EEE 473/573 Medical Imaging – Fall 2020-2021
Homework 4
Due 14 December 2020, Monday at 23:59

GUIDELINES FOR HOMEWORK SUBMISSION

1. NO submission via E-MAIL (all email submissions will be discarded).
2. Submit a PDF file. Other file types will not be accepted. If there are any handwritten parts, you can scan them (make sure they are legible) and insert into the PDF file. Unclear presentation of results will be penalized heavily. No partial credits to unjustified answers.
3. If your Matlab codes are not included at the end of the PDF file, your Matlab questions will NOT be graded.
4. This is a Turnitin submission. The Turnitin system requires the submitted file to contain at least 20 words in it. If you are submitting a Word file with scanned pages only, the file will be rejected by the system. You can type your name multiple times at the beginning of the file to overcome this problem.
5. Submission system will remain open for 1 day after the deadline. No points will be lost if you submit your assignment within 12 hours of the deadline. There will be a 50% penalty if you submit after 12 hours but within 24 hours past the deadline. No submissions beyond 24 hours past the deadline.

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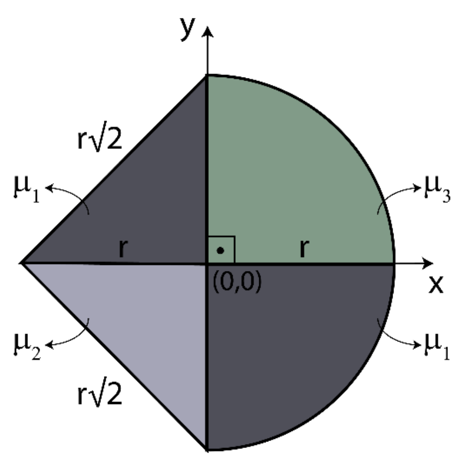
1) For each measured projection, $g(l,\theta)$, we can generate a backprojection image, $b_\theta(x,y)$. Given $g(l,15°) = e^{-l}$, answer the following.

- **a)** Write down an expression for $b_{15°}(x,y)$, i.e., the backprojected image at $\theta = 15°$.
- **b)** Based on the given information, can you determine $b_{165°}(x,y)$? If yes, what is it? If no, why not?
- **c)** Based on the given information, can you determine $b_{195°}(x,y)$? If yes, what is it? If no, why not?

2) As given in the previous homework, the following 2D object with $r = 15\ cm$ is being imaged with a CT scanner using parallel-ray geometry. The linear attenuation coefficients are $\mu_1 = 0.25\ cm^{-1}$, $\mu_2 = 0.05\ cm^{-1}$, and $\mu_3 = 0.35\ cm^{-1}$.

- **a)** Find and sketch $g(l,45°)$.
- **b)** Display $b_{45}(x,y)$, the backprojected image (without filtering) for the 45° case. Mark important points on this image. You may use MATLAB for this part, if you prefer.
- **c)** Assume the source-to-detector distance is 1 m. What is the smallest possible circular FOV to image the entire object shown in the figure? What is the shortest length of the detector array that will cover the FOV?
- **d)** Suppose the detector array has 256 elements. How many angles should be acquired? What is the pixel size (resolution) of the reconstructed image assuming the image covers the entire FOV?



3) Find the 2-D radon transform $g(l,\theta)$ of the following functions using Projection-Slice theorem. Simplify your answer as much as possible.

a) $f(x,y) = e^{-x^2}$

b) $f(x,y) = rect(x,y) - \delta(x,y)$

4) Given the projections, $g(l,\theta)$, find the associated objects, $f(x,y)$. Simplify your answer as much as possible.

a) $g(l,\theta) = \delta(l)$

b) $g(l,\theta) = \delta(l - a \cdot sin\theta)$

c) $g(l,\theta) = rect(l)$

d) $g(l,\theta) = rect(l - a \cdot sin\theta)$

5) **MATLAB Question:** As we covered in Chapter 6, the process of taking the projections of a 2D function is also called "Radon Transform", and the inverse process of reconstructing the images from projections is called the "Inverse Radon Transform". These transforms are available in MATLAB as built-in functions "*radon*" and "*iradon*".

Generate a "phantom" image in MATLAB using the following command:

```
P = phantom('Modified Shepp-Logan',256);
```

This digital phantom presents an axial crosscut of a human body, showing the lungs, the heart, and a few blood vessels. "P" is our "ideal" image.

a) Display image P.

b) Using Radon transform, take projections of P at sufficient number of angles. Reconstruct the image using inverse Radon transform function. Display the sinogram and the reconstructed image. Choose number of projections such that there are no visible artifacts in the reconstructed image.

c) Using the computed sinogram, plot the projections of P for the following angles: 0°, 45°, 90°, and 135°.

d) Repeat part (b), but for fewer number of projections. Make sure that there are some artifacts visible in the reconstructed image. Display the sinogram and the reconstructed image. What kind of an artifact are you seeing?

e) For the projection set in (b), reconstruct the image using three different filters:

(1) Default filter in "*iradon*" (cropped ramp filter, i.e., ramp filter multiplied with a rect function. This is called "Ram-Lak" filter in Matlab),

(2) Hamming windowed filter,

(3) No filter (this would be direct backprojection reconstruction, without any filtering). These are options available in "*iradon*" function, so type "*help iradon*" to see how you can use these filters. Display the resulting images. Comment on the differences that you see in the reconstructed images. Which filter provides the best image? Why?

f) Repeat part (e) for part (d). Display the resulting images. Comment on the differences. Which filter provides the best image? Why?