

# Parton Distribution Functions

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## 1 Introduction

We will call the LHAPDF library from python to evaluate parton distribution functions and play around with them a bit.

## 2 Evaluating PDFs

Let's have a look at one specific PDF set PDF4LHC21\_40

---

```
import lhapdf
import numpy as np

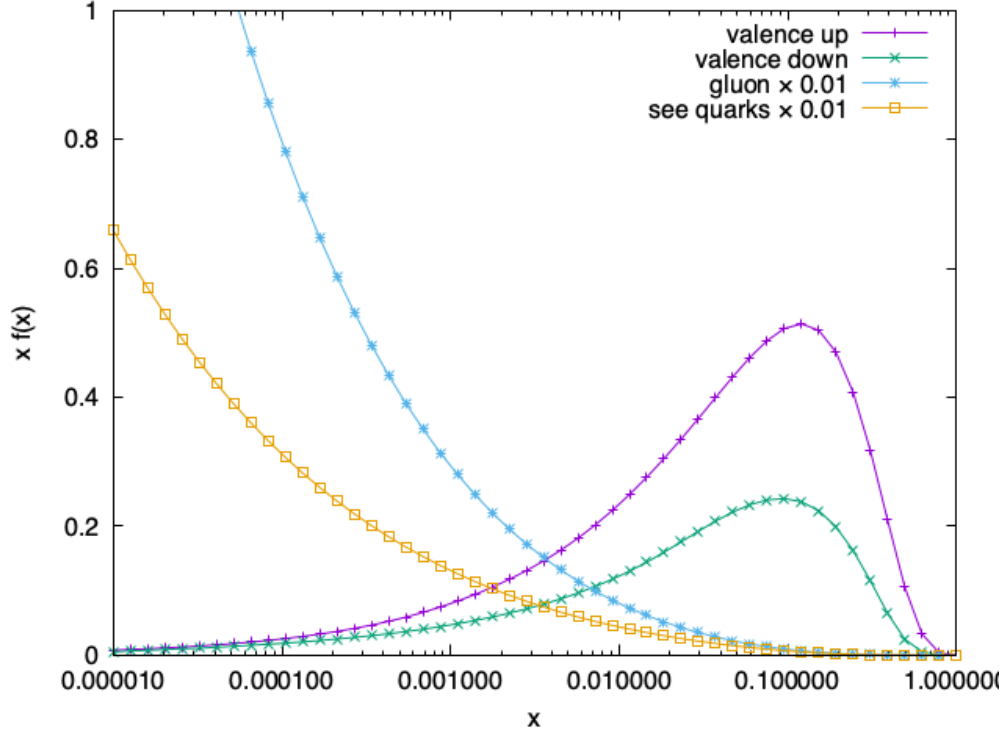
lhapdf.setVerbosity(0)
pdf = lhapdf.mkPDF("PDF4LHC21_40", 0)
if qlog:
    xs = [x for x in np.logspace(-5, 0, 50)]
else:
    xs = [x for x in np.linspace(0, 1, 50)]
res = np.empty((len(xs), 6))
for ix, x in enumerate(xs):
    fac = 1. # 1./x
    res[ix,0] = x
    res[ix,1] = q
    res[ix,2] = fac*(pdf.xfxQ(2, x, q) - pdf.xfxQ(-2, x, q)) # valence up-quark
    res[ix,3] = fac*(pdf.xfxQ(1, x, q) - pdf.xfxQ(-1, x, q)) # valence down-quark
    res[ix,4] = fac*(pdf.xfxQ(0, x, q)) # gluon (or `21`)
    res[ix,5] = fac*(
        2.*pdf.xfxQ(-2, x, q)
        +2.*pdf.xfxQ(-1, x, q)
```

```

+pdf.xfxQ(3, x, q)+pdf.xfxQ(-3, x, q)
+pdf.xfxQ(4, x, q)+pdf.xfxQ(-4, x, q)
+pdf.xfxQ(5, x, q)+pdf.xfxQ(-5, x, q)
) # sum over all sea quarks
return res.tolist()

```

---



- Change the scale in the figure from 10 to 100 GeV or even to 1 TeV; adjust the scaling factor as necessary. How does the distributions change? Does it correspond to the expectation of the QCD improved parton model?

### 3 The quantum numbers of the proton

The quantum numbers of the proton are determined by the valence quarks,  $p=(uud)$ . Given that PDFs are *number densities* for the constituent partons, the following flavour sum rules hold:

$$\int_0^1 dx \left( f_{u|p}(x) - f_{\bar{u}|p}(x) \right) = 2, \quad \int_0^1 dx \left( f_{d|p}(x) - f_{\bar{d}|p}(x) \right) = 1, \quad (1)$$

$$\int_0^1 dx \left( f_{q|p}(x) - f_{\bar{q}|p}(x) \right) = 0 \quad \forall q \notin \{u, d\} \quad (2)$$

Let's see if these hold for one of the global PDF sets:

---

```

import lhapdf
import math
import scipy
lhapdf.setVerbosity(0)
pdf = lhapdf.mkPDF("PDF4LHC21_40", 0)
res = list()
for id,label in [(1,"d"),(2,"u"),(3,"s"),(4,"c"),(5,"b")]:
    int_res = scipy.integrate.quad(lambda x : (pdf.xfxQ(id, x, q)-pdf.xfxQ(-id, x, q))/x, 1e-6, 1, limit=100, epsrel=1e-3)
    #print(int_res)
    res.append( (label, int_res[0]) )
return res

```

---

d	0.9867274422876253
u	1.9924213767549757
s	0.003708032343248529
c	-0.00018756880668405516
b	-8.819432616746102e-05

## 4 Momentum sum rules

The parton  $a$  carries a momentum fraction  $x_a$  of the parent hadron,  $p_a^\mu = x_a P^\mu$ . Therefore, the momentum density associated with that parton is given by  $x_{a|H}(x_a)$ . Since the sum over all parton momenta must sum back up to the parent hadron one, the PDF sets satisfy a *momentum sum rule* (in  $\overline{\text{MS}}$ ):

$$\sum_a \int_0^1 dx_a x_a f_{a|H}(x_a) = 1 \quad (3)$$

Let's see how the momenta are distributed across different flavours

---

```

import lhapdf
import math
import scipy
lhapdf.setVerbosity(0)
pdf = lhapdf.mkPDF("PDF4LHC21_40", 0)
res = list()
int_sum = [0.,0.]
for id,label in [(-5,"bb"),(-4,"cb"),(-3,"sb"),(-2,"ub"),(-1,"db"),(0,"g"),(1,"d"),(2,"u"),(3,"s"),(4,"c"),(5,"b")]:
    int_res = scipy.integrate.quad(lambda x : pdf.xfxQ(id, x, q), 1e-6, 1, limit=100, epsrel=1e-3)
    int_sum[0] += int_res[0]
    int_sum[1] += int_res[1]**2
    #print(int_res)
    res.append( (label, "{:.0f}%".format(int_res[0]*100.)) )
res.append( ("SUM", "{:.0f}%".format(int_sum[0]*100.)) )
return res

```

---

So the gluon actually carries almost 50% of the proton's momentum! The up quark, with  $\sim 20\%$ , has the second largest contribution, followed by the down-quark  $\sim$  half the size of the up (which makes sense as  $p=(uud)$ ).

bb	1%
cb	2%
sb	3%
ub	4%
db	4%
g	47%
d	11%
u	22%
s	3%
c	2%
b	1%
SUM	100%

- Vary the scale and see how the momentum composition of the proton changes. How robust are the numbers?