1 K-means algorithm

Algorithm 1 K-means

```
initialize the K cluster centroids μ_1, μ_2, ...μ_k
for i = 1 to m do
    c_i := index of cluster centroid satisfying ||x_i − μ_k|| = min_k||x_i − μ_k||
end for
for k = 1 to K do
    μ_k := average (mean) of points assigned to cluster k
end forconvergence
```

The first for loop is labeled the **cluster assignment** step, while the second is the **move centroid** step

2 Move centroid step

For given clusters, the move centroids step minimizes J with respect to $\mu_1, \mu_2 ..., \mu_k$. Rewriting the cost function as follows:

$$J(c_1, c_2, ...c_m, \mu_1, \mu_2 ..., \mu_k) = \frac{1}{m} \sum_{i=1}^{K} \sum_{j=1}^{s_k} ||x_{ij} - \mu_i||^2$$

where s_j is the number of training samples belonging to cluster j. and x_{ij} is the i-th sample belonging to the j-the cluster.

The sum $\sum_{j=1}^{s_k} ||x_{ij} - \mu_{c_i}||^2$ can be seen as function of μ_i. The derivative with respect to μ_i:

$$\frac{\delta}{\delta \mu_i} (\sum_{j=1}^{s_k} ||x_{ij} - \mu_i||^2) = 2 \cdot \sum_{j=1}^{s_k} (x_{ij} - \mu_i)$$

Setting the last equation to 0

$$\sum_{j=1}^{s_k} x_{ij} = \sum_{j=1}^{s_k} \mu_i \iff \mu_i = \frac{1}{s_i} \cdot \sum_{j=1}^{s_k} x_{ij}$$

Thus the average (mean) of points is the centroid realizing the minimum sum distance. Applying the same argument for each cluster, The move centroid minimizes J with respect to $\mu_1, \mu_2 ..., \mu_k$