0.1 One variable linear regression

The cost function is generally chosen as follows:

$$\text{J} = \sum_{i=1}^{m}(y_i - \dot{y}_i)^2 = \sum_{i=1}^{m}(y_i - \dot{a} * x_i - b)^2$$

The latter should be minimized. We will use the ***Least Squares*** method to minimize the sum in question. Taking into consideration that m refers to the number of examples, samples in out data set, we proceed with the method as follows:

0.1.1 Expand

we have J $= \sum(y_i^2 + a^2 * x_i^2 + b^2 + 2 * a * b * x_i - 2 * a * y_i * x_i - 2 * b * y_i)$
$= \sum y_i^2 + a^2 * \sum x_i^2 + m * b^2 + 2 * a * b * \sum x_i - 2 * a * \sum(x_i * y_i) - 2 * b * \sum(y_i)$

0.1.2 consider both derivative

$\frac{dJ}{da} = 2 * a \sum x_i^2 + 2 * b * \sum x_i - 2 * \sum(y_i * x_i)$
$\frac{dJ}{db} = 2 * b * m + 2 * a * \sum(x_i) - 2 * \sum(y_i)$
setting both derivative to 0, and solving the system of equations:

$$\begin{cases} \frac{dJ}{da} = 0 \\ \frac{dJ}{db} = 0 \end{cases} \tag{1}$$

The latter can be solved easily with simple algebraic manipulation:

$$\begin{cases} a = \frac{m \cdot \sum(x_i \cdot y_i) - \sum x_i \cdot \sum x_i}{m \cdot \sum x_i^2 - (\sum x_i)^2} \\ b = \frac{\sum y_i - a \cdot \sum x_i}{m} \end{cases} \tag{2}$$