# IE 313 – Supply Chain Management
# Homework 2, Spring 2022

## *“Lot Sizing”*

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# Implicit Formulation for the Optimal Solution

| | | | |
|---|---|---|---|
| $Sets$: | $\mathrm{i} = \{1, 2 \ldots N\}$ | : | Product index |
| | $t = \{0, 1 \ldots \mathrm{T}\}$ | : | Time index |
| $Parameters$: | $d_{it}$ | : | Demand for product i at period t |
| | $c_i$ | : | Inventory cost of item i |
| | $ii_i$ | : | Initial inventory of item i |
| | $b_i$ | : | Backlog cost of item i |
| | $s_i$ | : | Setup cost of item i |
| | $R$ | : | Weekly resource capacity of the facility |
| | $r_i$ | : | Capacity requirement of item i |
| $Decision\ variables$: | $x_{it}$ | : | $Amount\ of\ product\ i\ ordered\ at\ period\ t$ |
| | $I_{it}$ | : | $Stock\ level\ of\ product\ i\ at\ the\ end\ of\ period\ t$ |
| | $B_{it}$ | : | $Backlog\ of\ product\ i\ at\ the\ end\ of\ period\ t$ |
| | $y_{it}$ | : | $Whether\ product\ i\ is\ ordered\ at\ period\ t$ |

$$min \sum_{t=1}^{T} \sum_{i=1}^{N} b_i . B_{it} + \sum_{t=1}^{T} \sum_{i=1}^{N} c_i . (I_{it} + I_{i,t-1})/2 + \sum_{t=1}^{T} \sum_{i=1}^{N} s_i . y_{it}$$

(Backlog costs + Inventory costs + Setup costs)

$st$

$$I_{i,t-1} + B_{i,t-1} + x_{it} = d_{it} + I_{it} - B_{it} \quad , \forall_{it} \quad (Inventory\ balance)$$

$$\sum_{i=1}^{N} x_{it} . r_i \le R \quad , \forall_t \quad (Resource\ capacity, only\ for\ the\ forth\ part)$$

$$I_{i,0} = ii_i \quad , \forall_i \quad (Initial\ inventory\ levels)$$

$$B_{i,T} = 0 \quad , \forall_i \quad (No\ final\ backlog)$$

$$x_{it} \le M . y_{it} \quad , \forall_{it} \quad (Binary\ variable\ adjustment\ for\ fixed\ cost)$$

$$x_{it} \ge 0, I_{it} \ge 0, B_{it} \ge 0, y_{it} \in \{0,1\} \quad , \forall_{it}$$

(M can be set to total demand in the entire time horizon of the product that have the highest total demand.)

# Part I

*(Calculations of the first two parts can be found in the xlsx file.)*

| **Product 1, EOQ = 286** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| Demand | 226 | 117 | 5 | 11 | 211 | 91 | 147 | 182 | 92 | 36 | 188 | 11 |
| Order | 286 | 0 | 0 | 0 | 286 | 286 | 0 | 286 | 0 | 0 | 286 | 0 |
| Avg Inv | 258 | 86,5 | 25,5 | 17,5 | 192,5 | 327,5 | 208,5 | 330 | 193 | 129 | 303 | 203,5 |

| | |
|---|---|
| **Ordering Cost** | 3750 |
| **Inventory Carrying Cost** | 4549 |
| **Total Cost** | 8299 |

| **Product 2, EOQ = 356** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| Demand | 171 | 177 | 144 | 192 | 188 | 210 | 185 | 221 | 196 | 208 | 172 | 200 |
| Order | 356 | 0 | 356 | 0 | 356 | 0 | 356 | 356 | 0 | 356 | 0 | 356 |
| Avg Inv | 320,5 | 146,5 | 342 | 174 | 340 | 141 | 299,5 | 452,5 | 244 | 398 | 208 | 378 |

| | |
|---|---|
| **Ordering Cost** | 3150 |
| **Inventory Carrying Cost** | 10332 |
| **Total Cost** | 13482 |

# Part II

| Product 1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| Demand | 226 | 117 | 5 | 11 | 211 | 91 | 147 | 182 | 92 | 36 | 188 | 11 |
| Order | 274 | 0 | 0 | 0 | 302 | 0 | 457 | 0 | 0 | 0 | 199 | 0 |
| Avg Inv | 246 | 74,5 | 13,5 | 5,5 | 196,5 | 45,5 | 383,5 | 219 | 82 | 18 | 105 | 5,5 |

| | |
|---|---|
| **Ordering Cost** | 3000 |
| **Inventory Carrying Cost** | 2789 |
| **Total Cost** | 5789 |

| Product 2 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| Demand | 171 | 177 | 144 | 192 | 188 | 210 | 185 | 221 | 196 | 208 | 172 | 200 |
| Order | 121 | 321 | 0 | 192 | 188 | 210 | 185 | 221 | 196 | 208 | 172 | 200 |
| Avg Inv | 85,5 | 232,5 | 72 | 96 | 94 | 105 | 92,5 | 110,5 | 98 | 104 | 86 | 100 |

| | |
|---|---|
| **Ordering Cost** | 4950 |
| **Inventory Carrying Cost** | 3828 |
| **Total Cost** | 8778 |

# Part III

*(PuLP model of the third and fourth parts can be found in the ipynb file.)*

| **Optimal Solution** | |
|---|---|
| P1 | 3638 |
| P2 | 4787 |

| **% Deviation from Optimal** | | |
|---|---|---|
| | P1 | P2 |
| EOQ | 128,12% | 181,64% |
| Silver-Meal | 59,13% | 83,37% |

As can be seen in the above summary table, EOQ performs poorly for these two products due to their fluctuating demand rates. The reason that product 2 has a higher deviation from the optimal is that it has a 50% higher inventory cost than product 1, keeping in mind that inventory carrying cost constitutes a major part of the totals and that it has higher demand rates in general. The Silver-Meal algorithm performs significantly better since it focuses on approximating the inventory costs to the setup costs, which eliminates nearly half of the inventory cost for product one, and even more for product 2.

# Part IV

| Product 1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| Demand | 226 | 117 | 5 | 11 | 211 | 91 | 147 | 182 | 92 | 36 | 188 | 11 |
| Order | 0 | 265 | 0 | 0 | 0 | 0 | 640 | 0 | 0 | 0 | 327 | 0 |
| Avg Inv | 42,5 | 3,5 | 4,5 | 1,0 | 0,0 | 0,0 | 91,0 | 91,0 | 0,0 | 0,0 | 5,5 | 5,5 |
| Backlog | 141 | 0 | 0 | 9 | 220 | 311 | 0 | 0 | 92 | 128 | 0 | 0 |

| | |
|---|---|
| **Ordering Cost:** | 2250 |
| **Inventory Carrying Cost:** | 489 |
| **Backlog Cost:** | 901 |
| **Total Cost:** | 3640 |

| Product 2 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| Demand | 171 | 177 | 144 | 192 | 188 | 210 | 185 | 221 | 196 | 208 | 172 | 200 |
| Order | 196 | 0 | 246 | 192 | 246 | 246 | 0 | 246 | 246 | 229 | 120 | 246 |
| Avg Inv | 160,4 | 37,4 | 0,0 | 0,0 | 29,1 | 76,2 | 47,2 | 0,0 | 0,0 | 2,7 | 2,7 | 0,0 |
| Backlog | 0 | 102 | 0 | 0 | 0 | 0 | 91 | 66 | 15 | 0 | 46 | 0 |

| | |
|---|---|
| **Ordering Cost:** | 4500 |
| **Inventory Carrying Cost:** | 1067,06 |
| **Backlog Cost:** | 640 |
| **Total Cost:** | 6206,86 |

After adding the capacity constraints, when product 1 is considered, there is insignificant deviance from the previous optimal solution since there is only one week whose orders are adjusted. 2 products of type 1 shifted from week 28 to week 23, which has a minor effect on the result. But the capacity constraint forced product 2's plan to have fewer backlogs, more inventory, and more setups. The optimal solutions behavior for product 2 was ordering per two weeks, which is not possible this time since weekly capacities cannot be laid aside for the next weeks. For this reason, we face many weeks in the new solution that uses full capacity, resulting in 246 orders. Note that the solution for the second product group is a theoretical one, that is, it includes fractional values, and the possibility of requesting such orders should be analyzed when the plan is determined to be used in real life.