Lab 3 – Margin Analysis of a System

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1. Introduction

In this laboratory assignment, we have researched how to estimate gain, phase, delay margins by using mathematical modelling, and how we can verify these estimates. The first thing we do is to observe the three different controller gains that we have modelled in Preliminary Lab 3 and then chose the one that gave the least settling time. Then, we have modelled the plant also with the arithmetic and geometric mean operations, then we drawn the Bode plot. We have also calculated the gain and phase margins by using this plot and the crossover frequency.

Then we have configured the hardware and increased the gain experimentally until the DC Motor System became unstable. Then we have calculated the experimental and observed gain margins. After that we configured the hardware for another model and then increased the delay time until the system became unstable also showed the observed and calculated delay margins. We have plotted the calculated and observed margins on the same graph and then observed the differences and similarities in between.

2. Laboratory Content

2.1. Margin Estimation

We have picked the values that gave the least settling time and the transfer functions are defined an the bode plots are drawn. The MATLAB code written for this are given below.

```
%% Part 1
% 0.5 43
% 5.93 53
A1 = 44.1942;
B1 = 3.33541;

% 0.34 58
% 5.36 84
A2 = 55.706;
B2 = 3.16511597;

K1 = A1/B1;
tau1 = 1/B1;
K2 = A2/B2;
tau2 = 1/B2;
K = (K1 + K2) / 2;
tau = sqrt(tau1*tau2);

h = 0.005;
plnt_poles = [(tau*h) (tau+h) 1];
plnt_zeros = [(-h*K) (K)];
plnt = tf(plnt_zeros,plnt_poles);
int = -100:0.1:-1;

d_vals = [];
for z = int
    contr_poles = [1 0];
    contr_zeros = [-1/z 1];
    contr = tf(contr_zeros, contr_poles);
    [roots,ks] = rlocus(plnt*contr);
    max_re = max(real(roots));
    min_max_root = min(max_re);
    d_vals = [d_vals min_max_root];
end

[minimum_d,min_index] = min(d_vals);
z1 = int(min_index);
contr_poles = [1 0];
contr_zeros = [-1/z1 1];
contr = tf(contr_zeros, contr_poles);
figure();
bode(contr*plnt);
title('Bode for z1');
```

The Bode plot drawn is given below.

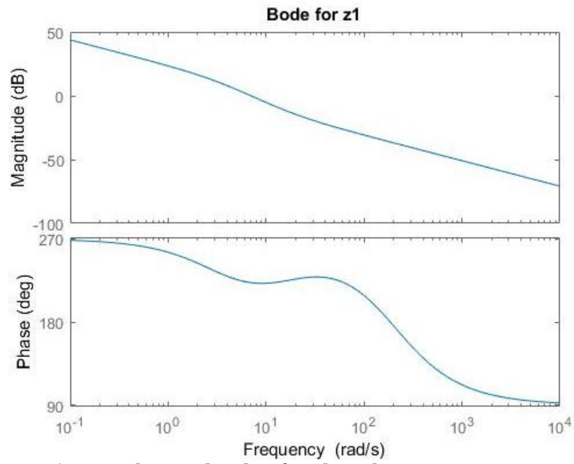

Fig. 1: The Bode plot for the Chosen System

Then, we have used the margin() and allmargin() plots in order to observe the margins. The plot and the output are given below.

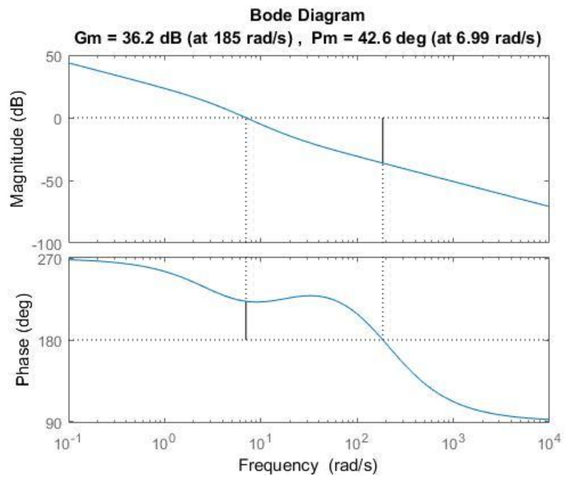

Fig. 2: The Bode Plot Drawn with margin() Command

```
    GainMargin: 64.714521292571277
   GMFrequency: 1.850850582462882e+02
   PhaseMargin: 42.584807404238298
   PMFrequency: 6.993497614477682
   DelayMargin: 0.106276593130307
```

```
    DMFrequency: 6.993497614477682
         Stable: 1
```

Also, we calculate the gain and phase margin by looking at the bode diagram plots. The plot that is marked for the gain margin is given below.

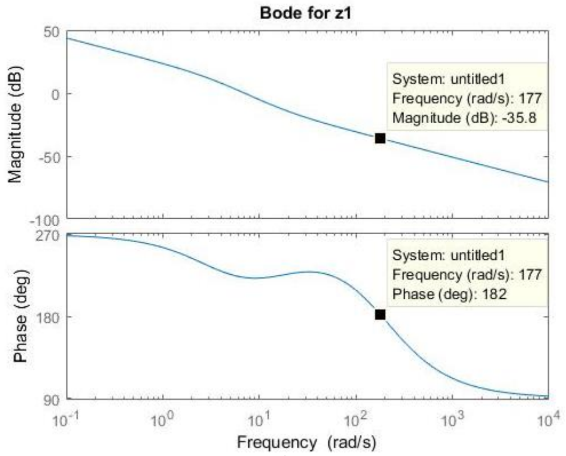

Fig. 3: Bode Plot for the Gain Margin

Here, we see that the gain margin becoming nearly 35.8dB. However, the difference between the values obtained by two different methods will be discussed in the conclusion. Then we look at the plot that is marked for the phase margin.

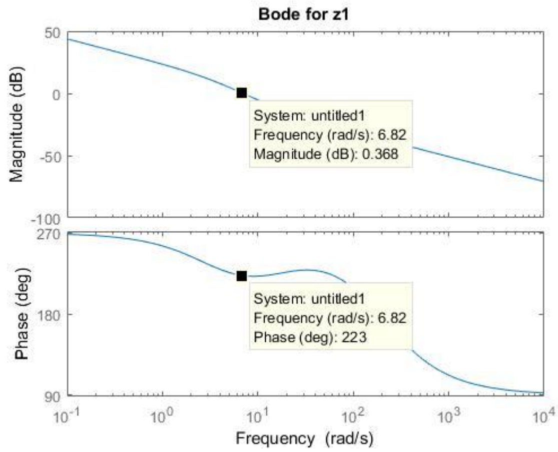

Fig. 4: Bode Plot for the Phase Margin

In this plot, we observe that the phase margin is 47 degrees, since it is the difference in the phase from the upper point of the plot to the corresponding point where the magnitude becomes zero.

Hence, we can calculate the delay margin as,

$$DM = \frac{PM}{Crossover\ Frequency} * \frac{\pi}{180}$$

Hence, as the phase margin is equal to 47 degrees and the and the crossover frequency is approximately 6.99 rad/sec. This results with a delay margin of 0.106 seconds, which is nearly the same with the allmargin command output.

2.2. Margin Verification

The second lab consisted of us experimentally creating the same system and compare the results with the mathematically obtained margin intervals. In the first part we have configured the lab3_step_GM.slz as given below.

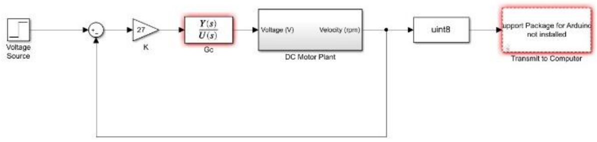

Fig. 5: Lab3_Step_GM Simulink Model

As observed from the schematic, the gain constant K is embedded separately. In this part of the lab, we have changed this gain value up until the system became unstable. The system output just before the system became unstable is given below.

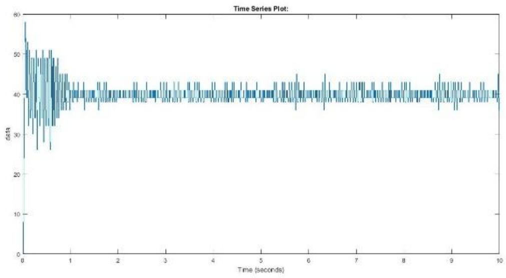

Fig. 6: System output at K=26

Here, we see that the system is stable. Hence, we look at the same system at K=27 and see that the system has become more unstable.

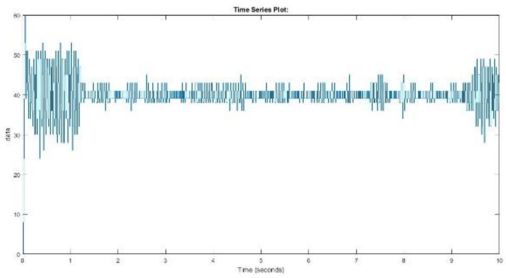

Fig. 7: System Output at K=27

Hence, we observe the output at K=28 and see that the system has become completely unstable.

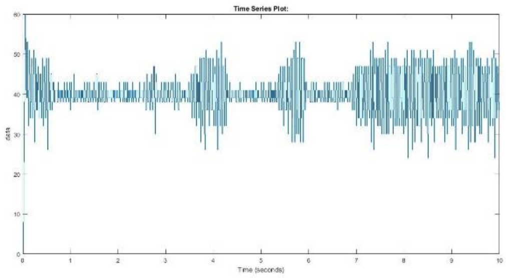

Fig. 8: System Output at K=28

Hence, we calculate the critical gain margin of the system to be 31.25dB, which is close to the experimental value that we have gathered.

The next thing we do is to is to apply the same procedure on Lab3_Step_DM, this time by changing the delay until the system has become unstable. At delay equals 0.07 seconds, we see the output being completely stable as given below with the Simulink schematic that we have configured.

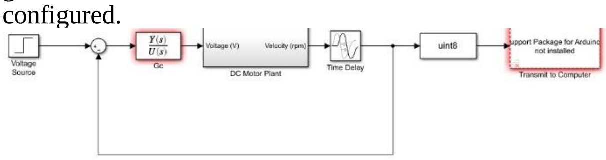

Fig. 9: Lab3_Step_DM Simulink Model

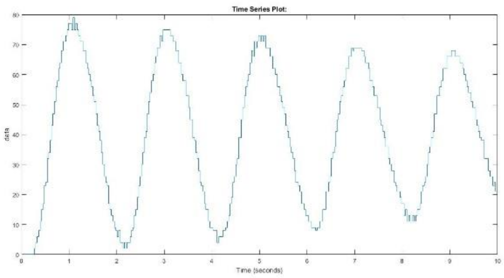

Fig.10: System output with delay t=0.07sec

Hence, we increase the delay to t=0.08 seconds and observe the output. We still see that the system is stable however as the integer values are hold in uint8 format, the maximum value it can store is 255 and then the value resets to zero. Hence, we see the types of irregularities that we see in the following graph.

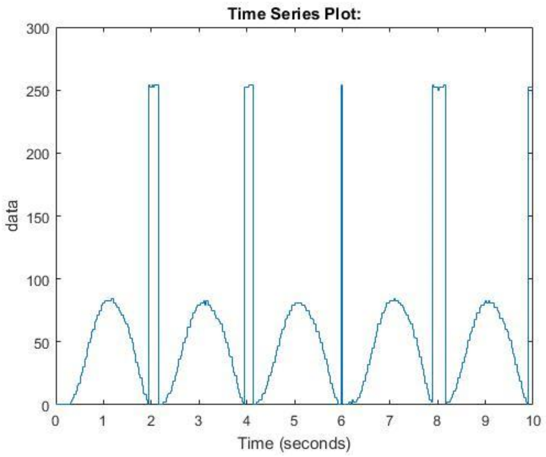

Fig. 11: System output with delay t=0.08sec

Hence, we increase the delay up to 0.1 sec and see that the system has become unstable.

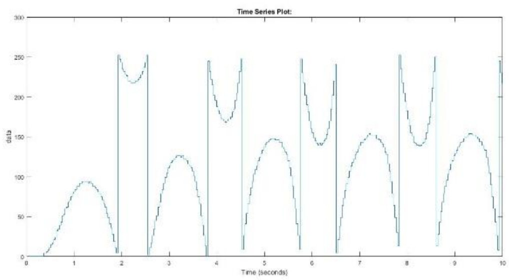

Fig. 12: System output with delay t=0.1sec

The critical delay margin in the calculation came up to be 0.106 seconds however, the experimental results came as 0.01 seconds, which is sufficiently enough.

3. Conclusion

In this lab assignment, we have progressed more into the preliminary work that we have done last week. From the Bode plot, we have configured the gain, phase and delay margins of the system and checked if the experimental values hold up with the mathematically obtained values. Overall, the lab assignment was an informative one in terms of learning what these margins are and what they represent and also using bode and margin commands of MATLAB.

REFERENCES

1. Richard C. Dorf, Rober H. Bishop, "Modern Control Systems" *Pearson Education Limited*, pp. 329–330, Harlow, England, 2017.