Intro to Statistics.

Chapter 5.1

Ex 7.

(a) First T time that a six is rolled up.

T-1 : rolled not six

$P(T) = \left(\frac{5}{6}\right)^{T-1} \cdot \frac{1}{6}$

(b) $P(T>3) = 1 - P(T\leq 3) = 1 - P(T=1) - P(T=2) - P(T=3)$

$= 1 - \frac{1}{6} - \frac{5}{6}\cdot\frac{1}{6} - \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$

$= \frac{216-36-30-25}{216} = \frac{125}{216}$

(c) $P(T>6 \mid T>3) = \frac{P(T>6 \wedge T>3)}{P(T>3)} = \frac{P(T>6)}{P(T>3)}$

$= \frac{1-P(T\leq 6)}{P(T>3)} = \frac{1 - \frac{1}{6} - \frac{5}{6^2} + \frac{25}{6^3} + \frac{75}{6^4} + \frac{75\cdot 5}{6^5} + \frac{75\cdot 25}{6^6}}{\frac{125}{216}}$

$= \frac{125}{216}.$

Ex. 10

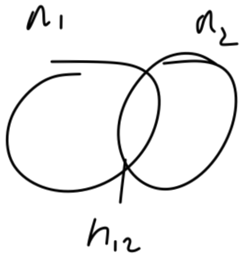
N = actual population

n_1 = people counted 1st time

n_2 = people counted 2nd time

n_{12} = people counted both time

$0 < k < n_2$.



① $P(X=k)$

$$\frac{\binom{n_1}{k}\binom{N-n_1}{n_2-k}}{\binom{N}{n_2}}$$

② Assume $X = n_{12}$

When
$$\frac{\binom{n_1}{n_{12}}\binom{N-n_1}{n_2-n_{12}}}{\binom{n_1}{n_{12}}\binom{N+1-n_1}{n_2-n_{12}}} = \frac{(N-n_1)!\,(N+1-n_1-n_2+n_{12})!}{(N-n_1-n_2+n_{12})!\,(N+1-n_1)!}$$

$$= \frac{N+1-n_1-n_2+n_{12}}{N+1-n_1}$$

$$= 1 - \frac{n_2-n_{12}}{N+1-n_1} \Rightarrow 0$$

$$N = n_2 - n_{12} + n_1 - 1$$

Ex. 1b

$n = 5 \cdot 60$

$P = 0.01$

$\lambda = 5 \cdot 60 \cdot 0.01 = 3$ calls missed per 5 min.

Poisson distribution.

$$P(X=k) = \frac{e^{-3}3^k}{k!}$$

$$P(X \leq 1) = P(X=0) + P(X=1)$$

$$= \frac{e^{-3}3^0}{0!} + \frac{e^{-3}3}{1!} = \frac{4}{e^3}$$

Ex 18.

(a) $\lambda_1 = \dfrac{600}{500} = 1.2$

$$P(X=k) = \frac{e^{-1.2}\,1.2^k}{k!}$$

$$P(X=0) = e^{-1.2} = 0.3.$$

(b) $\lambda_2 = \dfrac{400}{500} = 0.8$

$$P(Y=2) = \frac{e^{-0.8}\,0.8^2}{2!} = 0.14$$

(c)

$$1 - P(X+Y<2) = 1 - \big(P(X=1, Y=0) + P(X=0, Y=0) + P(X=0, Y=1)\big)$$

$$= 1 - (0.15 \times 0.45 + 0.05 \times 0.45 + 0.05 \times 0.36)$$

$$= 0.89.$$

Ex. 25

P(caught) = 0.05

cost(T=1) = 0

cost(T=2) = 2

cost(T≥3) = 5.

$$E = 2\binom{100}{2}0.05^{2}0.95^{98} + 7\binom{100}{3}0.05^{3}0.95^{97} + \cdots (7+5\cdot 98)\binom{100}{100}\cdot 0.05^{100}\cdot 0.98^{0}$$

Ex. 27.

$p = 0.001$

$$P(X\geq 1) = 1 - P(X=0) = 1 - e^{-0.001(100)} = 0.0\cancel{5}5$$

Ex. 28.

X denotes # people not shown up.

$$P(X\geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - \frac{e^{-4}4^{0}}{0!} - \frac{e^{-4}4^{1}}{1!} = 1 - 5e^{-4}$$

Ex. 38

(a) $P = \frac{5}{20} = 0.25$

$P(X=1) = \binom{5}{1} 0.25^1 0.75^4 = 0.4$ (binomial)

(b)

$$P(X=1) = \frac{\binom{5}{1}\binom{20-5}{5-1}}{\binom{20}{5}} = 0.44 \text{ (hypergeometric)}$$

Chapter 5.2.

Ex. 1

(a)

when $2 \le x \le 3$

$$f(x) = \frac{1}{3-2} = 1$$

$$F(x) = x-2 = \frac{x-2}{3-2} = x-2.$$

(b) $Y = g(U) = U^3 \Rightarrow U = Y^{\frac{1}{3}}$

$$f(y) = \frac{du}{dy} = \frac{1}{3} y^{-\frac{2}{3}}, \quad F(y) = \int_0^y \frac{1}{3} y^{-\frac{2}{3}} = y^{\frac{1}{3}}$$

Ex. 17

(a)

$$f(x) = \begin{cases} \pi \cos\left(\frac{\pi x}{2}\right) \sin\left(\frac{\pi x}{2}\right) & 0 \le x \le 1 \\ 0 & \text{o.w.} \end{cases}$$

(b)

$$\sin^2\left(\frac{\pi \cdot \frac{1}{4}}{2}\right) = 0.146.$$

Ex. 21.

$$\text{CDF of } F(X) = P(F(X) \le x)$$

$$= P(X \le F^{-1}(x)) = x.$$

Ex. 37.

$$f_X(x) = \frac{d}{dx} P(X \le x) = \frac{d}{dx} P(\log(X) \le \log(x))$$

$$= \frac{d}{dx} \text{CDF}\left(\frac{\log x - \mu}{\sigma}\right)$$

$$= \text{pdf}\left(\frac{\log x - \mu}{\sigma}\right) \frac{d}{dx}\left(\frac{\log x - \mu}{\sigma}\right)$$

$$= \text{pdf}\left(\frac{\log x - \mu}{\sigma}\right) \frac{1}{\sigma x}$$

$$= \frac{1}{x\sqrt{2\pi} \cdot \sigma} \cdot e^{-\frac{(\log x - \mu)^2}{2\sigma^2}} \qquad \sigma^2 = 1, \ \mu = 0$$

$$= \frac{1}{\sqrt{2\pi} x} e^{-\frac{\log^2 x}{2}}$$