Quantum Chromodynamics & Collider Phenomenology

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Outline

Ø The Beginnings

I Basics of QCD

II QCD in e^+e^- Collisions

III Hadron Colliders

- parton distribution functions
- DGLAP evolution
- Drell Yan process
- resummation

IV Jet Physics

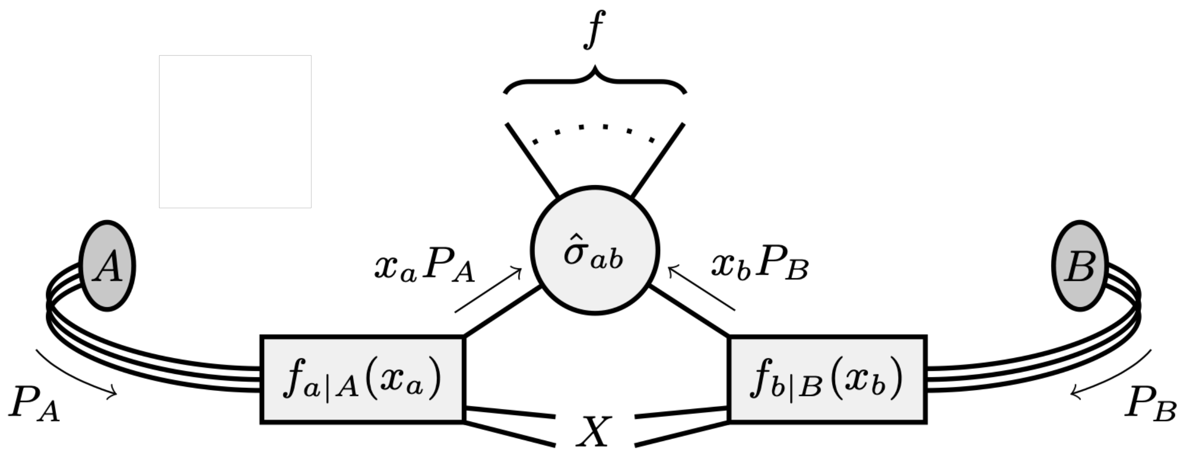
Hadron Colliders: The Parton Model

* time scale of interactions binding the proton $\sim 1/m_p = \tau_{proton}$

* hard interaction $Q \gg m_p$
  $\Rightarrow$ time scales $\sim 1/Q \ll \tau_{proton}$
  $\Rightarrow$ no time for struck parton to "negotiate" a response with others

* asymptotic freedom: at high energies
  $\Rightarrow$ collection of free partons

$\Rightarrow$ incoherent sum of possible partonic cross sections
$\rightsquigarrow$ sufficient to consider single parton



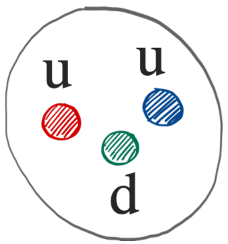
$$d\sigma_{A+B\to f}(P_A, P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_a(x_a)\, f_b(x_b)\, d\hat{\sigma}_{a+b\to f}(x_a P_A, x_b P_B)$$

momentum fraction

parton distribution function (PDF)
$\hat{=}$ number density for parton a w/ momentum fraction $[x_a, x_a + dx_a]$ of parent hadron A

Parton Distribution Functions

* just free quarks? ($p \simeq (uud)$)



- How would the PDFs look like for such a proton?

  PDF: number density of a parton flavour within the proton to carry the momentum fraction x.

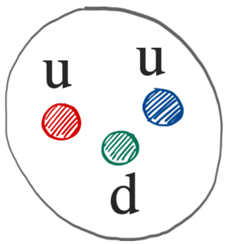
- what is the value of

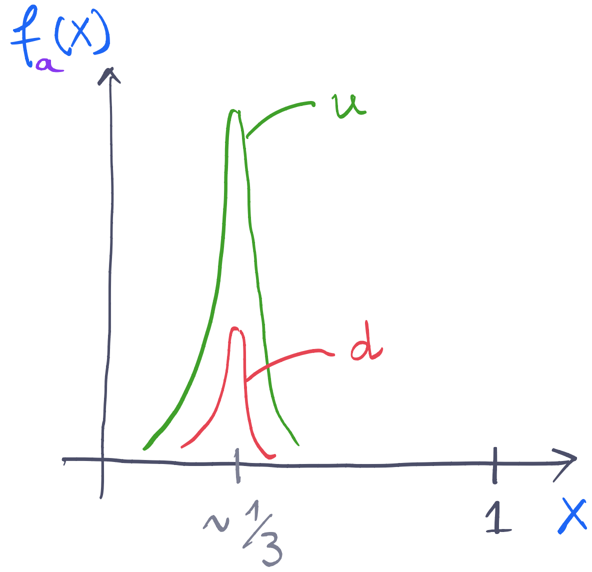
$$\int_0^1 dx \, [f_u(x) - f_{\bar{u}}(x)]$$

$$\int_0^1 dx \, [f_d(x) - f_{\bar{d}}(x)]$$

Parton Distribution Functions

* just free quarks? ($p \simeq (uud)$)





$$f_u(x) \sim 2\,\delta(x - 1/3)$$

$$f_d(x) \sim 1\,\delta(x - 1/3)$$

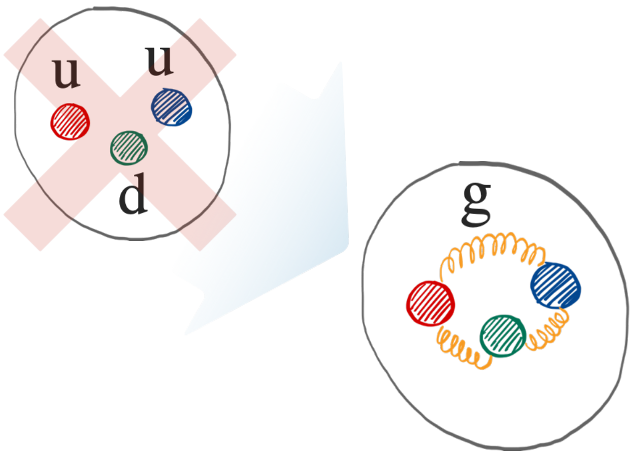
$$f_{etc}(x) \sim \emptyset$$

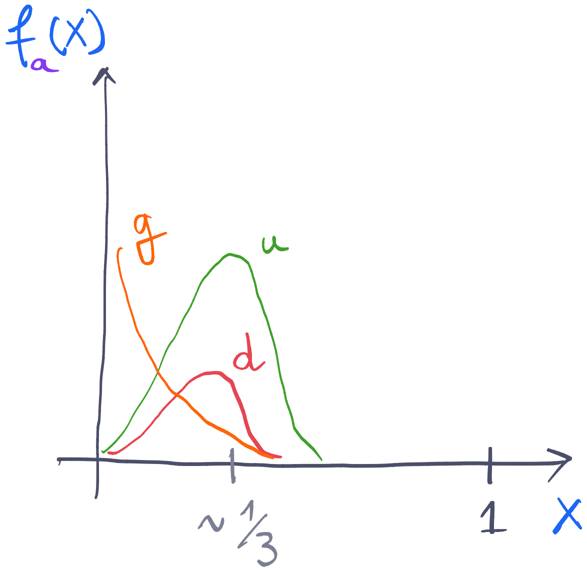
(+ some smearing)

Parton Distribution Functions

* just free quarks? ($p \simeq (uud)$)

* bound by gluons?





naive parton model

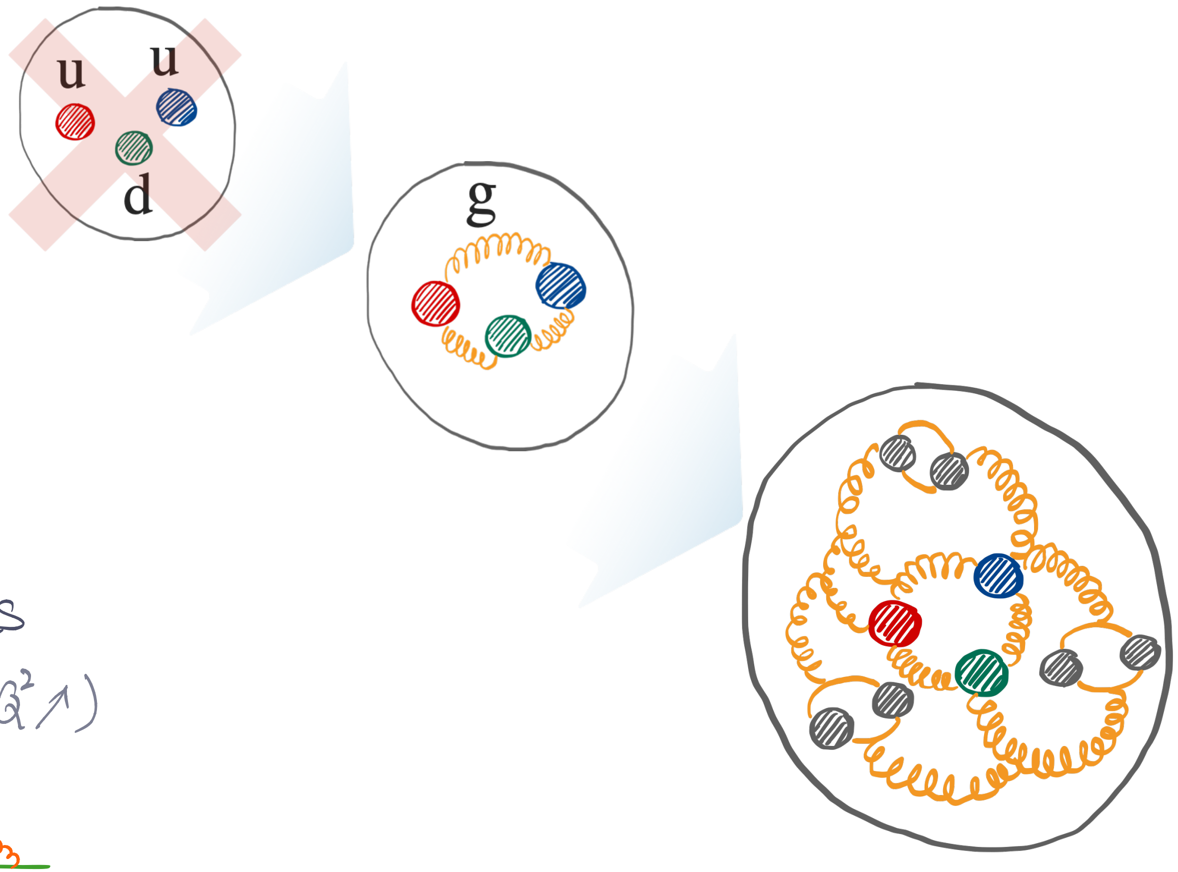
$\Leftrightarrow$ composition of point particles

$\rightsquigarrow$ zoom in ($Q^2 \nearrow$) $\leftrightarrow$ same composition: scaling

PDFs independent on scale, at which it is probed

(as long as $Q^2 \gg m_p^2$)

Parton Distribution Functions

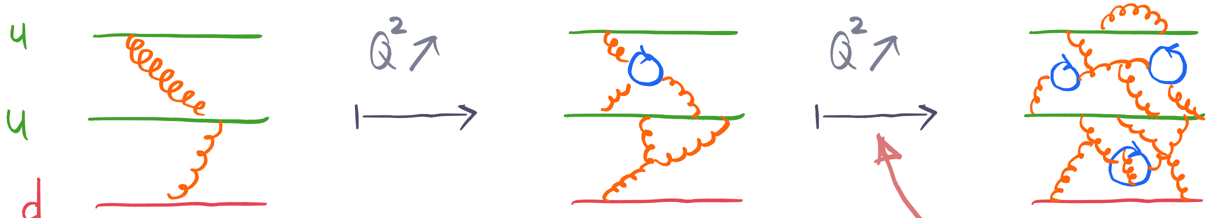


* just free quarks? ($p \simeq (uud)$)

* bound by gluons?

* QCD-improved parton model

  ⇒ quantum fluctuations → more g & $(q\bar{q})$ as we "zoom in" ($Q^2 \nearrow$)



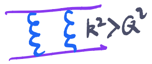
⇒ predominantly shifts partons from high-x to low-x

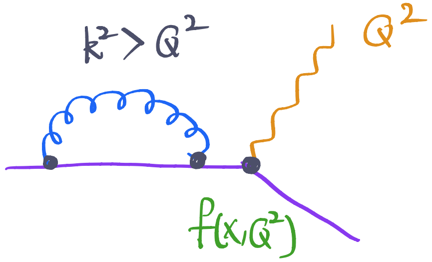
evolution is perturbatively calculable!
(test of QCD)

PDF Evolution

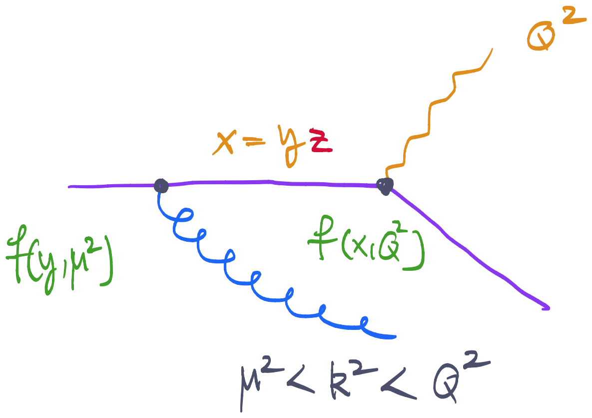
* interaction of high-energetic ($Q^2 \gg m_p^2$) probe γ with a parton in the proton
  ⇒ the probe "sees" a distribution $f(x, Q^2)$ for momentum fraction x

* any virtual corrections @ $k^2 > Q^2$ cannot be resolved

(hard exchange among partons suppressed  $\sim \frac{1}{Q^2}$)



* let's see what happens when Q^2 resolves a loop & how PDFs are affected

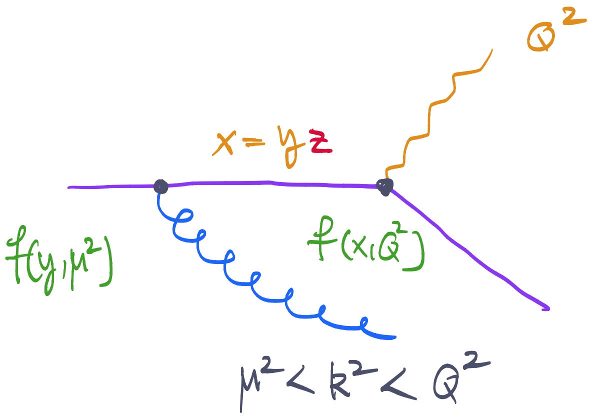


LO: no emission / evolution
⇒ scaling

$$f(x,Q^2) = f(x,\mu^2)$$

$$+ \int_x^1 dy\, f(y,\mu^2) \int_0^1 dz\, \delta(x - yz) \int_{\mu^2}^{Q^2} dk^2 \underbrace{P(z,k^2)}_{\mathcal{O}(\alpha_s)}$$

PDF Evolution



$$\underbrace{f(x,Q^2)}_{\text{independent of } \mu \text{ (arbitrary reference)}} = f(x,\mu^2) + \int_x^1 dy\, f(y,\mu^2) \int_0^1 dz\, \delta(x - yz) \int_{\mu^2}^{Q^2} dk^2\, P(z,k^2)$$

calculable in pQCD (we do it below)

one can show that $P(z,\mu^2) = \frac{\alpha_s}{2\pi} \frac{1}{\mu^2} P(z)$

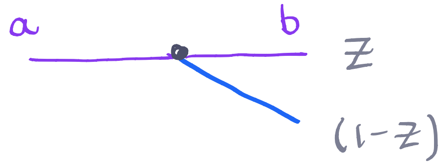
$$\frac{d f(x,Q^2)}{d\mu^2} = \emptyset \Rightarrow \frac{d f(x,\mu^2)}{d\mu^2} = \int_x^1 \frac{dz}{z} P(z,\mu^2)\, f\left(\frac{x}{z},\mu^2\right) + (\text{higher orders})$$

$\Rightarrow$ DGLAP evolution equations [Altarelli, Parisi; Gribov, Lipatov; Dokshitzer '77]

$$\frac{d f(x,\mu^2)}{d \ln \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z)\, f\left(\frac{x}{z},\mu^2\right)$$

PDF Evolution

* more than one flavour



$\simeq P_{ba}(z)$

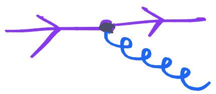
→ coupled integro-differential equations

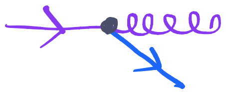
$$\frac{d f_a(x,\mu^2)}{d \ln \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ab}(z)\, f_b\left(\frac{x}{z}, \mu^2\right)$$

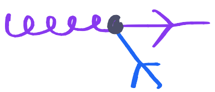
* we cannot compute PDFs, but predict the evolution in pQCD
* universality of Pab
  → measure in one process, use as input for another

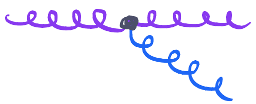
Splitting Functions

$z \leftrightarrow (1-z)$

P_{qq}  $C_F \left(\frac{1+z^2}{1-z}\right)_+$

P_{gq}  $C_F \frac{1+(1-z)^2}{z}$

P_{qg}  $T_R \left[z^2+(1-z)^2\right]$

P_{gg}  $2 C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right]$

$+ \delta(1-z)\left(\frac{11 N_c - 2 N_f}{6}\right)$

$z \to (1-z)$

* the plus distribution $(\ldots)_+$

→ regulates the soft-gluon pole

why no need to regulate $z=\phi$?

$$\int_0^1 dz \, [f(z)]_+ \, g(z) \equiv \int_0^1 dz \, f(z) \, [g(z) - g(1)]$$

think of it as the divergence from 

show: $\left(\frac{1+z^2}{1-z}\right)_+ = \frac{3}{2}\delta(1-z) + 2z\left(\frac{1}{1-z}\right)_+ + (1-z)$

PDF Evolution

[demo: PDFs]

* PDFs are universal

  - pick a common "starting scale"

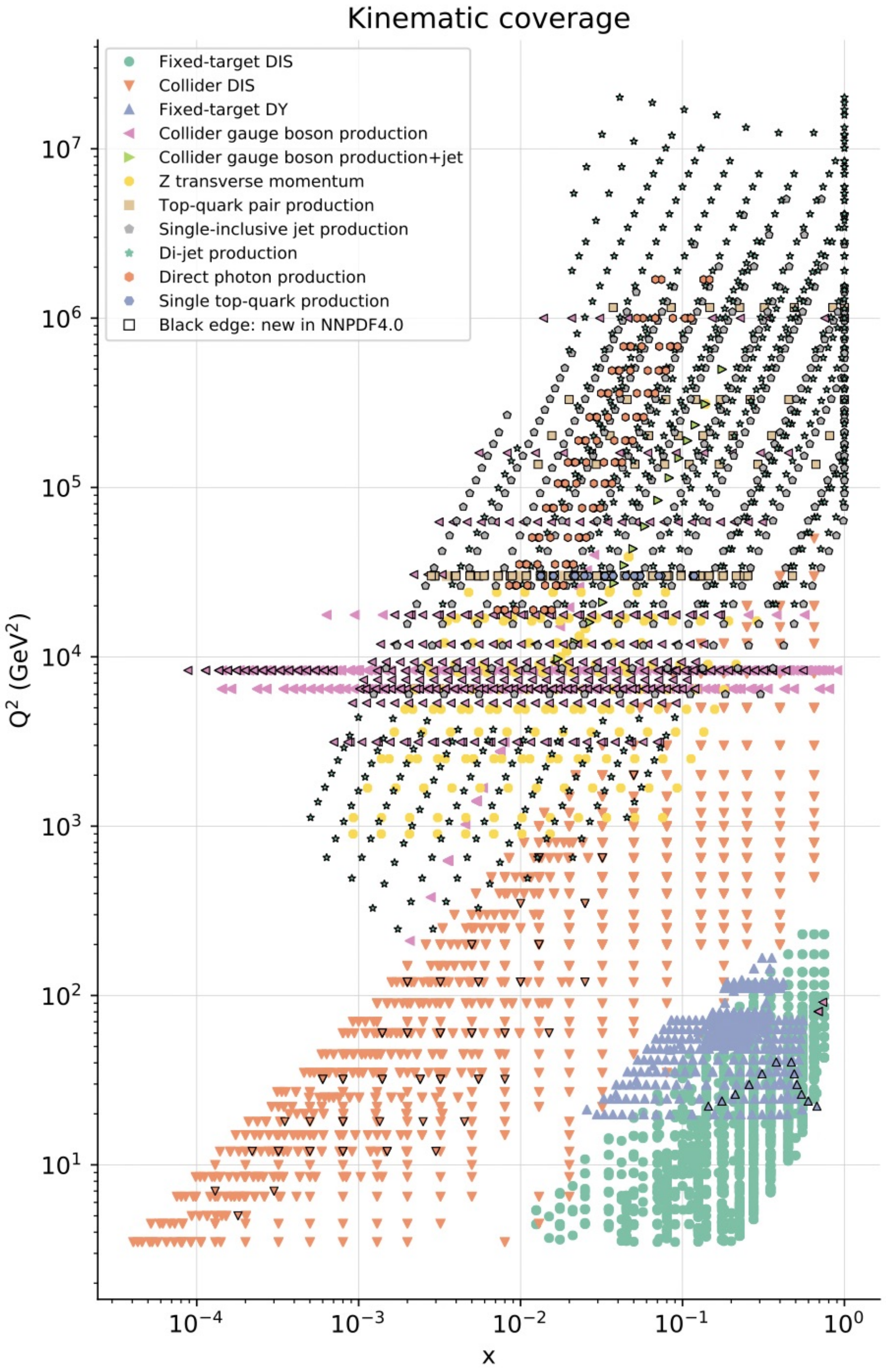
    $\mu_0^2 = \mathcal{O}(1\,\mathrm{GeV}^2) \longleftrightarrow m_p, m_c$

  - parametrise functions

    - $x f(x, \mu_0^2) = A x^B (1-x)^C [1 + Dx + Ex^2] - A' x^{B'} (1-x)^{C'}$ (HERAPDF)
    - Neural Network (NNPDF)
    - ...

  - fit the data

⇒ use evolution for predictions

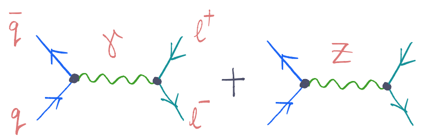


The Drell-Yan Process $\quad p+p \to \ell + \bar{\ell}$

[demo : Drell-Yan]

at LO only

$(a,b) \in \{(q,\bar{q}), (\bar{q},q)\}$



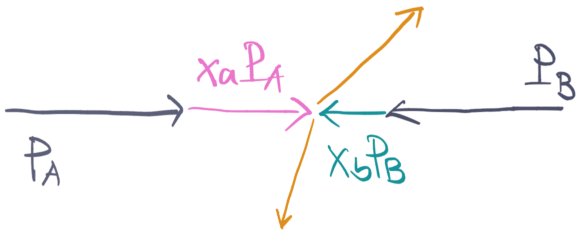
$$d\sigma_{DY}(P_A, P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \; f_a(x_a)\, f_b(x_b)\; d\hat{\sigma}_{a+b\to\ell^+\ell^-}(x_a P_A, x_b P_B)$$

* Integrate out $Z \to \ell^+\ell^-$ decay
* Observables of intermediate gauge boson $\quad q^\mu = (P_1+P_2)^\mu = (P_a+P_b)^\mu$

$$M_{\ell\ell} = \sqrt{q^2} \quad ; \quad Y_{\ell\ell} = \frac{1}{2}\ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$$

rapidity: $Y \longmapsto Y + \frac{1}{2}\ln\left(x_a/x_b\right)$

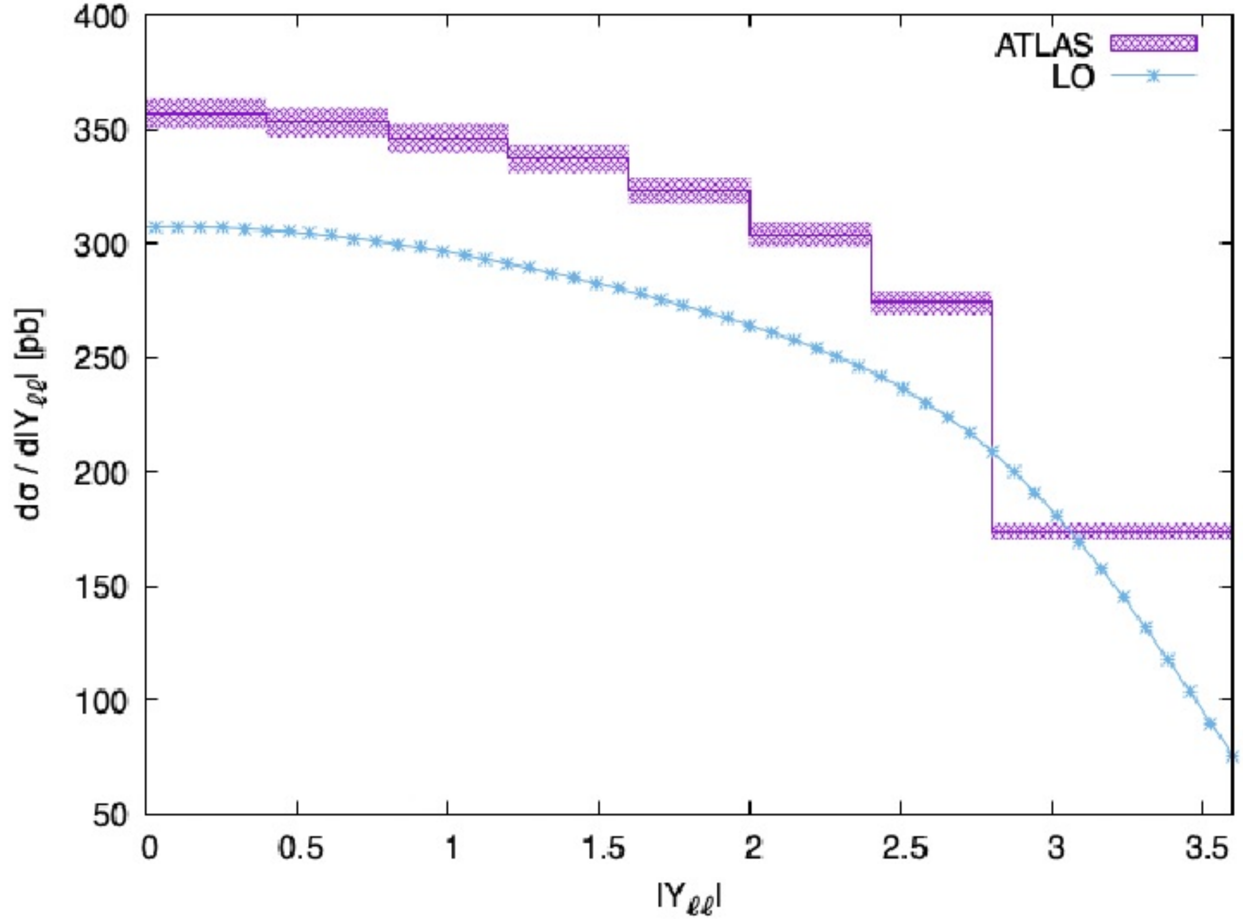
$$\Rightarrow \boxed{\frac{d^2\sigma_{DY}}{dM_{\ell\ell}\, dY_{\ell\ell}} = f_a(x_a)\, f_b(x_b)\, \frac{2M_{\ell\ell}}{E_{cm}^2}\, \hat{\sigma}_{a+b\to\ell^+\ell^-}\Bigg|_{x_{a/b} = \frac{M_{\ell\ell}}{E_{cm}} e^{\pm Y_{\ell\ell}}}}$$



DY @ LO

ATLAS $\sigma_Z = 1055.3 \ \pm 0.7\,(\text{stat.}) \ \pm 2.2\,(\text{syst.}) \ \pm 19.0\,(\text{lumi.})\,\text{pb}$

LO: #total 897.1018242299992 pb



* right ballpark
  $\alpha_s \sim 0.1$
  ↳ $\mathcal{O}(10\%)$ anticipated
  ↳ often much worse!
  (extreme: $gg \to H$ 100%)

@ LO only $q\bar{q}$ annihilation
  ↳ 50% of protons are g!

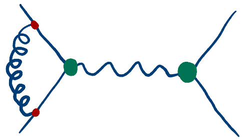
* no error estimate here
  ↳ no quantitative comp.

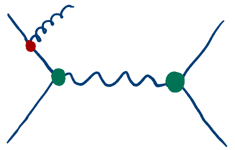
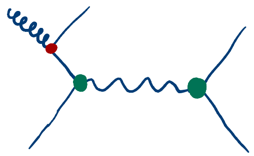
would scale variation help much?

⇒ need at least NLO!

Drell Yan @ NLO

* the virtual & real corrections

$\mathcal{M}^{V}_{2\to2} =$  $+ \;\ldots$

$\mathcal{M}^{R}_{2\to3} =$  $+ \;\ldots\;;$  $+ \;\ldots$

no UV divergences; why?
$\Rightarrow$ no renorm. needed

@ NLO, new partonic channels open up:
$qg, gq, \bar{q}g, g\bar{q}$

IR divergences all over the place!
$\Rightarrow$ need to do everything in $D = 4 - 2\epsilon$

* Let's first rewrite the differential cross section $\left(\xi_{a/b} = \frac{M}{\sqrt{s}}\, e^{\pm Y} \right)$

$$\frac{d^2\sigma}{dM\,dY} = \sum_{a,b} \underbrace{\frac{1}{n_a n_b}}_{\text{avg. fact.}} \frac{M}{\xi_a \xi_b s^2} \int_{\xi_a}^{1} \frac{dz_a}{z_a} \int_{\xi_b}^{1} \frac{dz_b}{z_b}\, f_a\!\left(\xi_a/z_a\right) f_b\!\left(\xi_b/z_b\right) F_{ab}\left(Q^2, z_a, z_b\right)$$

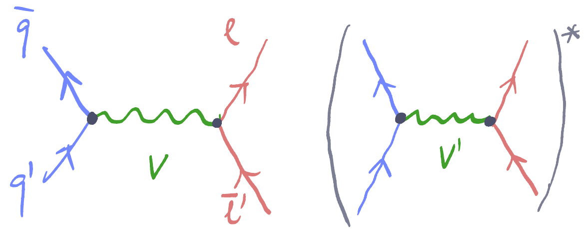
with a perturbative expansion for the formfactor $F_{ab} = F^{(0)}_{ab} + F^{(1)}_{ab} + \ldots$

Drell Yan @ NLO

* at LO, we only have $q\bar{q}$ & $\bar{q}q$ channels (can work in $D=4$)

$$F^{LO}_{q\bar{q}}(Q^2, z_a, z_b) = \delta(1-z_a)\,\delta(1-z_b) \sum_{VV'} L^{VV'}(Q^2)\cdot H^{VV'}(Q^2) \qquad (V,V' \in \{\gamma, Z, W^\pm\})$$

$$L^{VV'} = \frac{2}{3}\frac{\alpha}{Q^2}\left(v_\ell v_{\ell'} + a_\ell a_{\ell'}\right)\left(\frac{Q^2}{Q^2-M_V^2}\right)\left(\frac{Q^2}{Q^2-M_{V'}^2}\right)^*$$



$$H^{VV'} = 16\pi\,\alpha\,N_c\left(v_q v_{q'} + a_q a_{q'}\right) Q^2$$

$$F^{LO}_{\bar{q}q}(Q^2, z_a, z_b) = F^{(0)}_{q\bar{q}}(Q^2, z_b, z_a) = F^{(0)}_{q\bar{q}}(Q^2, z_a, z_b)$$

Drell Yan @ NLO

* at NLO, we have 6 channels, 2 distinct ones: $(q\bar{q} \,\&\, \bar{q}q)$, $(qg \,\&\, \bar{q}g) \xleftrightarrow{z_a \leftrightarrow z_b} (gq \,\&\, g\bar{q})$

* corrections w.r.t. LO formfactor:

$$F^{(1)}_{q\bar{q}} = \delta^{NLO}_{q\bar{q}}(Q^2, z_a, z_b)\, F^{(0)}_{q\bar{q}} \quad \& \quad F^{(1)}_{qg} = \delta^{NLO}_{qg}(Q^2, z_a, z_b)\, F^{(0)}_{q\bar{q}}$$

$$C_\epsilon \equiv \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}$$

⇒ let's just look at the divergent pieces

$$\delta^{R}_{q\bar{q}}(Q^2, z_a, z_b)\Big|_{\text{poles}} = \frac{\alpha_s}{\pi} C_F C_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \Bigg\{ \delta(1-z_a)\,\delta(1-z_b)\left[\frac{1}{\epsilon^2}\right] - \delta(1-z_a)\frac{1}{\epsilon}\left[\frac{1-z_b}{2} + z_b\left(\frac{1}{1-z_b}\right)_+\right] - \delta(1-z_b)\frac{1}{\epsilon}\left[\frac{1-z_a}{2} + z_a\left(\frac{1}{1-z_a}\right)_+\right] + \dots \Bigg\}$$

$$\delta^{V}_{q\bar{q}}(Q^2, z_a, z_b)\Big|_{\text{poles}} = \frac{\alpha_s}{\pi} C_F C_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \Bigg\{ \delta(1-z_a)\,\delta(1-z_b)\left[-\frac{1}{\epsilon^2} - \frac{3}{2}\frac{1}{\epsilon}\right] + \dots \Bigg\}$$

⇒ $\frac{1}{\epsilon^2}$ cancels between "real" & "virtual" but left-over $\frac{1}{\epsilon}$ poles ?!

Drell Yan @ NLO

* Let's take V+R and massage the expressions

$$\delta^{V+R}_{q\bar{q}} = -\frac{\alpha_s}{2\pi} C_F c_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \frac{1}{\epsilon} \left\{ \delta(1-z_a) \left(\frac{1+z_b^2}{1-z_b}\right)_+ + \delta(1-z_b) \left(\frac{1+z_a^2}{1-z_a}\right)_+ \right\}$$

$$= -\frac{\alpha_s}{2\pi} c_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \frac{1}{\epsilon} \left\{ \delta(1-z_a) P_{qq}(z_b) + \delta(1-z_b) P_{qq}(z_a) \right\}$$

$$\frac{N_c^2-1}{N_c} T_R$$

$$\delta^{V+R}_{qg} = \delta^{R}_{qg} = -\frac{\alpha_s}{2\pi} C_F c_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \frac{1}{\epsilon} \delta(1-z_a) \left[z_b^2 + (1-z_b)^2 \right]$$

$$= -\frac{\alpha_s}{2\pi} c_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \frac{1}{\epsilon} \underbrace{\frac{N_c^2-1}{N_c}}_{\text{changes the average factor } \frac{1}{n_g} \to \frac{1}{n_q}} \delta(1-z_a) P_{qg}(z_b)$$

this should give you a hint of what we were missing !

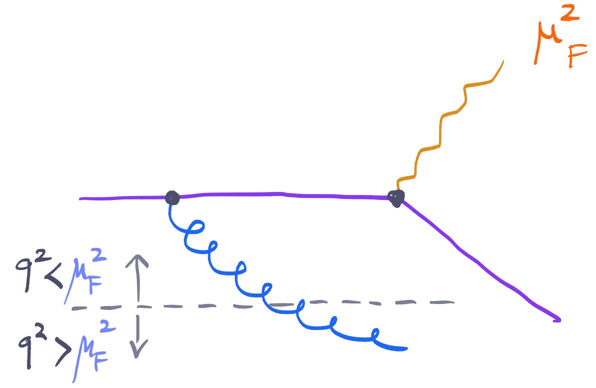
* left-over singularities $\rightsquigarrow$ collinear (universal!)

The NLO PDF

$$\frac{1}{\epsilon}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} = \frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{Q^2}\right) + \mathcal{O}(\epsilon) = \underbrace{\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{\mu_F^2}\right)}_{\substack{q^2 < \mu_F^2 \\ \text{long distance} \\ \to \text{ part of PDF}}} + \underbrace{\ln\left(\frac{\mu_F^2}{Q^2}\right)}_{\substack{q^2 > \mu_F^2 \\ \text{short distance} \\ \to \text{ part of XS}}} + \mathcal{O}(\epsilon)$$

* the NLO PDF (renormalization)

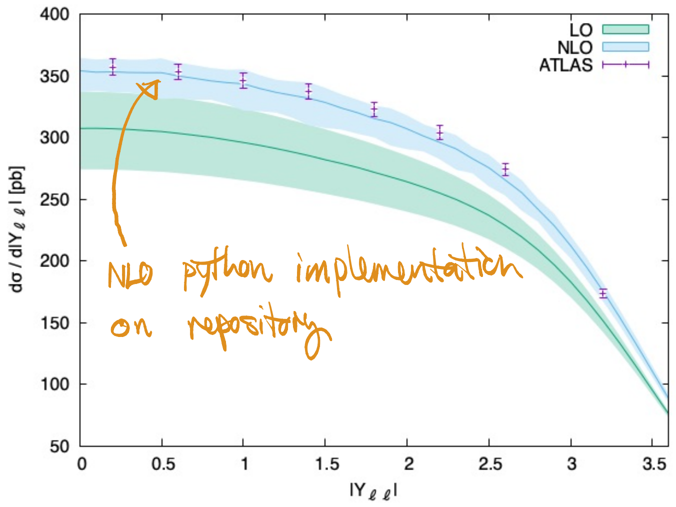
$$f_a^{(0)}(x) \longrightarrow f_a^{(0)}(x) + \frac{\alpha_s}{2\pi}\frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)}\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\frac{1}{\epsilon}\int_x^1 \frac{dz}{z}\, P_{ab}(z)\, f_b^{(0)}\left(\frac{x}{z}\right)$$



* with this, all divergences are gone!

[DY@NLO on repository]

Drell Yan @ NLO



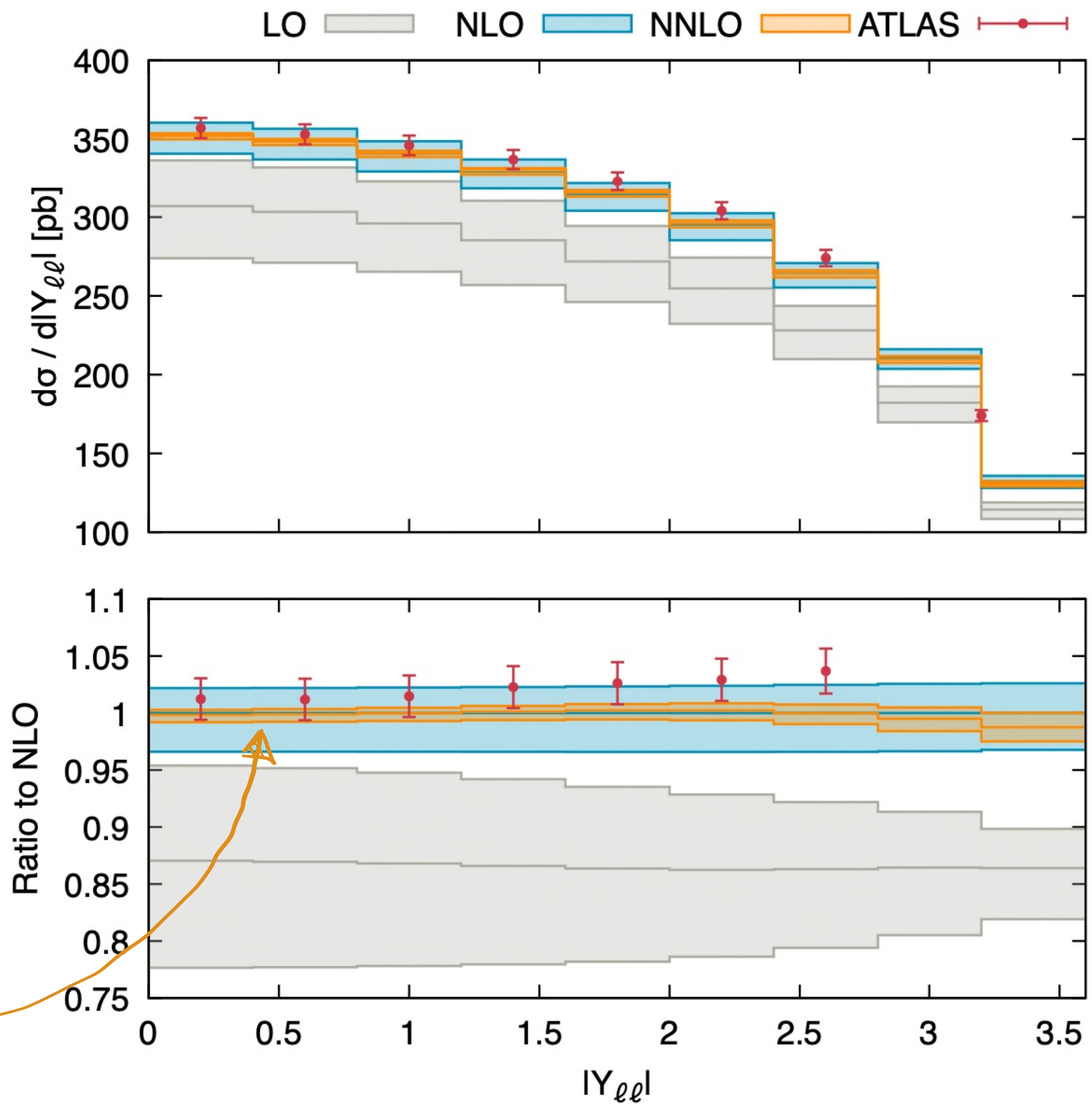
NLO python implementation on repository

* NLO not enough : $\Delta_{TH} \sim 2x \; \Delta_{EXP}$

→ need NNLO

no longer something you want to do in python

⇝ runcard for NNLOJET on repo



Accuracy of the Calculation v.s. Observables

using the Drell-Yan @ NLO calculation derived here, what is the formal perturbative accuracy of those observables?

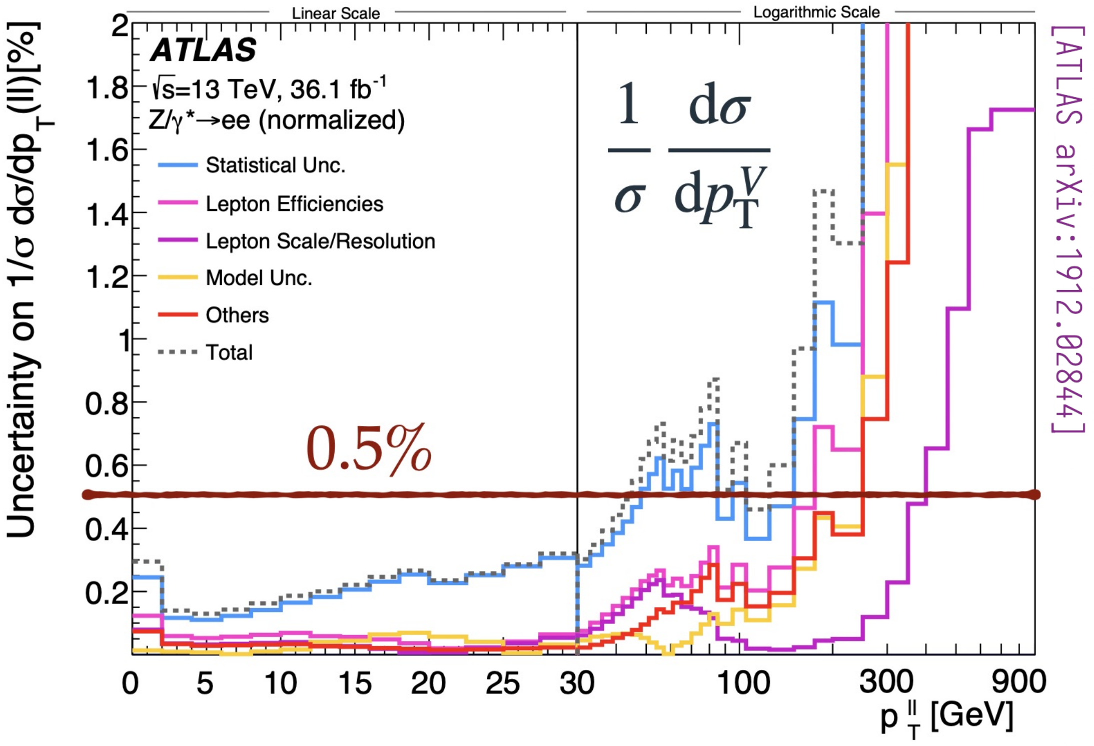
* σ^{fid}
* $\frac{d\sigma}{dY}$
* $\frac{d\sigma}{dp_{T,Z}}$
* $\frac{d\sigma}{d\eta_{e^+}}$
* $\frac{d\sigma}{dp_{T,e^+}}$

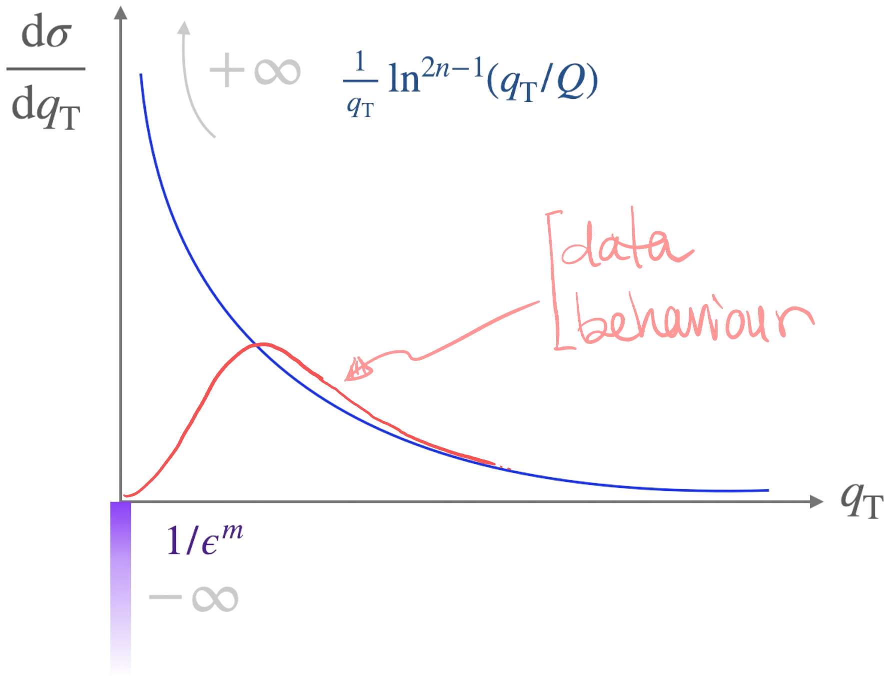
Drell-Yan @ small q_T

q_T distribution enters many important measurements: M_W, α_s, modeling, …

* extraordinary experimental precision

* fixed-order: completely off





Drell-Yan @ small q_T

q_T probes small scales in a high-energy process ($Q \sim M_Z$)

$\Rightarrow$ sensitivity to large logarithms $\ln(q_T/Q)$

* recall $d\omega_{X\to X+g}$ emission pattern in the soft-collinear limit

$$d\omega_{Z\to Z+g} = \frac{4\alpha_s}{\pi} C_F \ln\left(\frac{k_T}{Q}\right) \frac{dk_T}{k_T}$$

(2x from q & $\bar{q}$ leg)

* easiest to look at the cumulant $\Sigma(q_T)$ (prob for $p_{T,Z} < q_T$)

$$\Sigma(q_T) = \frac{1}{\sigma}\int_0^{q_T} \frac{d\sigma}{dp_{T,Z}}\, dp_{T,Z} = 1 + \frac{4\alpha_s}{\pi} C_F \int \frac{dk_T}{k_T} \ln\left(\frac{k_T}{Q}\right) \left[\underbrace{\Theta(k_T < q_T) - 1}_{-\Theta(k_T > q_T)}\right]$$

$p_{T,Z} = \emptyset$ @ LO

Z & emission back-2-back @ NLO

$p_{T,Z} = k_T$

"virtual" corrections via unitarity

$$= 1 - \frac{4\alpha_s}{\pi} C_F \int_{q_T}^{Q} \frac{dk_T}{k_T} \ln(k_T/Q) = 1 - \frac{2\alpha_s}{\pi} C_F \ln^2\left(Q/q_T\right)$$

can ruin the perturbative expansion!

(@ $q_T \simeq 5\,\text{GeV}$ $\alpha_s L^2 \simeq 1$)

Drell-Yan @ small q_T

the pattern of logarithmic divergences follows from soft and collinear factorization ($L = \ln Q/q_T$)

$\mathcal{O}(\alpha_s)$	$\alpha_s L^2$	$\alpha_s L$	α_s			
$\mathcal{O}(\alpha_s^2)$	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
$\mathcal{O}(\alpha_s^3)$	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$		$\dots$		α_s^3
$\vdots$						
$\mathcal{O}(\alpha_s^n)$	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$		$\dots$		α_s^n

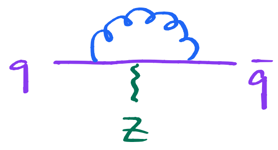
double-logarithmic (DL) tower of terms

$\Rightarrow$ need to account for arbitrary # of emissions in DL ($\alpha_s L^2 \sim 1$) counting

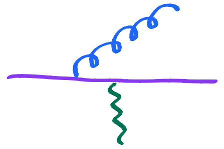
* convergence can be restored by reorganizing the perturbative series across orders: resummation

q_T Resummation

Let's first consider the virtual & real corrections separately with δ to regulate



$$= \Sigma^{(1)V}(q_T) = \frac{4\alpha_s}{\pi} C_F \int_\delta^Q \frac{dk_T}{k_T} \ln(Q/k_T)\,[-1] = -\frac{4\alpha_s}{\pi} C_F \frac{1}{2}\ln^2(Q/\delta)$$



$$= \Sigma^{(1)R}(q_T) = \frac{4\alpha_s}{\pi} C_F \int_\delta^Q \frac{dk_T}{k_T} \ln(Q/k_T)\,\Theta(k_T < q_T) = \cdots$$

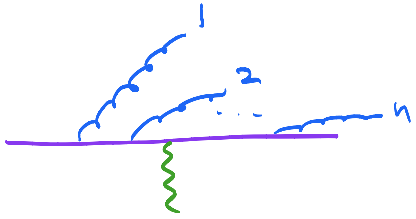
$$= \frac{4\alpha_s}{\pi} C_F \left[\frac{1}{2}\ln^2(q_T/\delta) + \ln(Q/q_T)\ln(q_T/\delta) \right]$$

$$\Sigma^{(1)}(q_T) = \Sigma^{(1)V}(q_T) + \Sigma^{(1)R}(q_T) = \frac{4\alpha_s}{\pi} C_F \frac{1}{2}\ln^2(Q/q_T)$$ same as before

q_T Resummation

arbitrary # of real emissions. For DL can assume strong ordering

$k_{T,1} \gg k_{T,2} \gg \dots \gg k_{T,n}$ $\longleftrightarrow$ the configuration ~~with~~ highest logarithmic enhancement



$$= \sum R^n (q_T)$$

$$= \left[\frac{4\alpha_s}{\pi} C_F\right]^n \int_\delta^{Q \to q_T} \frac{dk_{T,1}}{k_{T,1}} \ln\left(Q/k_{T,1}\right) \int_\delta^{k_{T,1}} \frac{dk_{T,2}}{k_{T,2}} \ln\left(Q/k_{T,2}\right) \cdots \int_\delta^{k_{T,n-1}} \frac{dk_{T,n}}{k_{T,n}} \ln\left(Q/k_{T,n}\right) \Theta\left(k_{T,1} < q_T\right)$$

1^{st} emission sets the $p_{T,Z}$

$$= \frac{1}{n!} \left[\frac{4\alpha_s}{\pi} C_F\right]^n \left[\int_\delta^{q_T} \frac{dk_T}{k_T} \ln\left(Q/k_T\right)\right]^n \Rightarrow \boxed{\overset{\Sigma_n R^n}{\sum} = \exp\left(\quad \right)}$$

q_T Resummation

analogously: arbitrary # of virtual emissions.

$$\boxed{\Sigma^{\Sigma_n V^n} = \exp\left(\right)}$$

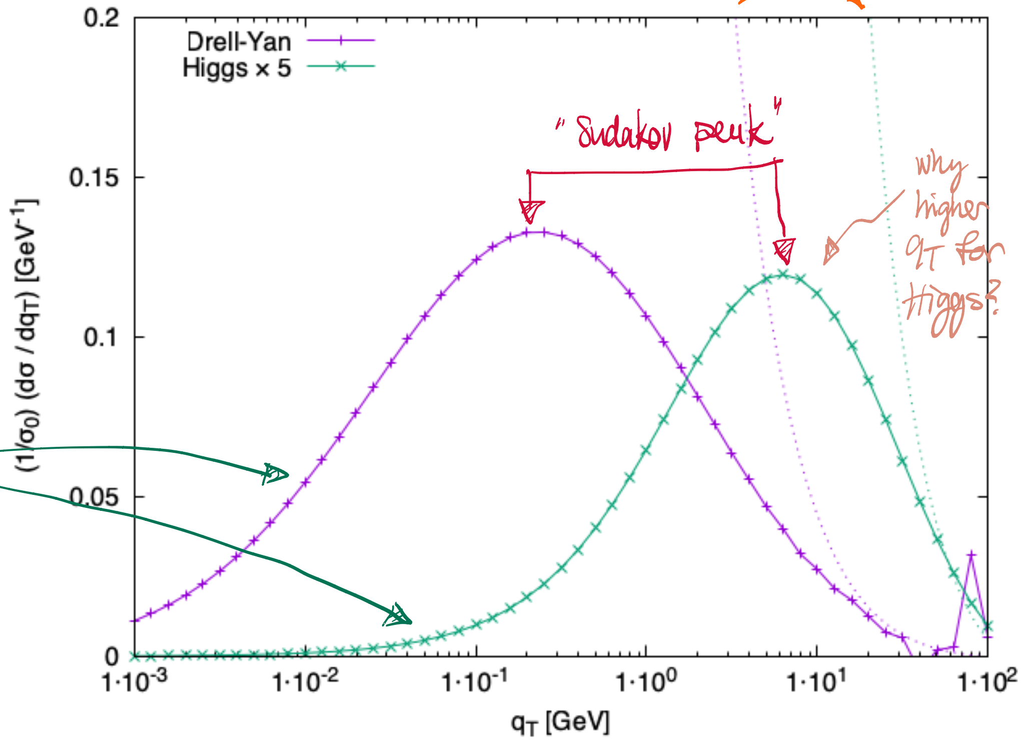
$\Rightarrow$ all emissions to all orders

$$\Sigma^{DL}(q_T) = \Sigma^{\Sigma_n V^n} \times \Sigma^{\Sigma_n R^n} = \exp\left(\quad + \quad \right)$$

$$= \exp\left(-\frac{2\alpha_s}{\pi} C_F \ln^2\left(Q/q_T\right)\right)$$

Sudakov formfactor

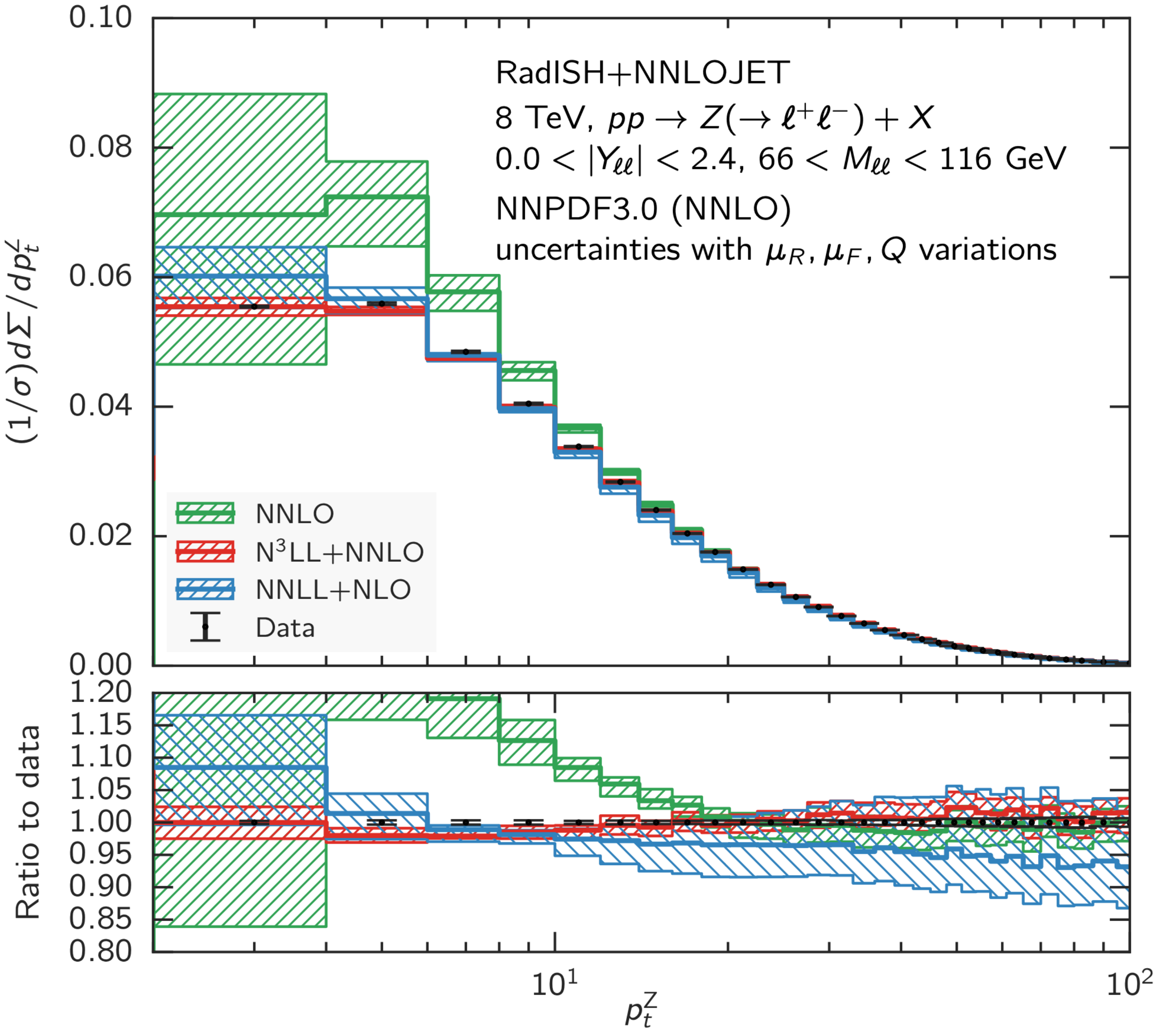
fixed order diverges



resummed prediction

q_T Resummation

* DL resummation can be extended to include additional tower of logarithms
  - ↳ drop strong ordering
  - ↳ b-space for
    $\delta^{(2)}(\vec{p}_{T,Z} + \sum_i \vec{k}_{T,i})$
  - ↳ hard-collinear limit, α_s running, DGLAP evolution h.o. corrections to ingredients
  - ↳ state of the art: N^3LL



Extra

PDF evolution