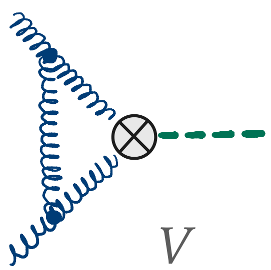
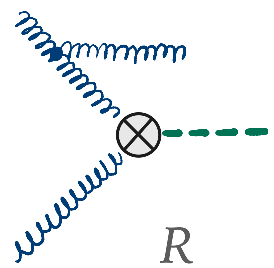
inclusive

Subtractions — A Toy Example



V



R

$$\frac{1}{\epsilon} + a$$

$$\int_0^1 \frac{1 + b x}{x^{1+\epsilon}}\, \mathrm{d}x$$

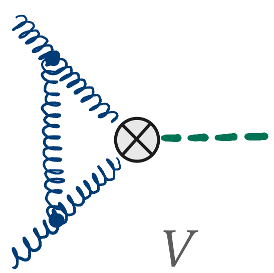
$$-\frac{1}{\epsilon} + b$$

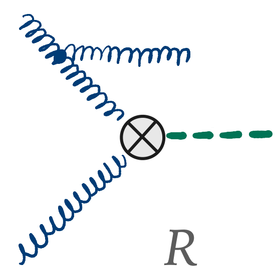
$$a + b$$

- *dimensionally regularized:*
  - $D = 4 - 2\epsilon$
  - everything up to $\mathcal{O}(\epsilon)$
- *emission "phase space":*
  - $x \in [0, 1]$
  - no emission $\leftrightsquigarrow$ $x \to 0$

SUBTRACTIONS — A TOY EXAMPLE

fiducial

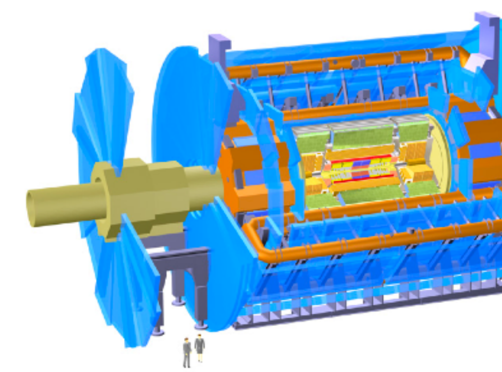




$$\left(\frac{1}{\epsilon} + a\right) \mathcal{J}(0)$$

$$\int_0^1 \frac{1+bx}{x^{1+\epsilon}}\, \mathcal{J}(x)\, \mathrm{d}x$$

- *measurement function* $\mathcal{J}(x)$
  - acceptance
  - jet algorithm
  - isolation
  - distributions
  - …



Very complicated / impossible(?) to do analytically

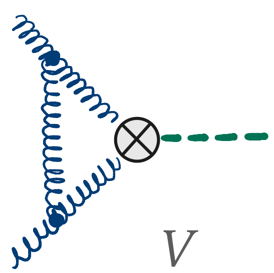
?

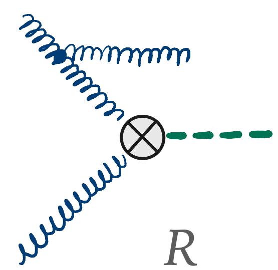
flexible *numerical* approach desired
↪ 2 strategies

(notebook: “toy-nlo”)

SUBTRACTIONS — A TOY EXAMPLE: SUBTRACTION

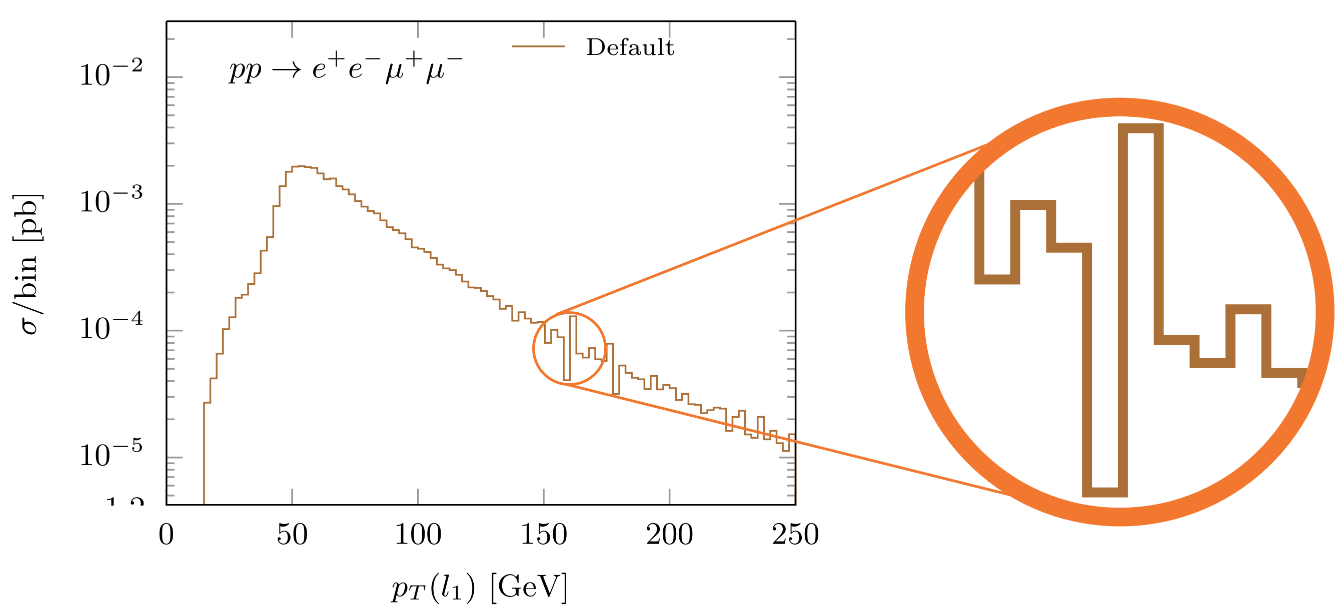
fiducial





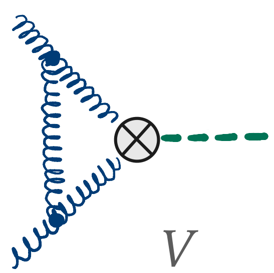
$$(a+b)\mathcal{J}(0) + \int_0^1 \frac{1+bx}{x}\left[\mathcal{J}(x) - \mathcal{J}(0)\right]\,\mathrm{d}x$$

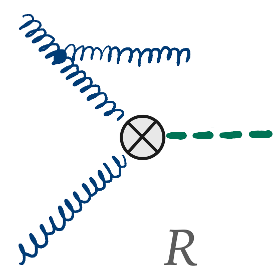
- *regulate divergence in the integrand*
  - can set $\epsilon = 0$
- *challenge: numerical cancellations*
  - floats not exact
  - outliers & "misbinning"

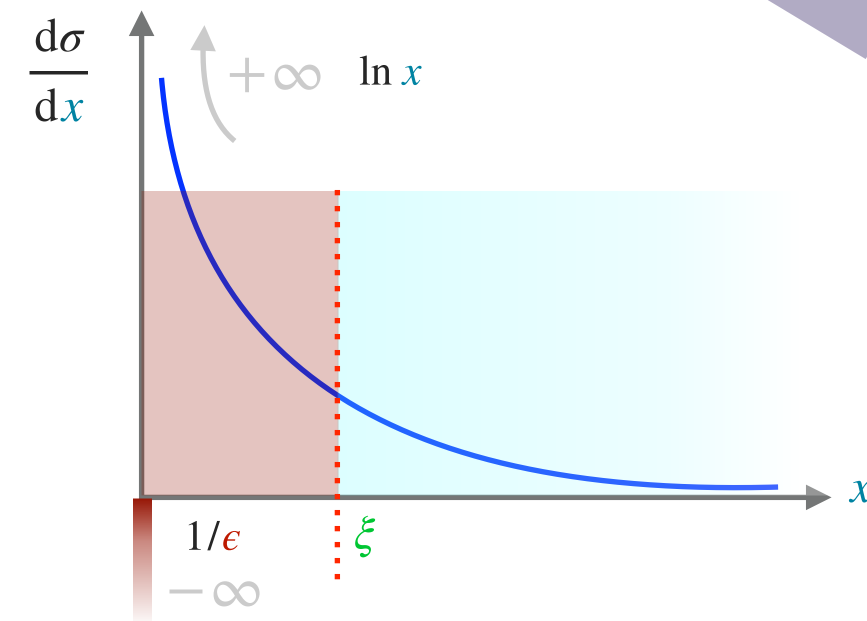


Subtractions – A Toy Example: Slicing

fiducial

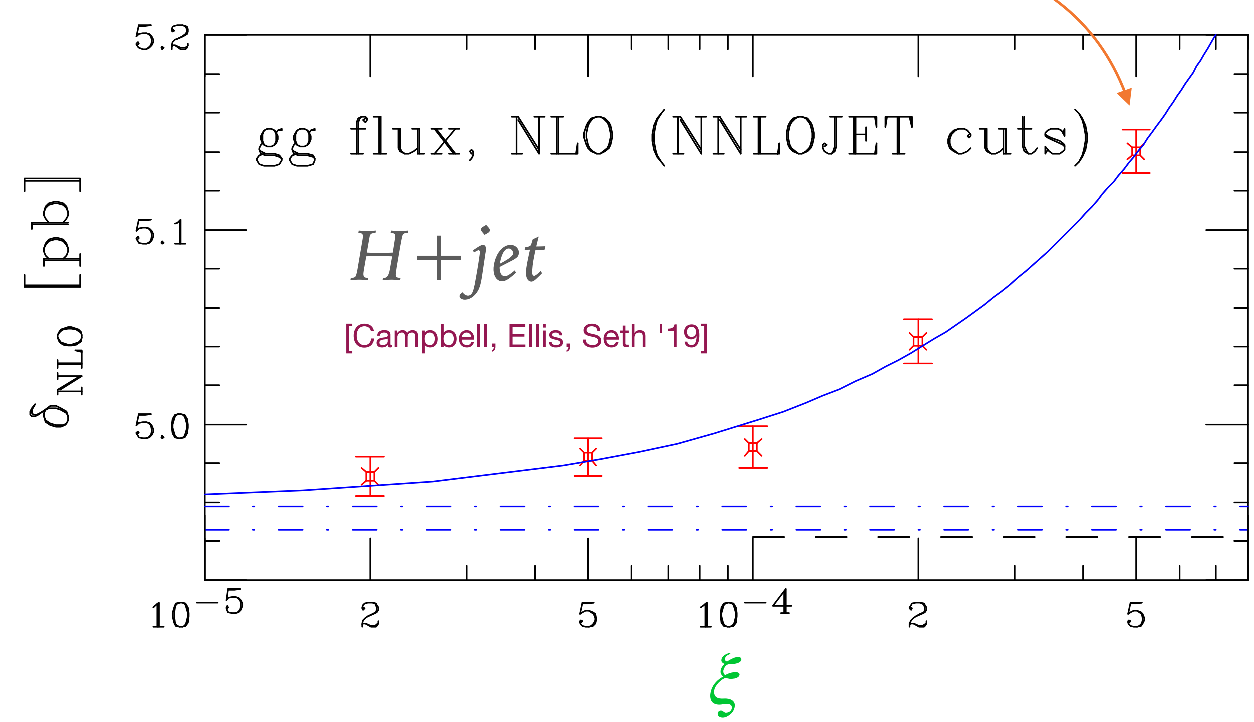






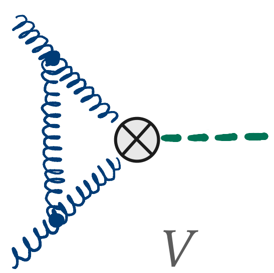
$$\left(a + \ln \xi\right) \mathcal{J}(0) + \int_{\xi}^{1} \frac{1 + bx}{x} \mathcal{J}(x)\, \mathrm{d}x + \mathcal{O}(\xi^n)$$

need to control the error!

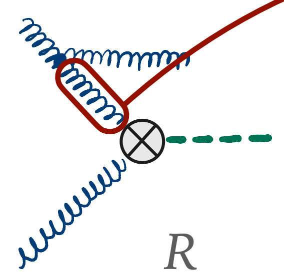


- *regulate divergence with cutoff*
  - error term $\mathcal{O}(\xi^n)$
- *challenge: numerical cancellations*
  - $\ln \xi$ cancels against 2nd term
  - higher target precision

SUBTRACTIONS – NLO



V



R

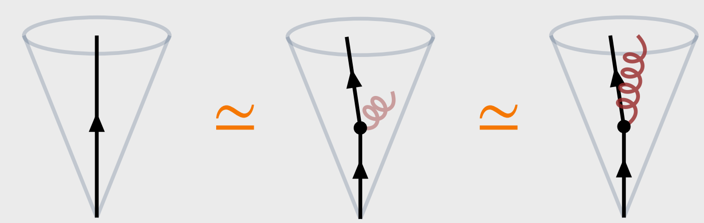
$$\sim \frac{1}{(p_a - k_1)^2} \sim \frac{1}{p_{T,1}}$$

- *$1/\varepsilon^2$, $1/\varepsilon$*

- *single unresolved*

  $p_T^H = p_{T,1} \to 0$

indistinguishable:



soft: $E_g \to 0$

collinear: $\theta_{qg} \to 0$

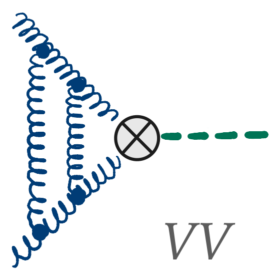
Infrared cancellation:

- subtraction: more complex integrands
  - correlated ME & counterterms
- slicing: higher precision target
  - non-local cancellations

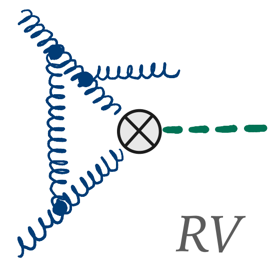
conceptually solved:

CS dipoles, FKS, …

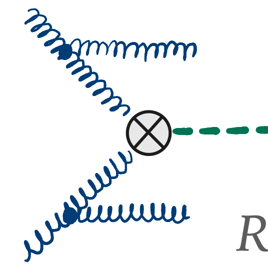
Subtractions – NNLO



VV



RV



RR

- *$1/\varepsilon^4$, $1/\varepsilon^3$, $1/\varepsilon^2$, $1/\varepsilon$*

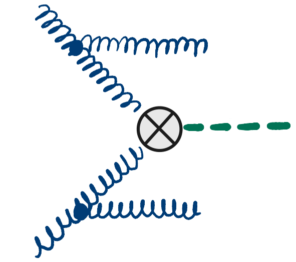
- *$1/\varepsilon^2$, $1/\varepsilon$*
- *single unresolved*

- *single unresolved*
- *double unresolved*

single unresolved $\simeq$ H+jet @ NLO

fully unresolved $\simeq$ H @ NNLO

SUBTRACTIONS – NNLO (DOUBLE-REAL)



- conceptual challenge NLO $\rightsquigarrow$ NNLO: overlapping singularities

$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \, \frac{1 + bx + cy + dxy}{x^{1+\epsilon}\, y^{1+\epsilon}} \, \mathcal{J}(x,y)$$

(toy double real)

local subtraction

$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \, \frac{1}{x\, y} \Big[(1 + bx + cy + dxy) \, \mathcal{J}(x,y) \\ - (1 + cy) \, \mathcal{J}(0,y) \\ - (1 + bx) \, \mathcal{J}(x,0) \\ + \mathcal{J}(0,0) \Big]$$

Sectors can disentangle counterterms but will induce large cancellations between integrated sectors

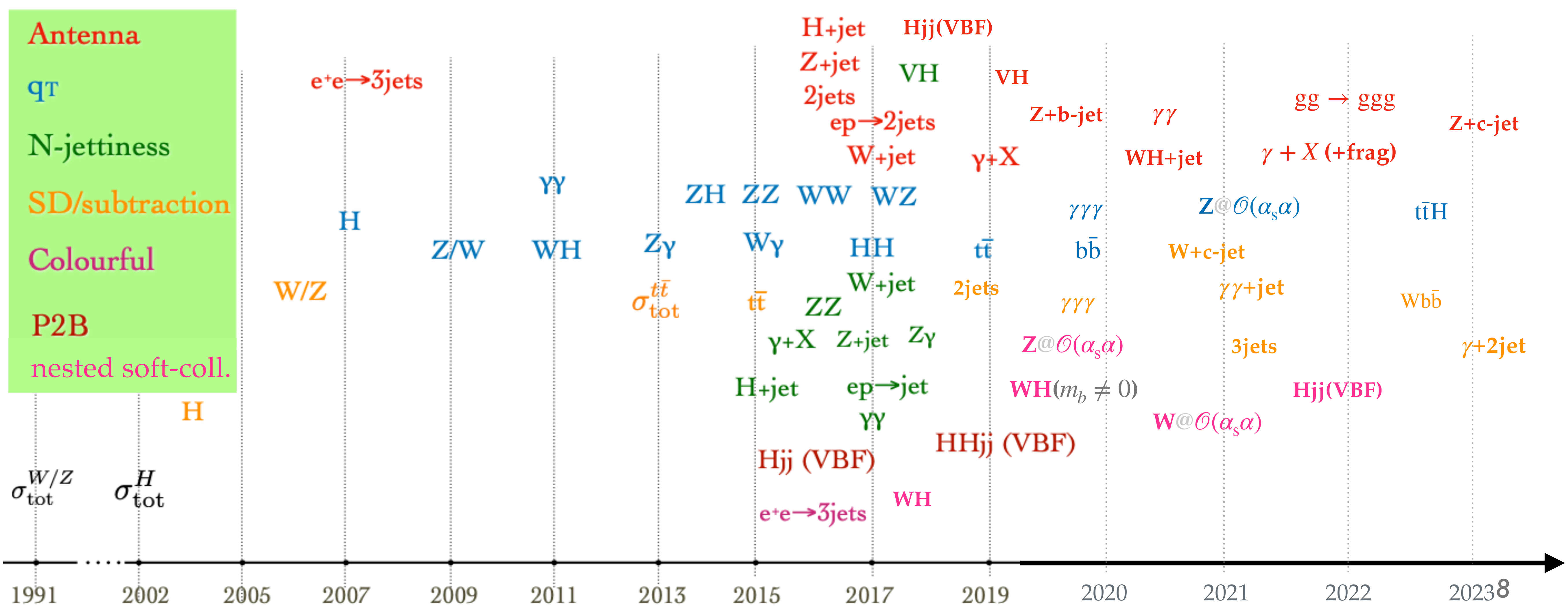
$x \to 0$ $\quad$ $y \to 0$ $\quad$ $x, y \to 0$

spurious limits

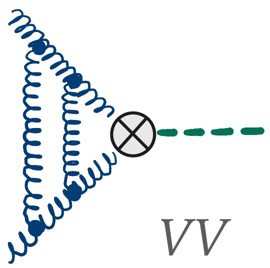
PREDICTIONS – NNLO

Tremendous progress in the past ~ 10 years!

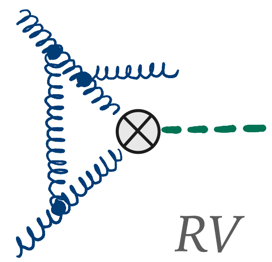
↪ $2 \to 2$ under good control; $2 \to 3$ steady progress



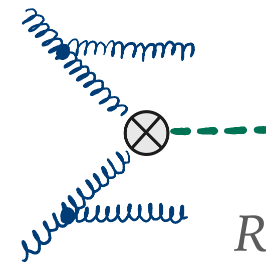
SUBTRACTIONS – NNLO



VV



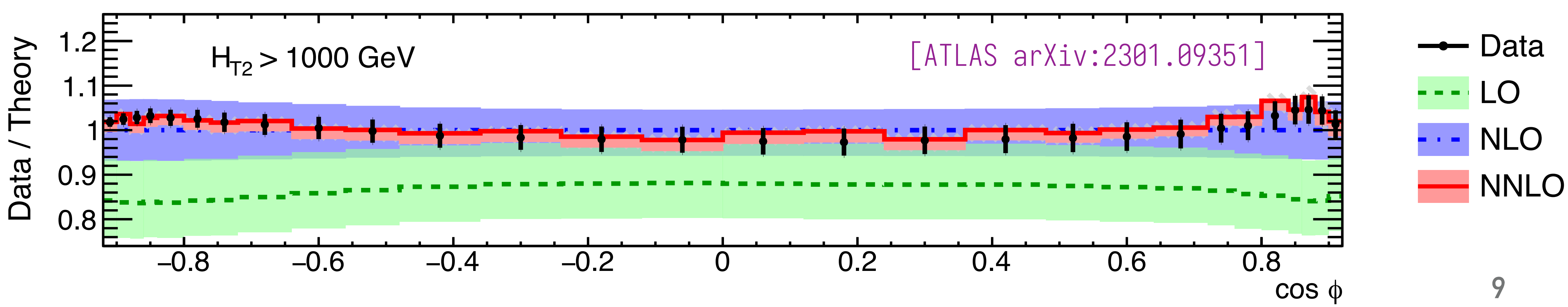
RV



RR

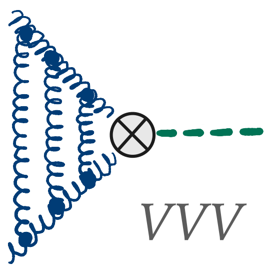
- typical runtime for $2 \to 2$ processes: $\mathcal{O}(100\text{k})$ CPU core hours
  - V+jet, di-jet, …
- an extreme $2 \to 3$ example: $\mathcal{O}(100\text{M})$ CPU core hours
  - tri-jet ⇝ $\simeq$ 700 tons of CO_2 $\simeq$ 2M CHF on AWS

$$\frac{1}{\sigma}\,\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi}$$

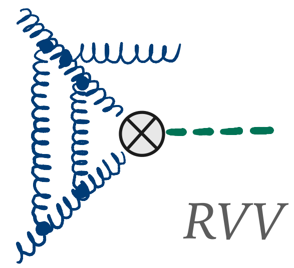


laregely unexplored

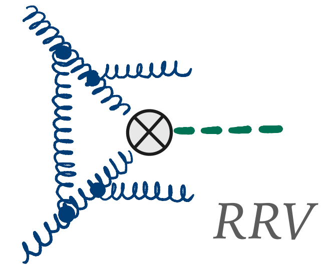
SUBTRACTIONS — N³LO



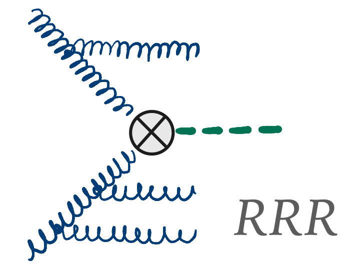
- $1/\varepsilon^6$, $1/\varepsilon^5$, …



- $1/\varepsilon^4$, $1/\varepsilon^3$, …
- *single unresolved*



- $1/\varepsilon^2$, $1/\varepsilon$
- *single unresolved*
- *double unresolved*



- *single unresolved*
- *double unresolved*
- *triple unresolved*

two methods for
"$2 \to 1$"

q_T-subtraction
Projection-to-Born

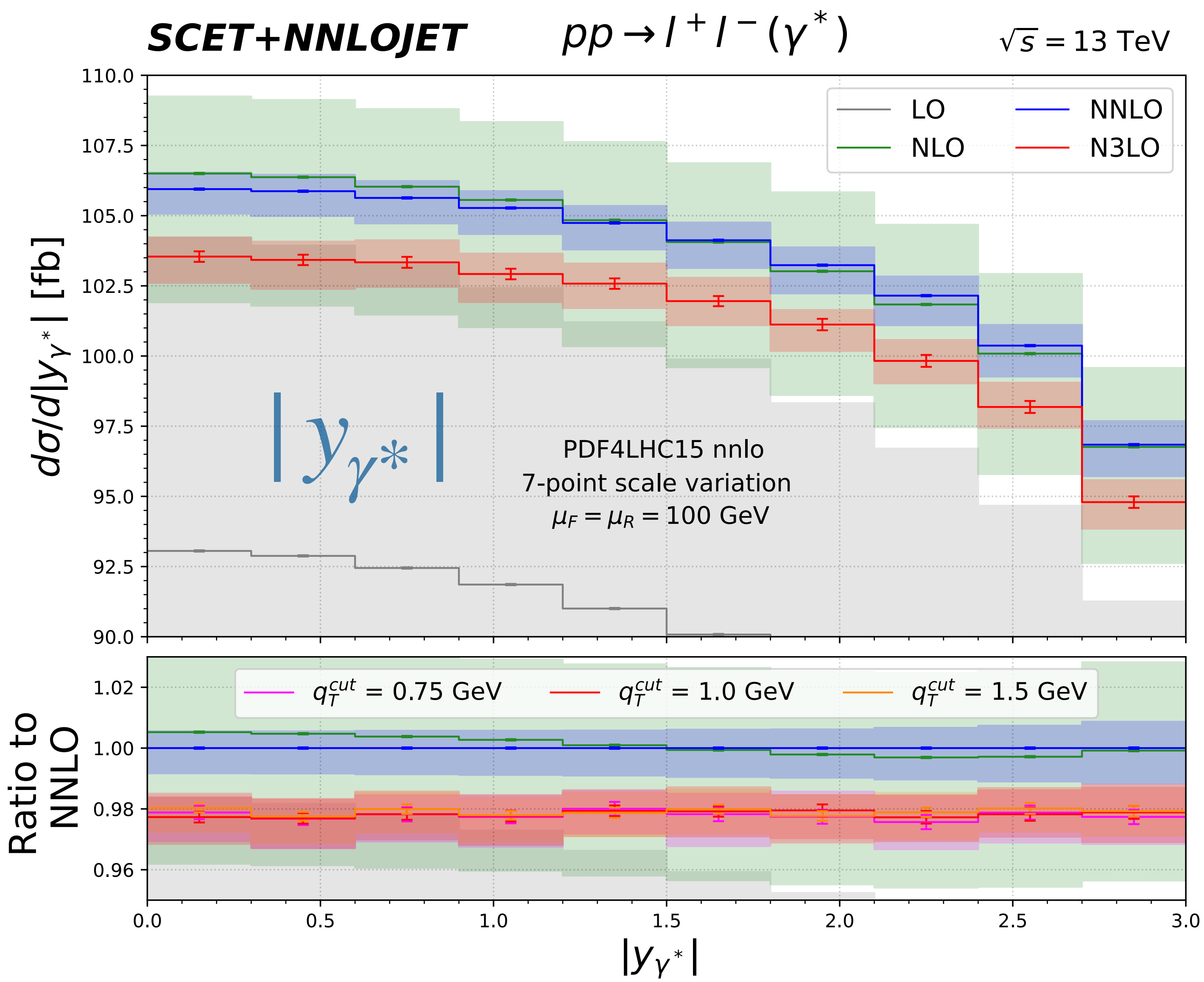
isolate "radiating" part

double unresolved $\simeq$ H+jet @ NNLO

fully unresolved $(\leftrightsquigarrow p_T^H \to 0)$ $\simeq$ H @ N³LO

SUBTRACTIONS – N^{3}LO: SLICING

[Chen, Gehrmann, Glover, AH, Yang Zhu '21]



- investment:
  ↪ $\mathcal{O}$(5M) CPU core hours
- in principle,
  fully differential
- experiments can measure DY *triply-differentially* in $\mathcal{O}$(500) bins!
- in practice, extrapolated $\mathcal{O}$(100M) CPU core hours is getting problematic

SUBTRACTIONS – N³LO: SUBTRACTION

[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

- a *local subtraction* can significantly improve the performance
- requires inclusive prediction (so far only ggH @ LHC)
- reduce cost to underlying H+jet @ NNLO level:
  ↪ $\mathcal{O}$(100k) CPU core hours