Quantum Chromodynamics & Collider Phenomenology

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Outline

Ø The Beginnings

I Basics of QCD

II QCD in e^+e^- Collisions

III Hadron Colliders

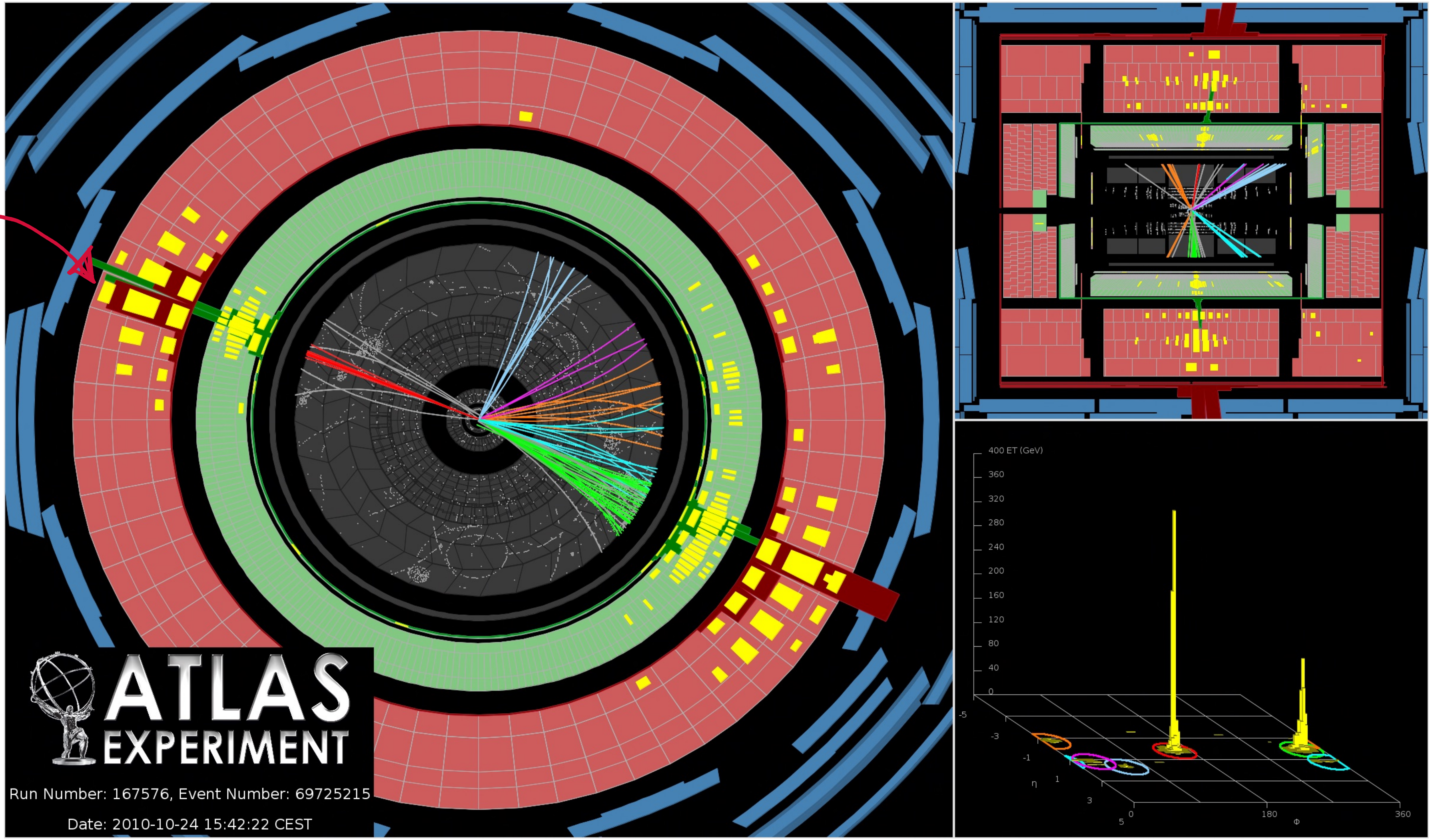
IV Jet Physics

- Jets: cone & sequential • IR safety • properties
- loss v.s. contamination • Parton Showers

Events at hadron colliders look more complex

[demo: diagrams]

$\mathcal{O}(100)$ tracks



Why? Any chance to compute this with what we did so far?

Nope ...

```
2->2 gluon scattering has 4 diagrams
2->3 gluon scattering has 25 diagrams
2->4 gluon scattering has 220 diagrams
2->5 gluon scattering has 2485 diagrams
2->6 gluon scattering has 34300 diagrams
2->7 gluon scattering has 559405 diagrams
2->8 gluon scattering has 10525900 diagrams
2->9 gluon scattering has 224449225 diagrams
2->10 gluon scattering has 5348843500 diagrams
2->11 gluon scattering has 140880765025 diagrams
```

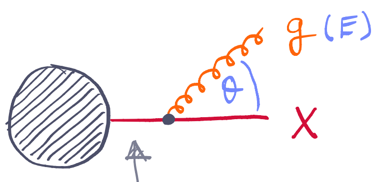
10^{11} !

* Even with more efficient recursive approaches, not the multiplicities you'd want to tackle $\#\dim(\Phi_n) = 3n-4$

1. define observables that "map back" the physics to fewer initiating objects } JETS

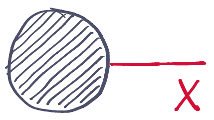
2. identify the relevant physics & model the full complexity approximately } Parton Showers

Jets – an Emergent Feature of QCD



gluon: soft & collinear

enhanced (divergent) $\sim \dfrac{1}{E_X\, E\, (1-\cos\theta)}$



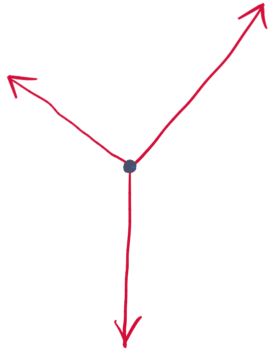
$\times\ d\omega_{X\to X+g}$

$$\boxed{2\,\frac{\alpha_s}{\pi}\, C_X\, \frac{dE}{E}\, \frac{d\theta}{\theta}}$$

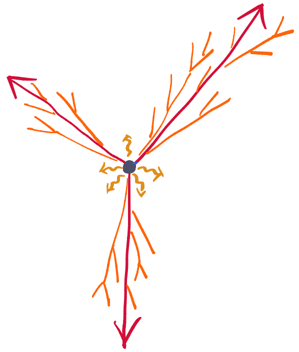
$$C_X \begin{cases} = C_F = 4/3 & \text{if } X = q \\ = C_A = 3 & \text{if } X = g \end{cases}$$

$\Rightarrow$ a collimated spray of hadrons

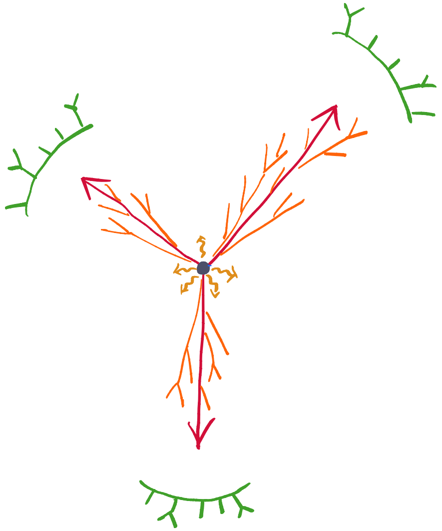
1. high energetic partons
   ↳ hard scattering



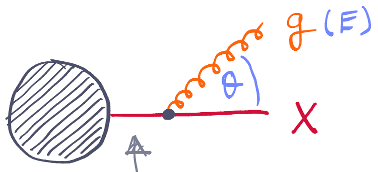
2. asymp. freedom & $d\omega$
   ↳ pert. parton shower



3. hadronization



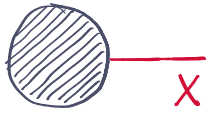
Jets – an Emergent Feature of QCD



gluon

soft & collinear

enhanced (divergent) $\sim \dfrac{1}{E_X\, E\, (1-\cos\theta)}$



$\times \; d\omega_{X\to X+g}$

$$\boxed{2\,\frac{\alpha_s}{\pi}\,C_X\,\frac{dE}{E}\,\frac{d\theta}{\theta}}$$

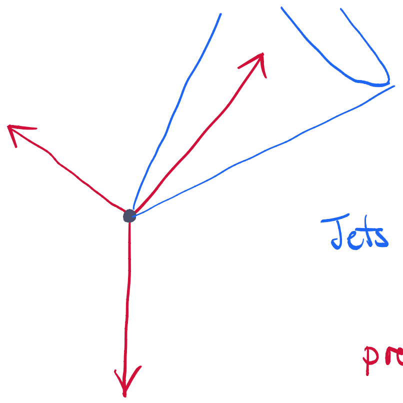
$$C_X \begin{cases} = C_F = 4/3 & \text{if } X = q \\ = C_A = 3 & \text{if } X = g \end{cases}$$

⇒ a collimated spray of hadrons

1. high energetic partons
   ↪ hard scattering

2. asymp. freedom & $d\omega$
   ↪ pert. parton shower

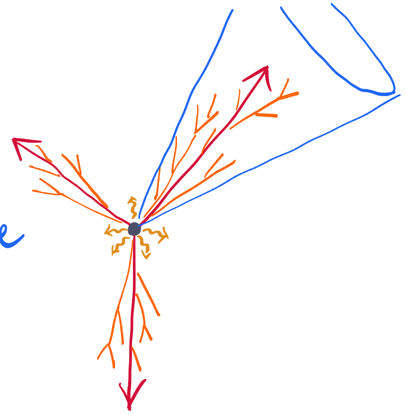
3. hadronization

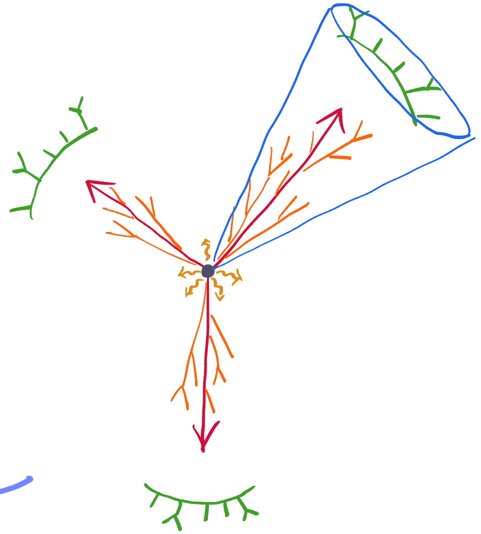


Jets act as an interface

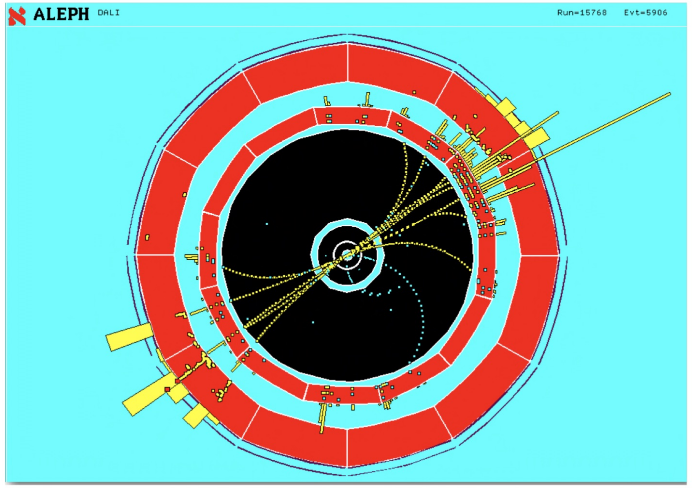
TH ⟷ EXP

proxy for a parton

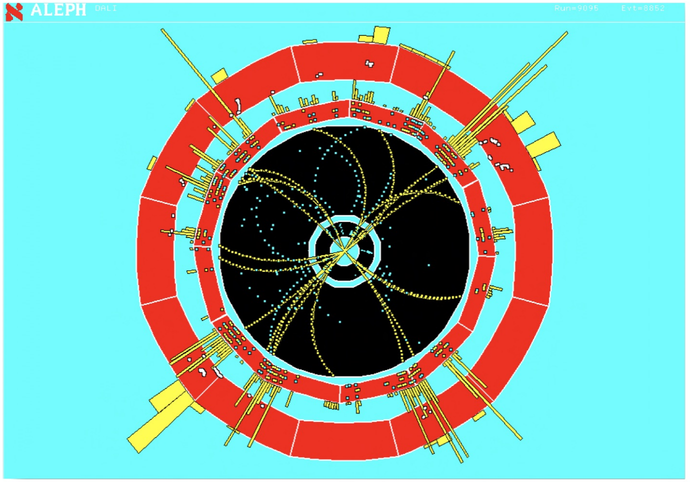




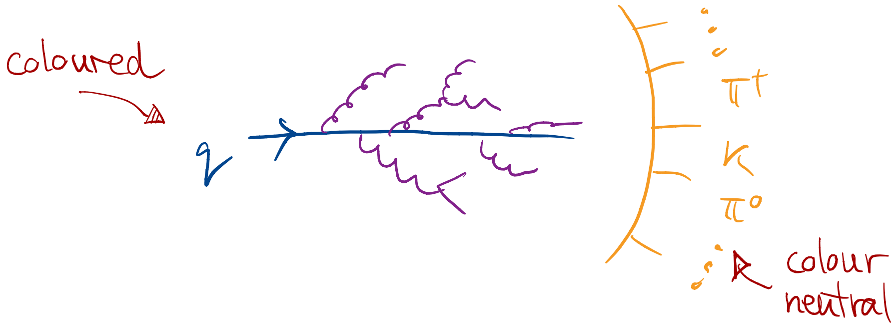
Jets are not unique ... ... and intrinsically ambiguous



\# jets = 2



\# jets = ?



* best we can hope for:

(clusters of partons)

$\simeq$

(clusters of hadrons)

up to $\mathcal{O}(\Lambda_{QCD}/Q)$

* Freedom
  (1) which particles to put together?
  (2) how to combine them?

$\hookrightarrow$ Jet definition

Jet Algorithms

FERMILAB-Conf-90/249-E
[E-741/CDF]

Toward a Standardization of Jet Definitions *

* To be published in the proceedings of the 1990 Summer Study on High Energy Physics, *Research Directions for the Decade*, Snowmass, Colorado, June 25 - July 13, 1990.

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis; performance
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory; IR safety
5. Yields a cross section that is relatively insensitive to hadronization.

Jet Algorithms

$\{p_i\} \Longrightarrow \{j\}$

[particles, momenta
calorimeter towers, ...]

A brief (incomplete) history of jets:

- Sterman-Weinberg jets '77
- k_T algorithm '93
- Cambridge / Aachen '97
- anti-k_T '08

Two main classes

① Cone [top down]

idea of directed energy flow

⇨ find coarse regions

(what we have been doing)

② Sequential recombination [bottom up]

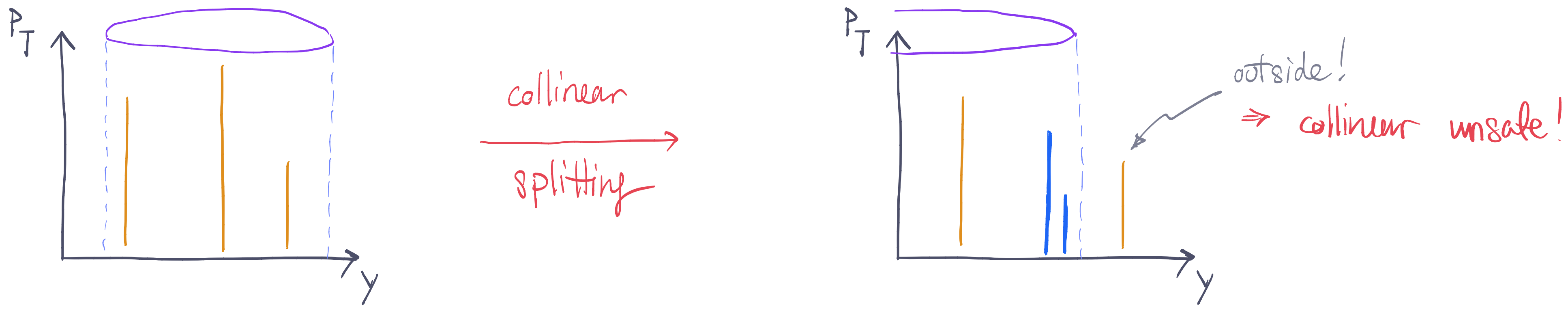
successively undo QCD branching

⇨ find "close" & aggregate

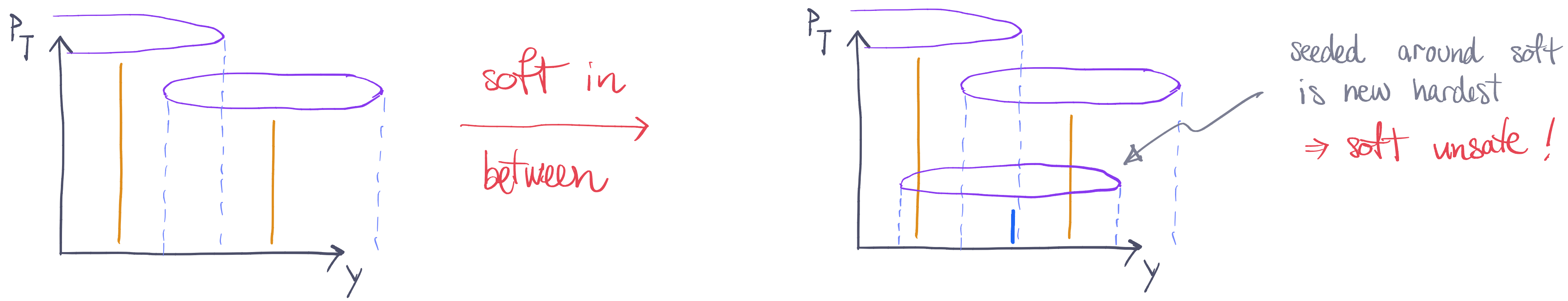
Potential IR issues when using (seeded) cones

$\varphi \equiv 0$
in examples

* start by placing cone around hardest particle



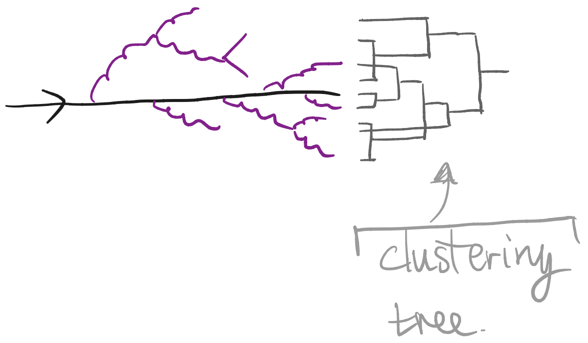
* try placing cones around all particles & look for hardest



* midpoint only postpones the issue ⇒ need to go seedless (naive: $\mathcal{O}(N\,2^N)$) ⇝ SISCone ✓

Sequential Recombination Algorithms

Try to work our way backwards through "branchings"



1) Compute distances between all particles

$$d_{ij} = ?$$

and to the beam [for hadron colliders]

$$d_{iB} = ?$$

2) Find the smallest of $\{d_{ij}\} \cup \{d_{iB}\}$

↪ $d_{ij} \Rightarrow$ merge i & j into a new "protojet"

E-scheme: $p^\mu_{(ij)} = p^\mu_i + p^\mu_j$

↪ $d_{iB} \Rightarrow$ remove i from the set & call it a "jet"

3) If particles left, goto step 1 & repeat

The k_T Algorithm

Try to work our way backwards through "branchings"

1) Compute distances between all particles

$$d_{ij} = \min\left(p_{T,i}^2, p_{T,j}^2\right) \frac{\Delta R_{ij}^2}{R^2}$$

and to the beam [for hadron colliders]

$$d_{iB} = p_{T,i}^2$$

[mimics the inverse of the S&C emission probability $\sim$ relative k_T]

remember: $(p_i + p_j)^2 \sim \underbrace{p_{T,i}\, p_{T,j}}_{}\, \Delta R_{ij}^2$

not so good (JADE)

2) Find the smallest of $\{d_{ij}\} \cup \{d_{iB}\}$

IR safe?

$\hookrightarrow$ $d_{ij} \Rightarrow$ merge i & j into a new "protojet"

E-scheme: $p^\mu_{(ij)} = p^\mu_i + p^\mu_j$

$\hookrightarrow$ $d_{iB} \Rightarrow$ remove i from the set & call it a "jet"

3) If particles left, goto step 1 & repeat

The k_T Algorithm

Try to work our way backwards through "branchings"

1) Compute distances between all particles

$$d_{ij} = \min\left(p_{T,i}^2, p_{T,j}^2\right) \frac{\Delta R_{ij}^2}{R^2}$$

→ soft first

⇝ irregular shapes

↔ exp. challenges

↔ collects "junk"

and to the beam [for hadron colliders]

$$d_{iB} = p_{T,i}^2$$

2) Find the smallest of $\{d_{ij}\} \cup \{d_{iB}\}$

↳ $d_{ij} \Rightarrow$ merge i & j into a new "protojet"

E-scheme: $p^\mu_{(ij)} = p^\mu_i + p^\mu_j$

↳ $d_{iB} \Rightarrow$ remove i from the set & call it a "jet"

3) If particles left, goto step 1 & repeat

4) Only retain jets above a minimum p_T threshold $\quad p_{T,\text{jet}} > p_{T,\text{cut}}$

The generalized k_T Algorithm

Try to work our way backwards through "branchings"

1) Compute distances between all particles

$$d_{ij} = \min\left(p_{T,i}^{2\alpha}, p_{T,j}^{2\alpha}\right) \frac{\Delta R_{ij}^2}{R^2}$$

and to the beam [for hadron colliders]

$$d_{iB} = p_{T,i}^{2\alpha}$$

$$\alpha = \begin{cases} +1 : & k_T \\ 0 : & \text{Cambridge/Aachen [geometric]} \\ -1 : & \text{anti-}k_T \end{cases}$$

anti-k_T : hard first

↳ nearly perfect cones

2) Find the smallest of $\{d_{ij}\} \cup \{d_{iB}\}$

↳ d_{ij} ⇒ merge i & j into a new "protojet"

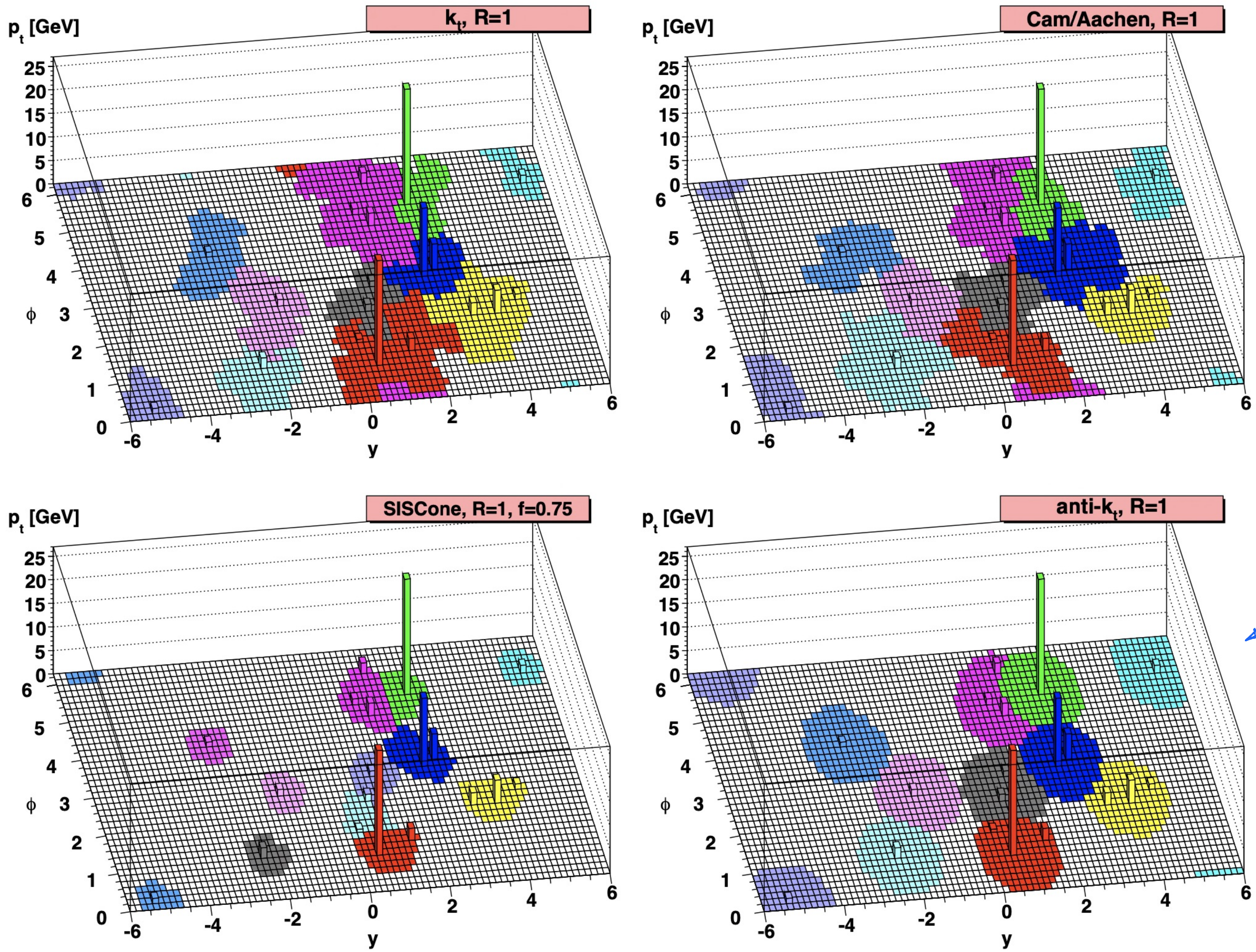
E-scheme: $p^\mu_{(ij)} = p^\mu_i + p^\mu_j$

↳ d_{iB} ⇒ remove i from the set & call it a "jet"

3) If particles left, goto step 1 & repeat

4) Only retain jets above a minimum p_T threshold $\quad p_{T,\text{jet}} > p_{T,\text{cut}}$

Comparison of Jet Algorithms

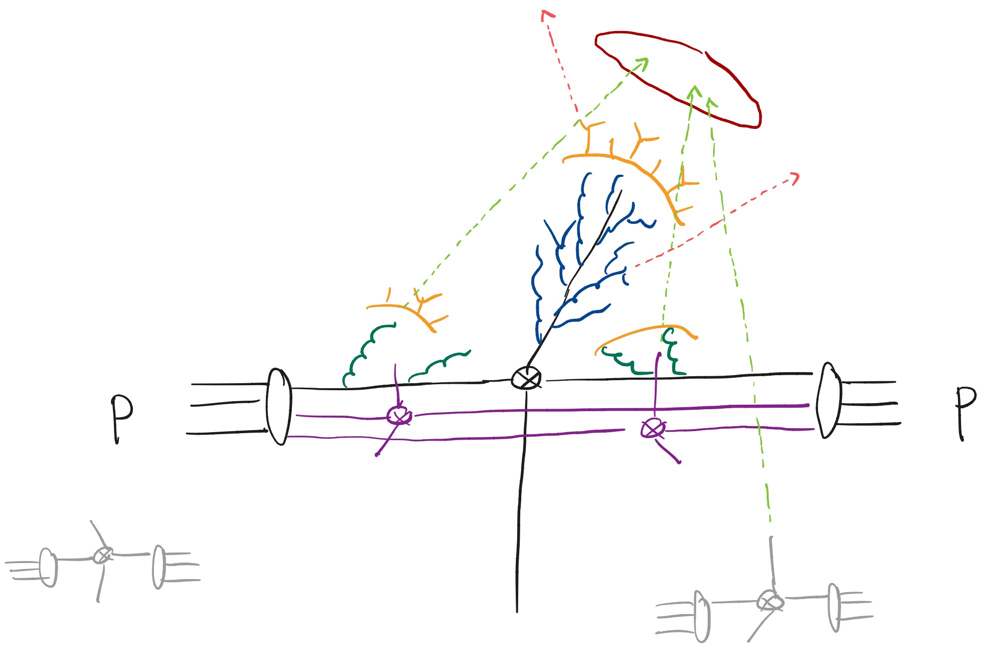


irregularities from
non- linear
behaviour w.r.t.
soft emissions

solved

nearly perfect cones!
(EXP happy)
* overlap → harder one

Jets at Hadron Colliders



- final-state radiation (FSR) $\sim Q/2$
- initial-state radiation (ISR) $\sim Q/2$
- multiple parton interactions (MPI) aka underlying event (UE) $\sim$ GeV
- pile up (PU) $\sim n_{PU} \cdot 0.5$ GeV
- hadronisation $\sim \Lambda_{QCD}$

$$\text{Jet} = \underbrace{\begin{pmatrix} \text{hard parton} \\ + \\ \text{radiation} \end{pmatrix}}_{\text{what we're after}} - \text{LOSS} + \text{CONTAMINATION}$$

R bigger?

R smaller?

⇒ there is no single "best" jet definition (trade-offs; depends on application)

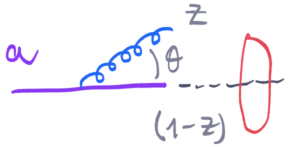
Final State Radiation v.s. the cone size

Let's consider the energy of a jet

LO:

$$E_{jet} = E_a$$

NLO:



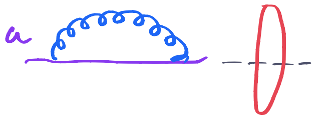
$$E_{jet} = \frac{2\alpha_s}{\pi} C_a \int \frac{d\theta}{\theta} \int dz\, P_{ga}(z) \Big\{ E_a \; \Theta(\theta < R)$$

$$+ E_a (1-z) \; \Theta(\theta > R) \; \Theta(z < 1/2)$$

$$+ E_a \, z \; \Theta(\theta > R) \; \Theta(z > 1/2) \Big\}$$

inside
↪ recovers energy

outside
↪ follow the more energetic branch



$$E_{jet} = \frac{2\alpha_s}{\pi} C_a \int \frac{d\theta}{\theta} \int dz\, P_{ga}(z) \Big\{ E_a \, [-1] \Big\}$$

virtual correction via "unitarity"

$$\Rightarrow E_{jet} = E_a \left[1 - \frac{\alpha_s}{\pi} \ln(1/R) \, L_a \right]$$

↳ $L_q = C_F \times 1.01129$

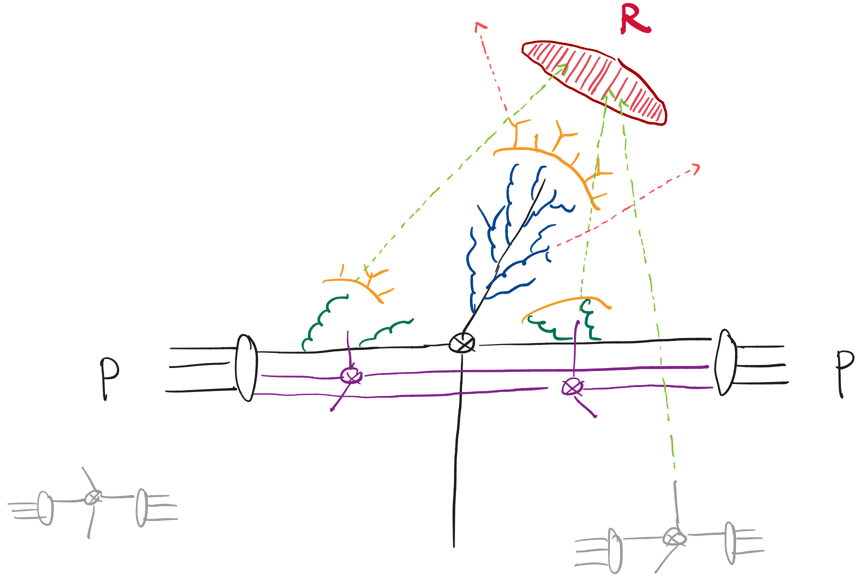
$L_g = C_A \times 0.94 + N_f \, 0.07$

R = 0.4

$\Delta E/E \sim -5\%$

$\Delta/E \sim -10\%$

Loss v.s. Contamination – the R dependence



$$\text{Jet} = \begin{pmatrix}\text{hard parton}\\ + \\ \text{radiation}\end{pmatrix}$$

$-$ LOSS

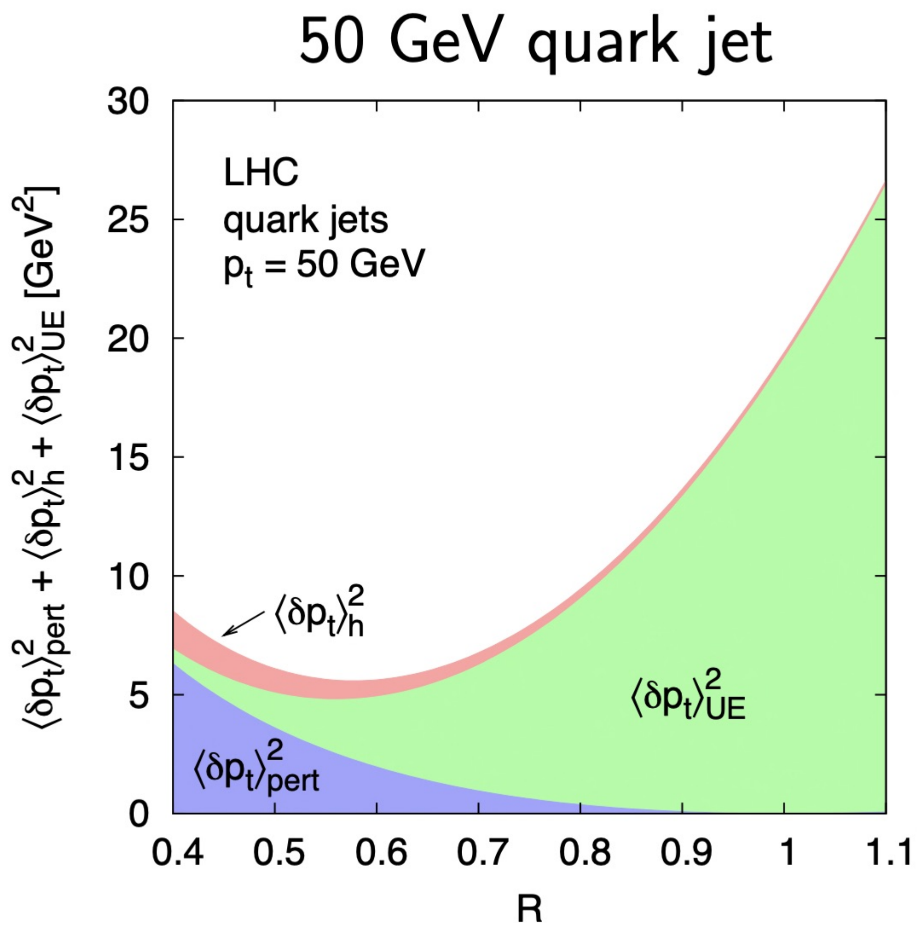
$+$ CONTAMINATION

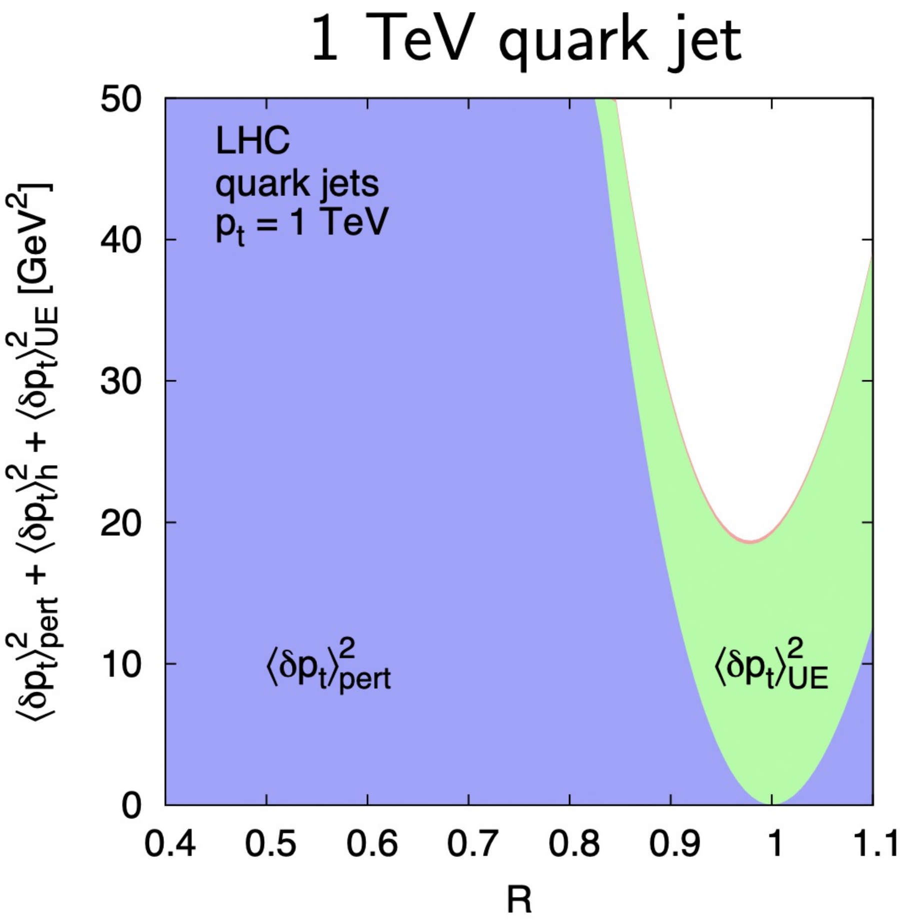
- final-state radiation (FSR) $\sim -\frac{\alpha_s}{\pi} C_i\, p_T \ln(1/R)$
- initial-state radiation (ISR) $\sim \frac{\alpha_s}{\pi} C_i\, p_T\, \pi R^2$
- multiple parton interactions (MPI) aka underlying event (UE) $\sim \rho^{MPI}\, \pi R^2$ $[\rho^{MPI} \sim \mathcal{O}(1\,\text{GeV})\ @\ \text{LHC}]$
- pile up (PU) $\sim \rho^{PU}\, \pi R^2$ $[\rho^{PU} \sim n_{PU} \times 0.5\,\text{GeV}]$ (n_{PU}: 100 – 1000)
- hadronisation $\sim -\Lambda_{QCD}\, \frac{1}{R}$

The "optimal" Cone size?

* get the different $\langle \delta p_T^2 \rangle$ to balance

["Towards Jetography" – G. Salam '09]

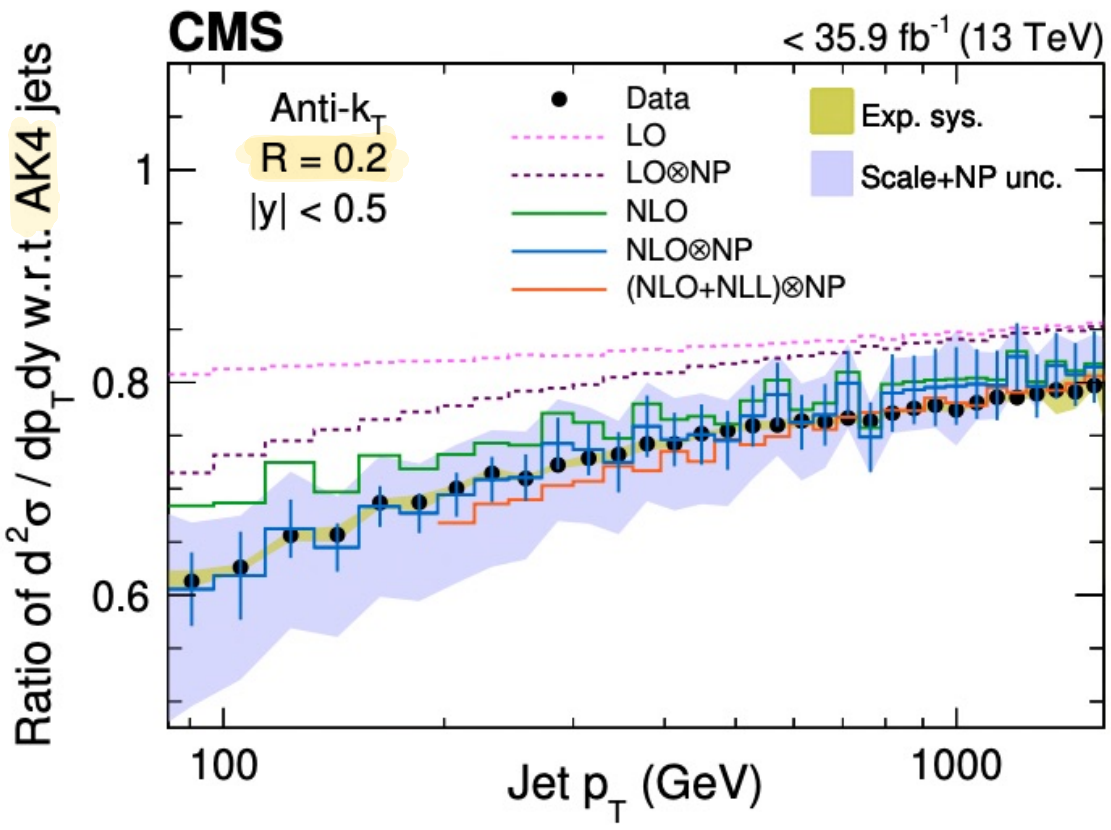




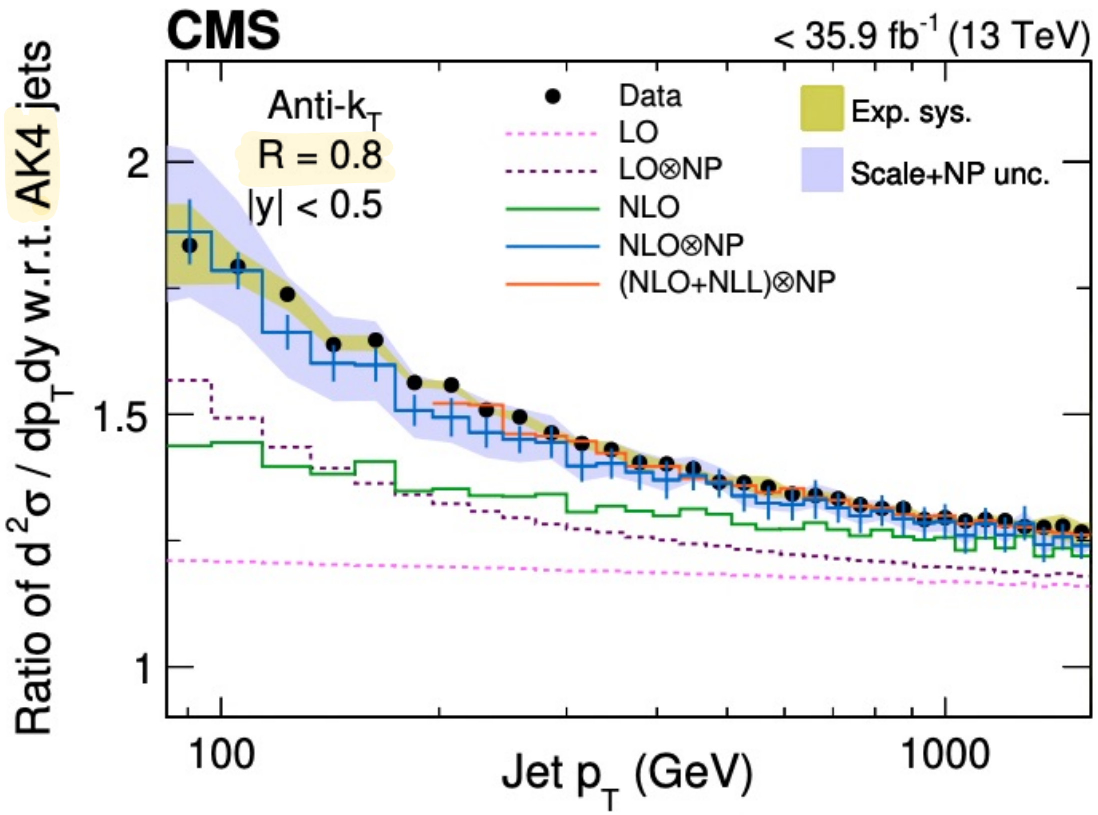
* R small to limit impact of UE

* R large to tame FSR

R variation

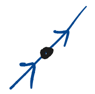


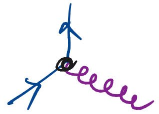
* "NP" $\hat{=}$ hadr. + MPI



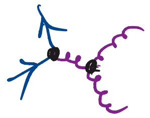
Jets are ...

* a proxy for high-energetic parton ⇝ interface between TH & EXP
* are everywhere (esp. @ LHC)
  ↳ study QFT's: $e^+e^- \to$ 2 jet, 3 jet, 4 jet





∃ gluon!

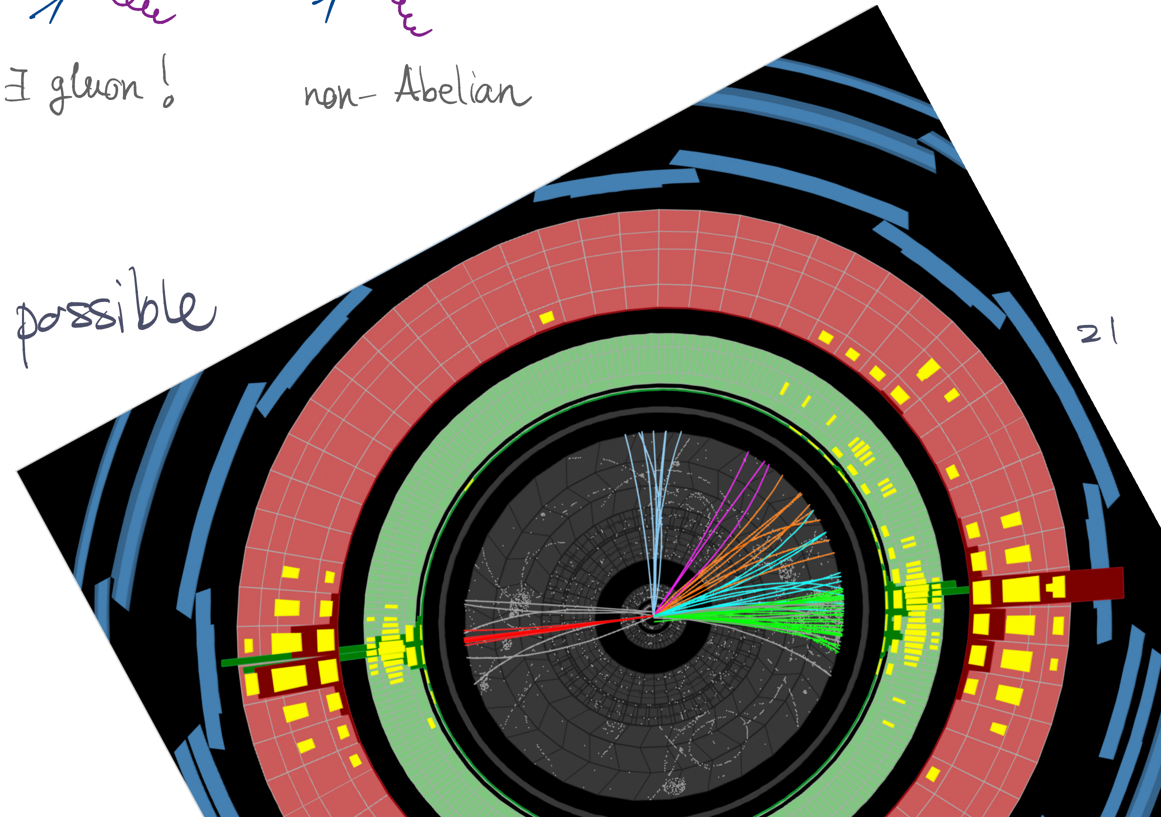


non-Abelian

  ↳ BSM searches (boosted!)

* are not unique & fundamentally ambiguous
  ↳ application-specific optimizations possible

⇝ what if I want to model the full complexity (exclusive) of an event?



Exclusive Emissions

* recall: exact calculation of $2 \to 100$ particle scattering: hopeless
  ⇨ but the relevant physics after the hard interaction is governed by soft & collinear QCD

$$d\omega_{X \to X+g} = 2\,\frac{\alpha_s}{\pi}\,C_X\,\frac{dE}{E}\,\frac{d\theta}{\theta}$$

integral diverges

* introduce a scale $q^2 > Q_0^2$ ↔ emission "resolved"

$$\Rightarrow P_X \simeq \frac{\alpha_s C_F}{2\pi} \underbrace{\ln^2\left(Q^2/Q_0^2\right)} + \mathcal{O}\left(\alpha_s \ln Q^2, \alpha_s^2\right)$$

probability to emit a resolved gluon

potentially a very large log

$$\left[\left.\begin{array}{l} Q_0 = \Lambda_{QCD} = 0.2\ \mathrm{GeV} \\ Q = 100\ \mathrm{GeV} \end{array}\right\} \Rightarrow \ln(\ldots) = \mathcal{O}(10)\right]$$

⇝ will want to "resum" these to all orders
while also retaining information on the emission kinematics

Parton Showers

* We wish to account for an arbitrary number of emissions ordered in our resolution variable $Q^2 > q_1^2 > q_2^2 > \dots > Q_0^2$ (stong ordering)

* current scale $q_n^2 \rightsquigarrow$ probability to have next emission @ q_{n+1}^2 ?

[demo: toy shower]

$$\underbrace{\left(\begin{array}{c}\text{probability of having}\\ \underline{\text{no emissions}}\quad q_n^2 \mapsto q_{n+1}^2\end{array}\right)} \times \underbrace{\left(\begin{array}{c}\text{emission}\\ @\ q_{n+1}^2\end{array}\right)}$$

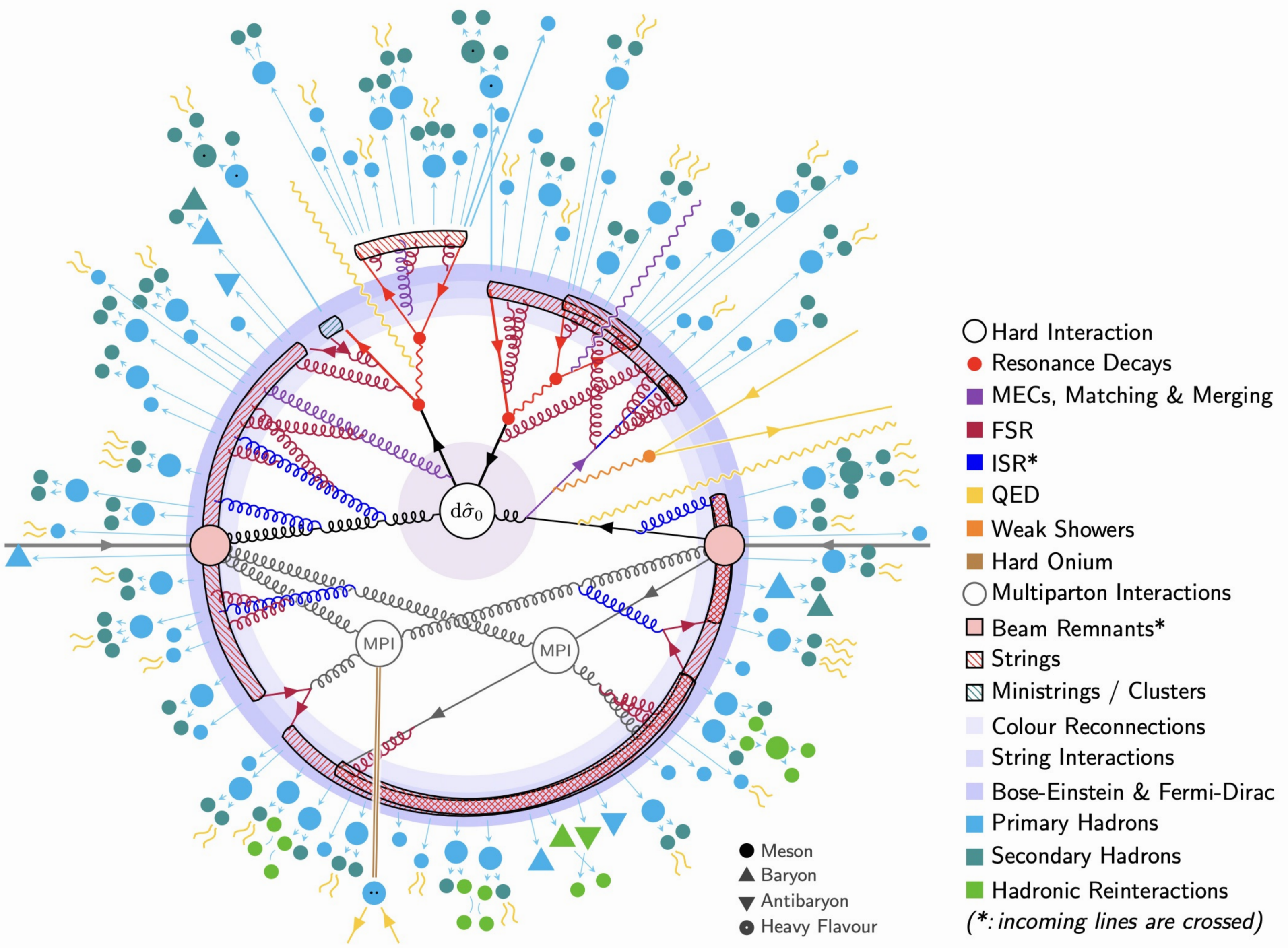
$$\Delta(q_n^2, q_{n+1}^2) \qquad \times \qquad \left.\frac{d\omega_{X\to X+g}}{dq^2}\right|_{q^2=q_{n+1}^2}$$

Sudakov formfactor

$$\hookrightarrow \quad \frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\,\frac{d\omega}{dq^2}$$

$$\left[\,\Delta(Q^2,q^2-dq^2) = \Delta(Q^2,q^2)\,\Delta(q^2,q^2-dq^2) = \Delta(Q^2,q^2)\left(1-\frac{d\omega}{dq^2}\right)\right]$$

Full Event Generator



+ pile-up

Conclusions

* covered general concepts in QCD that goes into hadron-collider predictions
  - ↪ asymptotic freedom ⟷ application of perturbation theory
  - ↪ seperation of scales ("factorization")

* Moment of comparing your predictions to data always exciting
  - ↪ learn to play with the tools ; break them (often interesting physics)

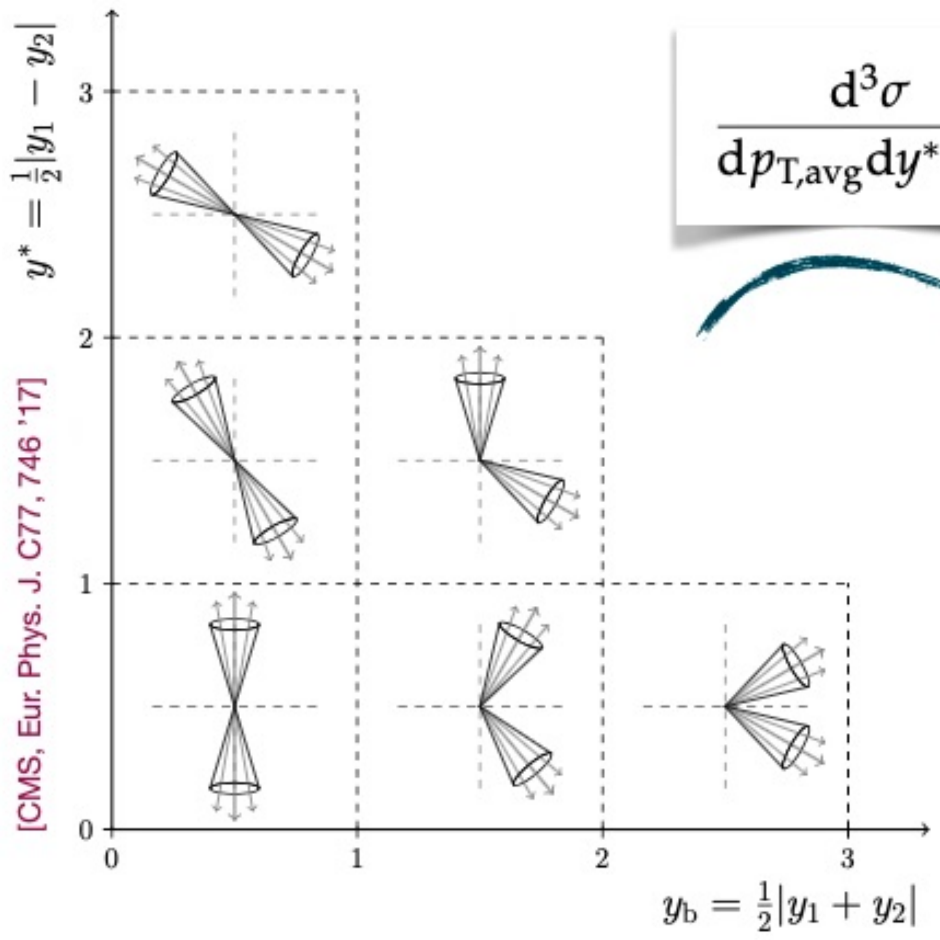
* Hope was able to lower the fear of entry for some of you, as it is sometimes perceived as very technical
  - ↪ pushing frontiers in precision can become arbitrary complex

    new ideas needed (maybe one of you?)

Thank you for your attention & participation!

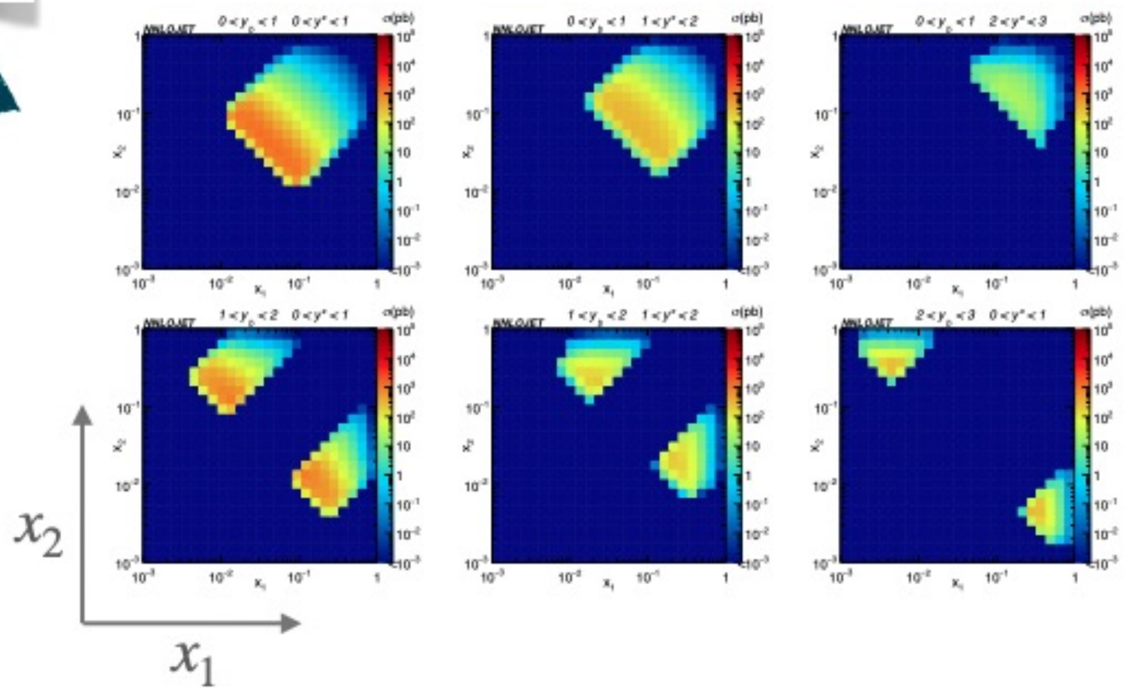
Extra

Triple - Differential Jet Production



$$\frac{d^3\sigma}{dp_{T,avg}\,dy^*\,dy_b}$$

$$x_{1,2} = \frac{2p_{T,avg}}{\sqrt{s}} e^{\pm y_b} \cosh(y^*)$$

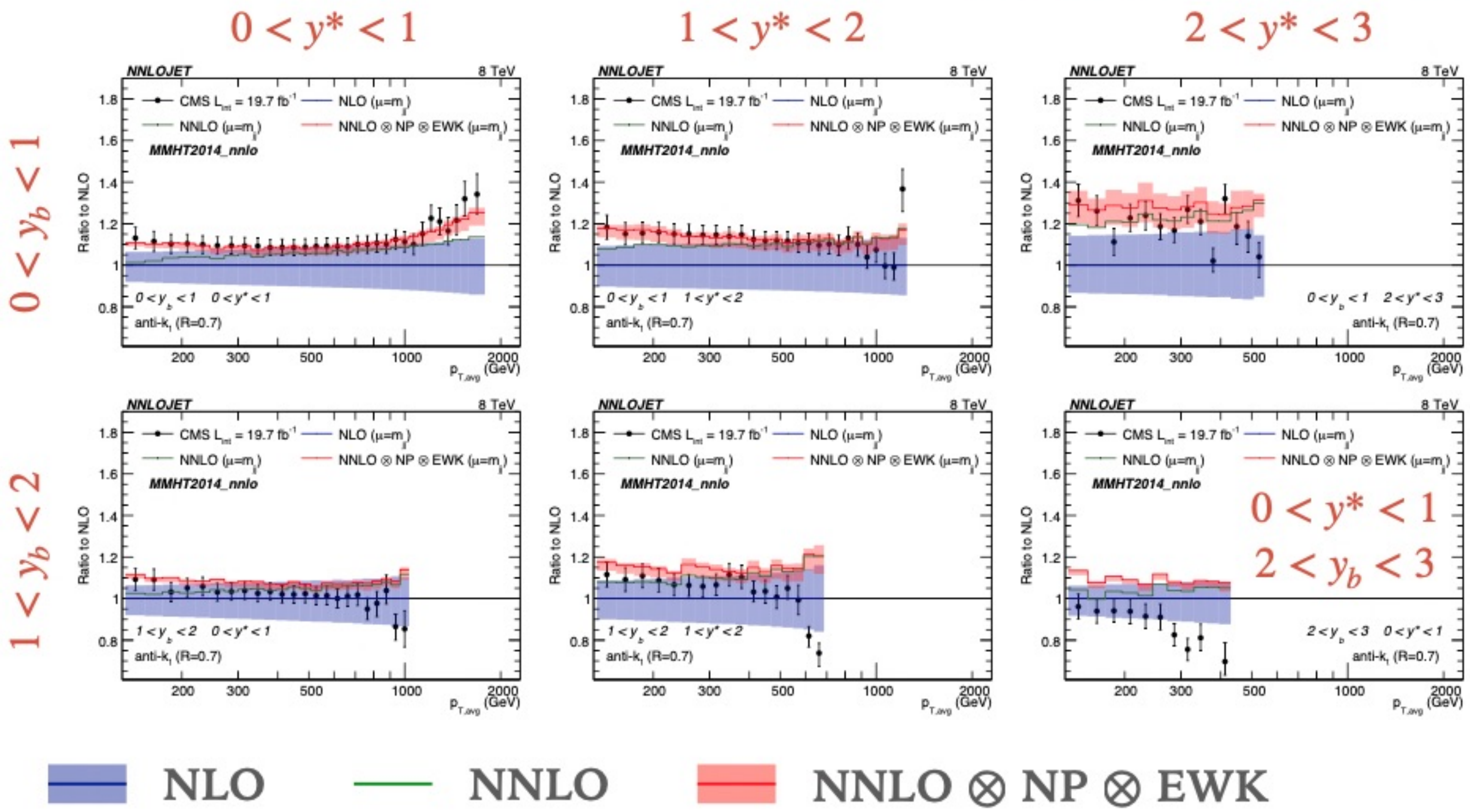


➤ study different **kinematic regimes**

➤ **disentangle** momentum fractions x_1 & x_2

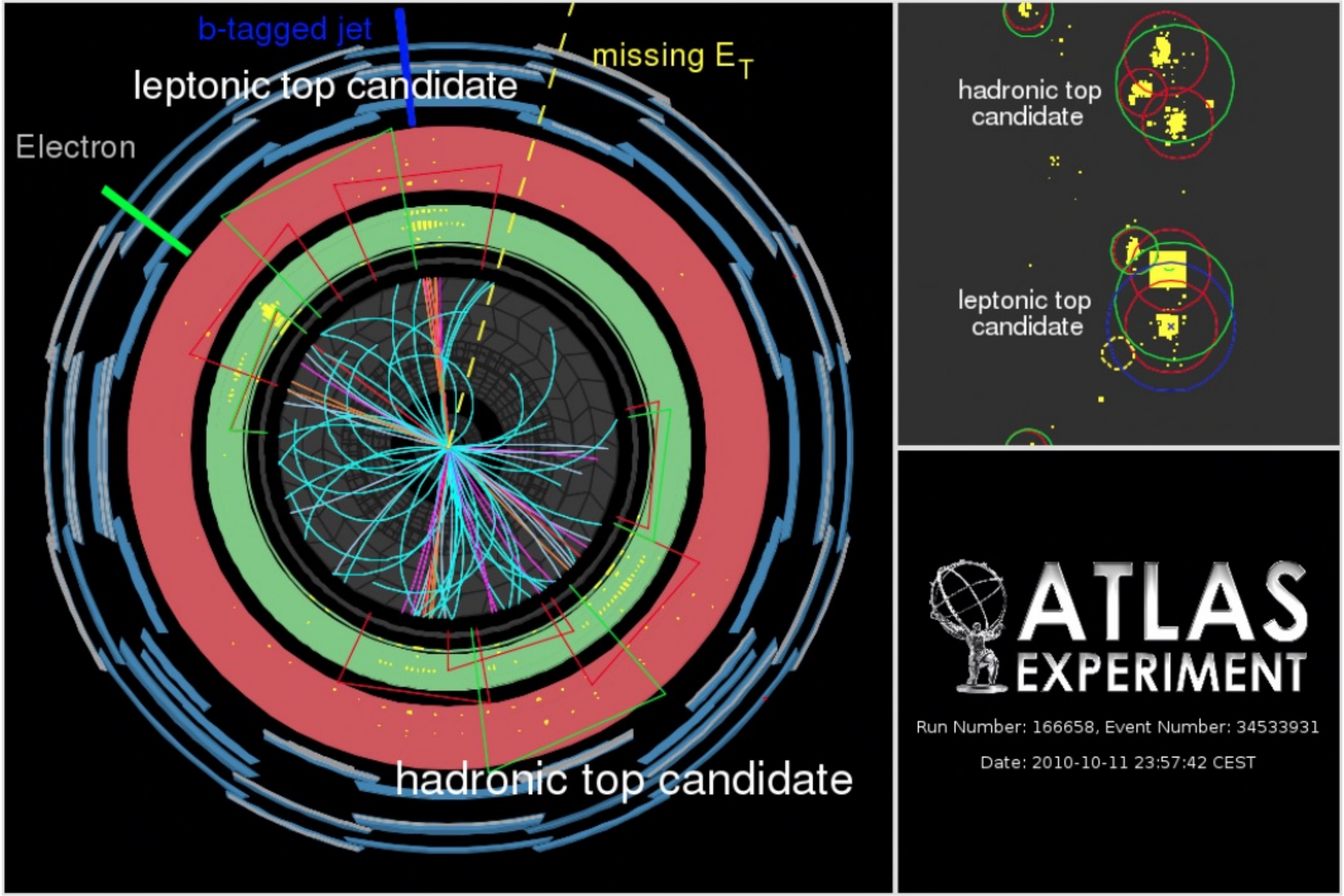
Triple - Differential Jet Production @ NNLO

[Gehrmann-De Ridder, Gehrmann, Glover, AH, Pires '19]



improved description of data & reduced uncertainties!

Jet Substructure



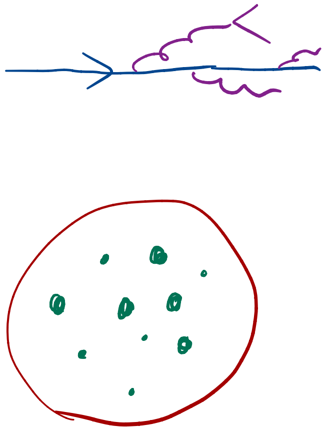
Looking inside Jets

* At the end of jet finding $\rightsquigarrow$ collection of constituents $\longleftrightarrow$ p^{μ}_{jet}

  $\hookrightarrow$ more information / physics than just the momentum

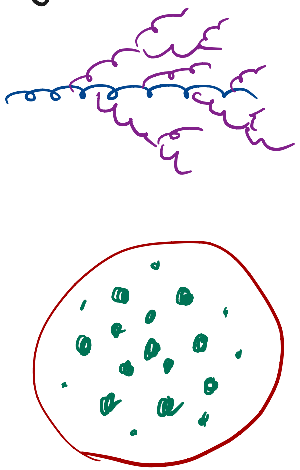
* what is the arrangement of the constituents inside the jets?

quark?



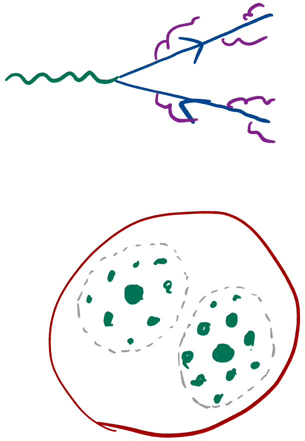
$$\frac{2\alpha_s}{\pi} C_F \frac{dE}{E} \frac{d\theta}{d\theta}$$

gluon?



$$\frac{2\alpha_s}{\pi} C_A \frac{dE}{E} \frac{d\theta}{\theta}$$

boosted object?

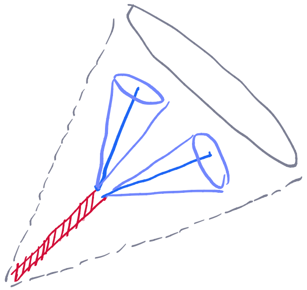


[H/W/Z $\longleftrightarrow$ 2 prongs

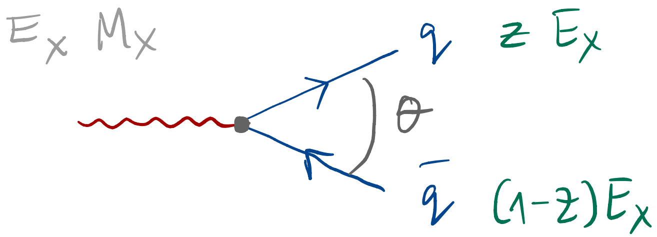
top quark $\longleftrightarrow$ 3 prongs]

Boosted Objects

* In extreme kinematic configurations, massive hadronically decaying object $\rightsquigarrow$ fat jets



* what cone sizes are we talking about?



$$m_J^2 = M_X^2 = 2E_X^2\, z(1-z)\, \overbrace{(1-\cos\theta)}^{\frac{1}{2}\theta^2}$$

$$\Rightarrow\ \theta = \frac{M_X}{E_X}\frac{1}{\sqrt{z(1-z)}} \overset{z\sim 1/2}{\sim} \frac{2M_X}{E_X}$$

put in some numbers:

$$\left.\begin{array}{l} M_X = M_W \simeq 80\ \text{GeV} \\ E_X \sim 1\ \text{TeV} \end{array}\right\} \theta \sim 0.15 \rightsquigarrow \text{likely end up in 1 jet}$$

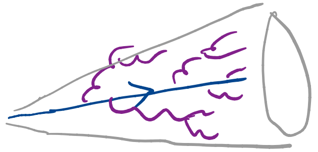
$\hookrightarrow$ how to distinguish this from a QCD jet?

The Jet mass

* naive expectation (common misconception)

  jet from "X" has mass M_X, whereas q/g jets are massless

* The jet mass of QCD partons



$$m^2 = \Big[\sum_{i \in jet} p_i\Big]^2$$

consider the cumulant :

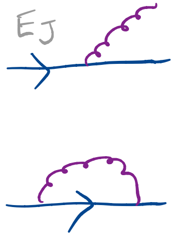
$\Sigma(m_J^2)$ = probability for the jet to have mass^2 < m_J^2

$$= \frac{1}{\sigma}\int dm'^2 \, \frac{d\sigma}{dm'^2}$$

@ LO :

$m^2 \equiv \emptyset \Rightarrow \Sigma(m_J^2) = 1$

@ NLO :



$$\Rightarrow \Sigma(m_J^2) = 1 - \frac{\alpha_s}{2\pi} C_F \ln^2\left(\frac{E_J^2 R^2}{m_J^2}\right)$$

do as exercise!

not good ($m_J \to \emptyset$)!
higher orders won't help either
$\sim \alpha_s^n \ln^{2n}\left(\frac{E_J^2 R^2}{m_J^2}\right)$

The resummed Jet mass

* need to account for these logs to all orders!

$$\Sigma_1(m_J^2) = \sum_{n=\emptyset}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left[\int_\delta \frac{d\theta_i^2}{\theta_i^2} \int_\epsilon \frac{dz_i}{z_i} \frac{\alpha_s C_F}{\pi} \Theta_{i\in \text{jet}} \right] \cdot \Theta\left(\left[\sum_{i=1}^{n} p_i\right]^2 < m_J^2 \right)$$

real emissions inside the jet

$$\sum_{m=\emptyset}^{\infty} \frac{1}{m!} \prod_{j=1}^{m} \left[\int_\delta \frac{d\tilde\theta_j^2}{\tilde\theta_j^2} \int_\epsilon \frac{d\tilde z_j}{\tilde z_j} \frac{\alpha_s C_F}{\pi} \left(\Theta_{j\notin \text{jet}} - 1 \right) \right]$$

$\leftrightarrow$ do not change m^2

real out of cone

virtual

* $\left[\sum_{i=1}^{n} p_i\right]^2 \simeq E_J^2 \sum_{i=1}^{n} z_i \theta_i^2$

* we're interested in the leading logs (LL) $\Rightarrow$ widely separate scales

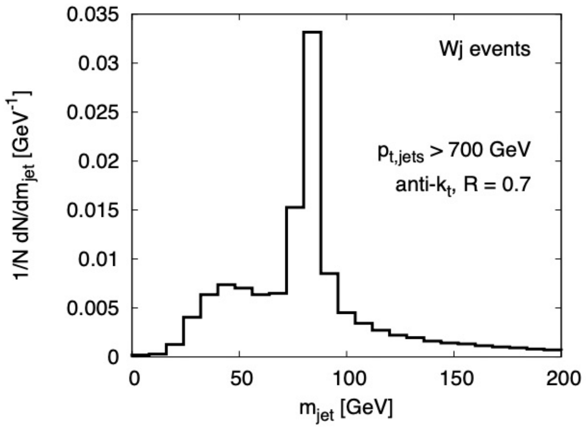
$\Rightarrow$ among all $z_i\theta_i^2$ one is dominant!

$$\Theta\left(E_J^2 \sum_i z_i\theta_i^2 < m_J^2\right) \simeq \Theta\left(E_J^2 \max\{z_i\theta_i^2\} < m_J^2\right) = \prod_{i=1}^{n} \Theta\left(E_J^2 z_i \theta_i^2 < m_J^2\right)$$

$$= \exp\left[-\int \frac{d\theta^2}{\theta^2} \int \frac{dz}{z} \frac{\alpha_s C_F}{\pi} \Theta(\theta < R)\, \Theta\left(E_J^2 z \theta^2 < m_J^2\right) \right] = \exp\left[-\frac{\alpha_s C_F}{2\pi} \ln^2\left(\frac{E_J^2 R^2}{m_J^2} \right) \right]$$

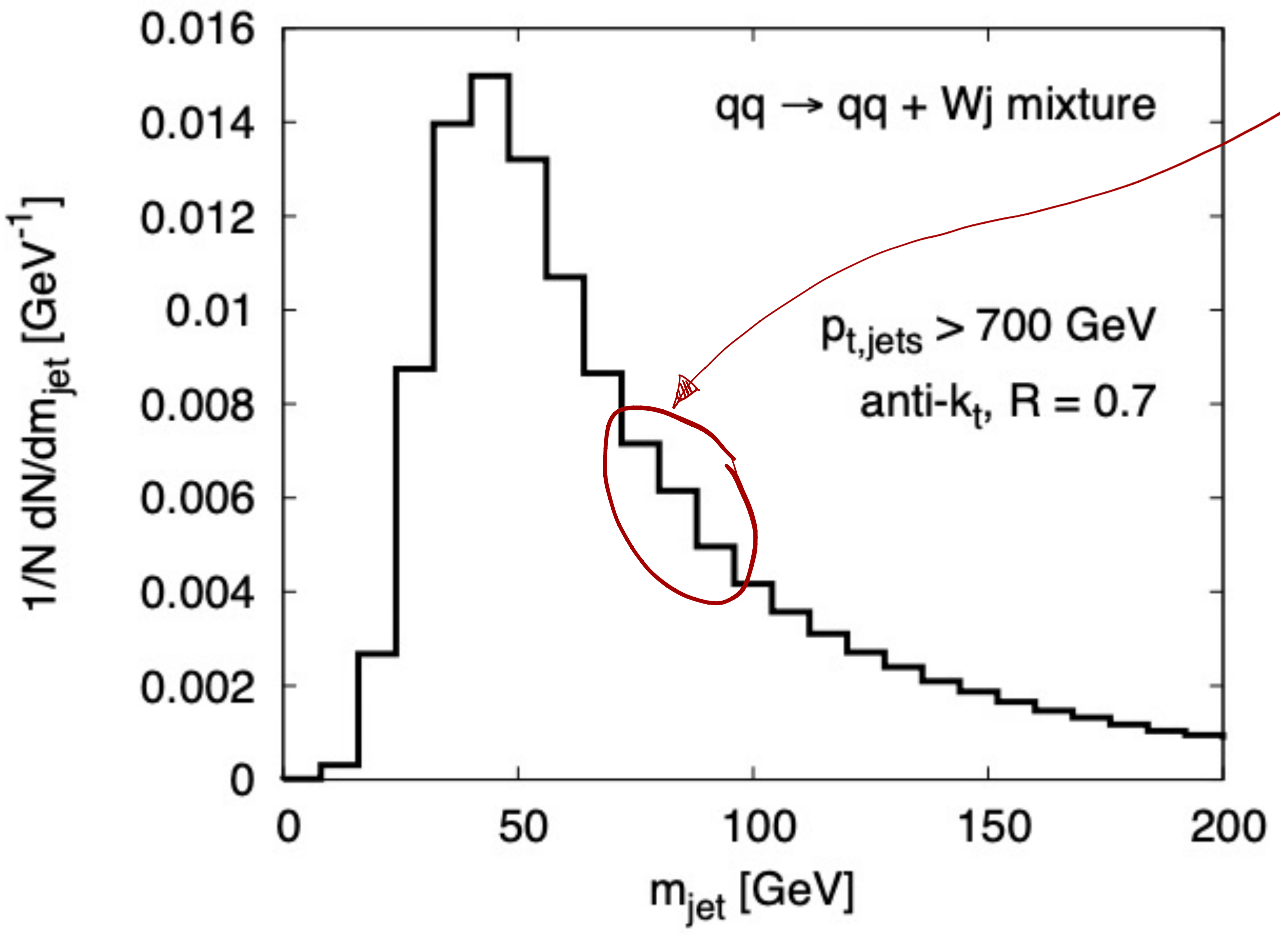
Jet Mass in Real Life

$$\frac{d\sigma}{dm_J^2} = \frac{d\Sigma}{dm_J^2} = \frac{1}{m_J^2}\frac{\alpha_s C_F}{\pi}\ln\left(\frac{E_J^2 R^2}{m_J^2}\right)\underbrace{\exp\left[-\frac{\alpha_s C_F}{2\pi}\ln^2\left(\frac{E_J^2 R^2}{m_J^2}\right)\right]}_{\text{"Sudakov"}}$$



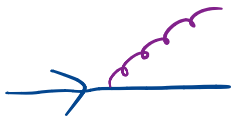
→ clear sign of W

→ but QCD jets massive too

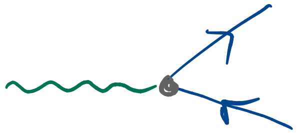


NEED TO REJECT QCD BACKGROUND & ENHANCE THE SIGNAL!

mMDT / Soft Drop



$\sim \frac{1}{z}$ [soft]



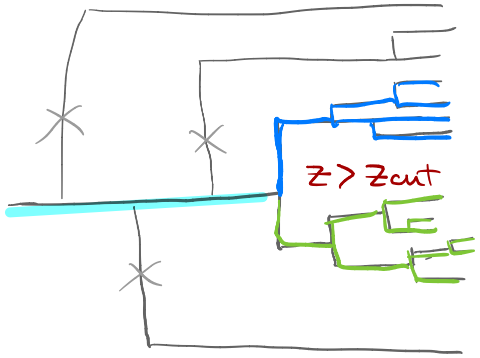
~ 1 [balanced momentum sharing]

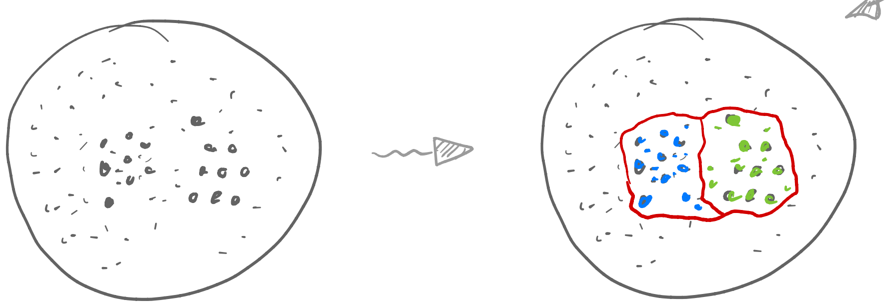
1. Take the constituents of the jet and recluster using C/A
   angular-ordered tree

2. Decluster jet, disregard the softer branch until

$$z = \frac{\min(E_i, E_j)}{E_i + E_j} > z_{cut}$$

mMDT / Soft Drop





k_T would keep the soft junk

* Removes soft radiation from periphery of jet
  [because Cambridge - Aachen for declustering]

* Dynamically shrinks jet radius to match hard core

Jet cleaning "grooming"

* Information on the 2-prong kinematics

Jet discrimination

* $$\Sigma_1(m_J^2) = \exp\left[-\frac{\alpha_s}{\pi} C_F \ln\left(1/z_{cut}\right) \ln\left(\frac{E_J^2 R^2}{m_J^2}\right)\right] \longleftrightarrow$$ [much smoother (& smaller) background

soft singularity regulated ⇒ single log!