

K23 - 0593

Muhammad Ayesha
Discrete Structures
Assignment #01.

Date: _____

1. Which of these structures are propositions? What are the truth values of those that are propositions?

(a) Boston is the capital of Massachusetts. YES TRUE

(b) Miami is the capital of Florida YES FALSE

(c) $2+3=5$ YES FALSE

(d) $5+7=10$ YES FALSE

(e) $x+2=11$ NO

(f) Answer this question NO.

2.

Smartphones	RAM	ROM	Resolution
Smartphone A	256 MB	32 GB	8 MP
Smartphone B	288 MB	64 GB	4 MP
Smartphone C	128 MB	82 GB	5 MP

(a) Smartphone B has the most RAM of these three smart
let, A represents "Smartphone A has the most RAM"
B represents "Smartphone B has the most RAM"
C represents "Smartphone C has the most RAM"

The statement can be expressed as: $B \wedge A \wedge \neg C$.

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TRUE. [Smartphone B indeed has the most RAM]

K23 - 0593

Muhammad Ayesha
Discrete Structures
Assignment H02.

Date:

1. Which of these structures are propositions? What are the truth values of those that are propositions?

(a) Boston is the capital of Massachusetts. YES : TRUE

(b) Miami is the capital of Florida YES FALSE.

(c) $2+3=5$ YES ~~FALSE~~ TRUE.

(d) $5+7=10$ YES FALSE

(e) $x+2=11$ NO

(f) Answer this question NO.

2.

Smartphones	RAM	ROM	Resolution
Smartphone A	256 MB	32 GB	8 MP
Smartphone B	288 MB	64 GB	4 MP
Smartphone C	128 MB	82 GB	5 MP

(a) Smartphone B has the most RAM of these three smartphones.

Let, A represents "Smartphone A has the most RAM".

B represents "Smartphone B has the most RAM".

C represents "Smartphone C has the most RAM".

The statement can be expressed as: B A \wedge A \wedge \neg C.

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TRUE. Smartphone B indeed has the most RAM.

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(b) Smartphone C has more ROM or a higher resolution camera than smartphone B.

~~Re Hc~~

Re represents "Smartphone C has more ROM than Smartphone B" (false)

Hc represents "Smartphone C has ~~higher resolution~~ camera than smartphone B (TRUE)"

RC V Hc

F V T

T

(c) Smartphone B has more RAM, more ROM, and a higher resolution camera than smartphone A.

Ram B represents "Smartphone B has more RAM than smartphone A" (True)

Ram B represents "Smartphone B has more ROM than smartphone A" (True)

~~HRB~~ HRB represents "Smartphone B has higher resolution than smartphone A" (False)

Ram B \wedge Ram B \wedge HRB

T \wedge T \wedge F

F Any

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(d) If smartphone B has more RAM and more ROM than smartphone C, then it also has a higher resolution camera.

Ram B represents "Smartphone B has more RAM than smartphone C" (True)

Rom B represents "Smartphone B has more ROM than smartphone C" (True)

H:B represents "Smartphone B has a higher resolution than smartphone C". (False).

$$(\text{Ram B} \wedge \text{Rom B}) \rightarrow \text{H.B.}$$

$$(\top \wedge \top) \rightarrow \text{F}$$

$$\top \rightarrow \text{F}$$

F Ans.

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Date:

- *) smartphone A has more RAM than smartphone B if and only if smartphone B has more RAM than smartphone A.

Ram A represents "Smartphone A has more RAM than Smartphone B"

Ram B represents "Smartphone B has more RAM than Smartphone A"

~~RAM B \leftrightarrow RAM A . RAM A \leftrightarrow RAM B~~
~~F \leftrightarrow T~~
~~P~~

Date: _____

3.

Revenue (billion) Net Profit (billion).

	Revenue (billion)	Net Profit (billion)
Acme	138	8
Nadir	87	5
Quixote	111	13

(a) Quixote Media had the largest annual revenue.

~~AR represents but A~~

QR represents "Quixote media had largest annual revenue". [False]

NR represents "Nadir Software had largest annual revenue". [False]

* AR represents "Acme Computer had largest annual revenue." [True].

$$QR \wedge (NR \wedge AR)$$

$$F A (F \wedge T)$$

Ans.

(b) Nadir software had the lowest net profit and Acme computer had the largest annual revenue.

NR represent "Nadir software had the lowest net profit" ~~[False]~~ [True].

AR represent "Acme had the largest annual revenue" ~~[False]~~. [True].

$$Np \wedge Ar$$

$$T \wedge T \quad \text{MIGHTY PAPER PRODUCT}$$

Ans.

(c) Acme computer had the largest net profit or Quixote Media had the largest net profit.

AP represents "Acme computer had the largest net profit. [False]

QP represents "Quixote media had the largest net profit". [True]

AP V QP

F V T

+n

(d) If Quixote Media had the smallest net profit, then Acme computer had the largest annual revenue.

QP represents "Quixote media had the smallest net profit". [False]

AR represents "Acme computers had the largest annual revenue". [True]

QP → AR

F → T

+n

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(e) Nadir software had the smallest net profit if and only if Acme computer had the largest annual revenue.

NP represents "Nadir software had the smallest net profit". [True].

AR represents "Acme computer had the largest annual revenue". [True].

$$NP \leftrightarrow AR$$

$$T \leftrightarrow T$$

$$\mathbb{B} T$$

4. Let P , q , and r be the propositions

P : You have the flu.

q : You miss the final examination.

r : You pass the course.

Express each of these propositions as an English sentence.

$$(a) P \rightarrow q.$$

If you have flu, then you miss the final examination.

$$(b) \neg q \leftrightarrow r$$

You do not miss the final examination if and only if you pass the course.

$$(c) q \rightarrow \neg r.$$

If you miss the final examination ~~if and only if~~ ^{then} you ~~fail~~ ^{pass} the course.

$$(d) P \vee q \vee r$$

You have the flu or miss the final examination or you pass the course.

$$(e) (P \rightarrow \neg r) \vee (q \rightarrow \neg r).$$

If you have flu then you fail the course, if you ~~miss~~ miss the final exam then you fail the course.

$$(f) (P \wedge q) \vee (\neg q \wedge r)$$

You have the flu and you miss the final examination or you didn't miss the final examination and pass the course.

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Or you didn't miss the final examination and pass the course.

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5. Let P , q and r be the propositions.

P : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in the class.

Write these propositions using P , q , and r and logical connectives.

(a) You get an A in this class, but you don't do every exercise in this book.

$$r \wedge \neg q.$$

(b) You get an A on the final, you do every exercise in this book, and you get an A in the class.

$$P \wedge q \wedge r.$$

(c) To get an A in this class, it is necessary for you to get an A on the final.

$$\rightarrow \text{Q} \quad r \rightarrow P \quad [\text{Form } q \text{ is necessary for } p]$$

(d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

$$(P \wedge \neg q) \wedge r$$

B.

(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

$$(P \wedge q) \rightarrow r \quad [\text{Form } P \text{ is sufficient for } q]$$

(f) You will get an A in this ~~final~~ class if and only if you either do every exercise in this book or you get an A on the final.

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$$r \leftrightarrow (q \vee P)$$

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6.

(R) P: I will remember

(a) $\neg P$: You send me an email message

q: I will remember to send you the address.

Form:- P only if q.

$\neg P \rightarrow q$: If you send me an email message, then I will remember to send you the address.

Form:- a

(b) P: You were born in the United State.

q: You are a citizen of this country.

Form: a sufficient condition for q, is p

$P \rightarrow q$: if you were born in the United state, then you are a citizen of this country.

~~Explain:~~

(c) P: If you keep your textbook

q: It will be a useful reference in your future courses

Form:- if P, q.

$P \rightarrow q$: If you keep your textbook, then it will a useful reference in your future courses.

(d) P: The Red wings' goalie plays well.

q: They will win the Stanley cup.

Form:- q if P.

$P \rightarrow q$: If the Red wings goalie plays well, then they will the stanley cup.

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(e) P: You get the job

q: You had the best credentials.

Form: P implies q.

$P \rightarrow q$: If you get the job, then you had the best credentials.

(f) P: ~~If~~ There is a storm.

q: The beach erodes.

Form: q whenever P.

$P \rightarrow q$: If there is a storm, then the beach erodes.

(g) ~~If~~ P: You have a valid password. You can log onto the server.

q: ~~To~~ log on to the server. You have a valid password.

Form:- q is necessary for p.

If you can log onto the server you have a valid password.

h) ~~If~~ P: You begin your climb too late.

q: You will reach the summit.

Form: q unless P.

If you will reach the summit, then you begin your climb too late.

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7. (a)

1. It is sunny tomorrow only if I will go for a walk tomorrow in the woods. " P only if q ".
2. ~~I want to~~ ^{necessary} go for a walk tomorrow in the woods if it is sunny tomorrow.
3. I will go for a walk in the woods if it is sunny tomorrow.
4. I will go for a walk in the woods when it is sunny tomorrow.
5. I will go for a walk in the woods unless it is not sunny tomorrow.

(b) 80

Converse $q \rightarrow P$ go for a walk
If I will ~~tomorrow~~ for a walk in the woods, then it is sunny tomorrow.

Inverse $\neg P \rightarrow \neg q$

If it is not sunny tomorrow, then I will not go for a walk in the woods.

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Contrapositive $\neg q \rightarrow \neg p$.

If I $\overset{\text{do not}}{\underset{\text{will}}{\cancel{\text{will}}}}$ go for a walk in the woods, then it is not sunny tomorrow.

(C)

→ Inverse of its inverse $p \rightarrow q$.

If I $\cancel{\text{will}}$ go for a walk in the woods, then it is sunny tomorrow.

Inverse of its converse $\neg q \rightarrow \neg p$

If I $\cancel{\text{will}}$ not go for a walk in the woods, then it is not sunny tomorrow.

Inverse of its contrapositive.

If I $\cancel{\text{will}}$ go for a walk tomorrow in the woods, then it is sunny tomorrow.

8.

(a) Jan is rich and happy.

 p : Jan is rich q : Jan is happy

$$\neg(p \wedge q) \equiv \neg p \vee \neg q. \sim \text{DeMorgan's Law}$$

 $\neg p \vee \neg q$: Jan is not rich or not happy.

(b) Carlos will bicycle or run tomorrow.

 p : Carlos will bicycle tomorrow q : Carlos will run tomorrow.

$$\neg(p \vee q) \equiv \neg p \wedge \neg q. \sim \text{DeMorgan's Law}$$

Carlos will not bicycle tomorrow and Carlos will not run tomorrow.

(c) The fan is slow or it is very hot.

 p : The fan is slow q : The fan is very hot.

$$\neg(p \vee q) \equiv \neg p \wedge \neg q.$$

 $\neg p \wedge \neg q$: The fan is not slow and the fan is not very hot.

(d) Akram is unfit and Saleem is injured.

 p : Akram is unfit q : Saleem is injured.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q.$$

Akram is fit ~~and~~^{or} Saleem is not injured.**MIGHTY PAPER PRODUCT**

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9.

- (a) Exclusive OR.
- (b) Inclusive OR.
- (c) Inclusive OR
- (d) Inclusive OR.

10.

$$(a) (P \wedge (\neg(\neg P \vee q))) \vee (P \wedge q) \equiv P.$$

Sol:

$$\Rightarrow (P \wedge (\neg(\neg P \vee q))) \vee (P \wedge q)$$

\therefore DeMorgan's law.

$$\Rightarrow (P \wedge (\neg\neg P \wedge \neg q)) \vee (P \wedge q)$$

\therefore Double Negation law.

$$\Rightarrow (P \wedge (P \wedge \neg q)) \vee (P \wedge q)$$

~~Distributive law.~~

$$\Rightarrow (P \wedge P \wedge \neg q) \vee (P \wedge q)$$

$\therefore P \wedge P \equiv P$ Idempotent law.

$$\Rightarrow (P \wedge \neg q) \vee (P \wedge q)$$

\therefore Distributive law

$$\therefore P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$$

$$\Rightarrow P \wedge (\neg q \vee q)$$

~~P~~ \therefore Negation Law $\therefore P \vee \neg P \equiv T$

$$P \wedge T$$

\therefore Identity law $\therefore P \wedge T \equiv P$

P

Hence Proved!

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$$(b) \neg(P \leftrightarrow q) \equiv (P \neq q)$$

SOL:

we have to express both conditions.

L.H.S =

$$\neg(P \leftrightarrow q) \equiv \neg((P \wedge q) \vee (\neg P \wedge \neg q)) \quad [P \leftrightarrow q \equiv (P \wedge q) \vee (\neg P \wedge \neg q)]$$

R.H.S

using DeMorgan's law.

$$\Rightarrow \neg(P \wedge q) \wedge \neg(\neg P \wedge \neg q) \quad \text{using DeMorgan's law.}$$

$$\Rightarrow \neg(P \wedge q) \wedge \neg(\neg P) \vee \neg(\neg q)$$

~~Double Negation Law~~

$$\Rightarrow \neg(P \wedge q) \wedge (P \vee q)$$

\therefore De Morgan's law

$$\Rightarrow (\neg P \vee \neg q) \wedge (P \vee q) \quad P \wedge (q \vee r) \equiv \neg q \vee \neg r$$

~~$\neg P \wedge P$~~ using distributive law.

$$\Rightarrow (\neg P \vee \neg q) \wedge P \vee (\neg P \vee \neg q) \wedge q$$

\therefore using distributive law.

$$\Rightarrow ((\neg P \wedge P) \vee (\neg q \wedge P)) \vee ((\neg P \wedge q) \vee (\neg q \wedge q))$$

$\therefore \neg P \wedge P \equiv F$ and $\neg q \wedge q \equiv F$ using Negation laws

$$\Rightarrow F \vee (\neg q \wedge P) \vee ((\neg P \wedge q) \vee F) \quad \therefore$$

\neg Distributive law

$$((F \vee \neg q) \wedge (F \vee P)) \vee ((\neg P \wedge F) \wedge (q \vee F)).$$

\therefore Identity law and \neg Commutation law.

$$(\neg q \wedge P) \vee (\neg P \wedge q)$$

\therefore Identity law \leftarrow

$$\Rightarrow (\neg q \wedge P) \vee (\neg P \wedge \neg q)$$

Hence Proved.

Date: _____

(c) $\neg P \leftrightarrow q \equiv P \leftrightarrow \neg q.$

$$\neg P \leftrightarrow q \equiv (\neg P \wedge q) \vee (P \wedge \neg q)$$

$$P \leftrightarrow \neg q \equiv (P \wedge \neg q) \vee (P \wedge q)$$

using commutative law.

$$(\neg P \wedge q) \vee (\neg P \wedge q)$$

Hence Proved.

(d) $(P \wedge q) \rightarrow (P \rightarrow q) \equiv T$

$$\neg P \rightarrow q \equiv \neg P \vee q$$

$$\neg(P \wedge q) \vee (\neg P \rightarrow q)$$

\therefore De Morgan's law and $P \rightarrow q \equiv \neg P \vee q.$

$$(\neg P \vee \neg q) \vee (\neg P \vee q)$$

\therefore Distributive law.

\therefore commutative law. \therefore associative law.

$$(\neg P \vee \neg q \vee \neg P) \vee q$$

\therefore Idempotent law $\neg P \vee \neg P \equiv \neg P$

$$(\neg P \vee \neg q) \vee q$$

\therefore Associative law.

$$\neg P \vee (\neg q \vee q)$$

\therefore Negation law.

$$\neg P \vee T$$

\therefore Domination law

$$T \equiv T$$

T \therefore ans.

$$(e) \neg(P \vee \neg(P \wedge q)) \equiv F$$

Sol:

$$\neg(P \vee \neg(P \wedge q))$$

\therefore De Morgan's Law.

$$\neg P \wedge \neg(\neg(P \wedge q))$$

~~$\neg\neg$~~ \therefore Double Negation Law.

$$\neg P \wedge (P \wedge q)$$

\therefore Associative Law.

$$(\neg P \wedge P) \wedge q$$

~~$\neg\neg$~~ \therefore Negation Law.

$$F \wedge q$$

\therefore Domination Law.

$$\boxed{F} \quad \text{Hence Proved!}$$

Date:

ii. Using truth table, show that these compound propositions are logically equivalent or not.

(a) $(P \rightarrow r) \wedge (q \rightarrow r)$ and $(P \vee q) \rightarrow r$

P	q	r	$P \rightarrow r$	$q \rightarrow r$	$(P \rightarrow r) \wedge (q \rightarrow r)$	$P \vee q$	$(P \vee q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
F	P	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	P	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Logically equivalent

(b) $(P \rightarrow q) \vee (P \rightarrow r)$ and $P \rightarrow (q \vee r)$

P	q	r	$P \rightarrow q$	$P \rightarrow r$	$(P \rightarrow q) \vee (P \rightarrow r)$	$q \vee r$	$P \rightarrow (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

Logically equivalent

mighty paper product

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(c) $(P \rightarrow q) \rightarrow (r \rightarrow s)$ and $(P \rightarrow r) \rightarrow (q \rightarrow s)$

P	q	r	s	$P \rightarrow q$	$r \rightarrow s$	$(P \rightarrow q) \rightarrow (r \rightarrow s)$	$P \rightarrow r$	$q \rightarrow s$	$(P \rightarrow r) \rightarrow (q \rightarrow s)$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	F	F
T	T	F	T	T	T	T	F	T	T
T	F	F	F	T	T	T	F	F	T
T	F	T	F	T	T	T	T	T	T
T	F	F	F	F	F	T	T	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	T	F
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

Not logically equivalent.

Let $P(m, n)$ be the statement "m divides n" where the domain for both variables consists of all positive integers.

(By "m divides n" we mean that $n = km$ for some integer k.)
Determine the truth values of each of these statements.

(a) $P(4, 5)$

Statement: 4 divides 5.

Explanation: 5 cannot be expressed as $4 \times k$ for any integer k.

Truth value: False.

(b) $P(2, 4)$

Statement: 2 divides 4

Explanation: 4 can be expressed as 2×2 for $k=2$

Truth value: True.

(c) $\forall m \forall n P(m, n)$

Statement: For all positive integers m and n, m divides n.

Explanation: This statement is false because not every integer m divides every integer n. For example 2 does not divide 3.

Truth value: False.

(d) $\exists m \forall n P(m, n)$

Statement: There exists a positive integer 'm' such that ~~that divides~~ for all the positive integers n, m divides n.

Explanation: For m to divide every n, must be like the only integer that divides all positive integers is 1.

Truth value: ~~False~~. True.

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13. determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

(a) $\exists x (x^2 = 2)$

sol:

Statement :- There exist x for which $x^2 = 2$.

Explanation:- The solution are $x=\sqrt{2}$ and $x=-\sqrt{2}$ they both are real nos.

Truth value : True.

(b) $\exists x (x^2 = -1)$

statement: There exist a real no x for which $x^2 = -1$

Explanation: x^2 will always give a positive real no hence, there is no such value of x which gives the -1 .

Truth value : False.

(c) $\forall x (x^2 + 2 > 1)$

Statement : For all

Explanation: Not all real numbers 'x' will be greater than 1.

Example $x=2 \rightarrow x^2 + 2 > 1 \rightarrow 4 + 2 > 1 \leftarrow$ This is wrong.

Truth value : False.

(d) $\exists x (x^2 = x)$

statement: There exists some real no x for which $x^2 = x$

Explanation:- If $x=1$ then the equation $1=1$ which is true.

Truth value : True.

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(c) $\forall x (x^2 + 2 > 1)$

Sol:-

Statement : For all ~~is~~ real nos x $x^2 + 2 > 1$

Explanation: ~~For~~ For all real number x , $x^2 > 0$ add '2' on both side $x^2 + 2 > 2$, hence for all real nos x $x^2 + 2 > 1$

Truth value : True.

Date:

14. Let $F(x, y)$ be the statement " x can fool y " where the domain consist of all people in the world. Use quantifiers to express each of these statements.

(a) Everybody can fool Bob.

$\forall x F(x, \text{Bob})$. For all people x , x can fool Bob.

(b) Alice can fool every body.

$\forall y F(\text{Alice}, y)$ For all people y , Alice can fool y .

(c) ~~Everybody~~ can fool somebody.

~~$\forall x \exists y F(x, y)$~~ For every person x , there exist a person y such that x can fool y .

(d) There is no one who can fool everybody.

$\neg \exists x \forall y F(x, y)$ There does not exist a person x such that x can fool everybody.

(e) Everyone can be fooled by somebody.

$\forall y \exists x F(x, y)$ For every person y , there exists a person x such that x can fool y .

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15.

(a) $\exists x (P(x) \wedge Q(x))$

(b) $\exists x (P(x) \wedge \neg Q(x))$

(c) $\forall x (P(x) \vee Q(x))$

(d) $\forall x (\neg P(x) \wedge \neg Q(x))$

16.

(a) $\exists x \exists y Q(x, y)$

There exist a student x and there exist a student y such that x has sent email to y .

(b) $\exists x \forall y P(x, y)$

There exist a student x such that x has sent a email to ~~all~~ every student y .

(c) $\forall x \exists y P(x, y)$

For all students x , there exist a student y such that every student x has sent an email to y .

(d) $\exists y \forall x Q(x, y)$

There exist ^a student y , ~~such~~ such that y has received an email from all students x .

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(e) $\forall y \exists x P(x, y)$

For all students y , there exists a student x such that all student y has received an email from x .

(f) $\forall x \forall y P(x, y)$

for all students x , and y such that all students x have sent email to all students y .

17.

(a) $\exists x \exists y P(x, y)$

There is a student in the class who has taken at least one computer science course.

(b) $\exists x \forall y P(x, y)$.

There is a student in the class who has taken all the computer science course.

(c) $\forall x \exists y P(x, y)$

All the students in the class has taken at least one CS course.

(d) $\exists y \forall x P(x, y)$

There is a CS course that every student in the class has taken.

(e)

Date: _____

(e) $\forall y \exists x P(x, y)$

All CS Co

for Every CS Course ~~is~~ taken by at least one student in the class.

(f) $\forall x \forall y P(x, y)$

Every student in the class has taken ~~one~~ all the CS courses.

Q8

18-

- (a) Addition.
- (b) Simplification.
- (c) Modus Ponens.
- (d) Modus Tollens.
- (e) Hypothetical Syllogism.

19-

P: Today is Tuesday.

Q: I have a test in Maths

R: I have a test in Econ

19.

- (A) P: Today is Tuesday.
 q: I have a test in Mathematics.
 r: I have a test in Economics.
 S: My economics professor is sick.

Premises:

$$P \rightarrow (q \vee r)$$

$$S \rightarrow \neg r$$

$$P \wedge S$$

 \textcircled{a}

Conclusion: q.

- | | |
|-------------------------------|-----------------------|
| 1. $P \wedge S$ | Premise |
| 2. P | Simplification on ①. |
| 3. $P \rightarrow (q \vee r)$ | Premise. |
| - 4. $q \vee r$ | Modus ponens on ② & ③ |
| 5. S | Simplification ④ |
| 6. $S \rightarrow \neg r$ | Premise |
| 7. $\neg r$ | modus ponens on ⑤ & ⑥ |
| 8. $r \vee q$ | commutative on ⑦ |
| 9. \boxed{q} | Distributive on ⑧ & ⑨ |

~~1. $P \wedge S$~~
~~Premises~~

1. (B) P: Ali is a lawyer.

q: He is ambitious.

r: Ali is an early riser.

S: He likes chocolates.

Conclusion Premises:

$$P \rightarrow q$$

$$r \rightarrow \neg s$$

$$q \rightarrow r$$

- | | |
|-----------------------------------|------------------------------------|
| 1. $P \rightarrow q$ | Premise |
| 2. $q \rightarrow r$ | Premise |
| 3. $P \rightarrow r$ | Hypothetical Syllogism
① & ② |
| 4. $q \rightarrow \neg s$ | Premise |
| 5. $\boxed{P \rightarrow \neg s}$ | Hypothetical Syllogism or
③ & ④ |

Conclusion: $P \rightarrow \neg s$.

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$$20(a) (A \cap B) \cap \bar{C}$$

$$A \cap B = \{1, 2, 4, 5\} \cap \{2, 3, 5, 6\}$$

$$A \cap B = \{2\}$$

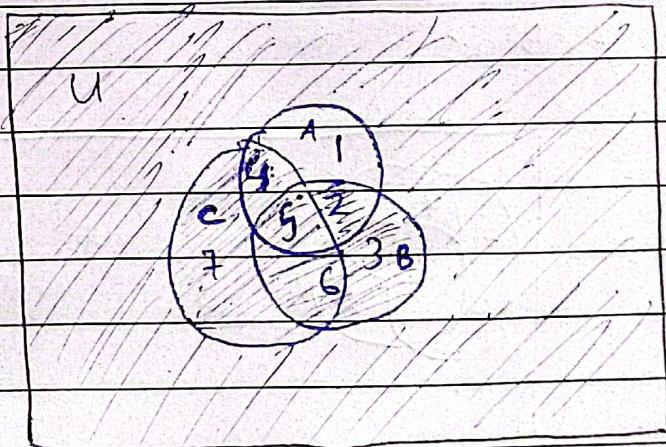
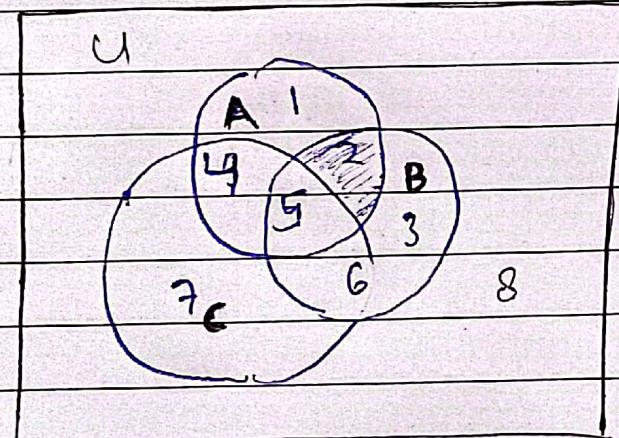
(~~A ∩ B~~)

$$\bar{C} = U - C = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 5, 6, 7, 8\}$$

$$\bar{C} = \{1, 2, 3\}$$

$$(A \cap B) \cap \bar{C} = \{2\} \cap \{1, 2, 3\}$$

$$(A \cap B) \cap \bar{C} = \{2\}$$



$$(b) \bar{A} \cup (B \cup C)$$

Sol:

$$B \cup C = \{2, 3, 5, 6\} \cup \{4, 5, 6, 7\}$$

$$B \cup C = \{2, 3, 4, 5, 6, 7\}$$



$$\bar{A} = U - A = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 2, 4, 5\}$$

$$\bar{A} = \{3, 6, 7, 8\}$$

MIGHTY PAPER PRODUCT

$$\bar{A} \cup (B \cup C) = \{2, 3, 4, 5, 6, 7, 8\}$$

$$\begin{aligned}(A - B) \cap C &= \{1, 2\} \cap \{4, 5, 6, 7\} \\ &= \{\}\end{aligned}$$

$$(c) (A - B) \cap C$$

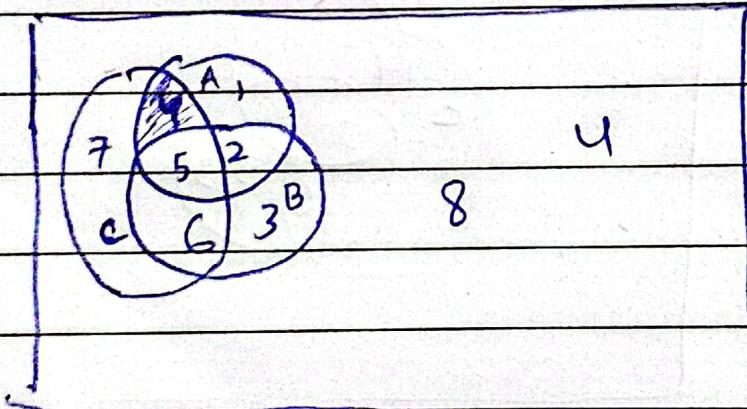
Sol:

$$A - B = \{1, 2, 4, 5\} \cap \{2, 3, 5, 6\}$$

$$A - B = \{1, 4\}$$

$$(A - B) \cap C = \{1, 4\} \cap \{4, 5, 6, 7\}$$

$$(A - B) \cap C = \{4\}$$



MIGHTY PAPER PRODUCT

Date:

$$(d) (A \cap B) \cup C$$

so h

$$(A \cap B) \cup C$$

$$B - U - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 3, 5, 6\}$$

$$\bar{B} = \{1, 4, 7, 8\}$$

$$A \cap \bar{B} = \{1, 2, 4, 8\} \cap \{1, 4, 7, 8\}$$

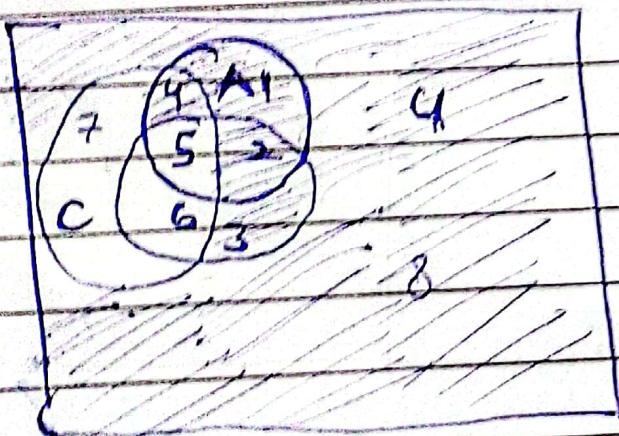
$$A \cap \bar{B} = \{1, 4\}$$

$$(A \cap \bar{B}) \cup \bar{C}$$

$$\bar{C} = U - C = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 5, 6, 7\}$$

$$\bar{C} = \{1, 2, 3, 8\}$$

$$(A \cap \bar{B}) \cup \bar{C} = \{1, 2, 3, 8\} \cup \cancel{\{4, 7\}}$$



Date: _____

$$(A \cap B) \cup C$$

Sol:

$$(A \cap B) \cup C$$

$$\bar{B} = U - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 3, 5, 6\}$$

$$\bar{B} = \{1, 4, 7, 8\}$$

$$A \cap \bar{B} = \{1, 2, 4, 5\} \cap \{1, 4, 7, 8\}$$

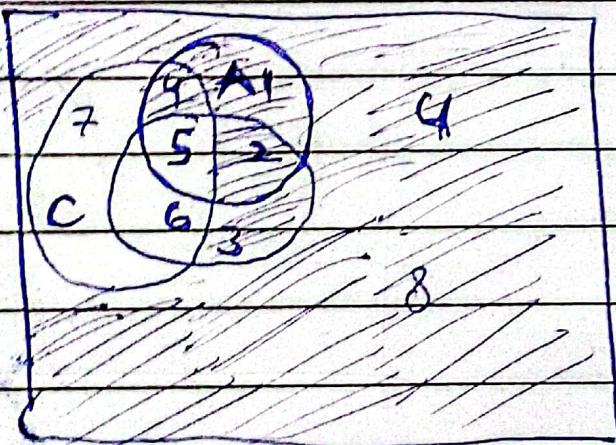
$$A \cap \bar{B} = \{1, 4\}$$

$$(A \cap \bar{B}) \cup \bar{C}$$

$$\bar{C} = U - C = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 5, 6, 7\}$$

$$\bar{C} = \{1, 2, 3, 8\}$$

$$(A \cap \bar{B}) \cup \bar{C} = \{1, 2, 3, \cancel{\{4, 5, 6, 7\}} \} \cup \{1, 2, 3, 8\}$$



Q1.

(a) $(A - (A \cap B)) \cap (B - (A \cap B)) = \emptyset$

Solution:-

$$(A \cap (\bar{A} \cap B)) \cap (B - (A \cap B))$$

\therefore Associative law.

$$(A \cap B) \cap ((\bar{A} \cap B) \cap (\bar{A} \cap B))$$

\therefore ~~AAA~~ Idempotent law.

$$(A \cap B) \cap (\bar{A} \cap B)$$

$\therefore A \cap \bar{A} = \emptyset$ complement law.

$\boxed{\emptyset}$

Hence proved!!!

(b) $(A - B) \cup (A \cap B) = A$

Sol:-

$$(A \cap \bar{B}) \cup (A \cap B) = A$$

\therefore Distributive law.

$$A \cap (\bar{B} \cup B)$$

\therefore Complementation law.

$$A \cap U$$

\therefore Identity laws.

\boxed{A}

Hence Proved!!!

Date:

$$(c) (A - B) - C = (A - C) - B$$

Sol:

$$(A \cap \bar{B}) - C$$

$$(A \cap \bar{B}) \cap \bar{C}$$

∴ associative

$$(A \cap \bar{C}) \cap \bar{B}$$

$$(A - C) \cap \bar{B}$$

$$\boxed{(A - C) - B} \quad \text{Hence Proved!}$$

$$(d) \overline{B \cup (\bar{B} - A)} = B$$

Sol:

$$\overline{B \cup (\bar{B} - A)}$$

∴ De Morgan's

$$\overline{\bar{B} \cup \overline{\bar{B} - A}}$$

∴ complementation law

$$B \cap \overline{\bar{B} - A}$$

∴ De Morgan's

$$\overline{B \cup \overline{\bar{B} \cup \bar{A}}}$$

∴ De Morgan's

$$B \cap \overline{\bar{B} \cup \bar{A}}$$

∴ complementation

$$\boxed{B \cap (B \cup A)}$$

∴ Absorbent

$$\boxed{B}$$

Hence Proved!!!

Date:

20

$$n(A) = 100$$

$$n(W) = 20$$

$$n(B) = 15$$

$$n(W \cap B) = 10.$$

$$n(W \cup B) = n(W) + n(B) - n(W \cap B) =$$

$$n(W \cup B) = \cancel{20} + 15 - 10 = 25$$

Number of apples that can be sold = $100 - n(W \cup B) = 100 - \boxed{25}$

= 75

Ans

The no of apples that can be sold is 75.

(b)

$$n(U) = 1000$$

$$n(CS) = 350$$

$$n(SE) = 450$$

$$n(CS \cap SE) = 100$$

$$\begin{aligned} n(CS \cup SE) &= n(CS) + n(SE) - n(CS \cap SE) \\ &= 350 + 450 - 100 \\ &= 700 \quad \text{like either.} \end{aligned}$$

$$\begin{aligned} n(U) - n(CS \cup SE) &= 1000 - 700 \\ &= 300 \quad \text{like neither.} \end{aligned}$$

$$n(M) = 78$$

$$n(I) = 32$$

$$n(E) = 57$$

~~$$n(M \cup I \cup E) = 13$$~~

~~$$n(M \cap I \cap E) = 21$$~~

~~$$n(M \cap I) = 16$$~~

~~$$n(I \cap E) = 5$$~~

~~$$n(E \cap M) = 5$$~~

$$n(M \cap I) = 13$$

$$n(I \cap T) = 21$$

$$n(E \cap M) = 16$$

$$n(M \cap I \cap T) = 5$$

$$n(M \cup I \cup T) = n(M) + n(I) + n(T) - n(M \cap I) - n(I \cap T) \\ - n(T \cap M) - n(T \cap M \cap I)$$

$$= 78 + 32 + 57 + 57 - 13 - 21 - 16 - 5$$

$$= 112$$

but 14 like none of 3 ~~free~~ flavors

$$n(U) = 112 + 14$$

$$= \boxed{136} \text{ ans}$$

(d)

MIGHTY PAPER PRODUCT

Date: _____

(d) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Left hand side:

$A \times (B \cap C)$ represents all ordered pairs (a, b) where
• $a \in A$
• $b \in B \cap C$ (i.e. b belongs to both B and C).

In set-builder notation, this can be written as:

$$A \times (B \cap C) = \{(a, b) \mid a \in A \text{ and } b \in B \cap C\}$$

since $b \in B \cap C$ means that $b \in B$ and $b \in C$.

$$A \times (B \cap C) = \{(a, b) \mid a \in A \text{ and } b \in B \text{ and } b \in C\}$$

Right hand side

$$(A \times B) \cap (A \times C)$$

• $A \times B$ represents all ordered pairs (a, b) where $a \in A$ and $b \in B$ i.e.

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

• $A \times C$ represents all ordered pairs (a, c) where $a \in A$ and $c \in C$.

$$A \times C = \{(a, c) \mid a \in A \text{ and } c \in C\}.$$

Thus the intersection $(A \times B) \cap (A \times C)$ consists of all ordered pairs (a, b) that are in both $A \times B$ and $A \times C$. In other words, the are the ordered pairs who **MIGHTY PAPER PRODUCT**

$$\begin{aligned} &: a \in A \\ &: b \in C \end{aligned}$$

Date: _____

Thus in set builder notation , we can express this as:

$$(A \times B) \cap (A \times C) = \{(a, b) \mid a \in A \text{ and } b \in B \text{ and } b \in C\}$$

Hence both sides are equal