

1 Convolution operator

The convolution operator is a binary matrix operator usually denoted by the symbol $*$. if A is of shape (m, n) and B is of shape a, b then $A * B$ is of shape $(m - a + 1, n - b + 1)$

1.1 Weighted sum

we first define the term weighted sum of two matrices. The latter should be of the exact same shape. The weighted sum is the sum of the element wise multiplication of two matrices.

1.2 Convolution

Let $A * B = C$. if A is of shape (m, n) and B is of shape a, b then there is $(m - a + 1, n - b + 1)$ matrices of B's shape that can fit in A. C_{ij} is the weighted sum of matrix of shape a, b with the A_{ij} being the top corner cell.

1.3 Example

let

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$$A = \begin{bmatrix} 3 & 0 & 1 & 2 & 7 & 4 \\ 1 & 5 & 8 & 9 & 3 & 1 \\ 2 & 7 & 2 & 5 & 1 & 3 \\ 0 & 1 & 3 & 1 & 7 & 8 \\ 4 & 2 & 1 & 6 & 2 & 8 \\ 2 & 4 & 5 & 2 & 3 & 9 \end{bmatrix}$$

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$$B = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

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$$A * B = \begin{bmatrix} -5 & -4 & 0 & 8 \\ -10 & -2 & 2 & 3 \\ 0 & -2 & -4 & -7 \\ -3 & -2 & -2 & -16 \end{bmatrix}$$

where the -5 in the top left corner is calculated as $1 * 3 + 1 * 1 + 1 * 2 + 0 * 0 + 0 * 5 + 0 * 7 + -1 * 1 + -1 * 8 + -1 * 2 = -5$