MA677 HW6
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Hypothesis Testing

1. $X \sim$ exponential(λ). Test $H_0 : \beta \geq 1$, $H_1 : \beta < 1$. Reject H_0 if $X \geq 1$.
   (a) Find the power function of the test.
   (b) Compute the size of the test.

a) Power function $\beta(\beta) = P_\beta(X \geq 1) = 1 - (1 - e^{-\lambda}) = e^{-\lambda}$

b) size $\alpha = \sup\limits_{\beta \geq 1} (\beta)$

2. The proportion p of defects in a large population of items is unknown.

   A random sample of 20 items is drawn from the population. Y is the number of defective items in the sample.

   Test the following hypotheses:

   $H_0 : p = 0.2$
   $H_1 : p \neq 0.2$

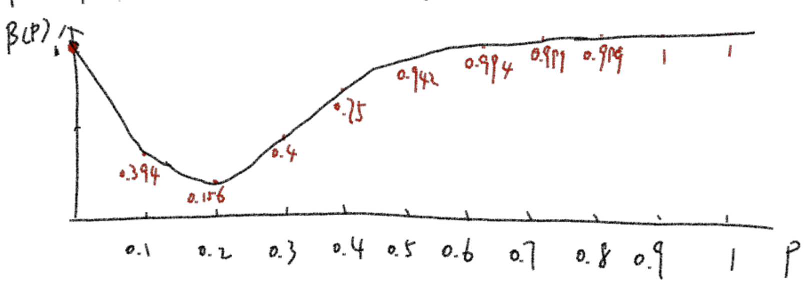
   The critical region is defined as $R = Y \geq 7$ or $Y \leq 1$.

   (a) Determine the value of the power function at the points p = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1; sketch the power function.
   (b) Determine the size of the test.

$f(y) = \binom{20}{y} p^y (1-p)^{20-y}$

$H_0 = p = 0.2 \Rightarrow$ reject when $Y \leq 1$, $Y \geq 7$.

$\beta = P(Y \leq 1) + P(Y \geq 7) = \sum_0^1 \binom{20}{i} p^i (1-p)^{20-i} + \left(1 - \sum_0^6 \binom{20}{i} p^i (1-p)^{20-i}\right)$



size $= \beta_{0.2}(p) = 0.156$

3. $X_1, \cdots, X_n \sim N(\mu, \sigma^2 = 1)$, μ unknown.

For a fixed μ_0, test the hypotheses
$H_0 : \mu = \mu_0$,
$H_1 : \mu \neq \mu_0$

Assume that the sample size $n = 25$. Use test statistics $T(X) = |\bar{X}_n - \mu_0|$ and reject H_0 if $T(X) > c$ Find value of c such that the size of the test is 0.05.

$$\beta(\mu) = P_\mu(\bar{x} > c)$$

$$= P\left(\frac{\sqrt{n}(\bar{x} - c)}{\sigma} > \frac{\sqrt{n}(c-\mu)}{\sigma}\right)$$

$$= P\left(Z > \frac{\sqrt{n}(c-\mu)}{\sigma}\right)$$

$$= 1 - \Phi\left(\frac{\sqrt{n}(c-\mu)}{\sigma}\right)$$

$$\alpha = 0.05 = 1 - \Phi\left(\frac{\sqrt{n}(c-\mu)}{\sigma}\right)$$

$$\Rightarrow c = \frac{\Phi^{-1}(0.975)}{\sqrt{25}} = 0.39$$

4. $X_1, \cdots, X_9 \sim \text{Bernoulli}(p)$. Let $Y = \sum_{i=1}^{9} X_i$

Test
$H_0 : p = 0.4$
$H_1 : p \neq 0.4$

(a) Find c_1 and c_2 such that
$P(Y \leq c_1 | p = .4) + P(Y \geq c_2 | p = .4)$
is as close as possible to 0.1 without being larger than 0.1.

(b) Assuming that H_0 is rejected if either $Y \leq c_1$ or $Y \geq c_2$, what is the size of the test?

a) $P(Y \leq c_1 | P = 0.4) + P(Y \geq c_2 | P = 0.4)$

$= \sum_{0}^{c_1} C_9^i \, 0.4^i \, 0.6^{9-i} + 1 - \sum_{0}^{c_2 - 1} C_9^i \, 0.4^i \, 0.6^{9-i}$

$c_1 = 1$. $c_2 = 7$, the equation $= 0.0956 \approx 0.1$

b) $\alpha_{0.4} = 0.0956$