EEE321 Lab Work 3

(Clearly justify all answers.)

(Due 13 Nov 2018 for Sec. 3, 14 Nov 2018 for Sec. 1 and 16 Nov 2018 for Secs. 2 and 4)

Work on these questions as a homework first: answer the analytical parts of the questions and write the answers on a paper, write a MATLAB program (or many such programs) to perform the tasks that need computation, print your MATLAB code(s), print your computer outputs (numerical and graphic) whenever needed; the collection of all those will be your lab report. Bring your code (in a computer readable form) to the lab; transfer your code to one of the lab computers; run and show your TA the results. Answer all the questions your TAs may ask. Modify your lab report, including any modifications needed in your MATLAB codes, during the lab hours in the lab. Finally, hand your TAs the lab report prepared as described above. Your report will get a grade based on your preparedness when you come to the lab, performance of your codes in the lab (any modifications needed and conducted during the lab hours included), your answers to the oral questions during your demo(s) in the lab, and the entire content of the submitted report at the end of the lab.

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In this work, you will learn the Fourier series expansion and some related approximations.

1- $y_a(t)$ is a rectangular waveform defined as:

$$y_a(t) = \begin{cases} 0 & t \in [0,4)\text{s.} \\ 5 & t \in [4,6)\text{s.} \\ 0 & t \in [6,12)\text{s.} \end{cases}$$

and $y_a(t)$ is periodic with a period of 12 seconds.

Any signal processing in a digital environment involves sampling; that includes your MATLAB environement, as well.

a) Discretize $y_a(t)$, using a sampling period $T_s = 1/5$ s. Plot (using MATLAB) $y[n] = y_a(nT_s)$, for $n \in [-40, 215]$.

b) Analytically find the Fourier series expansion of $y_a(t)$.

c) Plot (approximately by hand) the spectrum of $y_a(t)$.

d) Write a MATLAB code that computes the discrete function $z_N[n]$ using your results in (b), where,

$$z_N[n] = \sum_{k=-N}^{N} a_k e^{j\omega_0 knT_s}$$

and a_k's are the FSE coefficients found in (b). Plot the result for $n \in [-40, 215]$, and for $N = 70$.

Comment on the results: does the plot look like $y_a(t)$?

e) Repeat (e) for $N = 25$.

f) Repeat (e) for $N = 7$.

g) Repeat (e) for $N = 3$.

h) Repeat (e) for $N = 1$.

Consider your plots in (d) through (h) and comment on the quality of approximations $z_N[n]$, of $y_a(t)$, by taking only some of the FSE components during the synthesis; pay attention also to the Gibbs phenomenon.

i) Plot the zeroth, first, second, and third harmonics of $y_a(t)$, using the same scale for all these plots. (Hint: zeroth harmonic is also the DC component of $y_a(t)$ that is a constant function whose amplitude (value) is equal to a_0. $k'th$ harmonic of this real valued $y_a(t)$ is $a_{-k}e^{-j\omega_o kt} + a_k e^{j\omega_o kt}$ which is a real valued sinusoidal. Hint: Though you cannot plot a continuous function using MATLAB, you can get a very good approximation if the sampling rate is high enough.)

2- Replace $y_a(t)$ in (1), as,

$$y_a(t) = \left|4\cos\left(\frac{\pi}{6}t\right)\right| \quad .$$

Repeat (1). (Full-wave rectifier.) (You need to find the FSE analytically, first; to do that you need to find the fundamental period.)

3- Replace $y_a(t)$ in (1), as,

$$y_a(t) = \begin{cases} \left|4\cos\left(\frac{\pi}{6}t\right)\right| & t \in [-3, 3)\,\mathrm{s}\,. \\ 0 \quad t \in [3, 9)\mathrm{s}. \end{cases}$$

and it is periodic with period $T = 12s$. Repeat (1). (Half-wave rectifier.) (You need to find the FSE analytically, first; to do that you need to find the fundamental period.)