Risk-Informed Hourly Dispatch Optimization for Large Loads

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I. Introduction

Over the past few years, the energy mix in New York state has become increasingly diverse - providing generation from various sources, such as LNG, on-shore wind, solar, and in the future, etc. This increasingly diverse energy mix offers many opportunities but also comes with its challenges: how do we optimally utilize these various and highly dynamic energy sources in a cost-efficient way? As such, the main goal of this project is to optimize the daily energy supply contracts to ensure cost-effectiveness from different sources. This involves evaluating various options, including at-risk wind contracts, generator contracts offering baseload, peak load, and load-following services, on-site solar power, and participation in the spot market.

The key for this project is to determine the optimal energy portfolio that minimizes costs under different scenarios, considering predicted campus demand, real-time prices, and the capacity factors of wind and solar power. Our inputs are five scenarios, each with a unique solar power factor, wind power factor, real time price, and demand across a given day, in 15 minute increments (96 total time stamps). Using optimization to minimize expected cost as well as considering CVaR, we aim to determine the minimum expected cost and the worst-case cost to serve the campus demand and identify each corresponding contract portfolio. Moreover, our project also addresses the broader question of the feasibility of powering the campus entirely with renewable energy. By analyzing different scenarios and resource costs, we aim to outline potential pathways towards a sustainable campus, shedding light on the associated costs and offering insights to guide future decision-making in pursuit of renewable energy goals.

We will first dive into the three stages of optimization, where *stage two* and *three* are the most critical, whereas the first stage optimizes one scenario and one LNG contract at a time: this exercise is to build our intuition before a one-shot optimization process over all variables, across all scenarios. Stage two is a classic expected cost optimization, whereas stage three is an optimization considering tail risk with conditional value at risk (CVaR). Then we will discuss our results and present their significance on how to optimize an energy portfolio for a sustainable campus.

II. Methodology

To determine the minimum expected cost, minimize the worst-case scenario, and identify the associated contracts using data from all five scenarios, we divided our methodology into three stages: optimizing one contract at a time (stage one), optimizing all decision variables jointly (stage two), and CVaR optimization (stage three). Given the availability and no-cost of solar (with a 1 MW nameplate), our net demand was the difference in real-time demand and solar availability.

A. *Stage One*

We first began by optimizing one contract at a time, namely the base load contract first, the peak load contract second, and the generator contractor third. As seen in code block one, each contractor was toggled on or off and incorporated in the first stage of optimization. This process was critical in achieving a baseline understanding of how each contract behaves

TABLE I

Stage One Results

Optimal Objective Function by Scenario and Contract					
	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5
Base Load	**\$3,229**	**\$3,338**	**\$3,404**	**\$3,447**	**\$3,288**
Peak Load	\$4,523	\$5,399	\$4,547	\$4,983	\$4,684
Load Following	\$3,871	\$4,140	\$4,047	\$4,228	\$3,941

We can observe directly from our initial results that if we are forced to only pick one contract from the LNG source, the baseload contract would be ideal since it satisfies a great majority of our demand with stability and predictability. This hints at the importance of a stable baseload generation source, even for a campus looking to be as sustainable as possible. This is because renewable sources are often unpredictable and unable to handle the consistent demand from various activities.

Not only is our baseload contract able to handle a majority of the demand, we can also see that the baseload contracts are the most cost-efficient across various scenarios as well. This usually stems from the fact that LNG is (in general) an economically efficient source of generation.

As mentioned, this is only meant to build some intuition and a foundational understanding of how individual LNG contracts would be used in our optimization framework. The main body of our work will be provided in *Stage Two* and *Stage Three* analyses. The results of our Stage One analysis will be given in the appendix for reference.

B. *Stage Two*

During *Stage Two*, we are looking to jointly optimize all the decision variables at once, across all days, and optimize the expected cost. The provided optimization model represents a decision-making framework for achieving a sustainable campus energy system. It balances the use of wind energy, LNG contracts, and spot purchasing to meet the campus's electricity demand. Let's break down the key components of the model and discuss the reasoning behind each constraint.

In particular X_{w_i} is the wind contract quantity in scenario i in MW, X_{c1_i} is the "plan-ahead" the baseload generation contract quantity in scenario i in MW, X_{c2_i} is the "plan-ahead" peak generation contract in scenario i in MW, X_{c3_i} is the "intraday" options contract quantity in scenario i in MW, and lastly $X_{spt_{i,j}}$ is the spot purchase quantity during interval j (the jth 15 minute of the day) for scenario i in MW. $P_{rt_{i,j}}$ is the spot price during the jth interval in scenario i.

Now that we have an understanding of our variables, the objective function we are optimizing over, and the constraints are provided below:

Objective Function:

$$\begin{aligned}
\text{Min} \quad & \frac{1}{5}(\underbrace{250\sum_{i=1}^{5} X_{w_i}}_{\text{Wind Decision}} + \underbrace{360\sum_{i=1}^{5} X_{c1_i}}_{\text{LNG Contract 1 Decision}} + \underbrace{200\sum_{i=1}^{5} X_{c2_i}}_{\text{LNG Contract 2 Decision}} \\
& + \underbrace{\sum_{i=1}^{5}(50\max_i\{X_{c3_{ij}}\} + 18\sum_{j=1}^{96} \overbrace{(X_{c3_{ij}} \times \frac{1}{4}}^{\text{Normalized Units (MW)}}))}_{\text{LNG Contract 3 (Options) Decision}} \\
& + \underbrace{\sum_{i=1}^{5}\sum_{j=1}^{96}(P_{rt_{i,j}} \overbrace{\cdot X_{spt_{i,j}} \times \frac{1}{4}}^{\text{Normalized Units (MW)}})}_{\text{Spot Purchasing}}
\end{aligned} \tag{1}$$

In our objective function, we take the product of the complete objective function and $\frac{1}{5}$; this is because we are taking a simple expected value in this stage through the equation $E(C) = \frac{1}{n}\sum_{i=1}^{n} c_i$, where C is a random variable of cost, and c_i are minimized costs in scenario i.

Constraints:

1) Load Balance Constraint:

$$\begin{aligned}
& D_{ij} - \text{Sol}_{ij} \leq X_{w_i} \cdot \text{NCF}_{ij} \\
& \quad + X_{c1_i} \cdot I_{c1} + X_{c2_i} \cdot I_{c2} + X_{c3_{ij}} + X_{spt_{ij}} \\
& \quad \forall i = 1, \ldots, 5,\ j = 1, \ldots, 96
\end{aligned}$$

The load balance constraint comes directly from the need that generated supply must meet the demand on campus. Notice that we have relaxed the constraint from a hard equality to inequality. Here, we are assuming that, while we are unable to sell back to the grid, we can curtail generation should supply exceed demand.

2) Wind Generation Constraints:

$$0 \leq X_{w_i} \leq 20 \quad \forall i = 1, \ldots, 5$$

This is a simple constraint that comes directly from the fact that we cannot decide to buy more at-risk wind than the wind farm nameplate. Since wind is variable, this effectively means that our true wind purchase will be the purchased fraction of the 20 MW nameplate ($\frac{X_{w_i}}{20}$) multiplied by the true generating amount at scenario i, interval j: $20 \cdot \text{NCF}_{ij}$. Simplifying this expression, we find that we are simply purchasing $X_{w_i} \cdot \text{NCF}_{ij}$ MWs of capacity at scenario i, period j, where NCF is the net capacity factor for scenario i, period j. That is, we are purchasing a fraction of at-risk generation at any given time.

3) Generation Constraints:

$$X_{c1_i} + X_{c2_i} + \max_i\{X_{c3_{ij}}\} \leq 10 \quad \forall i = 1, \ldots, 5$$

This constraint stems directly from the fact that we have an LNG generator with a nameplate of 10MW with our base load, peak load, and options contracts; thus we cannot provision more generation than there is capacity. Notice the last term with $X_{c3_{ij}}$: this is simply the option capacity for that day.

4) No Shorting, No Sell Back:

$$X_i \geq 0 \forall X_i \in \{X_{w_i}, X_{c1_i}, X_{c2_i}, X_{c3_i}, X_{spt_{i,j}}\}$$

This is a non-negativity constraint on all of our generation sources: we cannot (especially in the spot market) short or sell back generation to the grid. That is to say, this campus is a pure consumer in the energy market.

5) Availability Indicator:

I_{c1} (and similarly, I_{c2})
is a fixed indicator vector to describe availability
(i.e., I_{c1} is a vector of all 1s for all-day availability)

This is a simple indicator for base load availability and peak load availability for our LNG contract, where the I_{c1} base load vector is a simple 1-only vector and the peak load vector I_{c2} is 1s for when the peak load is active, and 0 everywhere else.

C. Stage Three

The utilization of Conditional Value at Risk (CVaR) in our analysis has significantly enriched our understanding of the sustainable campus's energy procurement strategy, especially when considering tail-risk scenarios. The integration of cost parameters can refine our optimization approach, ensuring a robust and resilient energy system against tail risk.

Objective Function Modification

The modified objective function incorporates a novel cost parameter a and a penalty term M_i to balance both expected costs and the impact of tail-risk scenarios. The objective function now aims to minimize:

$$\text{Minimize} \quad a + \sum_{i=1}^{5} M_i$$

where M_i captures the tail-risk costs associated with each scenario.

Optimization Constraints:

The non-negativity constraint ensures that the introduced penalty term is non-negative:

$$M_i \geq 0 \quad \forall i = 1, \ldots, 5$$

Additional constraints enforce that the penalty term M_i captures positive deviations of actual costs (OBJ_i) from the baseline cost a:

$$M_i \geq \text{OBJ}_i - a \quad \forall i = 1, \ldots, 5$$

III. Results

A. Expected Costs Approach (Stage Two)

After the optimization process, we find that our optimized expected cost for our portfolio is $\$3207.92$.

A visualization of the decision variables that went into our process is provided below. Note that the overwhelming majority of the generation is taken up by the base load generation, supplemented by purchasing power from the spot market while accounting for the net generation provided by the solar installations.

This is an intuitive result, as we see that the bulk of generation is taken up by the baseload, and the peak is covered by demand-following options. since we see from stage one that our baseload generation *only* case gives us between $\$3,229$ to $\$3,447$ dollars, relaxing that constraint and adding flexibility will allow us to decrease the cost. Thus this result is moving in the correct direction.

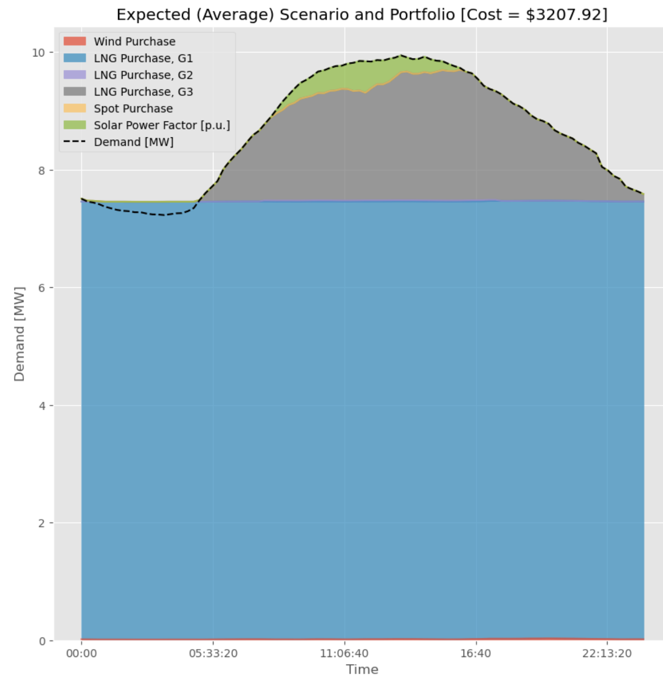

Fig. 1. Stage 2 Average Optimization Results

TABLE II
STAGE TWO RESULTS

Source	Quantity Purchased
Average Wind Quantity	0 MW
Average Base Load Quantity	7.44 MW
Average Peak Load Qunatity	0 MW
Average Option Quantity	1.13 MW
Average Spot Quantity	0 MW
Average Solar Quantity	0.1 MW
Average Demand	8.67 MW

To summarize: LNG contracts provide a stable and consistent baseload, while options allow flexibility to cover peak demand periods. The inclusion of spot purchasing further adds flexibility to the system, but it is less used due to the presence of an option (however, if we had more uncertainty, the cost of uncertainty from pre-committing to an option could shift the balance between buying from spot and buying from options).

What is also notable is that wind is effectively not present in our optimization process. This could signal a need to provide subsidies to wind generation, as subsidizing the risk to purchase will counteract the variability and make wind resource a more attractive option for optimization.

In summary, the optimization model balances the use of different energy sources based on their characteristics and costs. The results emphasize the importance of LNG contracts for providing a reliable baseload while utilizing demand-following options to meet additional demand and manage variability.

B. CVaR Approach (Stage 3)

CVaR is a way to measure the tail risk in an investment or situations where there is high variability in the outcomes of a particular action. The key difference between CVaR and a simple expected value is that CVaR takes into consideration the *amount* of value that could be lost under a certain threshold. This means that we can quantify what our generation should look like when considering this tail risk.

The CVaR optimized portfolio quantities are shown below, which gives us a CVaR-adjusted cost of $\$3345.85$, and once again, a snapshot of the decision variables that went into our process is provided below.

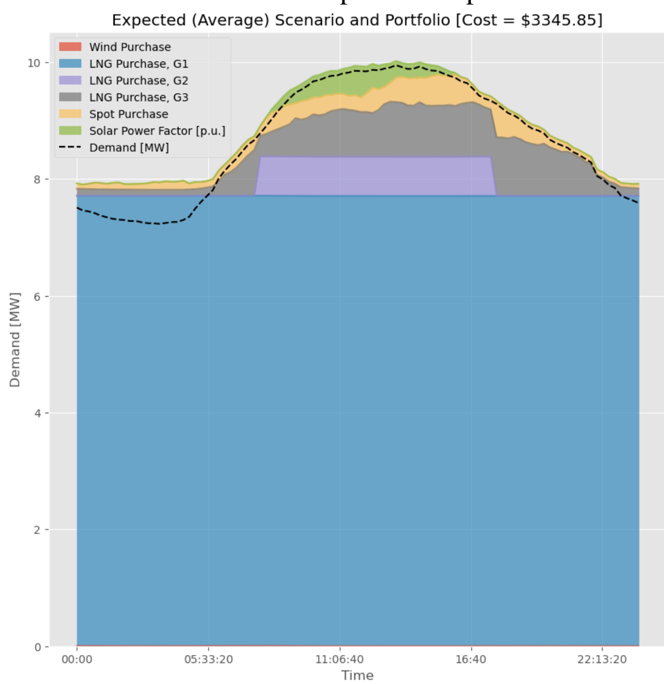

Fig. 2. Stage 3 Average Optimization Results

TABLE III
STAGE THREE RESULTS

Source	Quantity Purchased
Average Wind Quantity	0 MW
Average Base Load Quantity	7.71 MW
Average Peak Load Qunatity	0.28 MW
Average Option Quantity	0.55 MW
Average Spot Quantity	0.24 MW
Average Solar Quantity	0.1 MW
Average Demand	8.67 MW

We see that the incorporation of CVaR and the introduced penalty term refines our optimization strategy, emphasizing the importance of mitigating extreme costs associated with tail-risk scenarios. Here's how each observation is impacted:

Baseload Contract Importance: The heightened demand for the baseload contract gains even greater significance under tail-risk scenarios (notice the curtailment before sunrise is higher than that of stage two optimization). The stability and predictability of the baseload contract become crucial in mitigating extreme cost outcomes, reaffirming its role as a reliable anchor in the campus's energy system.

Demand-Following Contract Preferences: There is a trade-off between dynamic contracts (spot and options) for peak load generation (purple) from our LNG source in our CVaR optimization. We see that the peak load generation (in purple) pushes some of the demand-following sources out of the equation. In scenarios with less-than-perfect foresight, it is an intuitive and economically sound move to take a certainty than to bet on fluctuating prices (especially if the entity is a campus that needs to keep many critical instruments in research and education running)

Reduced Reliance on Wind Power: While a bit disappointing, we do not see any contribution from wind power, approaching negligible levels in certain scenarios. Wind power's intermittent nature becomes a potential risk factor under extreme conditions, highlighting the importance of stable and reliable energy sources to minimize the impact of tail-risk scenarios.

IV. SUSTAINABLE CAMPUS PATHWAYS

Based on the five scenarios and portfolio management results, a clear pathway towards a sustainable campus emerges, prioritizing solar energy coupled with battery storage.

Upstate New York receives good solar irradiance, making solar panels a viable and cost-effective option. Notably, the methodology approach enabled free solar from tits availability and power factor across all five scenarios, which aligns with its minimal ongoing costs after installation. Recent reports indicate an average cost of $2.50-4.00$ per watt for solar panel installation in New York, translating to roughly $1.25-2.00$ per watt-hour of energy generated over the system's lifespan (25-30 years) [1]. Compared to current grid electricity prices in upstate New York (around \$0.15/kWh), solar offers significant long-term saving, especially key for a large footprint such as a campus with readily available space such as roofs and surrounding areas.

While wind energy has potential, the five scenarios and optimization results suggest its feasibility is less clear and urgent [2]. The optimization results from both Stage Two and Stage Three showed wind being excluded in the worst-case scenario, indicating potential unreliability or cost inefficiency specific to the campus environment. Therefore, focusing on solar appears to be the more prudent initial step.

Battery energy storage becomes crucial to address the mismatch between solar generation and campus energy demand. By storing excess solar power during peak production times, the campus can tap into this reserve when solar output dips or nighttime demand rises. This reduces reliance on the grid and associated demand charges, further contributing to cost savings and sustainability. Battery storage costs are decreasing steadily, with estimates for commercial systems ranging from $150-200$ per kWh of storage capacity [3]. Combining this with solar panel costs, a rough estimate for a campus-scale system (e.g., 1 MW solar, 0.5 MWh storage) would be around $3.5-$5 million USD. However, specific costs will depend on the campus size, energy needs, and chosen approach.

V. CONCLUSION

In summary, the integration of CVaR and the introduced penalty term refines our optimization strategy, ensuring that the campus's energy system is not only cost-effective under expected conditions but also robust and resilient in the face of tail-risk scenarios. Each observed trend is intricately linked to the broader goal of creating a sustainable campus by balancing cost efficiency, reliability, and adaptability to unforeseen challenges. This approach ensures that the campus is well-prepared for a spectrum of scenarios, contributing to the overarching goal of a sustainable and resilient energy infrastructure.

A strategic approach prioritizing solar energy with battery storage offers the most promising pathway towards a sustainable campus in upstate New York. While wind energy may have future potential, solar's cost-effectiveness and reliability make it the current frontrunner. Integrating battery storage further enhances sustainability and reduces dependence on the grid. This path promises significant long-term cost savings and aligns with the campus's commitment to a greener future.

VI. Appendix

A. Figures

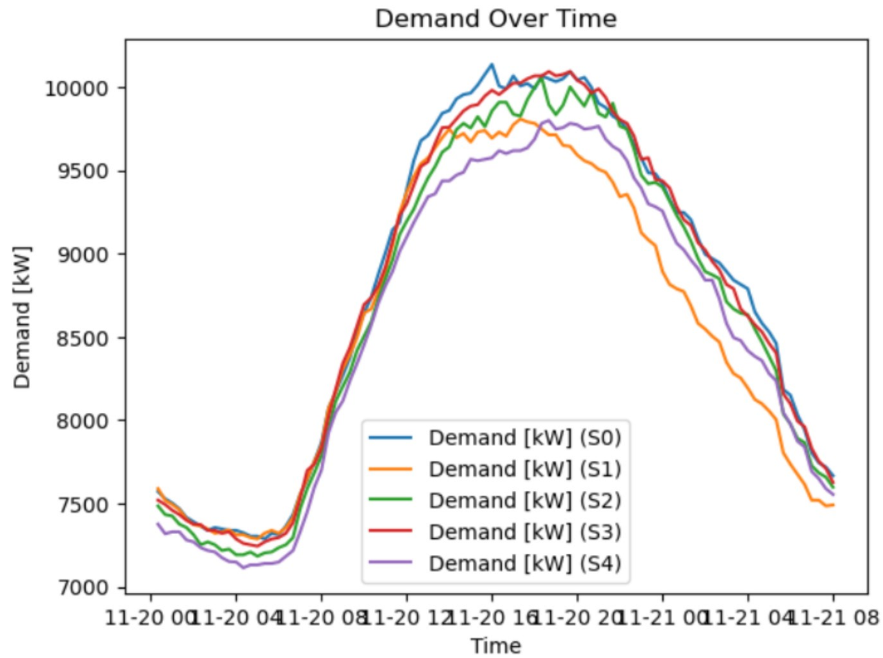

Fig. 3. Demand Profile - All Five Scenarios

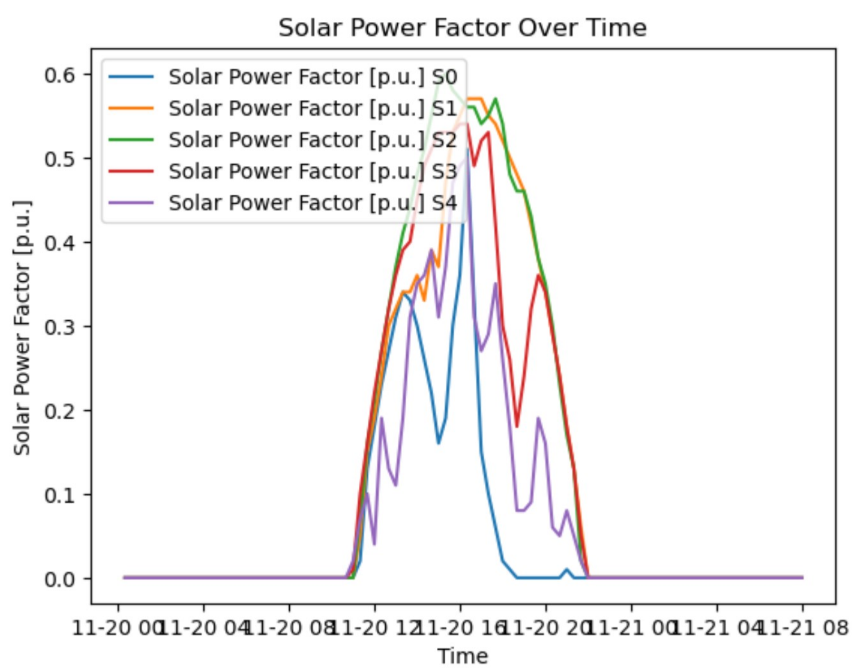

Fig. 4. Solar Power Factor - All Five Scenarios

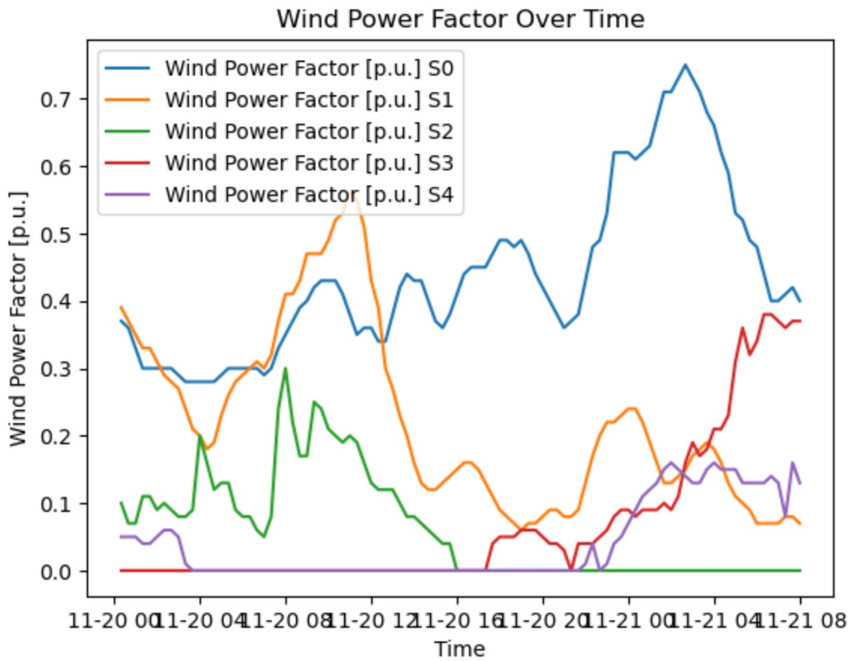

Fig. 5. Wind Power Factor - All Five Scenarios

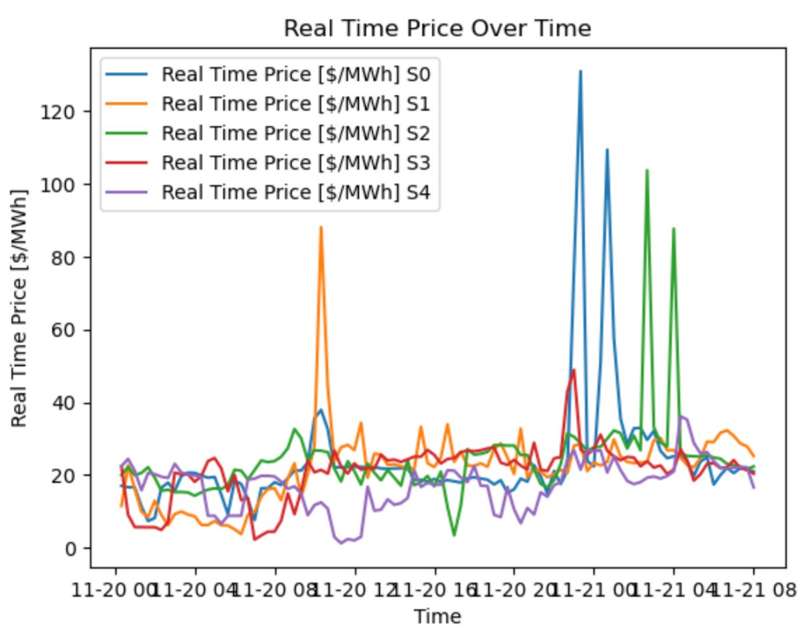

Fig. 6. Real Time Prices - All Five Scenarios

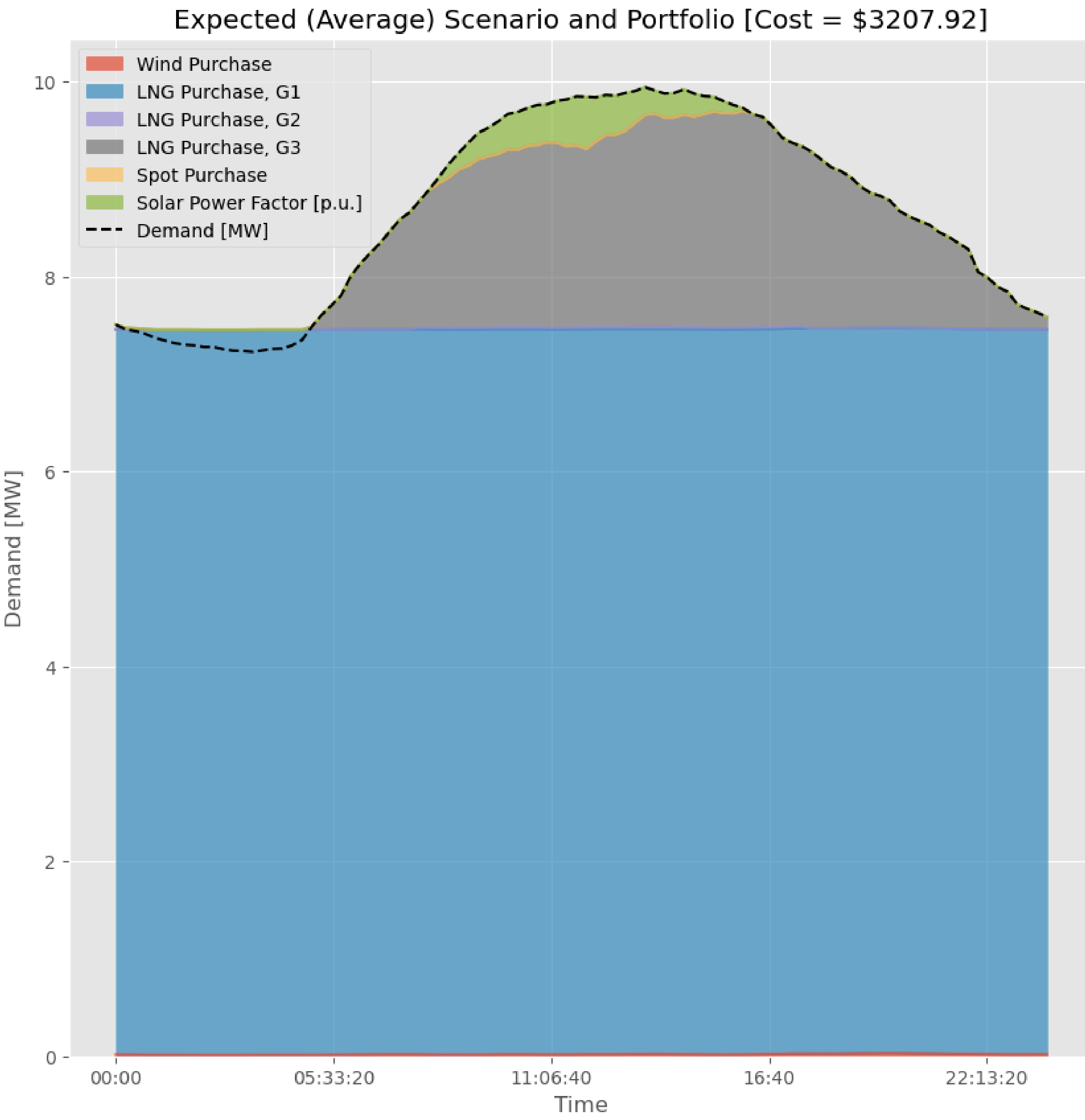

Fig. 7. Stage 2 Average Optimization Results

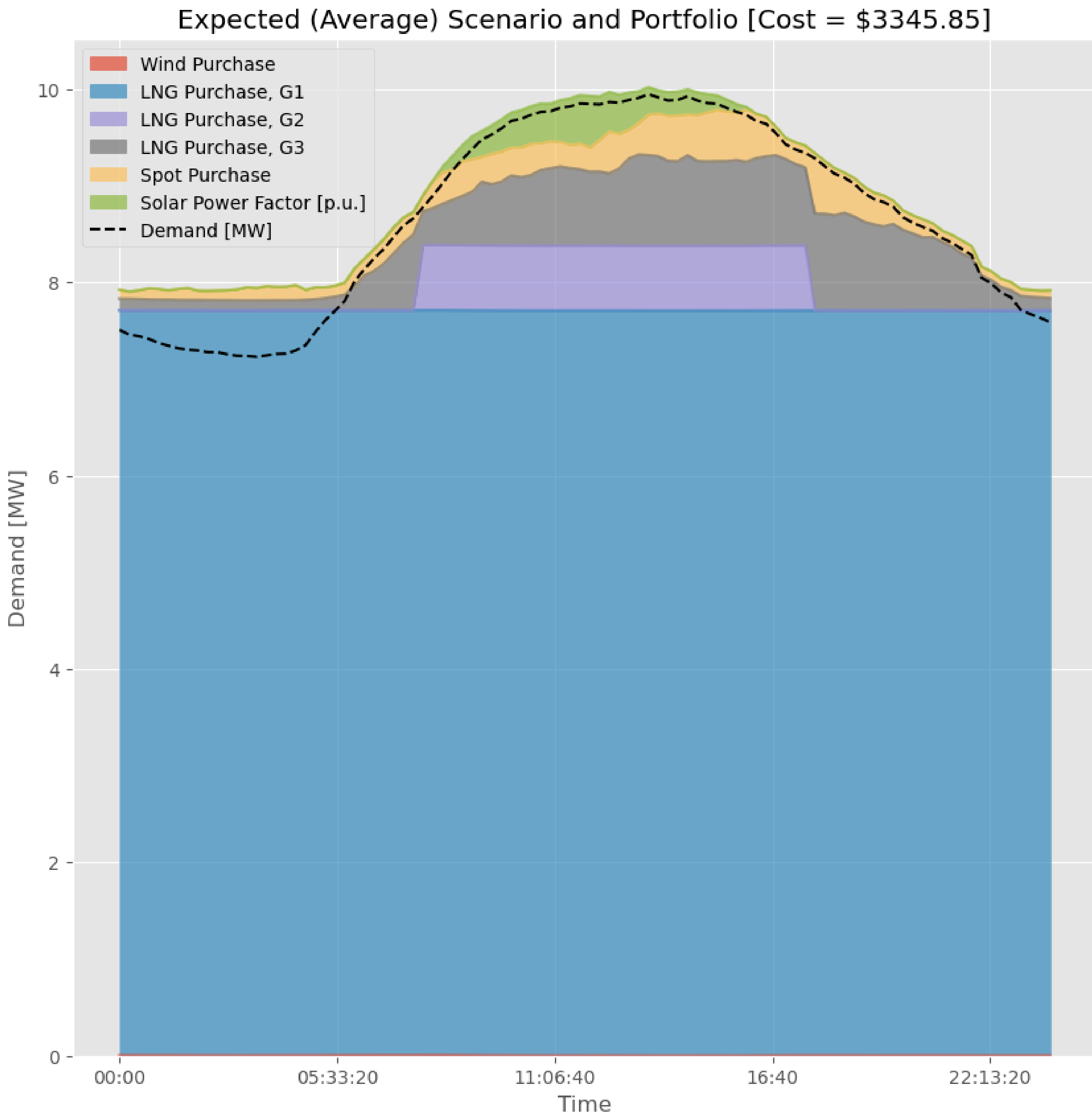

Fig. 8. Stage 3 Average Optimization Results

B. *Tables*

TABLE IV

STAGE ONE RESULTS

Optimal Objective Function by Scenario and Contract					
	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Base Load	**$3,229**	**$3,338**	**$3,404**	**$3,447**	**$3,288**
Peak Load	$4,523	$5,399	$4,547	$4,983	$4,684
Load Following	$3,871	$4,140	$4,047	$4,228	$3,941

TABLE V

OVERALL RESULTS

Best Case Scenario	**Worst Case Scenario**	**Expected Cost**	**CVaR Optimization**
$3,329	$3,447	$3,341	$3,345

TABLE VI

STAGE TWO RESULTS

Source	**Cost**
Average Wind Cost	$0
Average Base Load Cost	$2775.6
Average Peak Load Cost	$56
Average Option Cost	$32
Average Spot Cost	$16.45
Expected Cost	$3207.92

C. Mathematical Formulation

$$
\begin{aligned}
\text{Min}\quad & \frac{1}{5}(\underbrace{250\sum_{i=1}^{5} X_{w_i}}_{\text{Wind Decision}} + \underbrace{360\sum_{i=1}^{5} X_{c1_i}}_{\text{LNG Contract 1 Decision}} + \underbrace{200\sum_{i=1}^{5} X_{c2_i}}_{\text{LNG Contract 2 Decision}} \\
& + \underbrace{\sum_{i=1}^{5}(50X_{c3_i} + 18\sum_{j=1}^{96} \overbrace{(X_{c3_{ij}} \times \frac{1}{4}}^{\text{Normalized Units (MW)}}))}_{\text{LNG Contract 3 (Options) Decision}} \\
& + \underbrace{\sum_{j=1}^{96} P_{rt_j} \overbrace{\cdot X_{spt_j} \times \frac{1}{4}}^{\text{Normalized Units (MW)}}}_{\text{Spot Purchasing}}
\end{aligned}
$$

$$
\begin{aligned}
\text{Load Balance:}\quad & D_{ij} - \text{Sol}_{ij} \leq \frac{X_{w_i}}{20} \cdot \text{W\_max}_{ij} + X_{c1_i} \cdot I_{c1} + X_{c2_i} \cdot I_{c2} + X_{c3_{ij}} + X_{spt_{ij}} \\
& \forall i = 1, \ldots, 5,\ j = 1, \ldots, 96 \\
\text{Wind Constraints:}\quad & 0 \leq X_{w_i} \leq 20 \quad \forall i = 1, \ldots, 5 \\
\text{Generation Constraints:}\quad & X_{c1_i} + X_{c2_i} + X_{c3_i} \leq \text{gen\_max} \quad \forall i = 1, \ldots, 5 \\
& \text{Where } I_c \text{ is an fixed indicator vector to describe generation availability} \\
& \text{(i.e., } I_{c1} \text{ is a vector of all 1s to describe all day availability)}
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
\text{Minimize}\quad & a + \frac{1}{5} \cdot \frac{1}{20\%} \sum_{i=1}^{5} M_i \\
= \text{Minimize}\quad & a + \sum_{i=1}^{5} M_i
\end{aligned}
$$

$$
\begin{aligned}
\text{Non-negativity Constraint:}\quad & M_i \geq 0 \quad \forall i = 1, \ldots, 5 \\
\text{Additional Constraints:}\quad & M_i \geq \text{OBJ}_i - a \quad \forall i = 1, \ldots, 5 \\
\text{Load Balance Constraints:}\quad & \text{(same as in the previous code)}
\end{aligned}
$$

D. Code

```python
import numpy as np
import pandas as pd
import glob
import os
import matplotlib.pyplot as plt
import gurobi
import cvxpy as cp

for i in range(5):
    #setting the parameter arrays
    scenario = i #scenario index

    j = 0 #period index

    #setting default parameters
    D = list() #demand
    W_max = list() #max wind
    Sol = list() #solar production
    P_rt = list() #spot price
    gen_max = 10 #max LNG


    #column transformations
    for j in range(96):
        row = dfs[scenario].iloc[j]

        D.append((row['Demand [kW]'])/1000) #MW
        W_max.append(row['Wind Power Factor [p.u.]']*W_base) #MW
        Sol.append(row['Solar Power Factor [p.u.]']*S_base) #MW
        P_rt.append(row['Real Time Price [$/MWh]'] + 5) #$/MW


    #turning lists into arrays so that cvx can recognize them
    D = np.array(D)
    W_max = np.array(W_max)
    Sol = np.array(Sol)
    P_rt = np.array(P_rt)

    #contract constraints:
    #I -> indicator for when the generator can operate
    #essentially, multiply the constant result with the indicator
    I_c1 = [1]*96
    I_c2 = [0]*96

    for i in range(96):
        if i in range(31,71):
            I_c2[i] = 1
        else:
            I_c2[i] = 0

    #cast into arrays
    I_c1 = np.array(I_c1)
    I_c2 = np.array(I_c2)

    #decision variables
    #FOR THIS RUN, ASSUME IT IS THE BASELOAD CONTRACT

    X_c1 = cp.Variable(nonneg=True) #purchase for contract 1 (MW)
    X_c2 = cp.Variable(nonneg=True) #purchase for contract 2
    X_c3 = cp.Variable(96, nonneg=True) #purchase for contract 3
    X_w = cp.Variable(nonneg=True) #wind purchase (MW): has to be a single value
    X_spt = cp.Variable(96, nonneg=True) #spot purchase (MW)
```

```
    #objective

    obj = cp.Minimize(250*X_w #cost of x MW of wind
                      + 360*X_c1 #cost of x MW of LNG, contract 1
                      + 200*X_c2 #cost of x MW of LNG, contract 2
                      + ((cp.max(50*X_c3) + 18*cp.sum((X_c3*0.25)))) #cost of x MW of LNG,
    contract 3
                      + P_rt.T@X_spt*(1/4)) #cost of x MW of spot electricity


        #condition set
    con_set = []

    for j in range(96):
        con_set.append(D[j] - Sol[j] == (X_w/W_base)*W_max[j] + X_c1*I_c1[j] + X_c2*I_c2[j] +
     X_c3[j] + X_spt[j]) #Load Balance
        con_set.append(X_c3[j] <= gen_max) #cannot exceed capacity at each time

    con_set.append(X_w <= W_base) #Wind Constraints; MW
    con_set.append(0 <= X_w)
    con_set.append(X_c1 + X_c2 + X_c3 <= gen_max) #Generation Constraints, gen_max = 10


    #solve the problem
    prob = cp.Problem(obj,con_set)
    prob.solve(solver = "GUROBI")
    prob.solve();

    #print out

    m.append(prob.value)

    print(f"The end cost is: ${round(prob.value,2)}")
    print(f"Wind is: {round(X_w.value,2)}")
    print(f"Contract 1 is: {round(X_c1.value,2)}")
    print(f"Contract 2 is: {round(X_c2.value,2)}")
    print(f"Contract 3 max is: {round(np.max(np.array(X_c3.value)),2)}")
    print(f"Spot max is: {round(np.sum(np.array(X_spt.value)),2)}")
    print('__________')
```

Listing 1. Stage One

```
#setting key parameters
W_base = 20 #MW
S_base = 1 #MW

#setting the parameter arrays
D_registry = dict()
W_max_registry = dict()
Sol_registry = dict()
P_rt_registry = dict()

for scenario in range(5):

    j = 0 #period index

    #setting default parameters
    D = list() #demand
    W_max = list() #max wind
    Sol = list() #solar production
    P_rt = list() #spot price
    gen_max = 10 #max LNG

```

```python

    #column transformations
    for j in range(96):
        row = dfs[scenario].iloc[j]

        D.append((row['Demand [kW]'])/1000) #MW
        W_max.append(row['Wind Power Factor [p.u.]']*W_base) #MW
        Sol.append(row['Solar Power Factor [p.u.]']*S_base) #MW
        P_rt.append(row['Real Time Price [$/MWh]'] + 5) #$/MWh


    #turning lists into arrays so that cvx can recognize them
    D = np.array(D)
    W_max = np.array(W_max)
    Sol = np.array(Sol)
    P_rt = np.array(P_rt)

    if scenario not in D_registry.keys():
        D_registry[scenario] = D
    if scenario not in W_max_registry.keys():
        W_max_registry[scenario] = W_max
    if scenario not in Sol_registry.keys():
        Sol_registry[scenario] = Sol
    if scenario not in P_rt_registry.keys():
        P_rt_registry[scenario] = P_rt

#contract constraints:
#I -> indicator for when the generator can operate
#essentially, multiply the constant result with the indicator
I_c1 = [1]*96
I_c2 = [0]*96

for i in range(96):
    if i in range(31,71):
        I_c2[i] = 1
    else:
        I_c2[i] = 0

#cast into arrays
I_c1 = np.array(I_c1)
I_c2 = np.array(I_c2)

D_mtx = np.array(list(D_registry.values()))
W_max_mtx = np.array(list(W_max_registry.values()))
Sol_mtx = np.array(list(Sol_registry.values()))
P_rt_mtx = np.array(list(P_rt_registry.values()))

#decision variables
#FOR THIS RUN, ASSUME IT IS THE BASELOAD CONTRACT

X_c1 = cp.Variable(5, nonneg=True) #purchase for contract 1 (MW)
X_c2 = cp.Variable(5, nonneg=True) #purchase for contract 2
X_c3 = cp.Variable((5, 96), nonneg=True) #purchase for contract 3
X_w = cp.Variable(5, nonneg=True) #wind purchase (MW): has to be a single value
X_spt = cp.Variable((5,96), nonneg=True) #spot purchase (MW)

#objective

obj = (1/5)*cp.Minimize((250*cp.sum(X_w) #cost of x MW of wind
                  + 360*cp.sum(X_c1) #cost of x MW of LNG, contract 1
                  + 200*cp.sum(X_c2) #cost of x MW of LNG, contract 2
                  + (cp.sum(cp.max(50*X_c3) + 18*cp.sum((X_c3*0.25)))) #cost of x MW of LNG, contract 3
```

```
                  + cp.sum(P_rt_mtx.T@X_spt*(1/4)))) #cost of x MW of spot electricity

#condition set
con_set = []

for i in range(5):
    for j in range(96):
        con_set.append(D_mtx[i][j] - Sol_mtx[i][j] <= (X_w[i]/W_base)*W_max_mtx[i][j] + X_c1[
    i]*I_c1[j] + X_c2[i]*I_c2[j] + X_c3[i][j] + X_spt[i][j]) #Load Balance

    con_set.append(X_w <= W_base) #Wind Constraints; MW
    con_set.append(0 <= X_w)
    con_set.append(X_c1[i] + X_c2[i] + X_c3[i] <= gen_max) #Generation Constraints, gen_max =
     10

#solve the problem
prob = cp.Problem(obj,con_set)
#prob.solve(solver = "GUROBI")
prob.solve();

print(X_w.value.mean()*250)
print(X_c1.value.mean()*360)
print(X_c2.value.mean()*200)
print(X_c3.value.mean()*50+X_c3.value.mean()*(18*0.25))
print((P_rt_mtx.T@X_spt.value).mean())

print(f"The end cost is: ${round(prob.value,2)}")
```

Listing 2. Stage Two

```
a = cp.Variable()
M = cp.Variable(5)

obj_cvar = cp.Minimize(a + (1/5)*(1/0.2)*cp.sum(M))

con_cvar = []

con_cvar.append(M >= 0)
for i in range(5):
    con_cvar.append(M[i] >= (250*cp.sum(X_w[i]) #cost of x MW of wind
                        + 360*cp.sum(X_c1[i]) #cost of x MW of LNG, contract 1
                        + 200*cp.sum(X_c2[i]) #cost of x MW of LNG, contract 2
                        + (cp.sum(cp.max(50*X_c3[i]) + 18*cp.sum((X_c3[i]*0.25)))) #cost of x
    MW of LNG, contract 3
                        + cp.sum(P_rt_mtx[i].T@X_spt[i]*(1/4)))
                        - a
                    )
con_cvar.extend(con_set)

prob_cvar = cp.Problem(obj_cvar,con_cvar)
prob_cvar.solve()

#cast results into DF for better visualization
res_dfs = list()
for i in range(5):
    res_df = pd.DataFrame()
    names = ['Wind Purchase', 'LNG Purchase, G1', 'LNG Purchase, G2', 'LNG Purchase, G3', '
    Spot Purchase']
    res = [(X_w[i]/W_base)*W_max_mtx[i], X_c1[i]*I_c1, X_c2[i]*I_c2, X_c3[i], X_spt[i]]
    for name,x in zip(names,res):
        res_df[name] = x.value
        res_df[name] = res_df[name].apply(lambda x: round(x,4))

    res_df = res_df.join(dfs[i].iloc[:,1]/1000)
```

```
    res_df = res_df.join(dfs[i].iloc[:,3]*1)

    res_dfs.append(res_df)

#concat
df_concat = pd.concat((res_dfs[0], res_dfs[1], res_dfs[2], res_dfs[3], res_dfs[4]))
by_row = df_concat.groupby(df_concat.index)
df_means = by_row.mean()

#plotting
import matplotlib.pyplot as plt
fig, ax1 = plt.subplots(figsize=(10, 10))

res_df = df_means
# Plot the stacked area plot for the original variables
res_df.set_index(time_range,inplace=True)
res_df.reset_index(inplace=True)
res_df.plot(x="index", y=['Wind Purchase', 'LNG Purchase, G1', 'LNG Purchase, G2', 'LNG
    Purchase, G3', 'Spot Purchase', 'Solar Power Factor [p.u.]'],
             kind="area", stacked=True, alpha=0.7, ax=ax1)

# Create a secondary y-axis for 'Demand [kw]'
res_df.plot(x="index", y='Demand [kW]', kind="line", color='black', linestyle='dashed', ax=
    ax1)

# Set labels for the axes
ax1.set_xlabel('Time')
ax1.set_ylabel('MW')
ax1.set_ylabel('Demand [kW]')
ax1.set_title("CVaR Optimized Scenario and Portfolio [Cost = $3345.85]")
plt.show()
```

Listing 3.   Stage Three (CVaR)

References

[1] S. E. I. Association, "National solar panel price trends: Residential & commercial,"
[2] S. P. Authority, "Solar panel costs in new york,"
[3] Lazard, "Lazard levelized cost of energy analysis and insights,"