

QUANTUM CHROMODYNAMICS

& COLLIDER PHENOMENOLOGY

Alexander Huss

alexander.huss @ cern.ch

Outline

Ø The Beginnings

I Basics of QCD

II QCD in e^+e^- Collisions

III Hadron Colliders

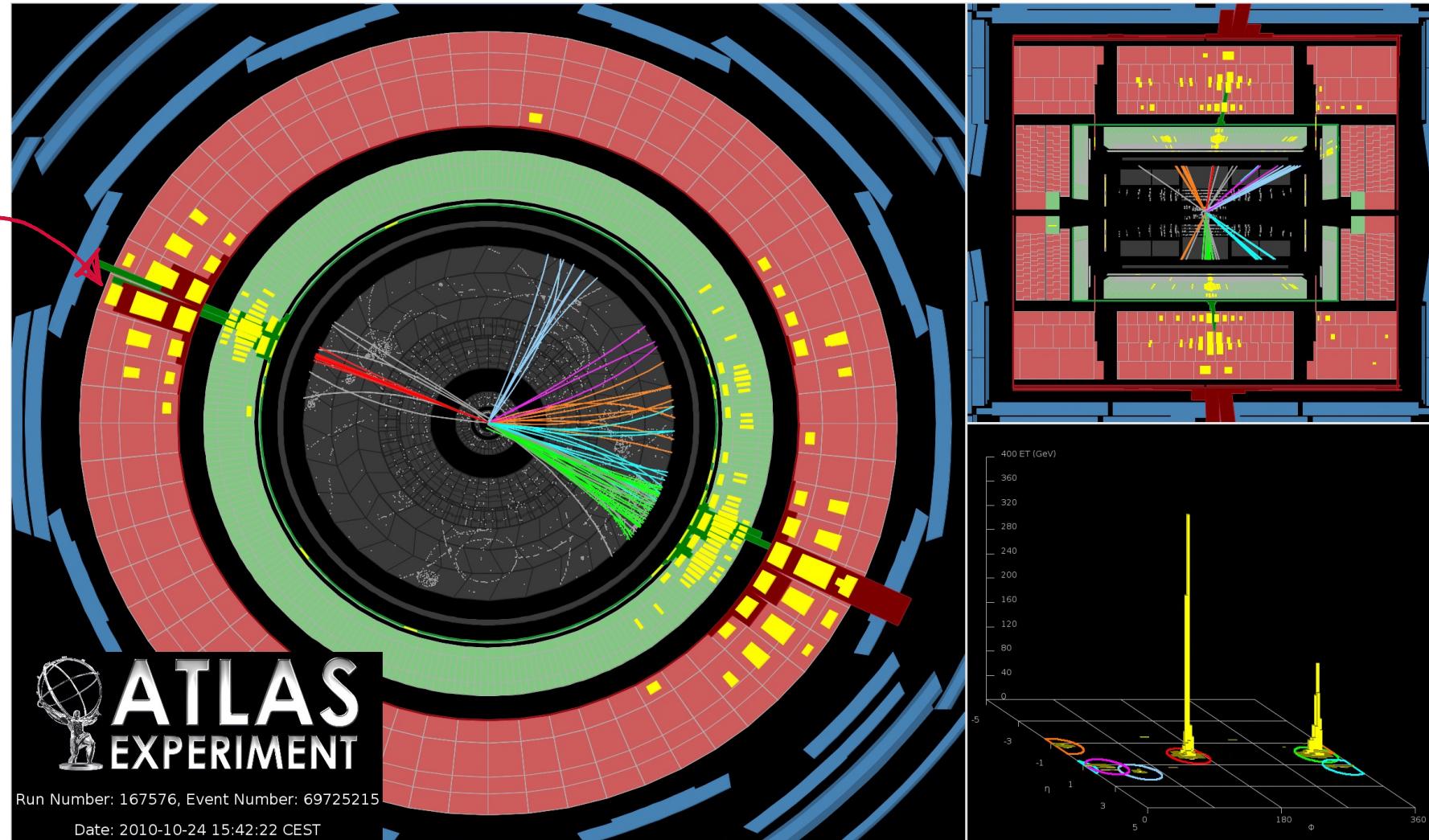
IV Jet Physics

- Jets : cone & sequential
- IR safety
- properties
- loss v.s. contamination
- Parton Showers

Events at hadron colliders look more complex

[demo: diagtams]

$\Theta(10\phi)$
tracks



Why? Any chance to compute this with what we did so far?

Nope...

2->2 gluon scattering has 4 diagrams

2->3 gluon scattering has 25 diagrams

2->4 gluon scattering has 220 diagrams

2->5 gluon scattering has 2485 diagrams

2->6 gluon scattering has 34300 diagrams

2->7 gluon scattering has 559405 diagrams

2->8 gluon scattering has 10525900 diagrams

2->9 gluon scattering has 224449225 diagrams

2->10 gluon scattering has 5348843500 diagrams

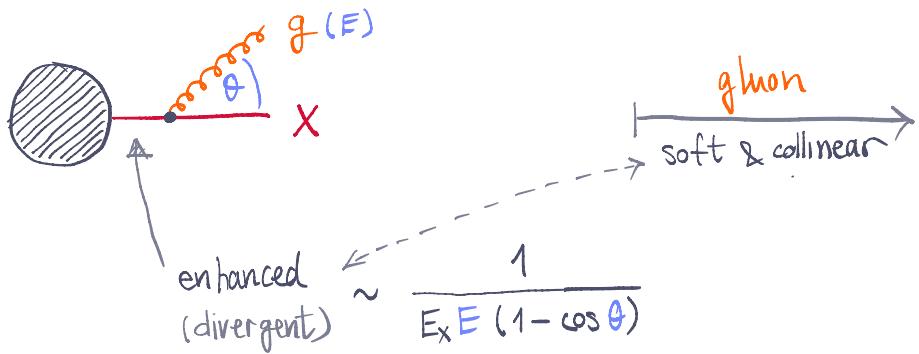
2->11 gluon scattering has 140880765025 diagrams

$10^M!$

* Even with more efficient recursive approaches,
not the multiplicities you'd want to tackle $\# \text{dim}(\Phi_n) = 3n - 4$

- ① define observables that "map back" the physics
to fewer initiating objects } JETS
- ② identify the relevant physics &
model the full complexity approximately } Parton Showers

Jets - an Emergent Feature of QCD



$\boxed{2 \frac{\alpha_s}{\pi} C_x \frac{dE}{E} \frac{d\theta}{\theta}}$

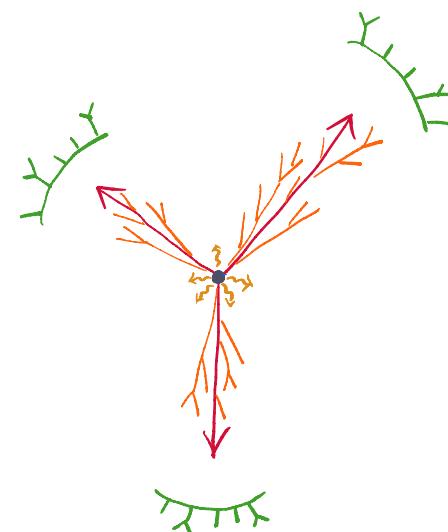
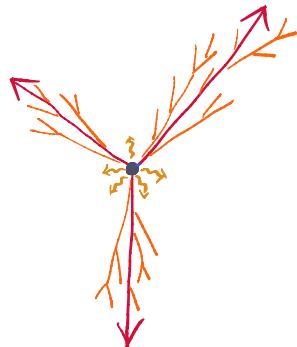
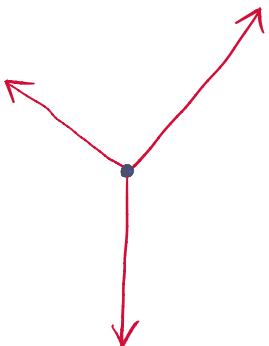
$\nabla \left\{ \begin{array}{l} = C_F = \frac{4}{3} \text{ if } X = g \\ = C_A = 3 \text{ if } X = g \end{array} \right.$

⇒ a collimated spray of hadrons

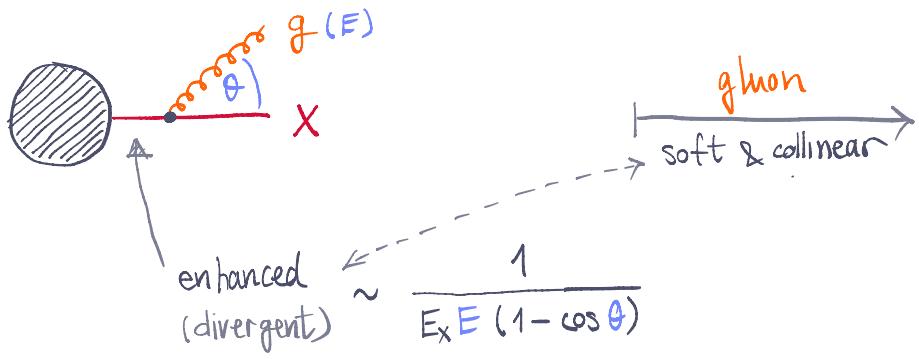
1. high energetic partons
↳ hard scattering

2. asymp. freedom & $d\omega$
↳ pert. parton shower

3. hadronization



Jets - an Emergent Feature of QCD

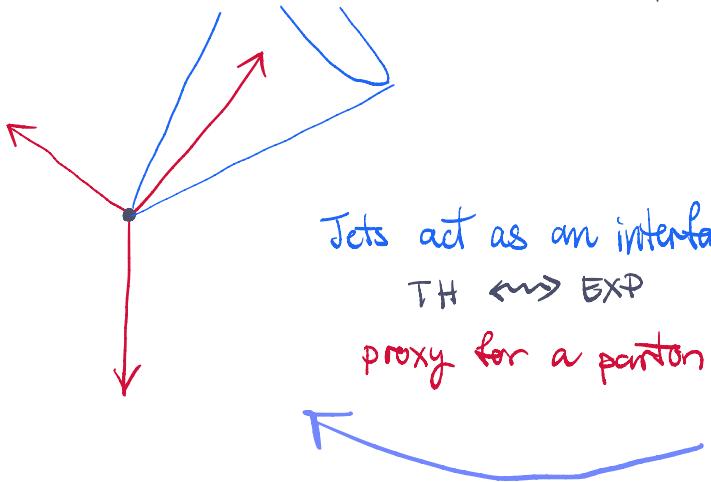


$\boxed{2 \frac{\alpha_s}{\pi} C_x \frac{dE}{E} \frac{d\theta}{\theta}}$

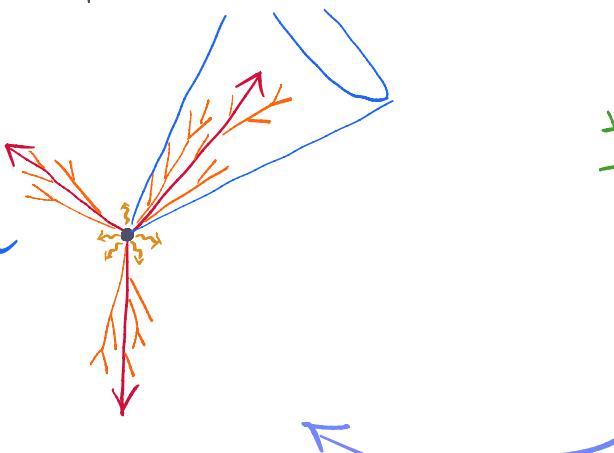
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⇒ a collimated spray of hadrons

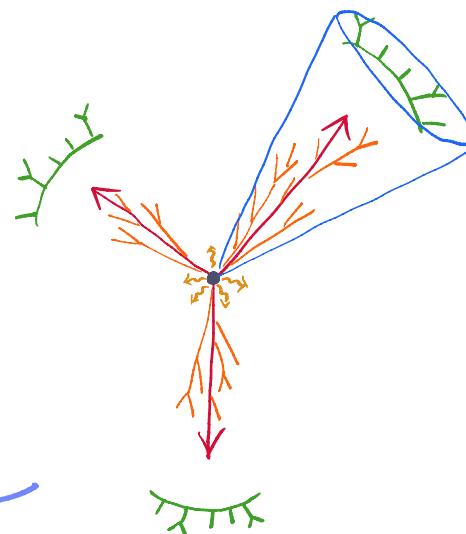
1. high energetic partons
↳ hard scattering



2. asympt. freedom & $d\omega$
↳ pert. parton shower

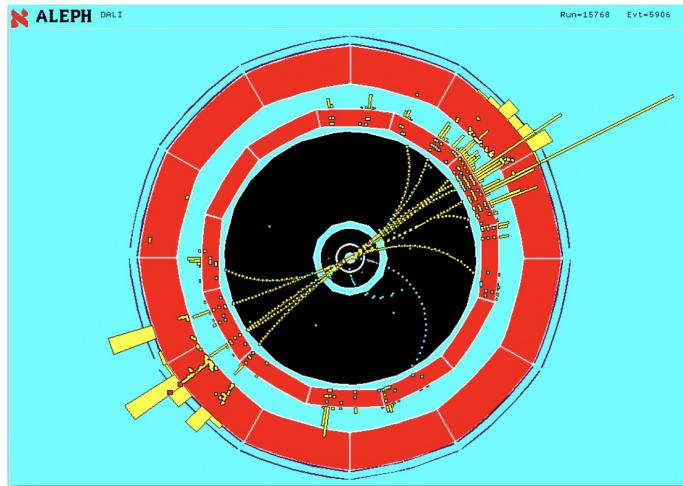


3. hadronization

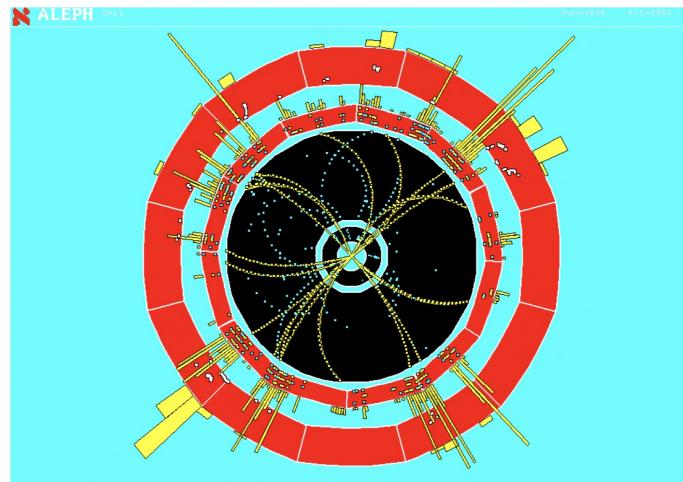


Jets are not unique ...

... and intrinsically ambiguous



jets = 2



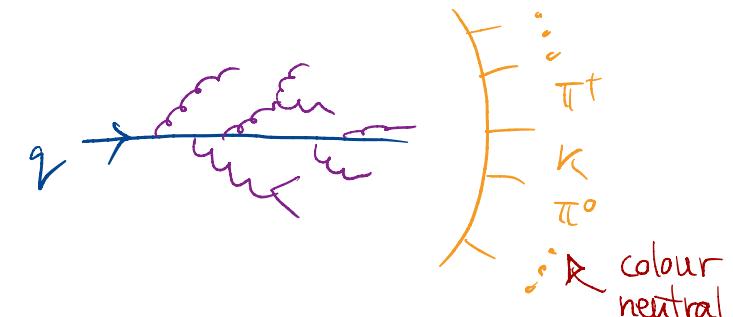
jets = ?

* Freedom

- (1) which particles to put together?
- (2) how to combine them?

➡ Jet definition

coloured



* best we can hope for:

(clusters of partons)

\simeq

(clusters of hadrons)

up to $\theta(1/\alpha_s)$

Jet Algorithms

FERMILAB-Conf-90/249-E
[E-741/CDF]

Toward a Standardization of Jet Definitions *

* To be published in the proceedings of the 1990 Summer Study on High Energy Physics, *Research Directions for the Decade*, Snowmass, Colorado, June 25 - July 13, 1990.

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;  performance
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;  IR safety
5. Yields a cross section that is relatively insensitive to hadronization.

Jet Algorithms

$$\{P_i\} \rightarrow \{j\}$$

[particles, momenta
[calorimeter towers, ...]]

Two main classes

① Cone [top down]

idea of directed energy flow

→ find coarse regions

(what we have been doing)

A brief (incomplete) history of jets:

- Sterman-Weinberg jets '77
- k_T algorithm '93
- Cambridge / Aachen '97
- anti- k_T '08

② Sequential recombination [bottom up]

successively undo GCD branching

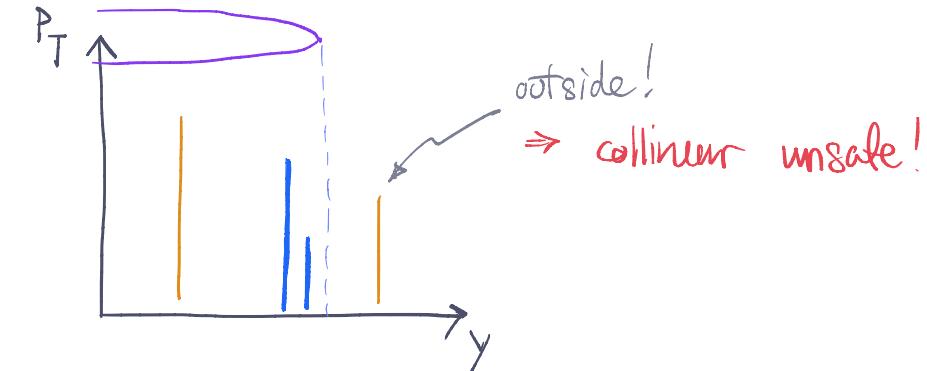
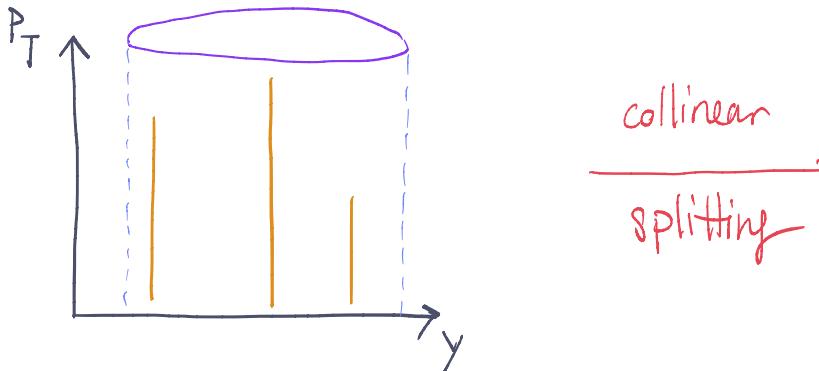
→ find "close" & aggregate

Potential IR issues when using (seeded) cones

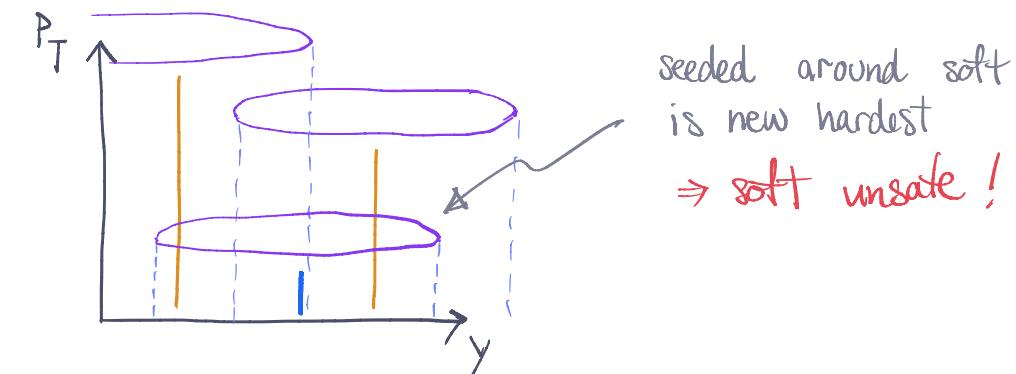
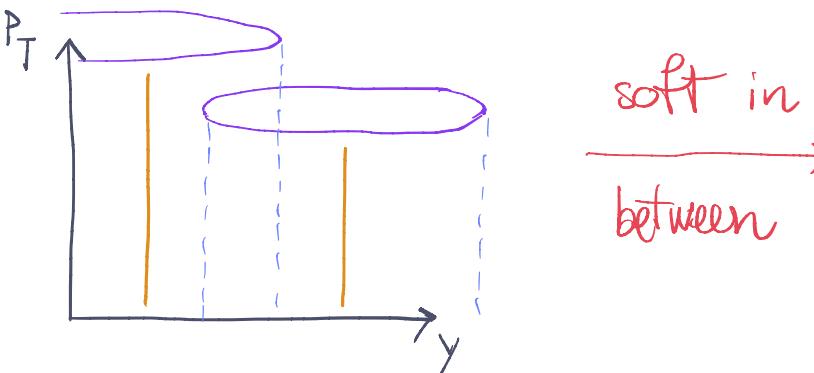
$$\varphi = \emptyset$$

in examples

- * start by placing cone around hardest particle



- * try placing cones around all particles & look for hardest



- * midpoint only postpones the issue \Rightarrow need to go seedless (naive: $O(N^{2^N})$) \rightsquigarrow SIScone ✓

Sequential Recombination Algorithms

Try to work our way backwards through "branchings"

- 1) Compute distances between all particles

$$d_{ij} = ?$$

and to the beam [for hadron colliders]

$$d_{iB} = ?$$

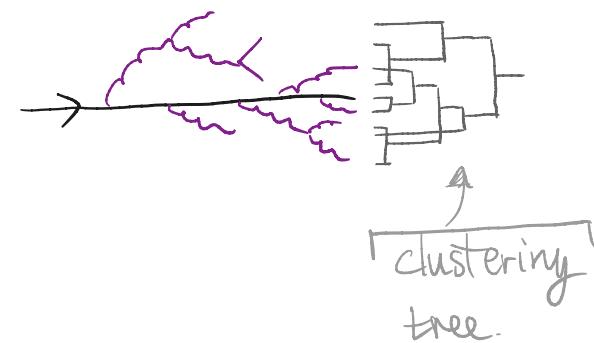
- 2) Find the smallest at $\{d_{ij}\} \cup \{d_{iB}\}$

↳ $d_{ij} \Rightarrow$ merge i & j into a new "protojet"

E-scheme: $P_{(ij)}^{\mu} = P_i^{\mu} + P_j^{\mu}$

↳ $d_{iB} \Rightarrow$ remove i from the set & call it a "jet"

- 3) If particles left, goto step 1 & repeat



The k_T Algorithm

Try to work our way backwards through "branchings"

- 1) Compute distances between all particles

$$d_{ij} = \min(P_{T,i}^2, P_{T,j}^2) \frac{\Delta R_{ij}^2}{R^2}$$

and to the beam [for hadron colliders]

$$d_{iB} = P_{T,i}^2$$

- 2) Find the smallest at $\{d_{ij}\} \cup \{d_{iB}\}$

IR safe?

➡ $d_{ij} \Rightarrow$ merge i & j into a new "protojet"

E-scheme: $P_{(ij)}^\mu = P_i^\mu + P_j^\mu$

➡ $d_{iB} \Rightarrow$ remove i from the set & call it a "jet"

- 3) If particles left, goto step 1 & repeat

mimics the inverse of
the S&C emission probability
 \sim relative k_T

remember: $(P_i + P_j)^2 \sim \underbrace{P_{T,i} P_{T,j}}_{\text{not so good (JADE)}} \Delta R_{ij}^2$

The k_T Algorithm

Try to work our way backwards through "branchings"

- 1) Compute distances between all particles

$$d_{ij} = \min(P_{T,i}^2, P_{T,j}^2) \frac{\Delta R_{ij}^2}{R^2}$$

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- 3) If particles left, goto step 1 & repeat

- 4) Only retain jets above a minimum P_T threshold $P_{T,jet} > P_{T,cut}$

soft first

↳ irregular shapes

↳ exp. challenges

↳ collects "junk"

The generalized k_T Algorithm

Try to work our way backwards through "branchings"

- 1) Compute distances between all particles

$$d_{ij} = \min(P_{T,i}^{2\alpha}, P_{T,j}^{2\alpha}) \frac{\Delta R_{ij}^2}{R^2}$$

and to the beam [for hadron colliders]

$$d_{iB} = P_{T,i}^{2\alpha}$$

- 2) Find the smallest at $\{d_{ij}\} \cup \{d_{iB}\}$

↳ $d_{ij} \Rightarrow$ merge i & j into a new "protojet"

E-scheme: $P_{(ij)}^\mu = P_i^\mu + P_j^\mu$

↳ $d_{iB} \Rightarrow$ remove i from the set & call it a "jet"

- 3) If particles left, goto step 1 & repeat

- 4) Only retain jets above a minimum P_T threshold

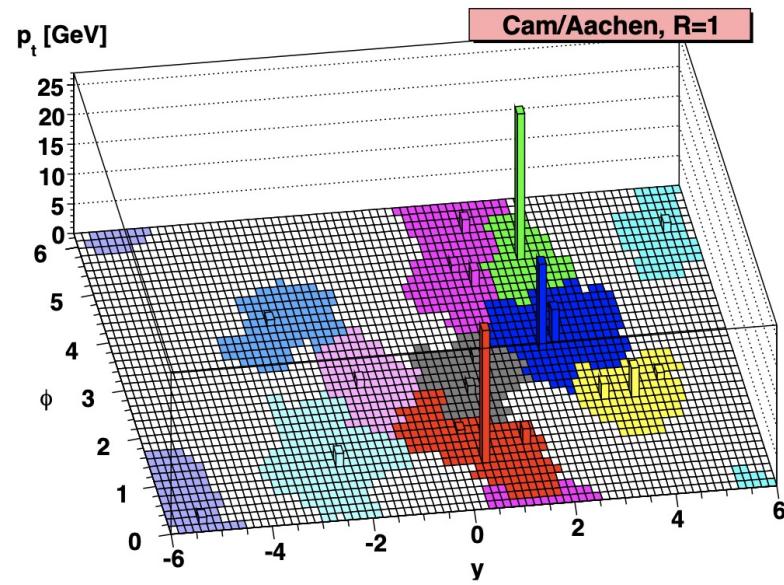
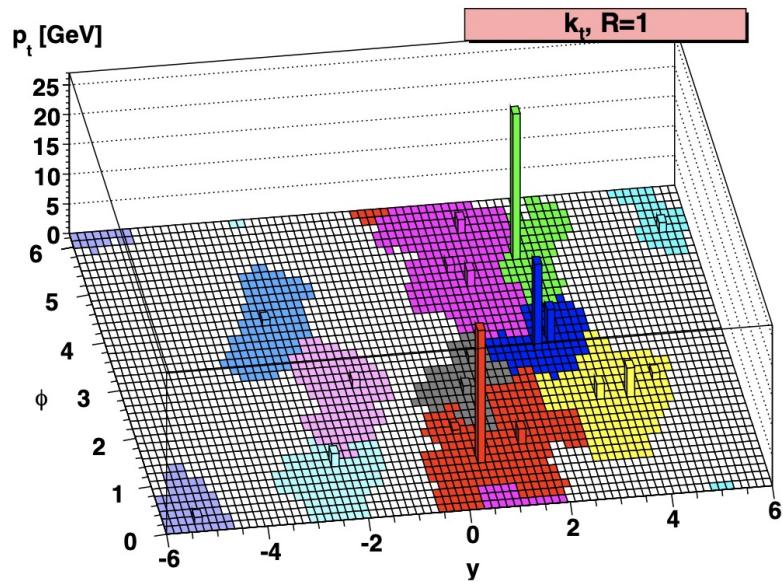
$$\alpha = \begin{cases} +1 : k_T \\ \emptyset : \text{Cambridge/Aachen} \\ \text{geometric} \\ -1 : \text{anti-}k_T \end{cases}$$

anti- k_T : hard first

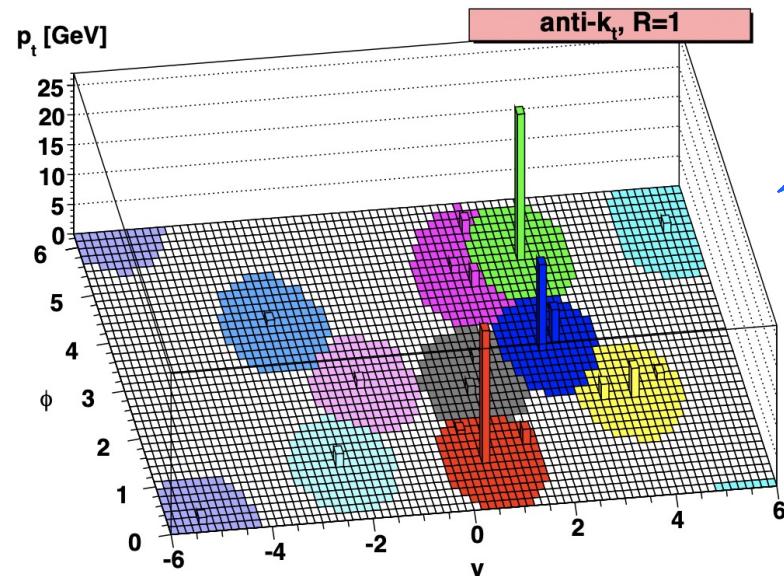
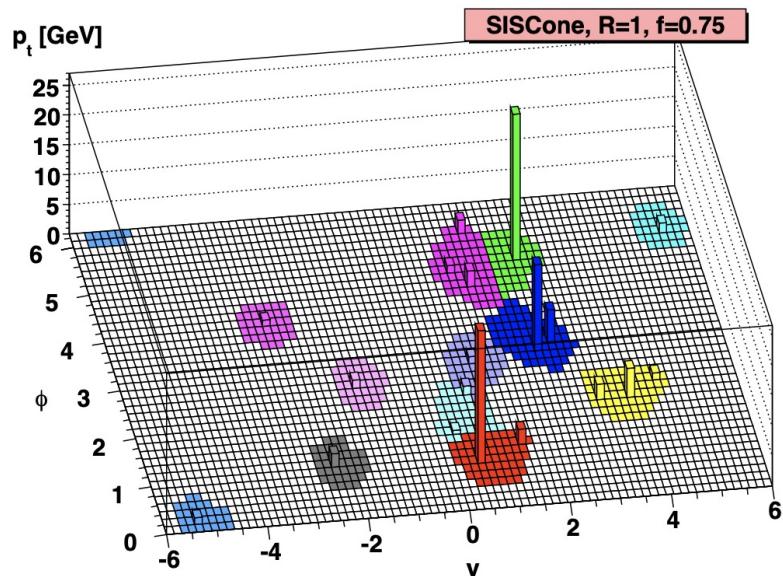
↳ nearly perfect cones

$$P_{T,\text{jet}} > P_{T,\text{cut}}$$

Comparison of Jet Algorithms



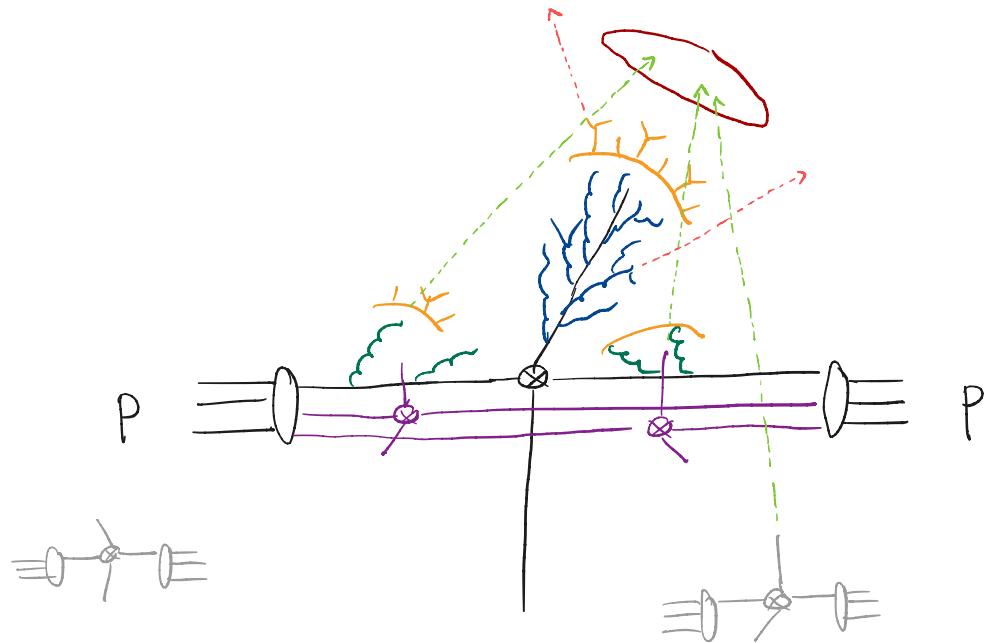
irregularities from
non-linear
behaviour wrt.
soft emissions



solved

nearly perfect cones!
(EXP happy)
* overlap \rightarrow harder one

Jets at Hadron Colliders



- final-state radiation (FSR) $\sim Q_2$
- initial-state radiation (ISR) $\sim Q_2$
- multiple parton interactions (MPI)
aka underlying event (UE) $\sim \text{GeV}$
- pile up (PU) $\sim n_{\text{pu}} \cdot 0.5 \text{ GeV}$
- hadronisation $\sim \Lambda_{\text{QCD}}$

$$\text{Jet} = \underbrace{\left(\begin{array}{c} \text{hard parton} \\ + \\ \text{radiation} \end{array} \right)}_{\text{what we're after}} - \text{LOSS} + \text{CONTAMINATION}$$

↑
R bigger?
↗ R smaller?

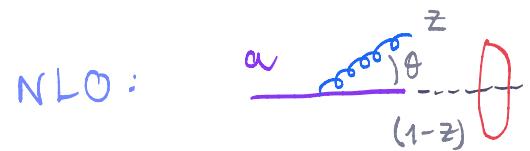
\Rightarrow there is no single "best" jet definition (trade-offs; depends on application)

Final State Radiation v.s. the cone size

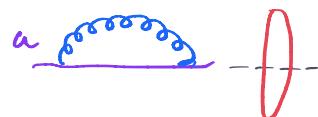
Let's consider the energy of a jet



$$E_{\text{jet}} = E_a$$



$$E_{\text{jet}} = \frac{2\alpha_s}{\pi} C_a \int \frac{d\theta}{\theta} \int dz P_{ga}(z) \left\{ \begin{array}{l} E_a \Theta(\theta < R) \\ + E_a (1-z) \Theta(\theta > R) \Theta(z < \frac{1}{2}) \\ + E_a z \Theta(\theta > R) \Theta(z > \frac{1}{2}) \end{array} \right. \quad \begin{array}{l} \text{inside} \\ \hookrightarrow \text{recovers energy} \end{array}$$



$$E_{\text{jet}} = \frac{2\alpha_s}{\pi} C_a \int \frac{d\theta}{\theta} \int dz P_{ga}(z) \left\{ E_a [-1] \right\} \quad \begin{array}{l} \text{virtual correction} \\ \text{via "unitarity"} \end{array}$$

$$\Rightarrow E_{\text{jet}} = E_a \left[1 - \frac{\alpha_s}{\pi} \ln(\frac{1}{R}) L_a \right]$$

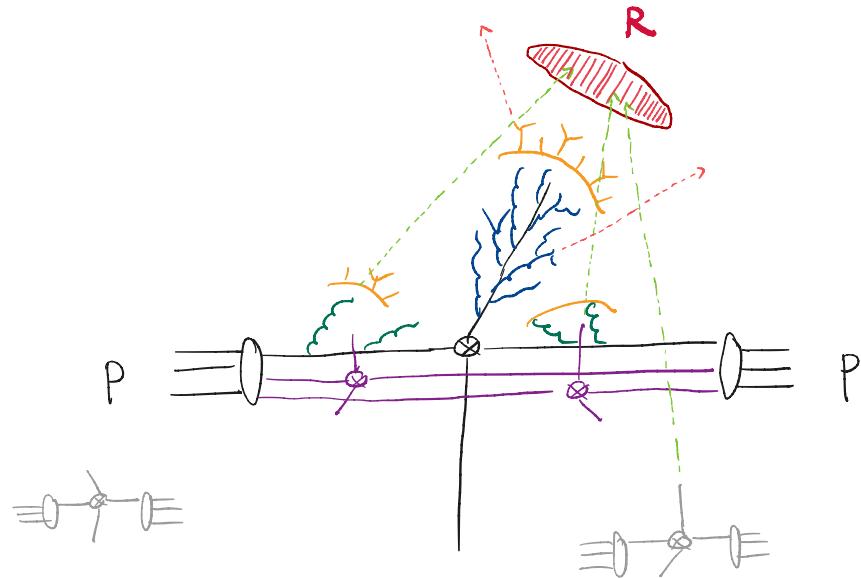
$$\hookrightarrow L_g = C_F \times 1.01129$$

$$L_g = C_A \times 0.94 + N_f \cdot 0.07$$

$$\Delta E/E \sim -5\% \quad \begin{array}{l} R=0.4 \\ \swarrow \end{array}$$

$$\Delta E/E \sim -10\%$$

Loss v.s. Contamination – the R dependence



Jet = (hard parton
+ radiation)

- LOSS
- + CONTAMINATION

- final-state radiation (FSR)
- initial-state radiation (ISR)
- multiple parton interactions (MPI)
aka underlying event (UE)
- pile up (PU)
- hadronisation

$$\sim -\frac{\alpha_s}{\pi} C_i P_T \ln(1/R)$$

$$\sim \frac{\alpha_s}{\pi} C_i P_T \pi R^2$$

$$\sim \beta^{MPI} \pi R^2 \quad [\beta^{MPI} \sim O(1 \text{ GeV}) @ LHC]$$

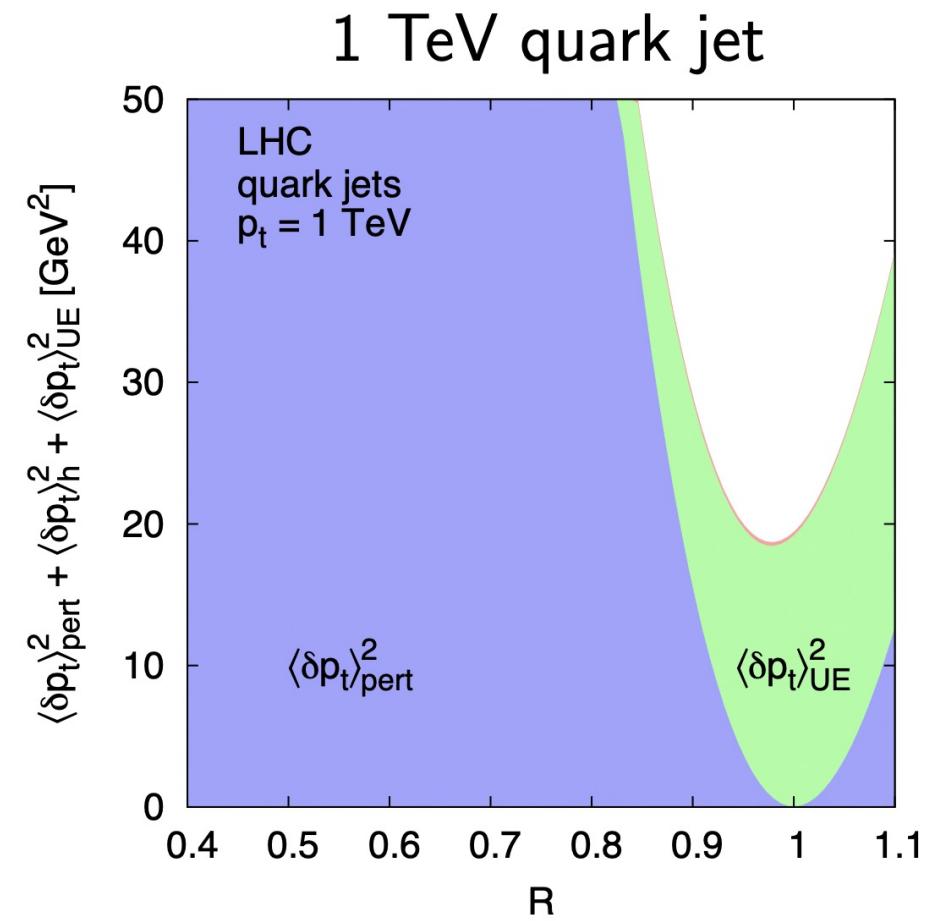
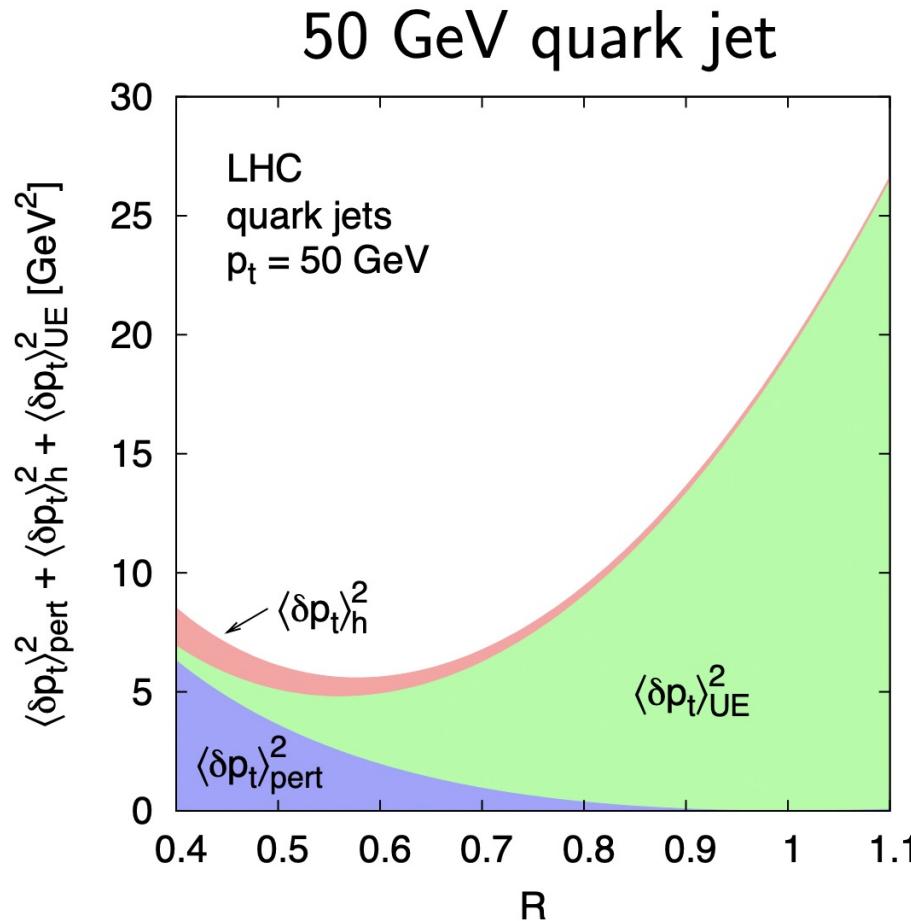
$$\sim \beta^{PU} \pi R^2 \quad [\beta^{PU} \sim \overset{100-1000}{n_{PU}} \times 0.5 \text{ GeV}]$$

$$\sim -\Lambda_{QCD} \frac{1}{R}$$

The "optimal" Cone size ?

* get the different $\langle \delta p_T^2 \rangle$ to balance

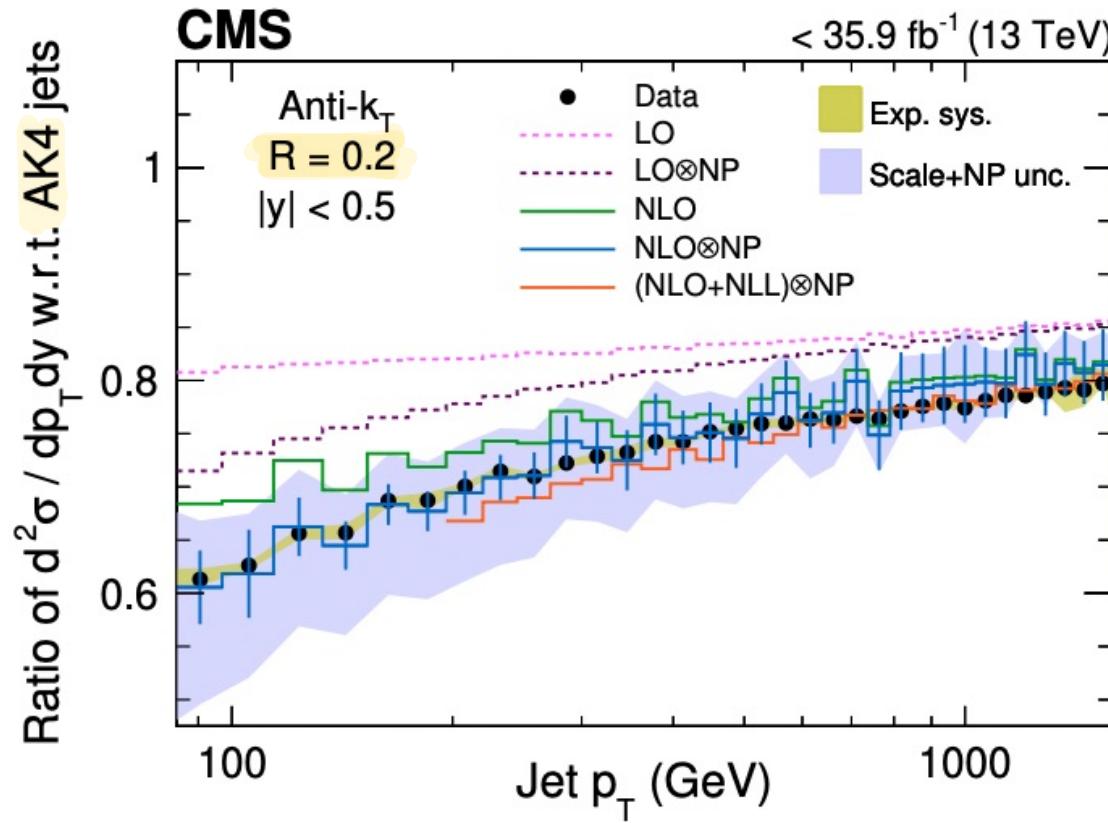
["Towards Jetography" - G. Salam '09]



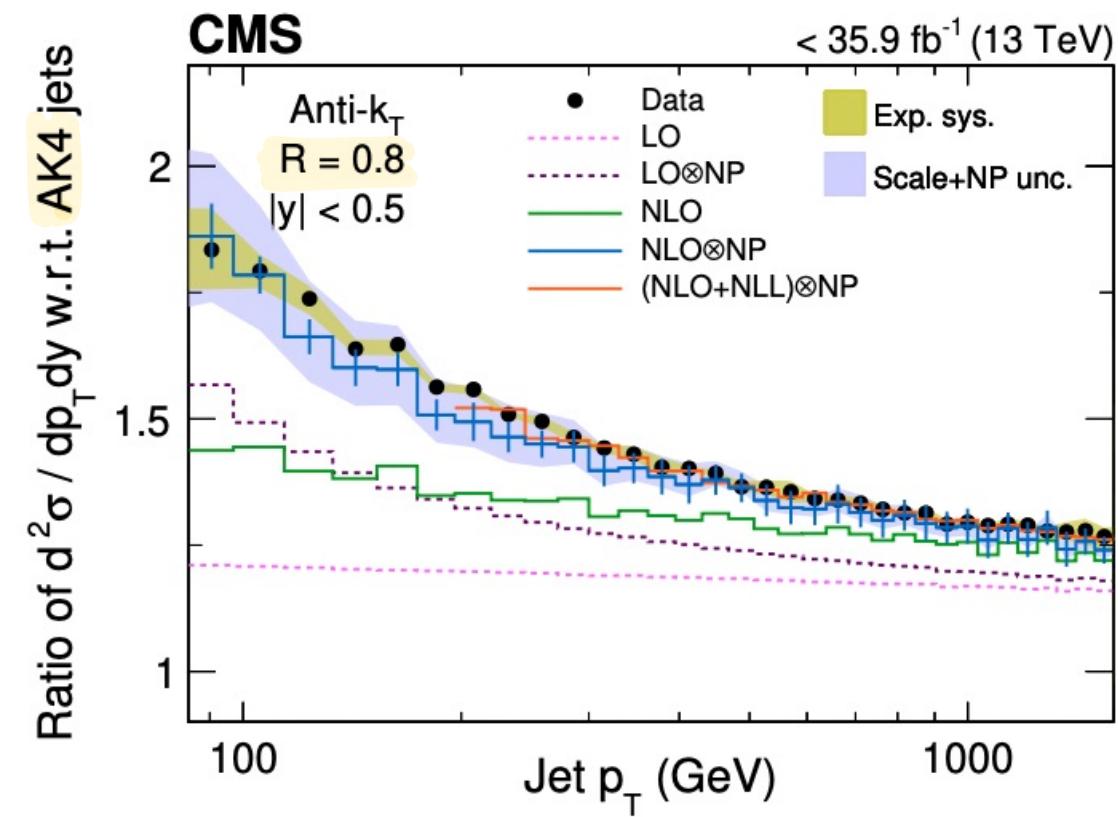
* R small to limit impact of UE

* R large to tame FSR

R variation



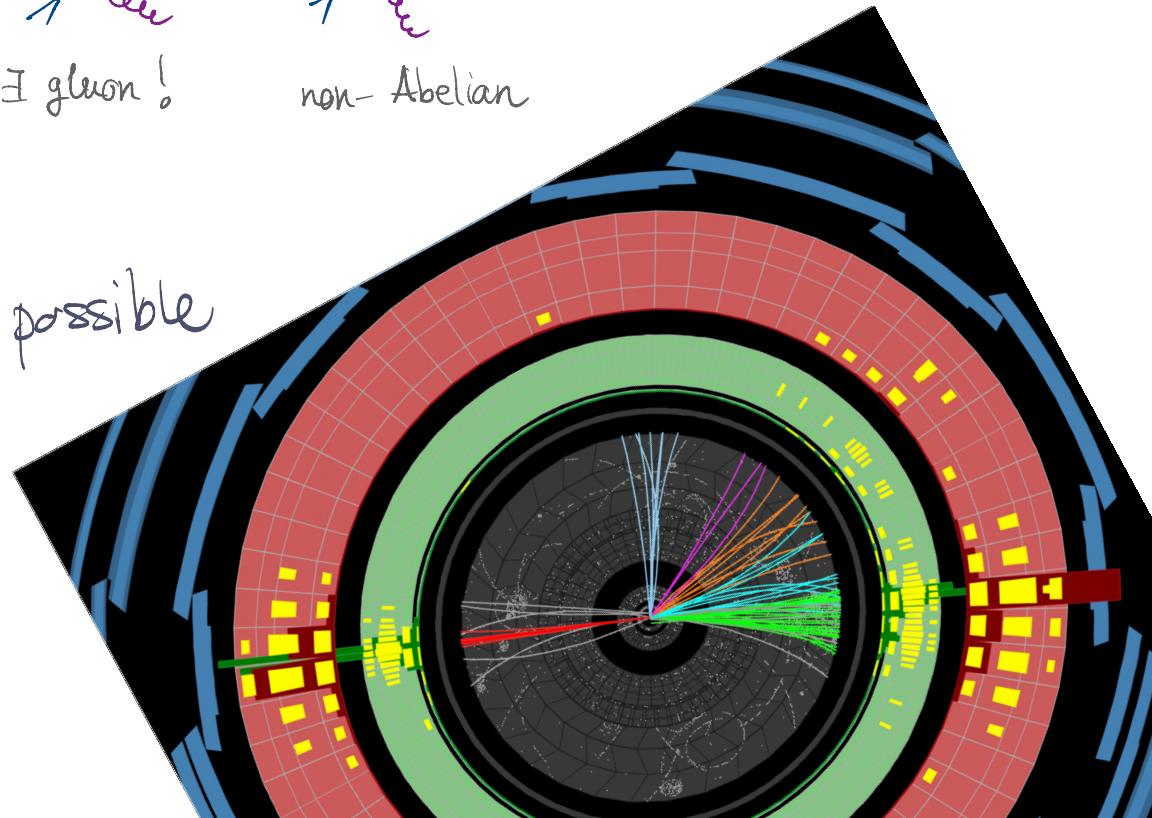
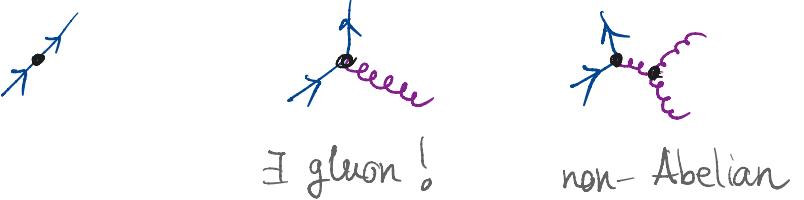
* "NP" \Rightarrow hadr. + MPI



Jets are ...

- * a proxy for high-energetic parton \rightsquigarrow interface between TH & EXP
- * are everywhere (esp. @ LHC)
 - \hookrightarrow study QFT's: $e^+e^- \rightarrow$ 2 jet , 3 jet , 4 jet
 - \hookrightarrow BSM searches (boosted!)
- * are not unique & fundamentally ambiguous
 - \hookrightarrow application-specific optimizations possible

\rightsquigarrow What if I want to model the full complexity (exclusive) of an event?



Exclusive Emissions

- * recall: exact calculation of $2 \rightarrow 100$ particle scattering: hopeless
↳ but the relevant physics after the hard interaction is governed by soft & collinear QCD

- * introduce a scale $q^2 > Q_0^2 \leftrightarrow$ emission "resolved"

$$d\omega_{x \rightarrow x+g} = 2 \frac{\alpha_s}{\pi} C_x \frac{dE}{E} \frac{d\theta}{\theta}$$

↑ integral
diverges

$$\Rightarrow P_x \simeq \frac{\alpha_s C_F}{2\pi} \ln^2 \left(\frac{Q^2}{Q_0^2} \right) + \mathcal{O}(\alpha_s \ln Q^2, \alpha_s^2)$$

probability to emit
a resolved gluon

potentially a very large log

$$\left[\begin{array}{l} Q_0 = 1 \text{ GeV} \\ Q = 100 \text{ GeV} \end{array} \right] \Rightarrow \ln(\dots) = \mathcal{O}(10)$$

→ will want to "resum" these to all orders
while also retaining information on the emission kinematics

Parton Showers

- * We wish to account for an **arbitrary** number of emissions ordered in our resolution variable $Q^2 > q_1^2 > q_2^2 > \dots > Q_0^2$ (strong ordering)
- * current scale $q_n^2 \Rightarrow$ probability to have next emission @ q_{n+1}^2 ?

$$\left(\begin{array}{l} \text{probability of having} \\ \text{no emissions } q_n^2 \mapsto q_{n+1}^2 \end{array} \right) \times \left(\begin{array}{l} \text{emission} \\ @ q_{n+1}^2 \end{array} \right)$$

[demo: toy shower]

$$\Delta(q_n^2, q_{n+1}^2)$$

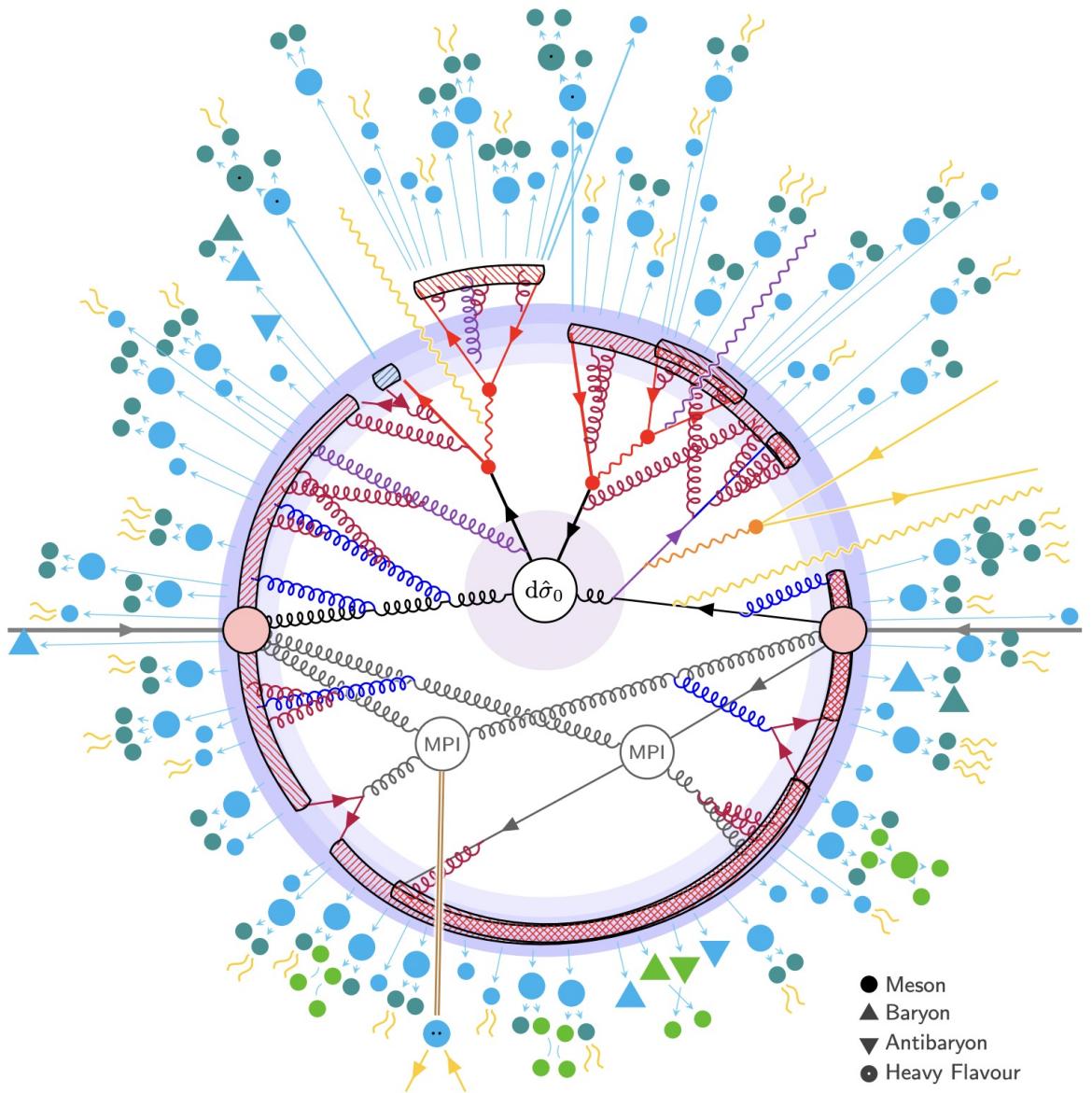
Sudakov form factor

$$x \quad \frac{d\omega_{x \rightarrow x+g}}{dq^2} \Big|_{q^2 = q_{n+1}^2}$$

$$\hookrightarrow \frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2) \frac{d\omega}{dq^2}$$

$$\left[\Delta(Q^2, q^2 - dq^2) = \Delta(Q^2, q^2) \Delta(q^2, q^2 - dq^2) = \Delta(Q^2, q^2) \left(1 - \frac{d\omega}{dq^2}\right) \right]$$

Full Event Generator



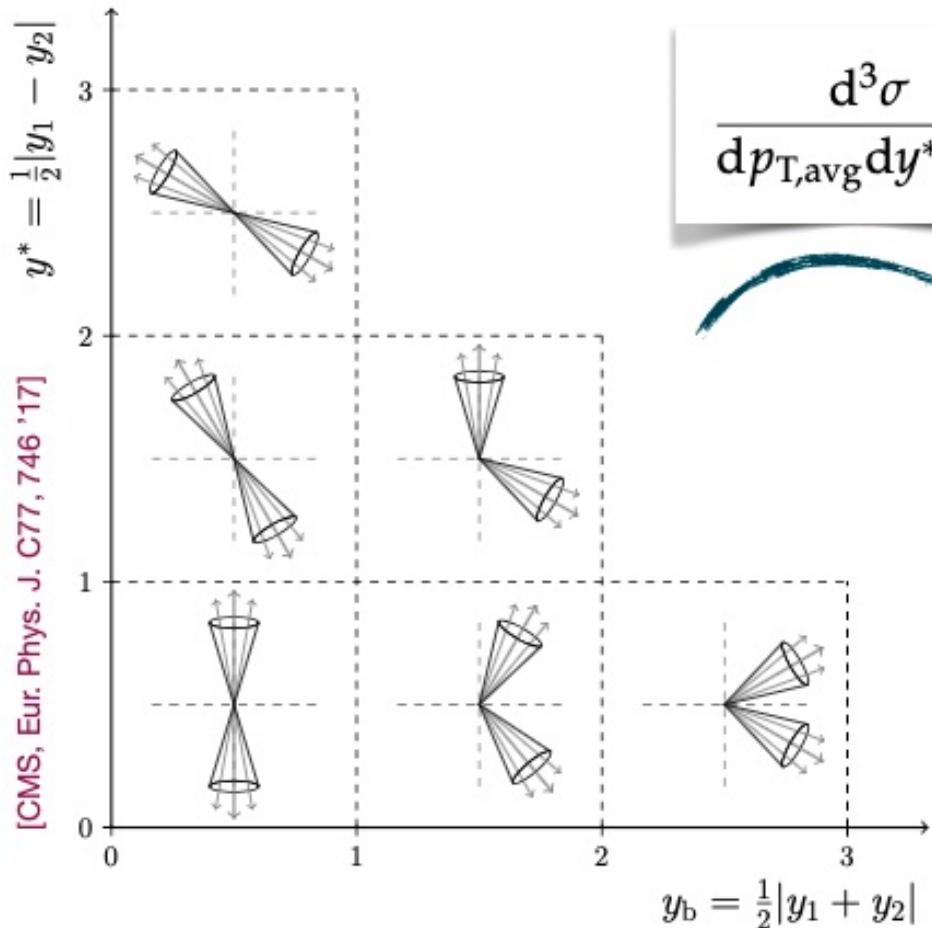
Conclusions

- * covered general concepts in QCD that goes into hadron-collider predictions
 - ↳ asymptotic freedom \leftrightarrow application of perturbation theory
 - ↳ separation of scales ("factorization")
- * Moment of comparing your predictions to data always exciting
 - ↳ learn to play with the tools ; break them (often interesting physics)
- * Hope was able to lower the fear of entry for some of you,
as it is sometimes perceived as very technical
 - ↳ pushing frontiers in precision can become arbitrary complex
new ideas needed (maybe one of you?)

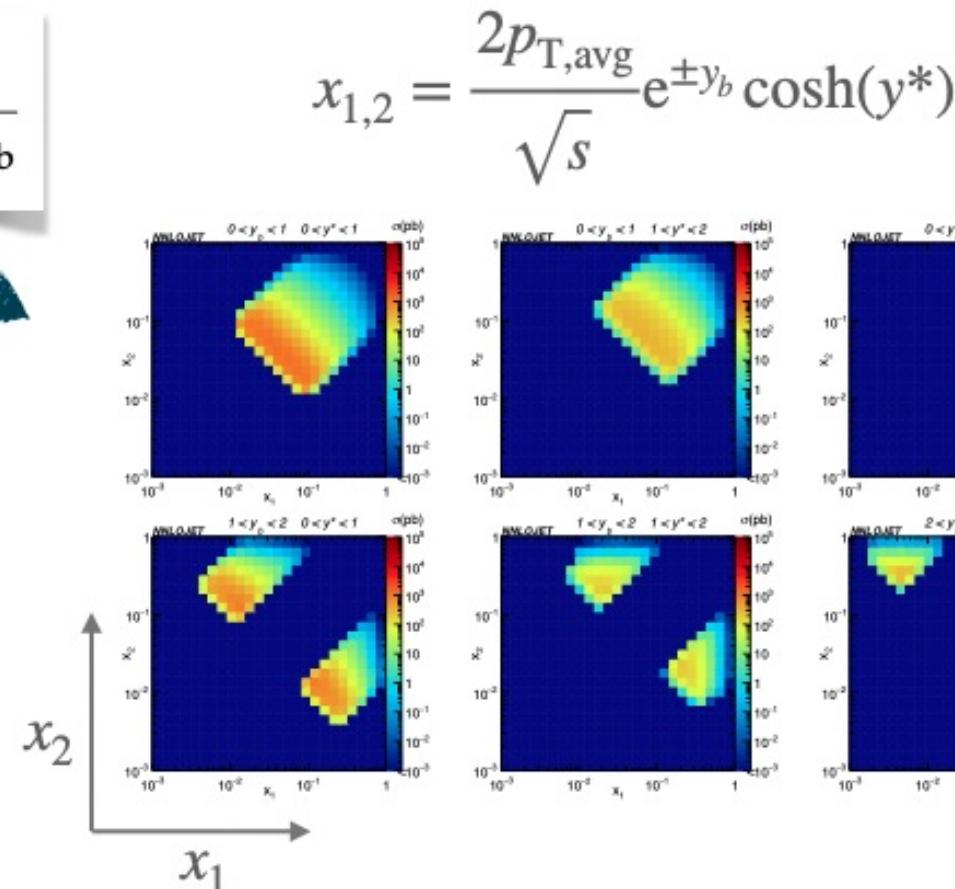
Thank you for your
attention & participation!

Extra

Triple - Differential Jet Production



$$\frac{d^3\sigma}{dp_{T,\text{avg}} dy^* dy_b}$$

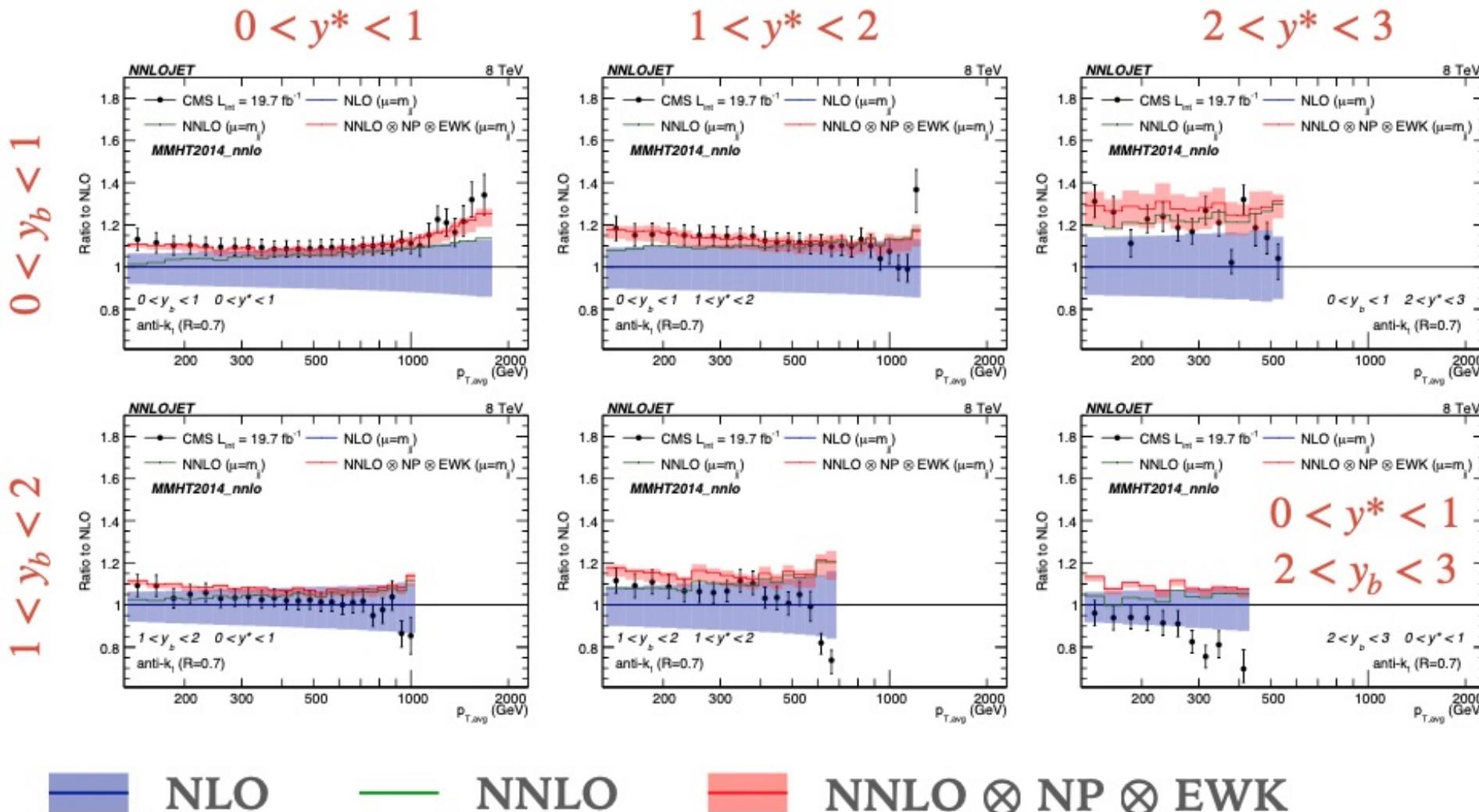


- study different kinematic regimes

- disentangle momentum fractions x_1 & x_2

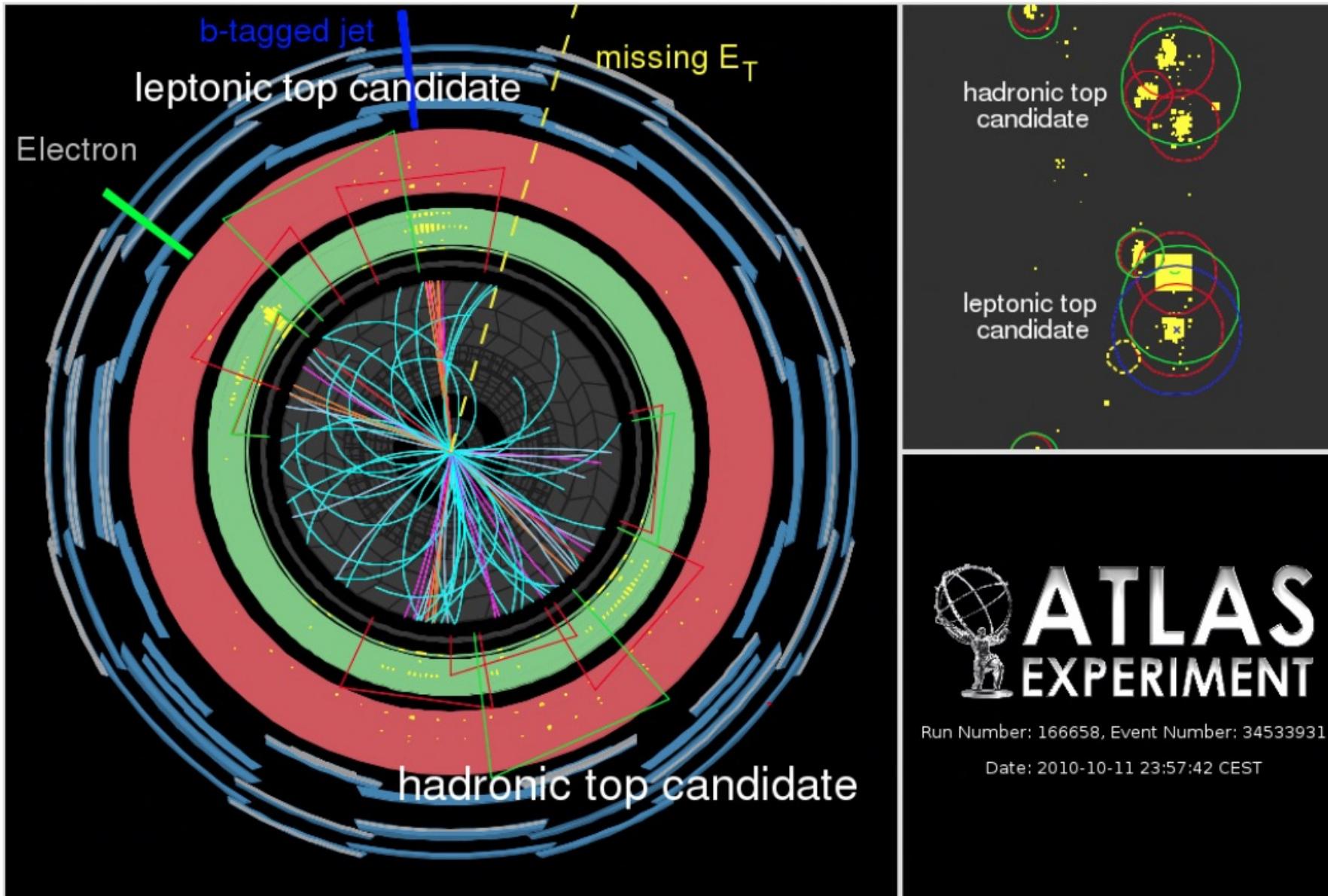
Triple - Differential Jet Production @ NNLO

[Gehrman-De Ridder, Gehrman, Glover, AH, Pires '19]



improved description of data & reduced uncertainties!

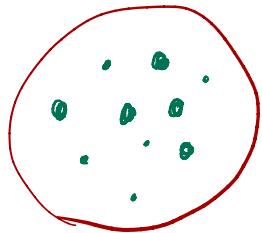
Jet Substructure



Looking inside Jets

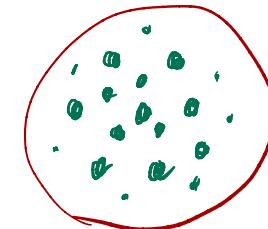
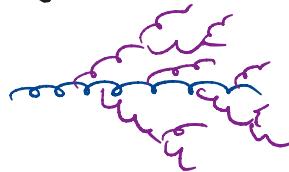
- * At the end of jet finding \Rightarrow collection of constituents $\rightarrow p_{\text{jet}}^M$
 \hookrightarrow more information / physics than just the momentum
- * What is the arrangement of the constituents inside the jets?

quark?



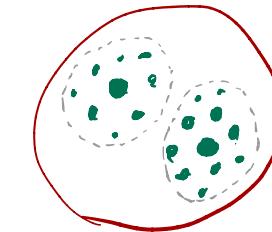
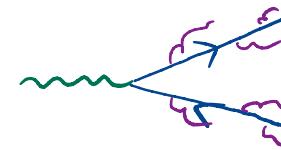
$$\frac{2\alpha_s}{\pi} C_F \frac{dE}{E} \frac{d\theta}{\theta}$$

gluon?



$$\frac{2\alpha_s}{\pi} C_A \frac{dE}{E} \frac{d\theta}{\theta}$$

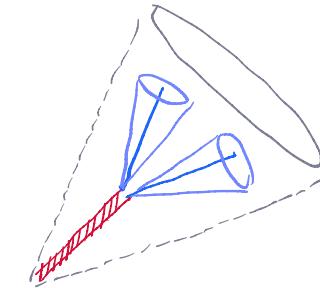
boosted object?



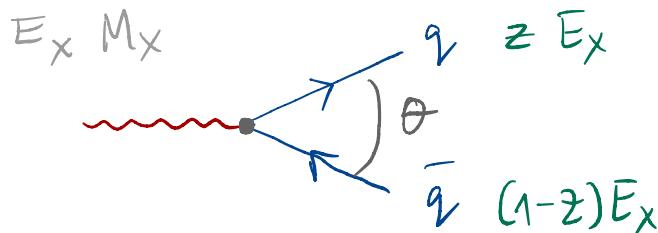
$$\left[\begin{array}{l} H/W/Z \leftrightarrow 2 \text{ prongs} \\ \text{top quark} \leftrightarrow 3 \text{ prongs} \end{array} \right]$$

Boosted Objects

- * In extreme kinematic configurations, massive hadronically decaying object \rightarrow fat jets



- * What cone sizes are we talking about?



$$m_J^2 = M_X^2 = 2 E_X^2 z(1-z) \underbrace{(1-\cos\theta)}_{\frac{1}{2}\theta^2}$$

$$\Rightarrow \theta = \frac{M_X}{E_X} \sqrt{\frac{1}{z(1-z)}} \stackrel{z \sim \frac{1}{2}}{\sim} \frac{2 M_X}{E_X}$$

put in some numbers :

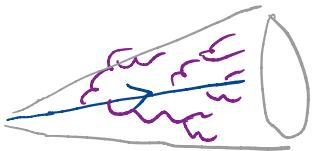
$$\left. \begin{array}{l} M_X = M_W \approx 80 \text{ GeV} \\ E_X \sim 1 \text{ TeV} \end{array} \right\}$$

$\theta \sim 0.15 \rightarrow$ likely end up in 1 jet

\hookrightarrow how to distinguish
this from a QCD jet?

The Jet mass

- * naive expectation (common misconception)
jet from "X" has mass M_X , whereas q/g jets are massless
- * The jet mass of QCD partons



$$m^2 = \left[\sum_{i \in \text{jet}} p_i \right]^2$$

consider the cumulant :

$$\Sigma(m_J^2) = \text{probability for the jet to have } m^2 < m_J^2$$

$$= \frac{1}{\sigma} \int dm^2 \frac{d\sigma}{dm^2}$$

@ LO: $\xrightarrow{E_J} m^2 = \phi \Rightarrow \Sigma(m_J^2) = 1$

@ NLO: $\xrightarrow{E_J}$

}

$$\Rightarrow \Sigma(m_J^2) = 1 - \frac{\alpha_s}{2\pi} C_F \ln^2 \left(\frac{E_J^2 R^2}{m_J^2} \right)$$

do as exercise!

not good ($m_J \rightarrow 0$)!
 higher orders
 won't help either
 $\sim \alpha_s^n \ln^{2n} \left(\frac{E_J^2 R^2}{m_J^2} \right)$

The renormalized Jet mass

* need to account for these logs to all orders!

$$\sum_1(m_J^2) = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^n \left[\int \frac{d\theta_i^2}{\theta_i^2} \int \frac{dz_i}{z_i} \frac{\alpha_s C_F}{\pi} \Theta_{i \in \text{jet}} \right] \cdot \Theta \left(\left[\sum_{i=1}^n p_i \right]^2 < m_J^2 \right)$$

real emissions inside the jet

$$\sum_{m=0}^{\infty} \frac{1}{m!} \prod_{j=1}^m \left[\int \frac{d\tilde{\theta}_j^2}{\tilde{\theta}_j^2} \int \frac{d\tilde{z}_j}{\tilde{z}_j} \frac{\alpha_s C_F}{\pi} \left(\Theta_{j \notin \text{jet}} - 1 \right) \right]$$

do not change m^2

real out of cone
 virtual

$$* \left[\sum_{i=1}^n p_i \right]^2 \simeq E_J^2 \sum_{i=1}^n z_i \theta_i^2$$

* we're interested in the leading logs (LL) \Rightarrow widely separate scales

\Rightarrow among all $z_i \theta_i^2$ one is dominant!

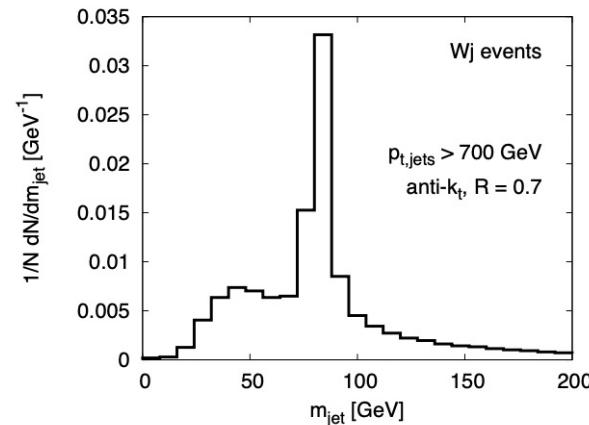
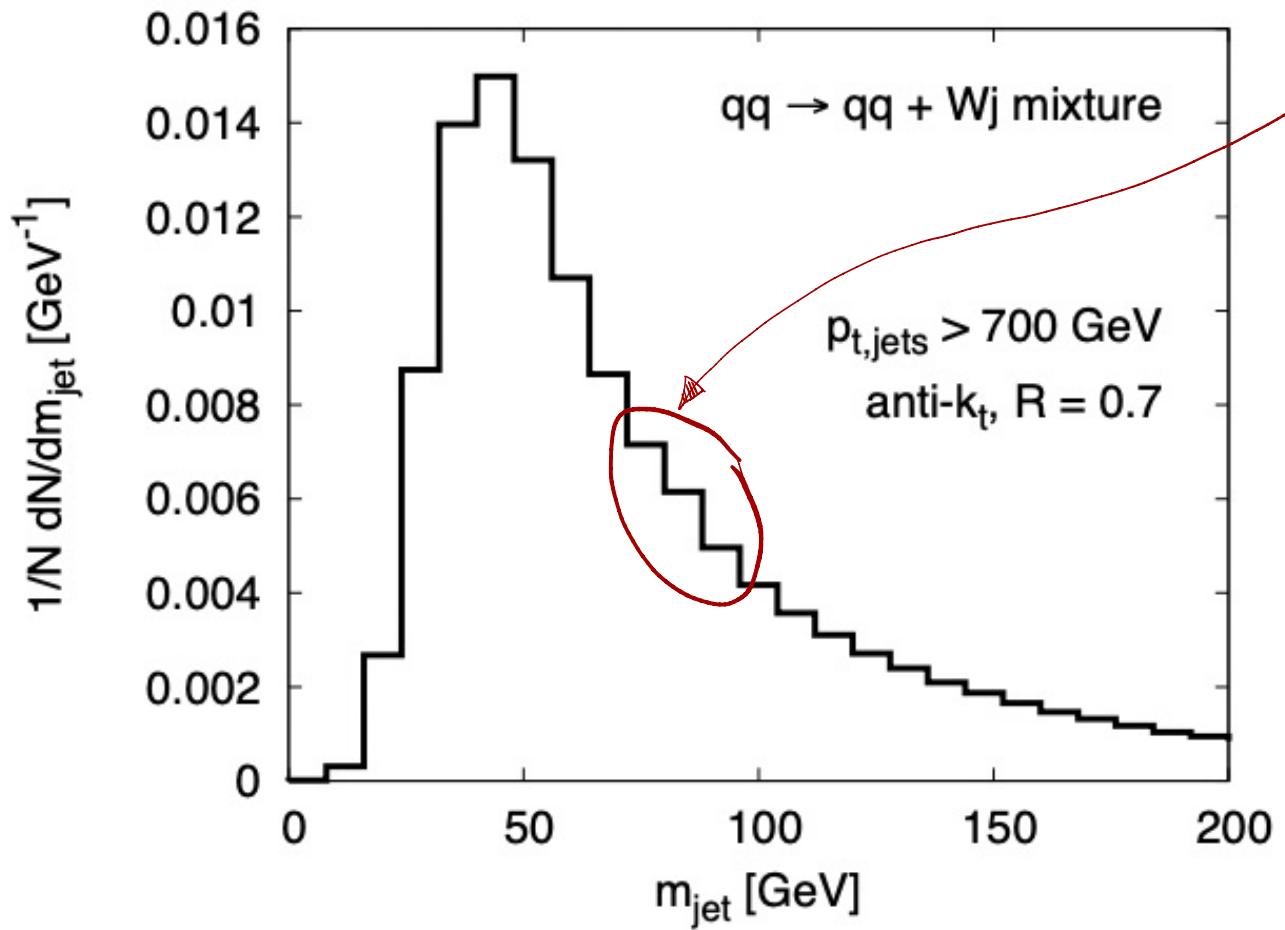
$$\Theta \left(E_J^2 \sum_i z_i \theta_i^2 < m_J^2 \right) \simeq \Theta \left(E_J^2 \max \{ z_i \theta_i^2 \} < m_J^2 \right) = \prod_{i=1}^n \Theta \left(E_J^2 z_i \theta_i^2 < m_J^2 \right)$$

$$= \exp \left[- \int \frac{d\theta^2}{\theta^2} \int \frac{dz}{z} \frac{\alpha_s C_F}{\pi} \Theta(\theta < R) \Theta(E_J^2 z \theta^2 < m_J^2) \right] = \exp \left[- \frac{\alpha_s C_F}{2\pi} \ln^2 \left(\frac{E_J^2 R^2}{m_J^2} \right) \right]$$

Jet Mass in Real Life

$$\frac{d\sigma}{dm_J^2} = \frac{d\Sigma}{dm_J^2} = \frac{1}{m_J^2} \frac{\alpha_S C_F}{\pi L} \ln\left(\frac{E_J^2 R^2}{m_J^2}\right) \exp\left[-\frac{\alpha_S C_F}{2\pi} \ln^2\left(\frac{E_J^2 R^2}{m_J^2}\right)\right]$$

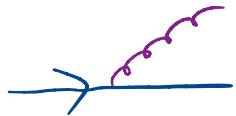
"Sudakov"



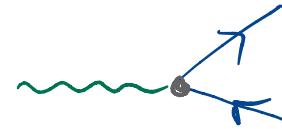
→ clear sign of W
 → but QCD jets massive too

NEED TO REJECT QCD
 BACKGROUND & ENHANCE
 THE SIGNAL !

mMDT / Soft Drop



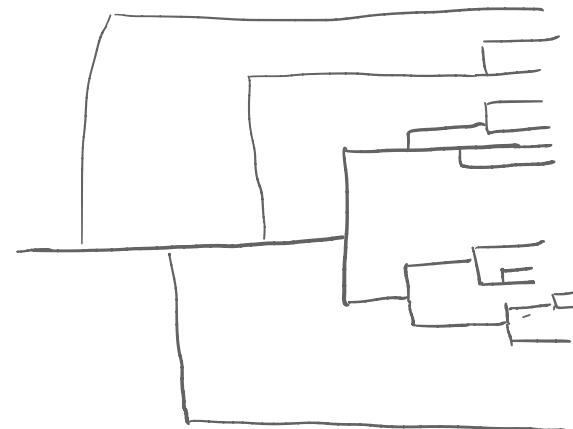
$$\sim \frac{1}{z} \text{ [soft]}$$



$$\sim 1$$

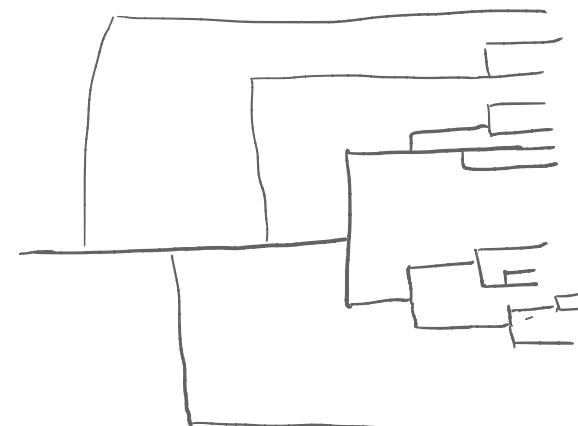
[balanced momentum Sharing]

1. Take the constituents of the jet and **recluster** using C/A
→ angular-ordered tree

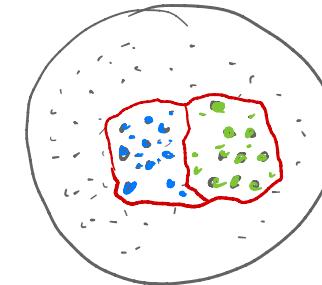
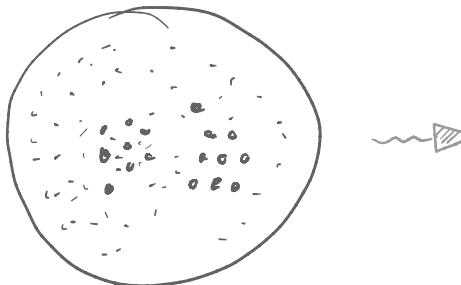
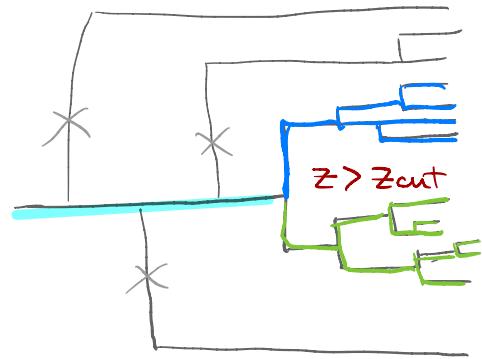


2. Decluster jet, disregard the softer branch until

$$z = \frac{\min(E_i, E_j)}{E_i + E_j} > z_{\text{cut}}$$



mMDT / Soft Drop



k_T would keep
the soft junk

- * Removes soft radiation from periphery of jet
[because Cambridge-Aachen for declustering]
 - * Dynamically shrinks jet radius to match
hard core
 - * Information on the 2-prong kinematics
 - * $\sum_i (m_j^2) = \exp \left[-\frac{\alpha_s}{\pi} C_F \ln \left(\frac{1}{z_{\text{cut}}} \right) \ln \left(\frac{E_J^2 R^2}{m_j^2} \right) \right] \leftrightarrow \begin{cases} \text{much smoother (& smaller)} \\ \text{background} \end{cases}$
- soft singularity regulated \Rightarrow single log!

} Jet cleaning
"grooming"

} Jet discrimination