EEE 473/573 Medical Imaging – Fall 2020-2021
Homework 5
Due 23 December 2020, Monday at 23:59

GUIDELINES FOR HOMEWORK SUBMISSION

1. NO submission via E-MAIL (all email submissions will be discarded).
2. Submit a PDF file. Other file types will not be accepted. If there are any handwritten parts, you can scan them (make sure they are legible) and insert into the PDF file. Unclear presentation of results will be penalized heavily. No partial credits to unjustified answers.
3. If your Matlab codes are not included at the end of the PDF file, your Matlab questions will NOT be graded.
4. This is a Turnitin submission. The Turnitin system requires the submitted file to contain at least 20 words in it. If you are submitting a Word file with scanned pages only, the file will be rejected by the system. You can type your name multiple times at the beginning of the file to overcome this problem.
5. Submission system will remain open for 1 day after the deadline. No points will be lost if you submit your assignment within 12 hours of the deadline. There will be a 50% penalty if you submit after 12 hours but within 24 hours past the deadline. No submissions beyond 24 hours past the deadline.

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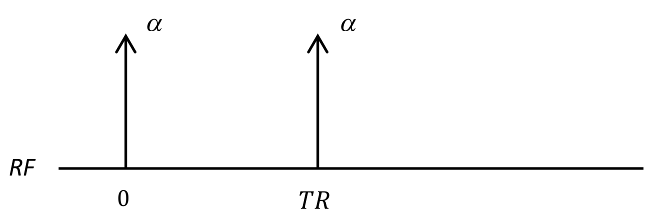
1) Consider the following MR sequence where two subsequent RF pulses are applied with tip angles α. Assume that $M_z(0^-) = M_0$ and $M_{xy}(0^-) = 0$, where M_0 is the equilibrium magnetization. Also assume that $TR \gg T_2$. Do NOT assume that $TR \gg T_1$.

a) Find $M_z(0^+)$ and $M_{xy}(0^+)$.

b) Find $M_z(TR^-)$ and $M_{xy}(TR^-)$.

c) Find $M_z(TR^+)$ and $M_{xy}(TR^+)$.

d) Find $M_z(t)$ and $M_{xy}(t)$ for $t > TR$.



2) If we repeat the RF pulses in Question 1 for a sufficient number of times (waiting for TR in between subsequent RF pulses), the expression for $M_{xy}(t)$ right after an RF pulse will reach what is called a "steady-state" value, which can be expressed as follows:

$$M_{xy}(t) = M_z^{SS}\, sin\alpha\; e^{j\phi} e^{-t/T_2}$$

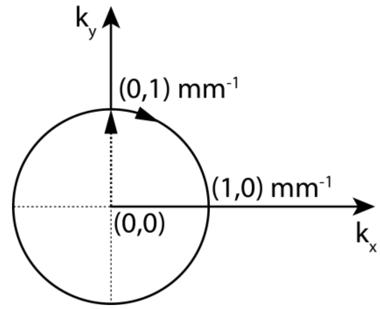
Note that M_z^{SS} is the steady-state value for $M_z(t)$ right before each RF pulse. Show that M_z^{SS} for the equation above can be expressed as:

$$M_z^{SS} = M_0 \frac{1-e^{-TR/T1}}{1-cos\alpha\; e^{-TR/T1}}\,.$$

Hint: Start at $t = nTR^-$, and use the fact that the following should be satisfied in steady-state:

$$M_z(nTR^-) = M_z((n+1)TR^-) = M_z^{SS}$$

3) Draw a pulse sequence diagram where the data is acquired on a circular trajectory in k-space, following an RF excitation pulse that is not slice-selective. Right after RF excitation, start from (0,0) in k-space and go up to (0, 1) mm^{-1}. Then, make a full circle as shown on the right, with constant velocity in k-space.



Assume that G_{max} = 4 G/cm is the maximum gradient strength for the MRI scanner. Choose the gradient/timing parameters so that the k-space trajectory is completed in the shortest time possible.

Assume that the data acquisition takes place only during the circular part of the trajectory. Mark the data acquisition window in your pulse sequence.

Label your timing diagram including the amplitudes of the gradients.

4) **MATLAB Question:** Include your MATLAB codes and related images in your solution.

$\boldsymbol{T_1}$ **Mapping:** If you estimate the T_1 for every pixel in an MRI image and display this as an image, it is called a "T_1 map". So, every pixel in this "T_1 map" image would correspond to the estimated T_1 of the corresponding pixel in the MRI image.

Download the files *brainT1_mri.mat* and *roiellipse.m* posted on Moodle. The file *brainT1_mri.mat* contains two sets of simulated brain MRI images:

- *image1* and *image2*: Simulated T_1-weighted MRI images with $\alpha = 45°$ and $\alpha = 90°$, respectively. For both images $TE = 15\ ms$ and $TR = 600\ ms$. No noise is added.
- *image1_noisy* and *image2_noisy*: Same as above, with a very small amount of noise added to both images.

a) The following imaging equation gives the signal level during echo time TE for a steady-state pulse sequence as in Question 2. Here, T_1 and T_2 are functions on (x, y).

$$f(x,y) = AM_0 \sin\alpha\, e^{-TE/T_2} \frac{1 - e^{-TR/T_1}}{1 - \cos\alpha\, e^{-TR/T_1}}$$

Show that this expression can be re-written as:

$$\frac{f(x,y)}{\sin\alpha} = e^{-TR/T_1} \frac{f(x,y)}{\tan\alpha} + AM_0 e^{-TE/T_2}\left(1 - e^{-TR/T_1}\right)$$

b) Assume that two images $f_1(x,y)$ and $f_2(x,y)$ are acquired with different tip angles α_1 and α_2, while keeping TE and TR the same. Show that an estimate of T_1 can be obtained from these two images using the following relation:

$$\widetilde{T_1}(x,y) = \frac{TR}{\ln\left(\frac{f_2}{tan\,\alpha_2} - \frac{f_1}{tan\,\alpha_1}\right) - \ln\left(\frac{f_2}{\sin\alpha_2} - \frac{f_1}{\sin\alpha_1}\right)}$$

c) Display the noise-free images (i.e., *image1* and *image2*). Calculate and display the T_1 map using the equation in part (b) for the noise-free dataset.

d) Estimate T_1 for white matter using *roiellipse.m*. Here is the MATLAB code that you can use to do that:

```
figure, imshow(T1map,[])
BW = roiellipse; % type "help roiellipse" to see how to use
T1_est = mean(T1map(BW))
```

This code will create a figure displaying your T_1 map. It will then place a draggable and scalable ellipse on it. Drag this ellipse on a part of the image that contains ***white matter only***. Maximize the window containing the figure first, so that you can move/scale the ellipse more carefully. When you are ready, click the "Continue" button on the bottom left of the image. This will create a mask of the selected elliptical region (BW), and then calculate the mean T_1 value in that region using the mask. In your solution, include the image with ellipse showing the selected region, together with the estimated T_1 value.

e) Estimate T_1 for gray matter and CSF (cerebrospinal fluid) using the steps in part (b). Include the images with ellipse in your solution, together with the estimated T1 values.

f) Display the noisy images (i.e., *image1_noisy* and *image2_noisy*). Calculate and display the T_1 map for the noisy dataset. Because these images are noisy, the computed T_1 values may be unreasonably high (or even complex valued). Do the following to restrict the range of T_1 values displayed to between 0-3000 ms, and to force T_1 to be real-valued:

```
figure, imshow(abs(T1map),[0 3000])
```

g) Estimate T_1 for white matter, gray matter, and CSF using the noisy T_1 map. Because of the noise in the T_1 map, it helps to select a reasonably large ellipse (rather than a very small one), so that the estimation of mean T_1 is more accurate. Include the images with ellipse in your solution, together with the estimated T_1 values.

h) Which noisy T_1 estimate showed the biggest deviation from its noise-free version? Why?

Hint: If you are not sure which part of the brain is gray/white matter or CSF, here is a segmented version of the simulated MRI image: