ISLR Ch.3 Exercise

MSSP MA679

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3.7.1 Describe the null hypotheses to which the p-values given in Table 3.4 correspond. Explain what conclusions you can draw based on these p-values. Your explanation should be phrased in terms of *sales*, *TV*, *radio*, and *newspaper*, rather than in terms of the coefficients of the linear model.

Intercept coefficient: The null hypothese to intercept coeffient is that we would expect an average sales of $0 with no budgets in TV ads, radio ads or newspaper ads. With p-value < 0.00001, we would reject this null hypothesis at least 99.99\% confidence level, meaning that the average sales would not be $0 with no budgets in TV ads, radio ads or newspaper ads.

TV coefficient: The null hypothese to TV coeffient is that we would expect an average sales change of $0 with a unit increase in TV ads budget, holding budgets in radio ads and newspaper ads constant. With p-value < 0.0001, we would reject this null hypothesis at least 99.99\% confidence level, meaning that the average sales change would not be $0 with a unit increase in TV ads budget, holding budgets in radio ads and newspaper ads constant.

Radio coefficient: The null hypothese to radio coeffient is that we would expect an average sales change of $0 with a unit increase in radio ads budget, holding budgets in TV ads and newspaper ads constant. With p-value < 0.0001, we would reject this null hypothesis at least 99.99\% confidence level, meaning that the average sales change would not be $0 with a unit increase in radio ads budget, holding budgets in TV ads and newspaper ads constant.

Newspaper coefficient: The null hypothese to newspaper coeffient is that we would expect an average sales change of $0 with a unit increase in newspaper ads budget, holding budgets in TV ads and radio ads constant. With p-value = 0.8599, we would fail to reject this null hypothesis 85.99\% confidence level, meaning that the average sales change would be $0 with a unit increase in newspaper ads budget, holding budgets in TV ads and radio ads constant.

3.7.2 Carefully explain the differences between the KNN classifier and KNN regression methods.

KNN classifier **classifies** the test observation to the most common class of K nearest neighbers. KNN regression makes an estimated **quantitative** value by averaging the value of K nearest neighbers.

3.7.5 Consider the fitted values that result from performing linear regression without an intercept. In this setting, the _ith fitted value takes the form $\hat{y}{i} = x{i} \hat{\beta}, where\hat{\beta} = \frac{\sum{i=1}^{n}\left (x{i} y{i} \right)}{\sum{i'=1}^{n}\left (x{i'}^2 \right)}. Show that we can write\hat{yi} = \sum{i'=1}^{n}\left(a{i'}y{i'}\right). What is a_{i'}$?

$$\hat{y_i} = x_i \frac{\sum_{i=1}^{n} \left(x_i y_i\right)}{\sum_{i'=1}^{n} \left(x_{i'}^2\right)} = \frac{\sum_{i=1}^{n} \left(x_i y_i \frac{x_i}{n}\right)}{\sum_{i'=1}^{n} \left(x_{i'}^2\right)} = \frac{\sum_{i=1}^{n} \left(\frac{x_i^2}{n} y_i\right)}{\sum_{i'=1}^{n} \left(x_{i'}^2\right)} = \sum_{i'=1}^{n} \left(\frac{\frac{x_{i'}^2}{n} y_{i'}}{x_{i'}^2}\right) = \sum_{i'=1}^{n} \left(\frac{1}{n} y_{i'}\right) = \sum_{i'=1}^{n} \left(a_{i'} y_{i'}\right)$$

Therefore, $a_{i'} = \frac{1}{n}$

3.7.6 Using (3.4), argue that in the case of simple linear regression, the least squares line always passes through the point ($\bar{x}, \bar{y}$).

In order to check if ($\bar{x}, \bar{y}$) is a point on $y = \beta_0 + \beta_1 x$, we put $x = \bar{x}, y = \bar{y}$ this line and see if the left and the right are equal.

We obtain $\bar{y} = \beta_0 + \beta_1 \bar{x}$

Using equation (3.4), $\hat{\beta_0} = \bar{y} - \hat{\beta_1}\bar{x}$, we obtain

$\bar{y} = \bar{y} - \hat{\beta_1}\bar{x} + \hat{\beta_1}\bar{x} = \bar{y}$

3.7.11

part a

In [1]:

```
import numpy as np
import pandas as pd
import statsmodels.api as sm
import matplotlib.pyplot as plt
import math
from sklearn.preprocessing import scale
import sklearn.linear_model as skl_lm
from sklearn.metrics import mean_squared_error, r2_score
import statsmodels.formula.api as smf
from numpy import corrcoef
from pandas.plotting import scatter_matrix
from statsmodels.stats.outliers_influence import OLSInfluence
from statsmodels.graphics.regressionplots import plot_leverage_resid2

np.random.seed(2019)
x=np.random.normal(0,1,100)
y=2*x+np.random.normal(0,1,100)




m1=sm.OLS(y,x).fit()
m1.summary()
```

Out[1]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.771
Model:	OLS	**Adj. R-squared:**	0.769
Method:	Least Squares	**F-statistic:**	333.2
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	1.89e-33
Time:	23:39:13	**Log-Likelihood:**	-149.22
No. Observations:	100	**AIC:**	300.4
Df Residuals:	99	**BIC:**	303.0
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
x1	1.9563	0.107	18.253	0.000	1.744	2.169

Omnibus:	0.149	**Durbin-Watson:**	2.053
Prob(Omnibus):	0.928	**Jarque-Bera (JB):**	0.044
Skew:	-0.051	**Prob(JB):**	0.978
Kurtosis:	2.997	**Cond. No.**	1.00

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The coeffient is 1.9563, which means if x increases by one unit, y will be increased by 1.9563 on average. The standard error of this coeffient estimate is 0.107, t-value is 18.253, p-value is very close to 0, and the 95% confidence interval is [1.744,2.169], which means the coeffient of x will fall in to this interval with 95% change. Since 0 is not in the interval and p-value is very small, we would conclude that x is statistically significant to y.

part b

In [2]:

```
m2=sm.OLS(x,y).fit()
m2.summary()
```

Out[2]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.771
Model:	OLS	**Adj. R-squared:**	0.769
Method:	Least Squares	**F-statistic:**	333.2
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	1.89e-33
Time:	23:39:13	**Log-Likelihood:**	-69.105
No. Observations:	100	**AIC:**	140.2
Df Residuals:	99	**BIC:**	142.8
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
x1	0.3941	0.022	18.253	0.000	0.351	0.437

Omnibus:	1.765	**Durbin-Watson:**	2.128
Prob(Omnibus):	0.414	**Jarque-Bera (JB):**	1.474
Skew:	0.297	**Prob(JB):**	0.479
Kurtosis:	3.035	**Cond. No.**	1.00

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The coeffient is 0.3941, which means if x increases by one unit, y will be increased by 0.3941 on average. The standard error of this coeffient estimate is 0.022, t-value is 18.253, p-value is very close to 0, and the 95% confidence interval is [0.351, 0.437], which means the coeffient of x will fall in to this interval with 95% change. Since 0 is not in the interval and p-value is very small, we would conclude that y is statistically significant to x.

part c

Theoratically, $\hat{\beta}_x = \frac{1}{\hat{\beta}_y}$, $\frac{1}{1.9563} = 0.51 \approx 0.3941$

part d

$$t = \frac{\beta}{SE(\beta)} = \frac{\sum x_i y_i}{\sum x_i^2} \sqrt{\frac{(n-1)\sum x_i^2}{\sum (y_i - x_i\beta)^2}}$$

$$= \frac{\sqrt{n-1}\sum x_i y_i}{\sqrt{\sum x_i^2 \sum (y_i - x_i\beta)^2}}$$

$$= \frac{\sqrt{n-1}\sum x_i y_i}{\sqrt{\sum x_i^2 \sum (y_i^2 - 2\beta x_i y_i + x_i^2\beta^2)}}$$

$$= \frac{\sqrt{n-1}\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2 - \sum x_i^2 \beta(2\sum x_i y_i - \beta \sum x_i^2)}}$$

$$= \frac{\sqrt{n-1}\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2 - \sum x_i y_i (2\sum x_i y_i - \sum x_i y_i)}}$$

$$= \frac{\sqrt{n-1}\sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2 - (\sum x_i y_i)^2}}$$

In [3]:

```
a=np.dot(x, x)
b=np.dot(x, y)
c=np.dot(y, y)

math.sqrt(len(x) - 1) * sum(x*y) / math.sqrt(sum(x**2)*sum(y**2) - sum(x*y)**2
)
```

Out[3]:

```
18.253237805600786
```

part e

plugging (x,y) and (y,x) into part d equation, we would obtain the same result

part f

In [4]:

```
m3 = sm.OLS(y,sm.add_constant(x)).fit()

m3.summary()
```

Out[4]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.771
Model:	OLS	**Adj. R-squared:**	0.769
Method:	Least Squares	**F-statistic:**	330.6
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	3.64e-33
Time:	23:39:13	**Log-Likelihood:**	-148.78
No. Observations:	100	**AIC:**	301.6
Df Residuals:	98	**BIC:**	306.8
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.1015	0.109	0.928	0.356	-0.116	0.319
x1	1.9709	0.108	18.182	0.000	1.756	2.186

Omnibus:	0.160	**Durbin-Watson:**	2.075
Prob(Omnibus):	0.923	**Jarque-Bera (JB):**	0.061
Skew:	-0.060	**Prob(JB):**	0.970
Kurtosis:	2.992	**Cond. No.**	1.16

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [5]:

```
m4 = sm.OLS(x,sm.add_constant(y)).fit()
m4.summary()
```

Out[5]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.771
Model:	OLS	**Adj. R-squared:**	0.769
Method:	Least Squares	**F-statistic:**	330.6
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	3.64e-33
Time:	23:39:13	**Log-Likelihood:**	-67.950
No. Observations:	100	**AIC:**	139.9
Df Residuals:	98	**BIC:**	145.1
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-0.0732	0.048	-1.514	0.133	-0.169	0.023
x1	0.3914	0.022	18.182	0.000	0.349	0.434

Omnibus:	1.807	**Durbin-Watson:**	2.178
Prob(Omnibus):	0.405	**Jarque-Bera (JB):**	1.497
Skew:	0.299	**Prob(JB):**	0.473
Kurtosis:	3.047	**Cond. No.**	2.26

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The t statistics are both 18.182

3.7.12

part a

$\sum x_i^2 = \sum y_i^2$

part b

In [6]:

```
print("The coefficient estimate for the regression of x onto y is",m1.params)
print("the coefficient estimate for the regression of y onto x is",m2.params)
```

```
The coefficient estimate for the regression of x onto y is [1.9562
9319]
the coefficient estimate for the regression of y onto x is [0.3940
7649]
```

In [7]:

```
np.random.seed(20191)

p = np.random.normal(0,1,1000)
q = np.random.normal(0,1,1000)


m5 = sm.OLS(p,q).fit()
m6 = sm.OLS(q,p).fit()

print("The coefficient estimate for the regression of q onto p is",m5.params)
print("The coefficient estimate for the regression of p onto q is",m6.params)
```

```
The coefficient estimate for the regression of q onto p is [-0.026
9477]
The coefficient estimate for the regression of p onto q is [-0.026
58923]
```

3.7.13

part a

In [8]:

```
np.random.seed(20192)

x13 = np.random.normal(0,1,100)
```

part b

In [9]:

```
eps = np.random.normal(0,0.25,100)
```

part c

In [10]:

```
y13 = -1 + 0.5*x13 + eps

print("The length of y13 is",len(y13))
```

```
The length of y13 is 100
```

$\beta_0 = -1$ $\beta_1 = 0.5$

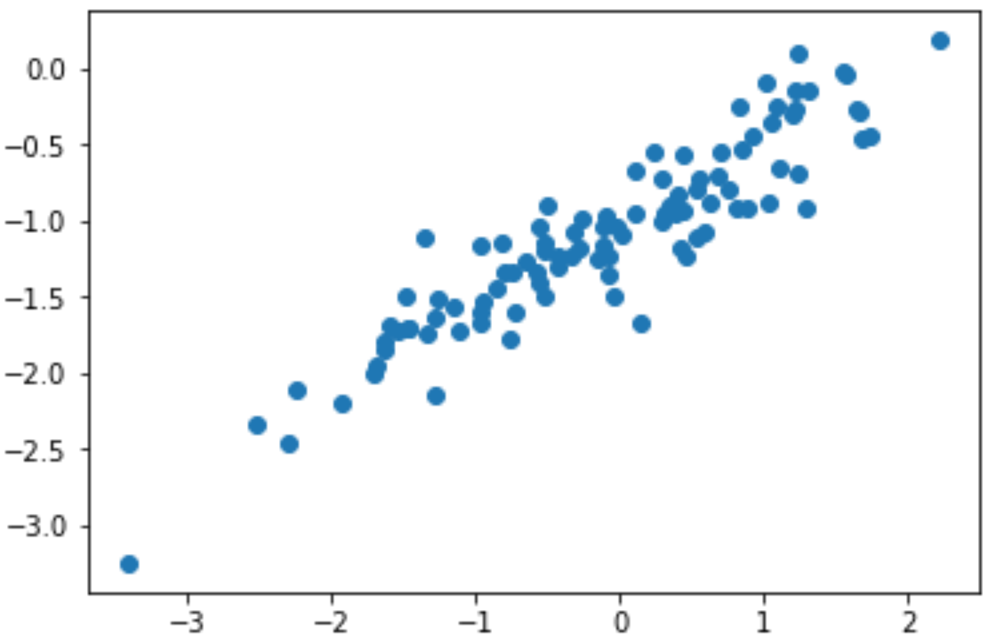
part d

In [11]:

```
plt.scatter(x13,y13)
```

Out[11]:

```
<matplotlib.collections.PathCollection at 0x1c1ab597b8>
```



From the plot above we could see that x13 and y13 are positively correlated.

part e

In [12]:

```
x131= sm.add_constant(x13)

m7 = sm.OLS(y13,x131).fit()


m7.summary()
```

Out[12]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.848
Model:	OLS	**Adj. R-squared:**	0.847
Method:	Least Squares	**F-statistic:**	548.4
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	6.26e-42
Time:	23:39:13	**Log-Likelihood:**	2.3777
No. Observations:	100	**AIC:**	-0.7553
Df Residuals:	98	**BIC:**	4.455
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-1.0492	0.024	-43.738	0.000	-1.097	-1.002
x1	0.5084	0.022	23.418	0.000	0.465	0.551

Omnibus:	1.209	**Durbin-Watson:**	1.797
Prob(Omnibus):	0.546	**Jarque-Bera (JB):**	0.772
Skew:	-0.190	**Prob(JB):**	0.680
Kurtosis:	3.201	**Cond. No.**	1.15

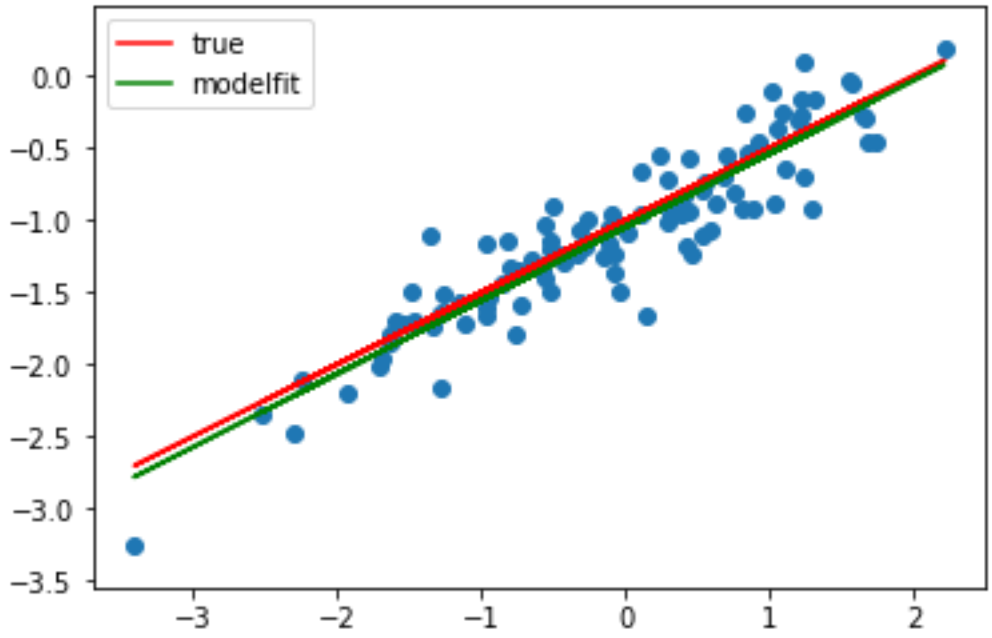
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The estimated coefficients are very close to the $\beta_0 = -1$ and $\beta_1 = 0.5$

part f

In [13]:

```
fig, ax = plt.subplots()
ax.scatter(x13,y13)
ax.plot(x13,-1 + 0.5*x13,'r',label='true')
ax.plot(x13, -1.0492 + 0.5084*x13, 'g',label='modelfit' )
ax.axis('equal')
leg = ax.legend();
```



part g

In [14]:

```
data13 = {"y13": y13, "x13": x13}

m8 = smf.ols(formula = 'y13 ~ np.power(x13,2) + x13', data = data13).fit()
m8.summary()
```

Out[14]:

OLS Regression Results

Dep. Variable:	y13	**R-squared:**	0.850
Model:	OLS	**Adj. R-squared:**	0.847
Method:	Least Squares	**F-statistic:**	275.5
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	9.92e-41
Time:	23:39:13	**Log-Likelihood:**	3.0167
No. Observations:	100	**AIC:**	-0.03348
Df Residuals:	97	**BIC:**	7.782
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-1.0290	0.030	-34.252	0.000	-1.089	-0.969
np.power(x13, 2)	-0.0174	0.016	-1.117	0.267	-0.048	0.014
x13	0.4988	0.023	21.397	0.000	0.453	0.545

Omnibus:	1.127	**Durbin-Watson:**	1.814
Prob(Omnibus):	0.569	**Jarque-Bera (JB):**	0.664
Skew:	-0.157	**Prob(JB):**	0.717
Kurtosis:	3.247	**Cond. No.**	2.97

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

R^2 improved from 0.848 to 0.85 and the quadratic term has non significant coefficient

No significant improvement. Because the data is pretty linear.

part h

In [15]:

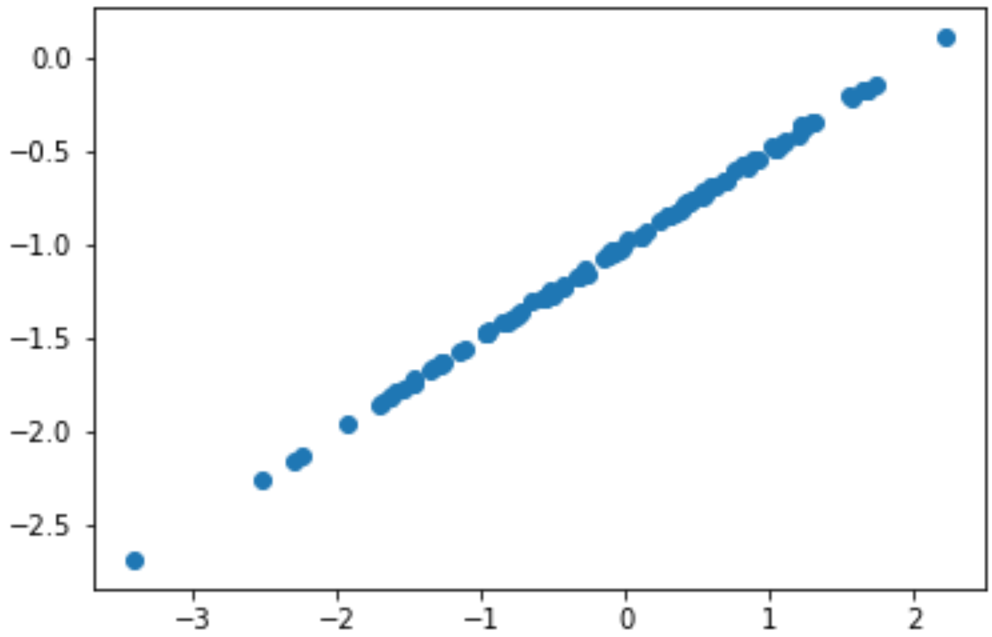
```
np.random.seed(20193)

xh=x13
epsh = np.random.normal(0,0.01,100)
yh = -1 + 0.5*xh + epsh

plt.scatter(xh,yh)
```

Out[15]:

```
<matplotlib.collections.PathCollection at 0x1c1ad7d3c8>
```



In [16]:

```
m9 = sm.OLS(yh,sm.add_constant(xh)).fit()


m9.summary()
```

Out[16]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	1.000
Model:	OLS	**Adj. R-squared:**	1.000
Method:	Least Squares	**F-statistic:**	2.655e+05
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	4.87e-170
Time:	23:39:13	**Log-Likelihood:**	313.26
No. Observations:	100	**AIC:**	-622.5
Df Residuals:	98	**BIC:**	-617.3
Df Model:	1		
Covariance Type:	nonrobust		

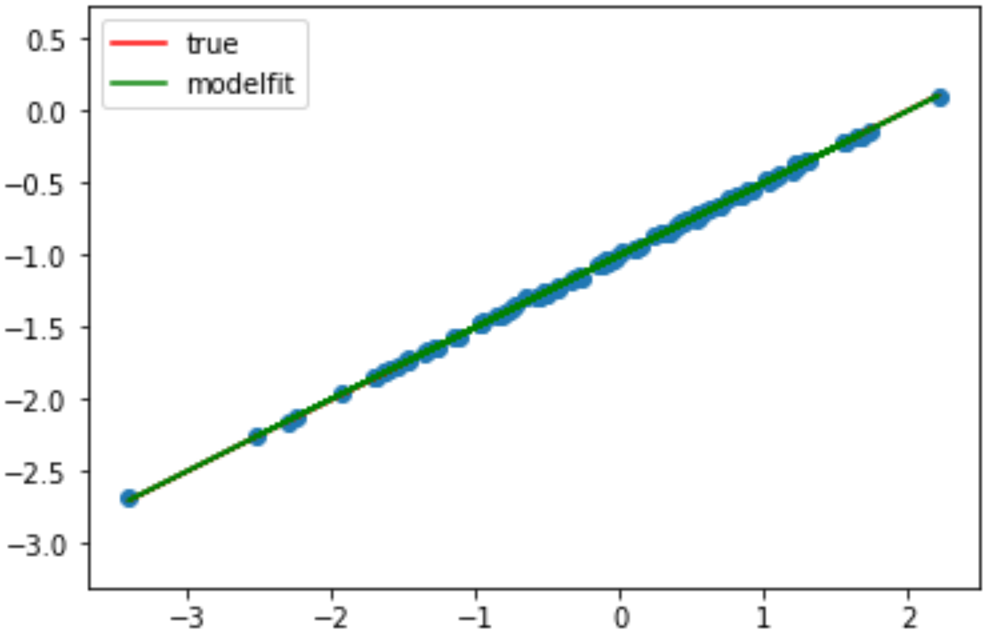
	coef	std err	t	P>|t|	[0.025	0.975]
const	-0.9999	0.001	-933.430	0.000	-1.002	-0.998
x1	0.4995	0.001	515.280	0.000	0.498	0.501

Omnibus:	0.380	**Durbin-Watson:**	2.168
Prob(Omnibus):	0.827	**Jarque-Bera (JB):**	0.483
Skew:	0.137	**Prob(JB):**	0.785
Kurtosis:	2.798	**Cond. No.**	1.15

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [17]:

```
figh, axh = plt.subplots()
axh.scatter(xh,yh)
axh.plot(xh,-1 + 0.5*xh,'r',label='true')
axh.plot(xh, -0.9999 + 0.4995*xh, 'g',label='modelfit' )
axh.axis('equal')
leg = axh.legend();
```



The two lines are almost overlapping completely

part i

In [18]:

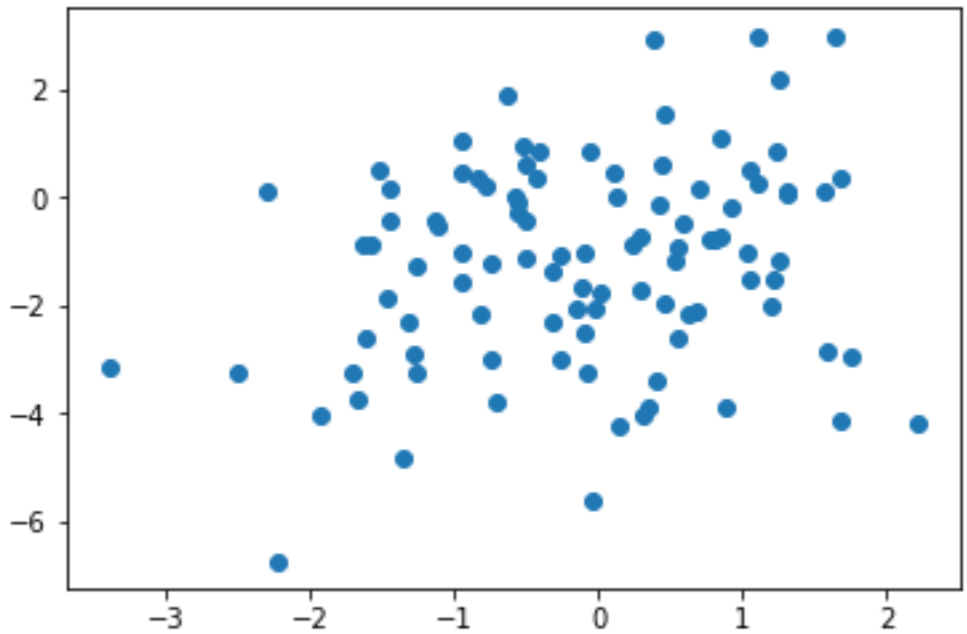
```
np.random.seed(20194)

xi=x13
epsi = np.random.normal(0,2,100)
yi = -1 + 0.5*xi + epsi

plt.scatter(xi,yi)
```

Out[18]:

```
<matplotlib.collections.PathCollection at 0x1c1aeb5898>
```



In [19]:

```
m10 = sm.OLS(yi,sm.add_constant(xi)).fit()


m10.summary()
```

```
Out[19]:
```

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.048
Model:	OLS	**Adj. R-squared:**	0.038
Method:	Least Squares	**F-statistic:**	4.905
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	0.0291
Time:	23:39:13	**Log-Likelihood:**	-201.77
No. Observations:	100	**AIC:**	407.5
Df Residuals:	98	**BIC:**	412.8
Df Model:	1		
Covariance Type:	nonrobust		

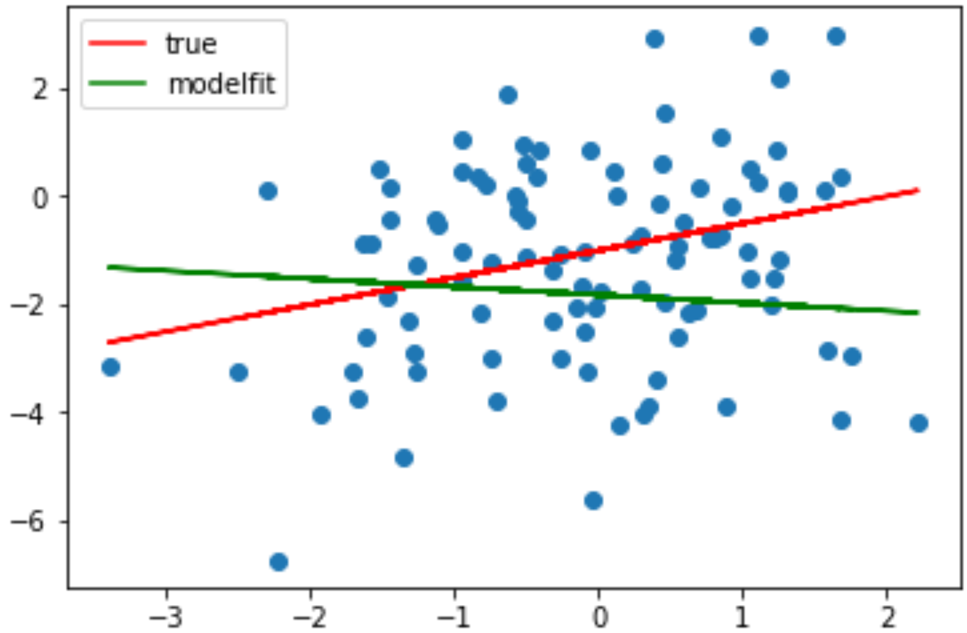
	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-1.1647	0.185	-6.304	0.000	-1.531	-0.798
x1	0.3703	0.167	2.215	0.029	0.039	0.702

Omnibus:	1.419	**Durbin-Watson:**	2.053
Prob(Omnibus):	0.492	**Jarque-Bera (JB):**	1.461
Skew:	-0.272	**Prob(JB):**	0.482
Kurtosis:	2.765	**Cond. No.**	1.15

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [20]:

```
figi, axi = plt.subplots()
axi.scatter(xi,yi)
axi.plot(xi,-1 + 0.5*xi,'r',label='true')
axi.plot(xi, -1.8233 -0.1485*xi, 'g',label='modelfit' )
axh.axis('equal')
leg = axi.legend();
```



The two lines have slopes in bigger difference

part j

The confidence interval of β_0 for the original data set is [-1.097, -1.002]

The confidence interval of β_0 for the less noisy data set is [-1.002, -0.998]

The confidence interval of β_0 for the more noisy data set is [-1.531, -0.798]

The confidence interval of β_1 for the original data set is [0.465,0.551]

The confidence interval of β_1 for the less noisy data set is [0.498, 0.501]

The confidence interval of β_1 for the more noisy data set is [0.039, 0.702]

The confidence interval for the less noisy data set is narrow and having the true β in the middle. The confidence interval for the original data set is wider and not very far off the true β, while the confidence interval for the more noisy data set is very wide and far from the true β

3.7.14

part a

In [21]:

```
np.random.seed(20195)

c1 = np.random.random(100)

c2 = 0.5*c1 + np.random.randn(100)/10

d =2+2* c1 +0.3* c2 + np.random.randn(100)

data14 = {"d":d , "c1": c1, "c2": c2}
```

part b

In [22]:

```
print(corrcoef(c1,c2))

df14 = pd.DataFrame(np.column_stack((d,c1,c2)), columns=['d','c1','c2'] )
df14.head()
```

```
[[1.         0.76171502]
 [0.76171502 1.        ]]
```

Out[22]:

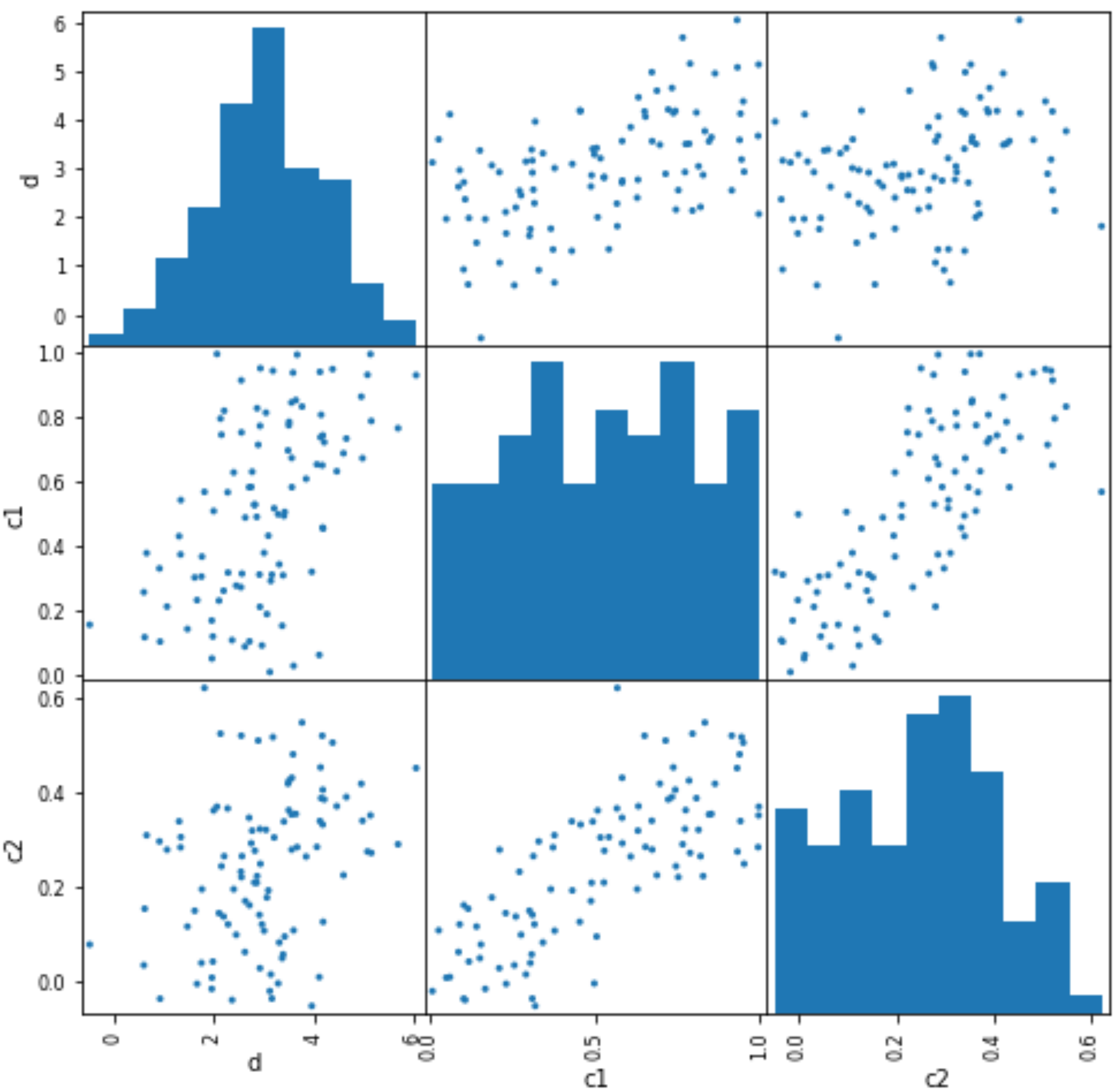
	d	c1	c2
0	0.913956	0.331116	0.297105
1	3.043108	0.815345	0.321567
2	3.385680	0.309791	0.058966
3	5.132308	0.997439	0.352063
4	4.169054	0.651828	0.519775

In [23]:

```
scatter_matrix(df14, figsize = (8,8),alpha=1)
```

Out[23]:

```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x1c1b008
a58>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c1b0b9
be0>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c1b0e9
198>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x1c1b10f
710>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c1b138
c88>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c1b166
240>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x1c1b191
7b8>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c1b1b8
d68>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x1c1b1b8
da0>]],
      dtype=object)
```



In [24]:

```
### part c
```

In [25]:

```
m11 = smf.ols(formula = 'd ~ c1 + c2', data = data14).fit()
m11.summary()
```

Out[25]:

OLS Regression Results

Dep. Variable:	d	**R-squared:**	0.283
Model:	OLS	**Adj. R-squared:**	0.268
Method:	Least Squares	**F-statistic:**	19.13
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	9.90e-08
Time:	23:39:14	**Log-Likelihood:**	-141.77
No. Observations:	100	**AIC:**	289.5
Df Residuals:	97	**BIC:**	297.3
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	1.8754	0.215	8.720	0.000	1.449	2.302
c1	2.7981	0.560	4.994	0.000	1.686	3.910
c2	-1.4160	0.984	-1.439	0.153	-3.368	0.536

Omnibus:	0.689	**Durbin-Watson:**	2.364
Prob(Omnibus):	0.709	**Jarque-Bera (JB):**	0.799
Skew:	-0.103	**Prob(JB):**	0.671
Kurtosis:	2.613	**Cond. No.**	12.4

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The intercept coefficient $\beta_0 = 1.8754$ and β_1 = 2.7981 are statistically significant with very small p-value and 95% confidence intervals containing no zero. We can reject the null hypothesis of $\beta_1 = 0.$ while β_2 = -1.416 is not very statistically significant at a 95% confidence level, which has a p-value = 0.15 and a 95% confidence interval containg zero. We cannot reject the null hypothesis of $\beta_2 = 0.$

part d

In [26]:

```
m12 = smf.ols(formula = 'd ~ c1', data = data14).fit()

m12.summary()
```

Out[26]:

OLS Regression Results

Dep. Variable:	d	**R-squared:**	0.268
Model:	OLS	**Adj. R-squared:**	0.260
Method:	Least Squares	**F-statistic:**	35.80
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	3.58e-08
Time:	23:39:14	**Log-Likelihood:**	-142.82
No. Observations:	100	**AIC:**	289.6
Df Residuals:	98	**BIC:**	294.9
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	1.8470	0.215	8.578	0.000	1.420	2.274
c1	2.1838	0.365	5.983	0.000	1.460	2.908

Omnibus:	0.889	**Durbin-Watson:**	2.313
Prob(Omnibus):	0.641	**Jarque-Bera (JB):**	0.946
Skew:	-0.113	**Prob(JB):**	0.623
Kurtosis:	2.581	**Cond. No.**	4.61

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The intercept coefficient $\beta_0 = 1.8470$ and β_1 = 2.1838 are statistically significant with very small p-value and 95% confidence intervals containing no zero. We can reject the null hypothesis of $\beta_1 = 0$.

part e

In [27]:

```
m13 = smf.ols(formula = 'd ~ c2', data = data14).fit()

m13.summary()
```

Out[27]:

OLS Regression Results

Dep. Variable:	d	**R-squared:**	0.098
Model:	OLS	**Adj. R-squared:**	0.089
Method:	Least Squares	**F-statistic:**	10.71
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	0.00147
Time:	23:39:14	**Log-Likelihood:**	-153.21
No. Observations:	100	**AIC:**	310.4
Df Residuals:	98	**BIC:**	315.6
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	2.4108	0.208	11.592	0.000	1.998	2.823
c2	2.3263	0.711	3.272	0.001	0.915	3.737

Omnibus:	0.694	**Durbin-Watson:**	2.239
Prob(Omnibus):	0.707	**Jarque-Bera (JB):**	0.531
Skew:	-0.178	**Prob(JB):**	0.767
Kurtosis:	2.998	**Cond. No.**	6.67

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The intercept coefficient $\beta_0 = 2.4108$ and β_1 = 2.3263 are statistically significant with very small p-value and 95% confidence intervals containing no zero. We can reject the null hypothesis of $\beta_1 = 0$.

part f

In part c, we find that the coeffecient of c1 is significant and the coeffecient of c2 is not significant.

In part d, we find that the coeffecient of c1 is significant by itself and in part e, the coeffecient of c2 is significant by itself.

This is not contradicted because c1 and c2 are both influential to d independently, and there exists correlation between c1 and c2.

part g

In [28]:

```
df14miss = pd.DataFrame([[6,0.1,0.8]], columns=['d','c1','c2'])

df14g = df14.append(df14miss)

m14 = smf.ols(formula = 'd ~ c1 + c2', data = df14g).fit()
m14.summary()
```

Out[28]:

OLS Regression Results

Dep. Variable:	d	**R-squared:**	0.218
Model:	OLS	**Adj. R-squared:**	0.202
Method:	Least Squares	**F-statistic:**	13.67
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	5.79e-06
Time:	23:39:14	**Log-Likelihood:**	-150.22
No. Observations:	101	**AIC:**	306.4
Df Residuals:	98	**BIC:**	314.3
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	1.9530	0.230	8.507	0.000	1.497	2.409
c1	1.6546	0.514	3.218	0.002	0.634	2.675
c2	0.8200	0.863	0.951	0.344	-0.892	2.532

Omnibus:	0.253	**Durbin-Watson:**	2.151
Prob(Omnibus):	0.881	**Jarque-Bera (JB):**	0.091
Skew:	0.071	**Prob(JB):**	0.956
Kurtosis:	3.035	**Cond. No.**	10.1

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [29]:

```
m15 = smf.ols(formula = 'd ~ c1', data = df14g).fit()

m15.summary()
```

Out[29]:

OLS Regression Results

Dep. Variable:	d	**R-squared:**	0.211
Model:	OLS	**Adj. R-squared:**	0.203
Method:	Least Squares	**F-statistic:**	26.47
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	1.36e-06
Time:	23:39:14	**Log-Likelihood:**	-150.68
No. Observations:	101	**AIC:**	305.4
Df Residuals:	99	**BIC:**	310.6
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	1.9916	0.226	8.819	0.000	1.543	2.440
c1	1.9789	0.385	5.145	0.000	1.216	2.742

Omnibus:	3.183	**Durbin-Watson:**	2.136
Prob(Omnibus):	0.204	**Jarque-Bera (JB):**	2.674
Skew:	0.248	**Prob(JB):**	0.263
Kurtosis:	3.624	**Cond. No.**	4.57

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [30]:

```
m16 = smf.ols(formula = 'd ~ c2', data = df14g).fit()

m16.summary()
```

Out[30]:

OLS Regression Results

Dep. Variable:	d	**R-squared:**	0.136
Model:	OLS	**Adj. R-squared:**	0.127
Method:	Least Squares	**F-statistic:**	15.53
Date:	Thu, 31 Jan 2019	**Prob (F-statistic):**	0.000152
Time:	23:39:14	**Log-Likelihood:**	-155.29
No. Observations:	101	**AIC:**	314.6
Df Residuals:	99	**BIC:**	319.8
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	2.3439	0.204	11.503	0.000	1.940	2.748
c2	2.6609	0.675	3.940	0.000	1.321	4.001

Omnibus:	0.787	**Durbin-Watson:**	2.189
Prob(Omnibus):	0.675	**Jarque-Bera (JB):**	0.712
Skew:	-0.202	**Prob(JB):**	0.701
Kurtosis:	2.924	**Cond. No.**	6.35

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

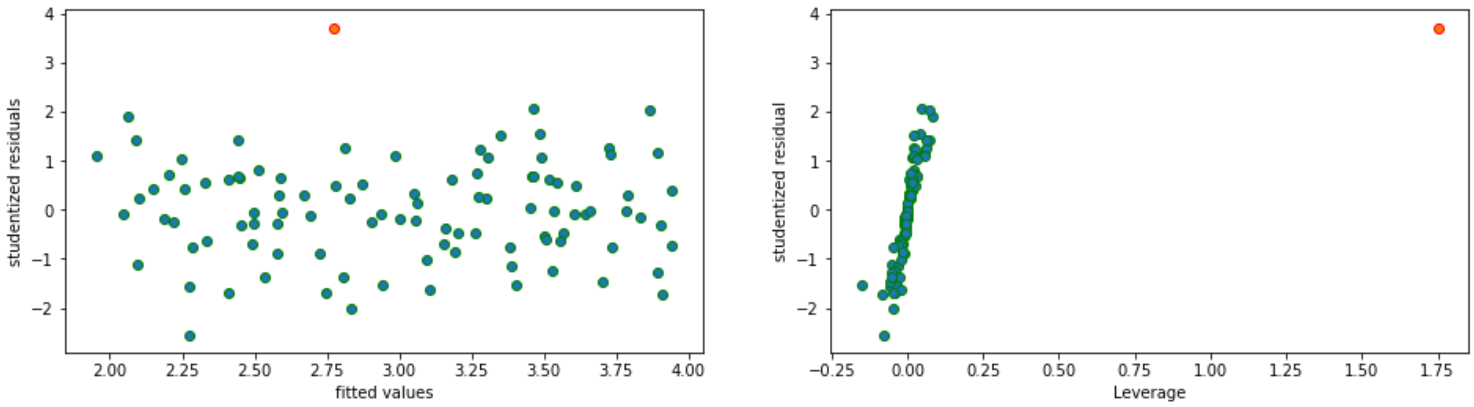
In [31]:

```
#model 1
fitted_values14 = m14.fittedvalues.values
residuals14 = m14.resid.values
studentized_residuals14 = OLSInfluence(m14).resid_studentized_internal
leverages14 = OLSInfluence(m14).influence

fig, (ax11,ax21) = plt.subplots(1,2,figsize=(16,4))

ax11.scatter(fitted_values14[:-1], studentized_residuals14[:-1],  edgecolors='
g');
ax11.scatter(fitted_values14[-1], studentized_residuals14[-1],  edgecolors='r'
);
ax11.set_xlabel('fitted values');
ax11.set_ylabel('studentized residuals');

ax21.scatter(leverages14[:-1], studentized_residuals14[:-1],  edgecolors='g');
ax21.scatter(leverages14[-1], studentized_residuals14[-1],  edgecolors='r');
ax21.set_xlabel('Leverage');
ax21.set_ylabel('studentized residual');
```



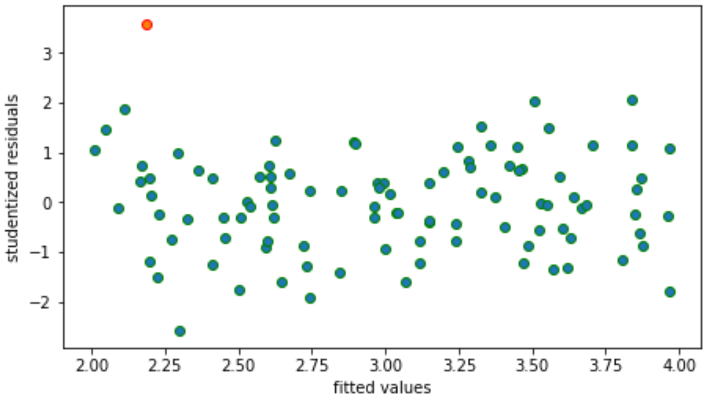
In [32]:

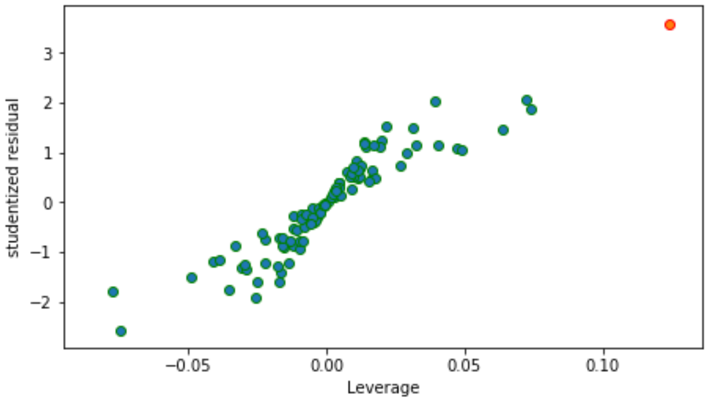
```
#model 2
fitted_values15 = m15.fittedvalues.values
residuals15 = m15.resid.values
studentized_residuals15 = OLSInfluence(m15).resid_studentized_internal
leverages15 = OLSInfluence(m15).influence

fig, (ax12,ax22) = plt.subplots(1,2,figsize=(16,4))

ax12.scatter(fitted_values15[:-1], studentized_residuals15[:-1],  edgecolors='
g');
ax12.scatter(fitted_values15[-1], studentized_residuals15[-1],  edgecolors='r'
);
ax12.set_xlabel('fitted values');
ax12.set_ylabel('studentized residuals');

ax22.scatter(leverages15[:-1], studentized_residuals15[:-1],  edgecolors='g');
ax22.scatter(leverages15[-1], studentized_residuals15[-1],  edgecolors='r');
ax22.set_xlabel('Leverage');
ax22.set_ylabel('studentized residual');
```





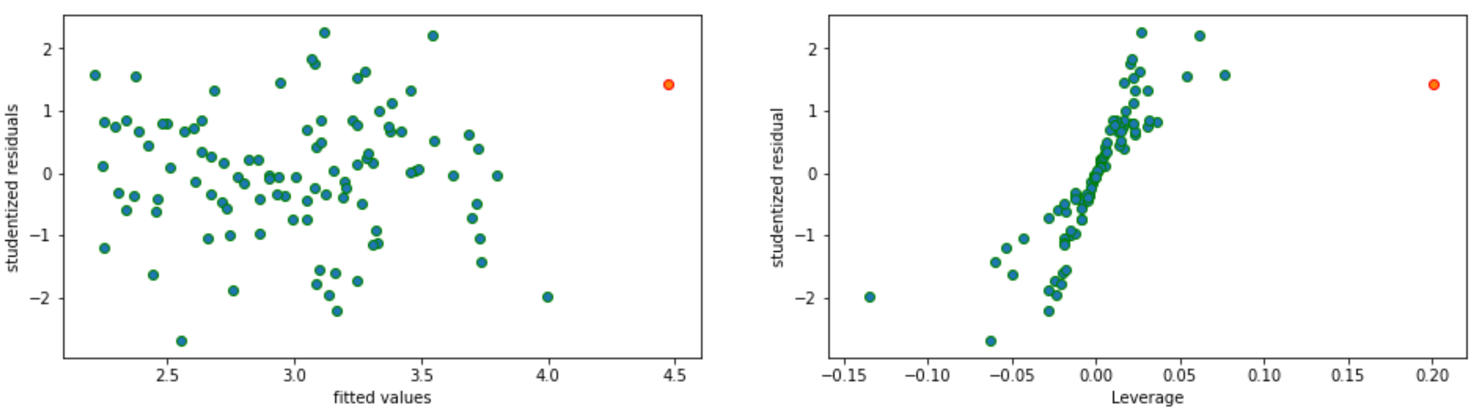
In [33]:

```
#model 3
fitted_values16 = m16.fittedvalues.values
residuals16 = m16.resid.values
studentized_residuals16 = OLSInfluence(m16).resid_studentized_internal
leverages16 = OLSInfluence(m16).influence

fig, (ax13,ax23) = plt.subplots(1,2,figsize=(16,4))

ax13.scatter(fitted_values16[:-1], studentized_residuals16[:-1],  edgecolors='
g');
ax13.scatter(fitted_values16[-1], studentized_residuals16[-1],  edgecolors='r'
);
ax13.set_xlabel('fitted values');
ax13.set_ylabel('studentized residuals');

ax23.scatter(leverages16[:-1], studentized_residuals16[:-1],  edgecolors='g');
ax23.scatter(leverages16[-1], studentized_residuals16[-1],  edgecolors='r');
ax23.set_xlabel('Leverage');
ax23.set_ylabel('studentized residual');
```



This observation(c1=0.1,c2=0.8,d=6) is outlier in all three models.