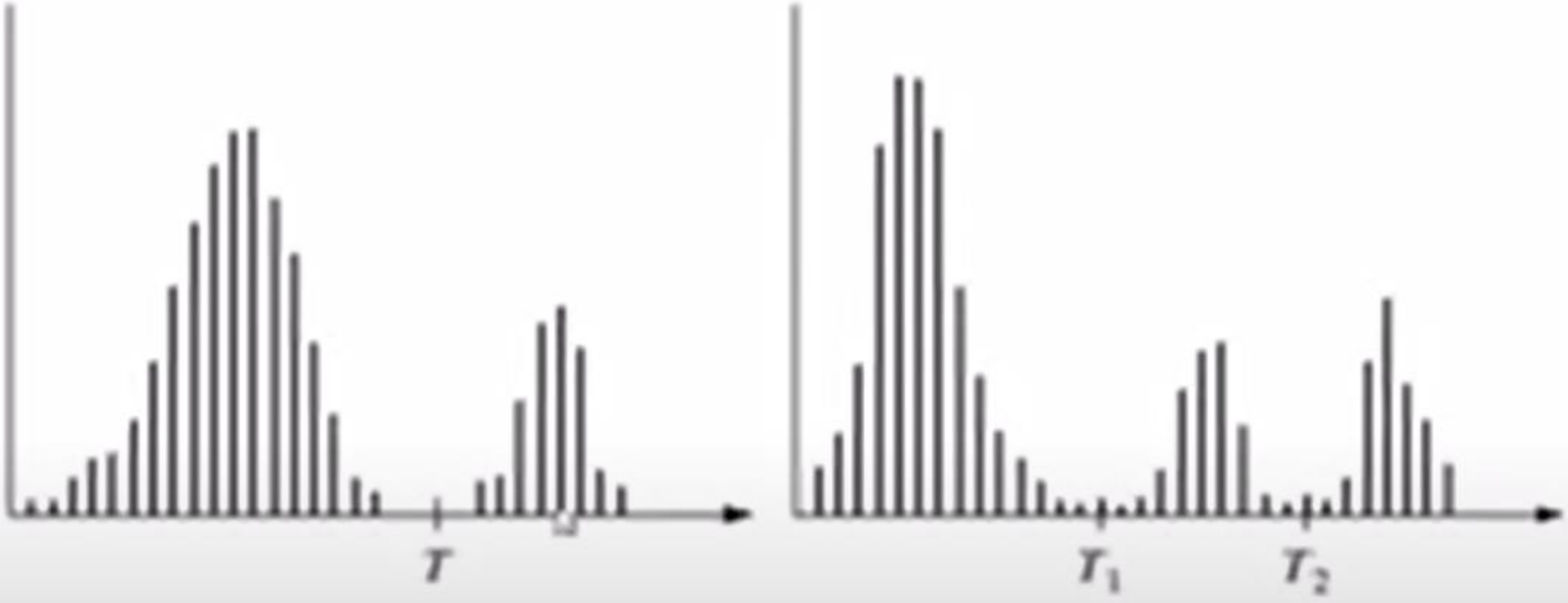
(a) Source image. (b) Global. (c) Local.

Thresholding Contd… Simple Way *Global Thresholding*

The simplest thresholding methods replace each pixel in an image with a black pixel if the image intensity $\mathbf{I}_{i,j}$ is is less than a fixed value called the threshold $\boldsymbol{T}$ or

A white pixel if the pixel intensity is greater than that threshold. In the example image on the right, this results in the dark tree becoming completely black, and the bright snow becoming completely white.

- Label pixel as belonging to one of two (or more) classes



Global Thresholding

A heuristic algorithm

1. Select an initial estimate for threshold T
2. Segment the image using T
   - G1 is all pixels with intensities > T
   - G2 is all pixels with intensities <= T
3. Compute averages m1 and m2 for the pixels in G1 and G2 4.
   Let T = (m1+m2)/2
5. Repeat steps 1-4 until no more change

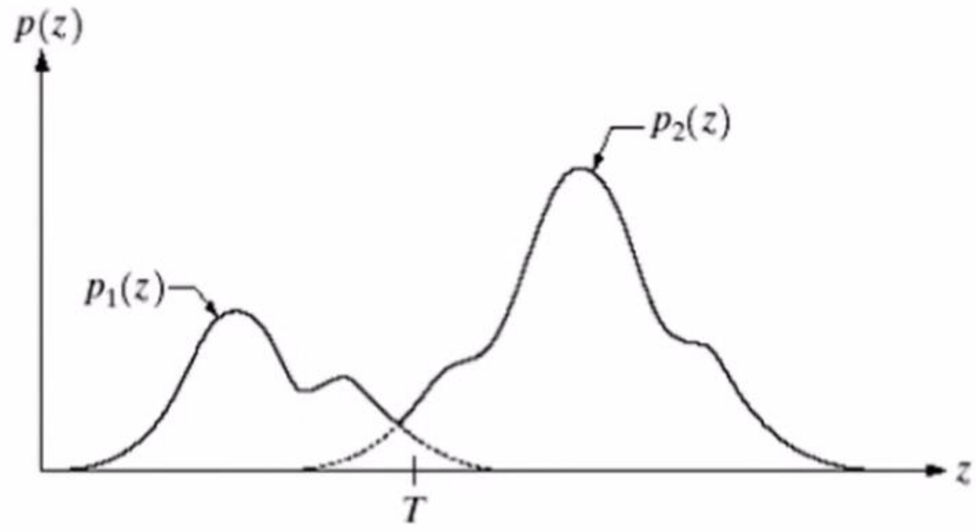
Note – you can use the histogram to compute the averages (very fast)

Optimal Global Thresholding

- Want to pick a threshold that minimizes the error of miss-classifying a point

$$E_1(T) = \int_{-\infty}^{T} p_2(z)\,dz \quad and \quad E_2(T) = \int_{T}^{\infty} p_1(z)\,dz$$

FIGURE 10.32 Gray-level probability density functions of two regions in an image.



Unfortunately, we usually don't know the pdf's $p_1(z)$, $p_2(z)$ in advance

Example

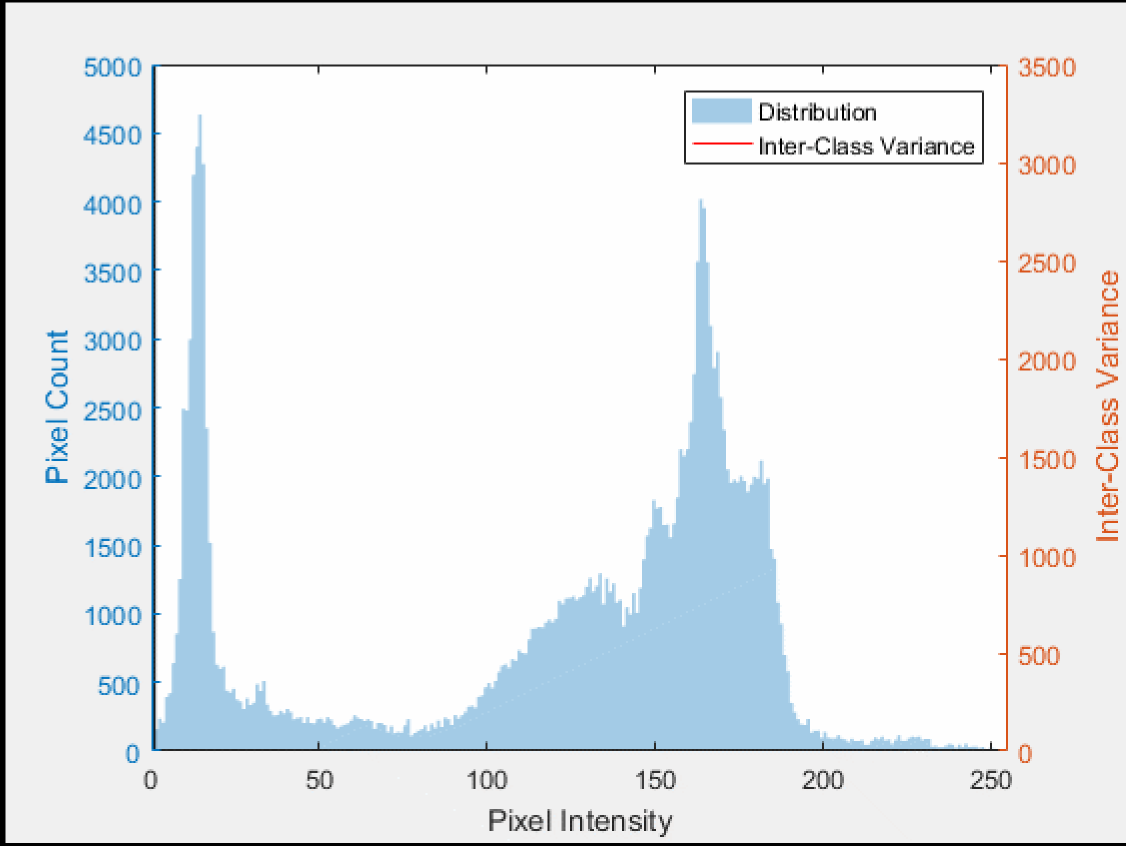
Otsu's method

In computer vision and image processing, **Otsu's method**, named after Nobuyuki Otsu (大津展之, *Ōtsu Nobuyuki*), is used to perform automatic image thresholding [1].

In the simplest form, the algorithm returns a single intensity threshold that separate pixels into two classes, foreground and background.

This threshold is determined by minimizing **intra-class intensity variance**, or equivalently, by **maximizing inter-class variance**.

1. *M. Sezgin & B. Sankur (2004). "Survey over image thresholding techniques and quantitative performance evaluation". Journal of Electronic Imaging. **13** (1): 146–165. Bibcode:2004JEI....13..146S. doi:10.1117/1.1631315.*


Distribution
Inter-Class Variance
Pixel Count
Inter-Class Variance
Pixel Intensity
0
50
100
150
200
250
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
0
500
1000
1500
2000
2500
3000
3500

The algorithm exhaustively searches for the threshold that minimizes the intra-class variance, defined as a weighted sum of variances of the two classes:

$$\sigma_w^2(t) = \omega_0(t)\sigma_0^2(t) + \omega_1(t)\sigma_1^2(t)$$

Variances of two classes

Sum of variance of two classes

Probabilities of two classes separated by the threshold T

$$\omega_0(t) = \sum_{i=0}^{t-1} p(i)$$

Probabilities of two classes for L bins of histogram

$$\omega_1(t) = \sum_{i=t}^{L-1} p(i)$$

For 2 classes, minimizing the intra-class variance is equivalent to maximizing inter-class variance:

$$\sigma_b^2(t) = \sigma^2 - \sigma_w^2(t) = \omega_0(t)(\mu_0 - \mu_T)^2 + \omega_1(t)(\mu_1 - \mu_T)^2$$
$$= \omega_0(t)\omega_1(t)\left[\mu_0(t) - \mu_1(t)\right]^2$$

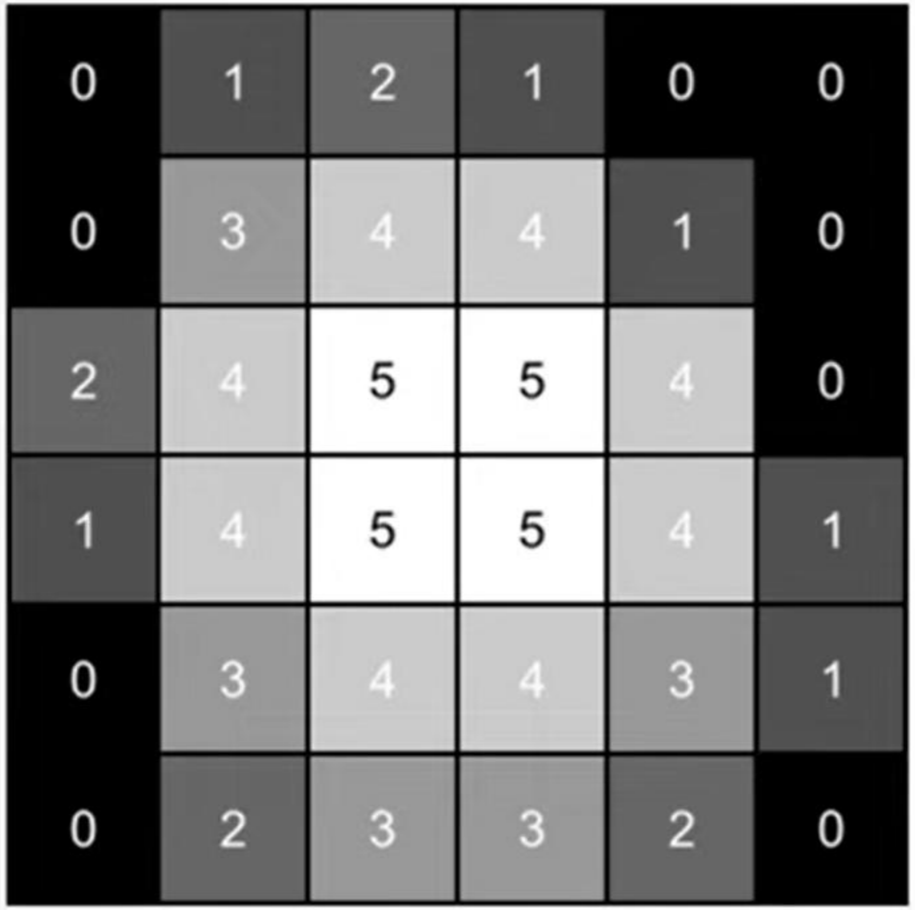
which is expressed in terms of class probabilities *w* and class means µ. And,

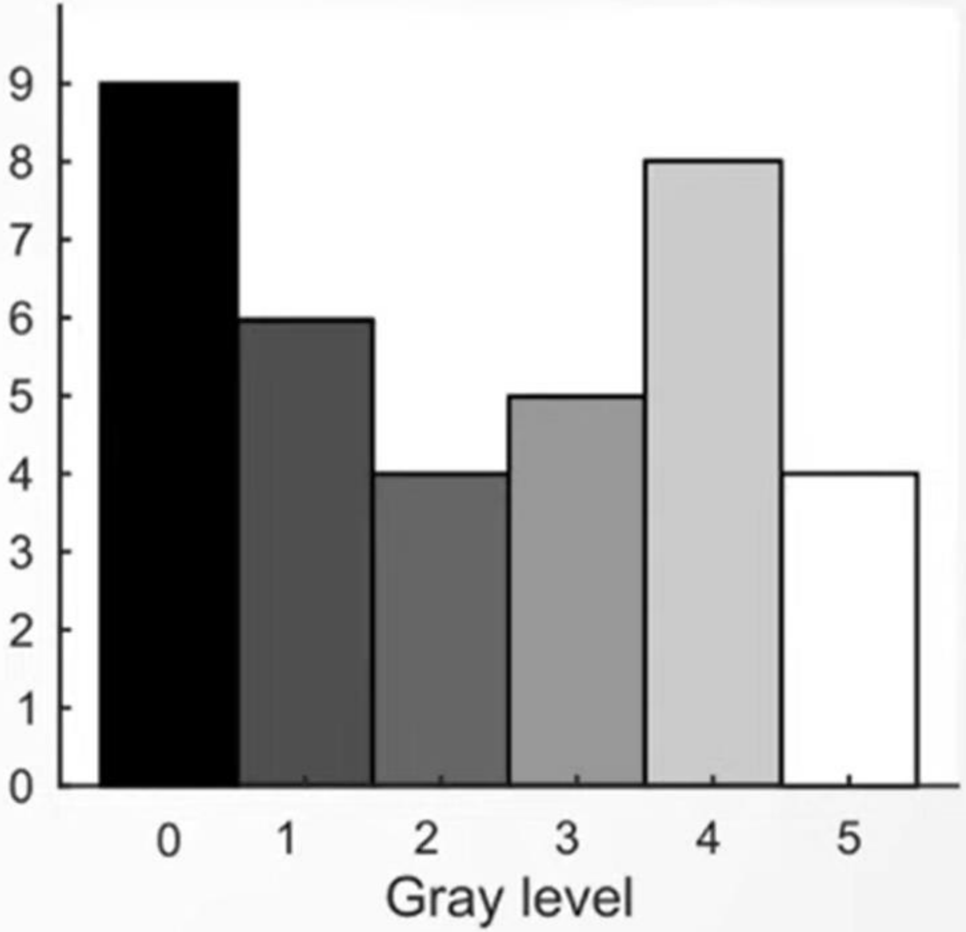
$$\mu_0(t) = \frac{\sum_{i=0}^{t-1} ip(i)}{\omega_0(t)}$$
$$\mu_1(t) = \frac{\sum_{i=t}^{L-1} ip(i)}{\omega_1(t)}$$
$$\mu_T = \sum_{i=0}^{L-1} ip(i)$$

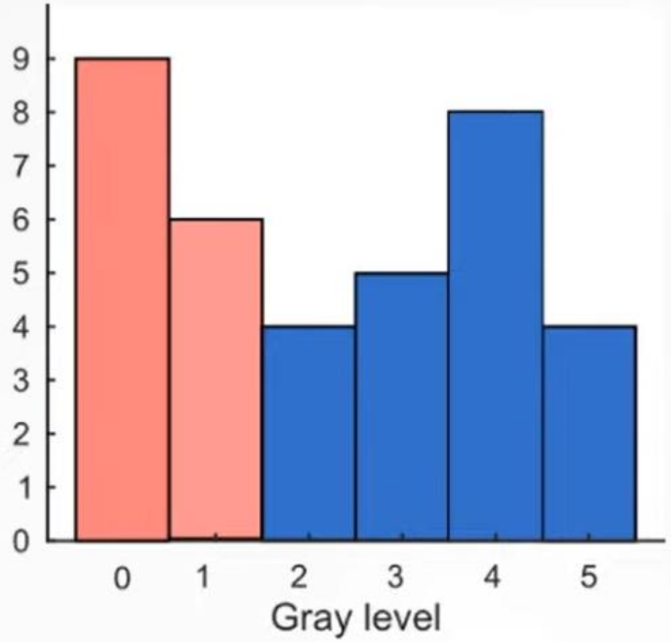
Algorithm

1. Compute histogram and probabilities of each intensity level
2. Set up initial $\omega_i(0)$ and $\mu_i(0)$
3. Step through all possible thresholds $t = 1, \ldots$ maximum intensity
    1. Update ω_i and μ_i
    2. Compute $\sigma_b^2(t)$
4. Desired threshold corresponds to the maximum $\sigma_b^2(t)$

Example


0 1 2 1 0 0
0 3 4 4 1 0
2 4 5 5 4 0
1 4 5 5 4 1
0 3 4 4 3 1
0 2 3 3 2 0


0 1 2 3 4 5 6 7 8 9
0 1 2 3 4 5
Gray level



Background

$$W_b = \frac{9+6}{36} = 0.42$$

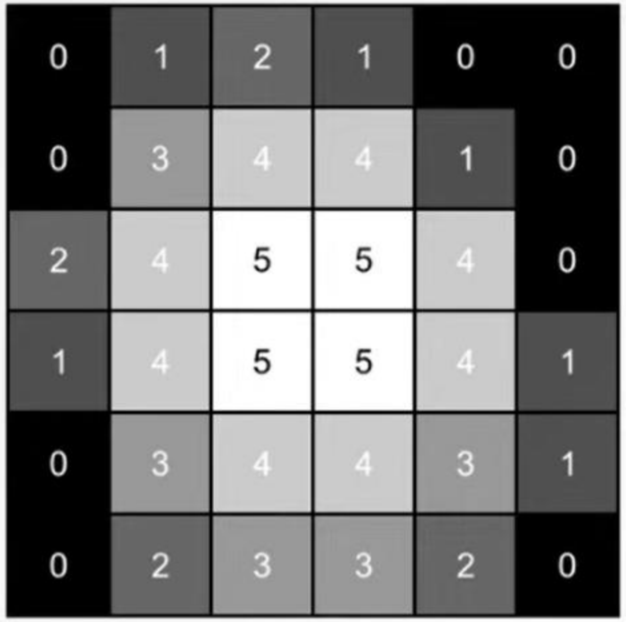
$$\mu_b = \frac{(9\times 0)+(6\times 1)}{9+6} = 0.4$$

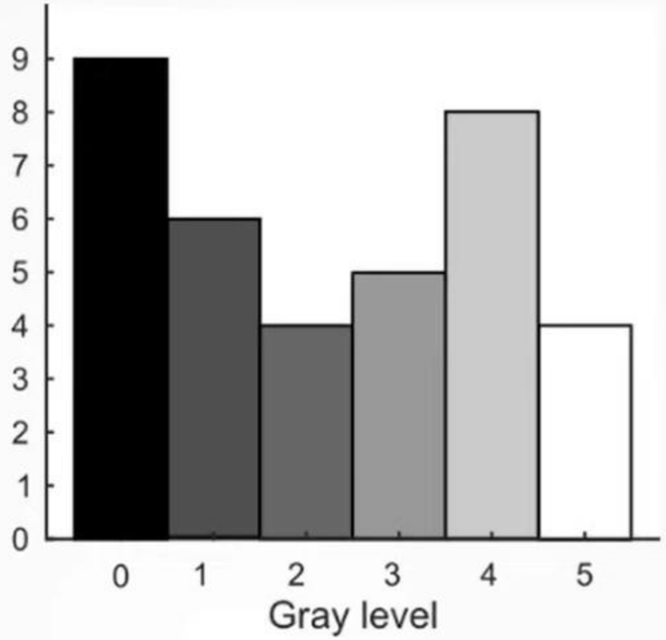
Foreground

$$W_f = \frac{4+5+8+4}{36} = 0.58$$

$$\mu_f = \frac{(4\times 2)+(5\times 3)+(8\times 4)+(4\times 4)}{4+5+8+4} = 3.57$$

$$\sigma_B^2 = W_b W_f \left(\mu_b - \mu_f\right)^2 = 2.44$$





I_t	0	1	2	3	4	5
W_b	0	0.25	0.42	0.53	0.67	0.89
μ_b	0	0	0.40	0.74	1.21	1.91
W_f	1	0.75	0.58	0.47	0.33	0.11
μ_f	2.25	3.00	3.57	3.94	4.33	5.00
σ_b^2	0	1.69	2.44	$\boxed{2.56}$	2.17	0.95

Key-Points

Otsu's method performs well when the histogram has a bimodal distribution with a deep and sharp valley between the two peaks.

Like all other global thresholding methods, Otsu's method performs badly in case of heavy noise, small objects size, inhomogeneous lighting and larger intra-class than inter-class variance.

In those cases, local adaptations of the Otsu method have been developed.

Moreover, the mathematical grounding of Otsu's method models the histogram of the image as a mixture of two Normal distributions with equal variance and equal size.

Otsu's thresholding may however yield satisfying results even when these assumptions are not met, in the same way statistical tests (to which Otsu's method is heavily connected) can perform correctly even when the working assumptions are not fully satisfied. However, several variations of Otsu's methods have been proposed to account for more severe deviations from these assumptions, such as the Kittler-Illingworth method.