United States International University-Africa (USIU-A)

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Course Title: MATHEMATICAL FINANCE
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Assignment One: Review Questions

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Mean-Variance Analysis : Question - one

S	P	RA	RB	P*ERA	P*ERB
O	0.3	0.20	0.10	0.06	0.03
B	0.5	0.12	0.08	0.06	0.04
P	0.2	-0.05	0.04	-0.01	0.008

1. $E[R_A] = \sum_i p_i [R_A] = 0.06 + 0.06 - 0.01 = 0.11 = 11\%$
2. $E[R_B] = \sum p_i R_{Bi} = 0.03 + 0.04 + 0.008 = 0.078 = 7.8\%$

It is true that expected return of A is better than expected return of A. But

Mean (expected return) is only half the story.

Risk - Variance

S	p_i	$(R_A)^2$	$(R_B)^2$	$p_i(R_{Ai})^2$	$p_i(R_{Bi})^2$
O	0.3	0.04	0.01	0.012	0.0030
B	0.5	0.0144	0.0064	0.0072	0.0032
P	0.2	0.0025	0.0016	0.0005	0.00032

3 $VAR(R_A) = E[(R_A)^2] - (E[R_A])^2$

$\delta^2_A = \sum p_i (R_{Ai})^2 - (E[R_A])^2$

$\delta^2_A = 0.012 + 0.0072 + 0.0005 - (0.11)^2$

$= 0.0197 - 0.121$

$\delta^2_A = 0.0076 = 0.0076$

$\delta_A = \sqrt{0.0076} = 0.08718 = 8.718\%$

$\delta^2_B = E[R_B^2] - (E[R_B])^2$

$= 0.0030 + 0.0032 + 0.00032 - (0.078)^2$

$= 0.00652 - 0.006084$

$\delta^2_B = 0.000436$

$\delta_B = \sqrt{\delta^2_B} = \sqrt{0.000436} = 0.02088 \approx 2.088\%$

Now we have

	E[R]	VAR	Std	So Expected return in rate
A	0.11 (11%)	0.0076 (76%)	8.718%	11% ± 8.718%
B	0.078 (7.8%)	0.000436 (4.36%)	2.088%	7.8% ± 2.088%

Worest case for A 2.282%
Worest case for B 5.712%

It means that
- Asset A has a higher expected return, but also much higher risk.
- Asset B has lower return, but lower risk.

Now what about Diversification?

i) $cov(A,B) = \sum_{i=1}^{3} P_i(R_{A,i} - E[R_A]) \cdot (R_{Bi} - E[R_B])$

$= 0.3[0.20 - 0.11)(0.1 - 0.078) + 0.5(0.12 - 0.11)(0.08 - 0.078)$
$0.2(-0.05 - 0.11)(0.04 - 0.078)$

$= \underline{0.00182}$
$= \underline{18.2\%^2}$

ii) $correlation(a,b) = \frac{cov(a,b)}{SA\,SB}$

$= \frac{0.00182}{(0.08718)(0.02088)} = 0.9998 \simeq \underline{1}$

$\rho_{A,B} = 1 \rightarrow$ perfect positive correlation

The two assets move together perfectly (their return always rise/fall in the same proportion.
- No diversification benefit
  * Risk-seeking – A
  * Risk-averse – B

1 Question - one part b

Given Required: $E[R_P]$ - 1

Assets	$E[R]$	w	$E[R_P]\cdot w$
A	0.11	0.6	$0.11 \times 0.6 = 0.066$
B	0.078	0.4	$0.078 \times 0.4 = 0.0312$

~~Required~~

$E[R_P] = \sum E[R_i]\cdot w_i = 0.066 + 0.0312 = 0.0972$

$E[R_P] = 0.0972$

$\approx$ 9.72%

ii) Portofilio ~~of~~ Variance

$\delta_P^2 = w_A^2\delta_A^2 + w_B^2\delta_B^2 + 2w_Aw_B\,Cov(R_A, R_B)$

$= (0.6)^2(0.0076)^2 + (0.4)^2(0.000436)^2 + 2(0.4)(0.6)(0.00182)$

$= 0.002736 + 0.00006976 + 0.0008736$

$= 0.00367936$

$= 0.00367936$

$\delta_P^2 \approx 36.79\ \%^2$

$\delta_P = \sqrt{\delta^2 P} = \sqrt{0.00367936} \approx 0.06066$

$\delta_P \approx 0.06066$

$= 6.07\%$

Summary

Asset	$E[R]$	VAR (%^{2})
A	11%	76%^{2}
B	7.8%	4.36
60/40 Portofolio	9.72%	36.79

So
- Risk Averse - Asset B
- Bellanced / Moderate
  ✓ 60/40 Portofolio
- Risk - seeking
  ✓ Asset B

$\rho_{A,B} = 1$

Question-two

Given P $S_0 = 110$, $K = 110$, $P_0 = 10$

S	P	S_T
P	0.2	130
B	0.5	100
O	0.3	80

Two-Assets

~~Scenario~~ - Asset A: Stock

- Asset B: European Put Opition

Solution

i) $E[R_A]$

$S_0 = 110$ ← 6 month → $S_6 = S_T$

Return depends on the economic scenarios

$\boxed{S}$	P	RA	RA(%)
P	0.2	$130-110 = 20$	$20/110 = 2/11$
B	0.5	$100-110 = 10$	$10/110 = -1/11$
O	0.3	$80-110 = -30$	$-30/110 = -3/11$

$\therefore E[R_A] = \sum P_i R_{A_i} = (0.2)(2/11) + (0.5)(-1/11) + (0.3(-3/11))$

$= 0.4/11 + 0.5/11 - 0.9/11$

$= -1/11 = -0.090909\ldots$

$\approx \underline{-9.0909\%}$ - Expected to lose

- The stock alone has negative expected return.

ii) $E[R_B]$

Put payoff $= \max(\cancel{\max}\ K - S_T, 0) = R_B$, $K = 110$

S	P	S_T	RB	RB(%)
P	0.2	130	$\max(110-130, 0) = 0$	$(0-10)/10 = -1$
B	0.5	100	$\max(110-100, 0) = 10$	$(10-10)/10 = 0$
O	0.3	80	$\max(110-80, 0) = 30$	$(30-10)/10 = 2$

$E[R_B] = (0.2)(-1) + (0.5)(0) + (0.3(2)) = 0.4 \approx \underline{40\%}$

Expected return of Asset B (European put opition) is better than a stock Asset. But we can't conclude that Asset B is better than Asset A. Next step variance!

Variance

S	P	RA	RB
P	0.2	2/11	-1
B	0.5	-1/11	0
O	0.3	-3/11	2

$VAR(A) = \sum P_i (R_{Ai} - E[R_A])^2$

$= (0.2)(2/11 - (-1/11))^2 + 0.5(-1/11 - (-1/11))^2 + 0.3(\frac{-3}{11} - (\frac{-1}{11}))^2$

$= (0.2)(\frac{3}{11})^2 + 0.5(0)^2 + 0.3(\frac{-2}{11})^2$

$= (0.2)(9/121) + 0 + (0.3)(\frac{4}{121})$

$= 0.024793388$

$\approx \underline{0.02479}$

$Std(A) = \sqrt{VAR(A)} = \sqrt{0.02479} \approx 0.157459166$

≈ 0.1575

$\approx \underline{15.75\%}$

$E(R_B) = \sum P_i R_{Bi} = (0.2)(-1) + (0.5)(0) + (0.3)(2)$

$= -0.2 + 0 + 0.6$

$= 0.4$

$VAR(B) = E[R_B]^2 - (E[R_B])^2$

$= \sum P_i (R_B)^2 - E[R_B]^2$

$= (0.2)(-1)^2 + (0.5)(0)^2 + (0.3)(2)^2 - (0.4)^2$

$= 0.2 + 0 + 1.2 - 0.16$

$\delta^2_B = \underline{1.24}$

$\delta_B = \sqrt{\delta^2 B} = \sqrt{1.24} = 1.11355287$

$\approx \underline{111.36\%}$

Even if the E[RB] is greater than E[RA], the variance (risk) of Asset Put option is higher than risk of Asset A.
✓ risk - seeking - Asset B
✓ risk - averse - Asset A

Diversification

Covariance between stock and put

$COV(A,B) = \sum P_i (R_{Ai} - E[R_A])(R_{Bi} - E[R_B])$

$= (0.2)(2/11 - (-1/11))(-1-0.4) + 0.5(-1/11 - (-1/11))(0-0.4) + 0.3(-3/11 - (-1/11))$

$= -0.16363636$

≈ -0.16364

The negative covariance between the two assets shows opposite co-movement.
But at this stage we can't talk the degree of correlation, so we have to find correlation coefficient (ρ)

$$\rho(A,B) = \frac{COV(A,B)}{\sigma_A \sigma_B} = \frac{-0.16364}{(0.1574)(1.1136)}$$

$= -0.9333$

A strong negative correlation.

Summary

* Individually

Stock = "Losing asset, but relatively stable"

Put opition = "Winning asset, but extermely risky"

Together : strong negative correlation, meaning that when the stock performs badly, the put performs well and vice versa.

- In other words
- If David holds only the stock, he expects losses.
- If David holds only the put, he expects big gain, but with massive risk.
- If David combines them, the negative correlation means that the risk of the overall portfolio will be much lower than the weighted average of the individual risks.

1 False
- Opitions can be used both ways
  - as hedge
hedge means protecting against downside risk with puts, or securing purchase price with calls.
- for speculation - betting on future movements without owning the underlying.

2 False
Because derivative pricing like (- the Black scholes model assumes that stock prices fellow GBW, not just any stochastic process.
- Wiener process underlines GBM, but prices can't be modeled directly with it since it can go negative.
- The ~~log~~ Normal distribution is used for log returns not prices.

3 False:
- A risk averse person prefers lower variance for the same level of expected return.
  High variance = high uncertianity = not attractive to risk-averse investors.

4 False
Negative correlation between assets helps with diversification - when one assets goes down, the other may go up, reducing the overall portfolio risk.
- Positively correlated asset move together and do not provide much risk reduction.