Parton Showers

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Contents

1 Introduction

We will investigate the emission probability of gluons off quarks and gluons and use that to implement a **very** simplified parton shower (only final state, primary branching, leading double-log, only virtuality q^2 and not proper kinematics, . . .).

2 Emission probability and the Sudakov form factor

In the leading double-log approximation (soft *and* collinear emission), we have seen in the lecture that the emission probability is given as

$$\mathrm{d}\omega_{X\to X+g} = 2\,\frac{\alpha_s}{\pi}\,C_X\,\frac{\mathrm{d}E}{E}\,\frac{\mathrm{d}\theta}{\theta}\,, \tag{1}$$

where E denotes the energy of the emitted gluon and θ the angle w.r.t. the parent particle. We denote the emitting particle by "X" and C_X is the associated colour factor. For quarks, $C_X = C_F = \frac{4}{3}$ and for gluons $C_X = C_A = 3$.

For any parton shower, we first need to fix the evolution variable w.r.t. which we want to generate emissions. To this end, we choose the virtuality q^2 associated with the emission for which we find

$$d\mathcal{P} = \frac{\alpha_s}{\pi}\,C_X\,\frac{\mathrm{d}q^2}{q^2}\,\ln\!\left(\frac{q^2}{Q_0^2}\right) \xrightarrow{\int \mathrm{d}q^2} \frac{\alpha_s C_X}{2\pi}\,\ln^2\!\left(\frac{q^2}{Q_0^2}\right) \tag{2}$$

where Q_0 denotes a cutoff below which emissions are considered unresolved. Note that this fixed-order result comes with a serious problem: For a power of α_s, we get two powers of a potentially large logarithm (the so-called “double logarithms” that appear frequently in higher-order calculations), a pattern that will continue to higher orders. For some representative values ($\alpha_s \sim 0.1$, $Q_0 \sim \Lambda_{\text{QCD}} \sim 0.2\,\text{GeV}$, $q \sim 100\,\text{GeV}$), we quickly realize that the large logarithm compensates the small value of the coupling, giving rise to a non-converging expansion. In such situations, where we are sensitive to large logarithms, we need to re-arrange the perturbative expansion in such a way to “re-sum” these large logarithms to all orders.

To accomplish this, we define the so-called Sudakov form factor $\Delta(Q^2, q^2)$, which is the probability for *no resolved emissions* to happen between the evolution $Q^2 \to q^2$. It satisfies a differential equation reminiscent of radiative decay with a simple solution

$$\frac{\mathrm{d}\Delta(Q^2,q^2)}{\mathrm{d}q^2} = \Delta(Q^2,q^2)\,\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}q^2}\,,$$

$$\Delta(Q^2,q^2) = \Delta(Q^2)/\Delta(q^2)\,, \quad \Delta(Q^2) \equiv \Delta(Q^2,Q_0^2) = \exp\left\{-\frac{\alpha_s C_X}{2\pi}\,\ln^2\left(\frac{q^2}{Q_0^2}\right)\right\}, \quad (3)$$

which now has the large logarithm in the exponent. This solution therefore accomplishes exactly what we wanted: sum up the problematic logarithms to all orders, and in doing so, tame the otherwise divergent behaviour ($Q_0 \to 0$). It turns out that we can use the Sudakov form factor to sample successive emissions (it’s a Markovian process), which we discuss in the next section.

3 Implementation

3.1 Interlude: Sampling using the inversion method

asdf

3.2 Our Toy Shower

With the Sudakov form factor in Eq. (3) at hand, we can easily iterate the sampling of emissions using the following steps:

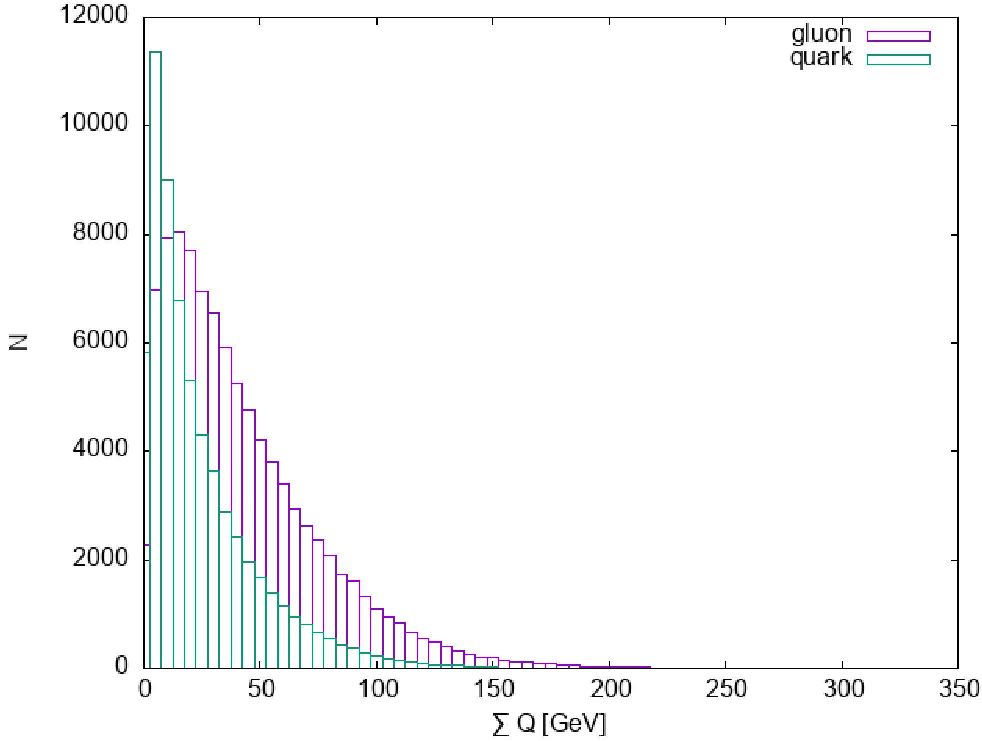
1. set $Q = Q_{\text{start}}$
2. draw a uniform random number r in the range [0, 1]
3. if $r < \Delta(Q^2)$, no resolvable emission can be generated ($< Q_0$): Terminate loop.
4. solve $r = \Delta(Q^2)/\Delta(Q^2_{\text{new}})$ for Q_{new}, which is the new emission scale.
5. “generate” the emission at Q_{new}, set $Q = Q_{\text{new}}$ and go back to step 2.

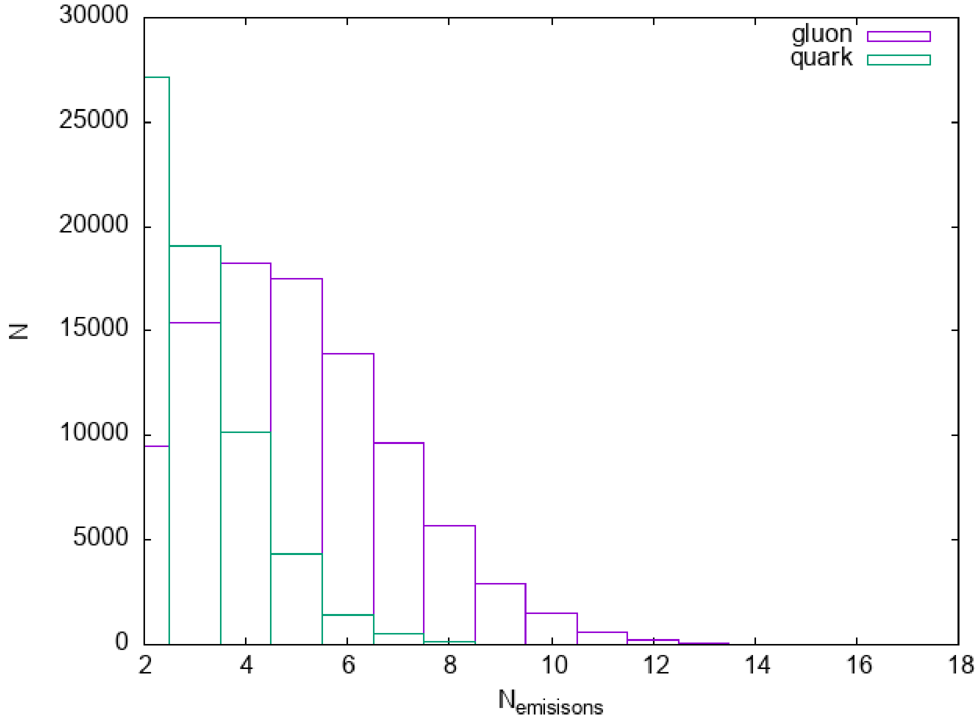
```python
#!/usr/bin/env python

import math
import random
import sys

random.seed(42)
alphas = 0.118

def generate_event(Q2_start: float, Q2_cutoff: float, CX: float):
  sudakov = 1.  # initialize Sudakov to the starting scale
  fac = alphas*CX/(2.*math.pi)
  Qlist = []
  while True:
    r = random.uniform(0.,1.)
    sudakov *= r
    #> sudakov = exp( -[alphas*CX/(2.*pi)] * log^2[Q2/Q2_start] )
    #> determine Q2 from the associated sudakov
    L2 = - math.log(sudakov) / fac
    Q2 = Q2_start * math.exp(-math.sqrt(L2))
    if Q2 < Q2_cutoff:
      break
    Qlist.append( math.sqrt(Q2) )
  if len(Qlist) > 1:
    print("#summary2 {} {} {} {}".format(len(Qlist),sum(Qlist),Qlist[0],Qlist))

if __name__ == "__main__":
  if len(sys.argv) < 3:
    raise RuntimeError("I expect at least two arguments:  Q_start [g|q]")
  Q_start = float(sys.argv[1])  # the hard scale
  Q_cutoff = 1  # shower cutoff (PS stops -> hand over to hadronization)
  if sys.argv[2] == "q":
    CX = 4./3.  # quark
  elif sys.argv[2] == "g":
    CX = 3.     # gluon
  else:
    raise RuntimeError("unrecognised parton: {}".format(sys.argv[2]))
  if len(sys.argv) >= 4:
    alphas = float(sys.argv[3])
  if len(sys.argv) >= 5:
    nevents = int(sys.argv[4])
  else:
    nevents = 1000
  for i in range(nevents):
    print("# event {} [{} {} {} {} {}]".format(i,Q_start,sys.argv[2],CX,alphas,nevents))
    generate_event(Q_start**2, Q_cutoff**2, CX)
```

Let's use the implementation to generate some "events"

```
python main.py 100 g 0.118 100000 > data_g.dat
python main.py 100 q 0.118 100000 > data_q.dat
```

We can see that the all-order description damps the divergent behaviour of a pure fixed-order prediction for $Q \to 0$. Given $C_A > C_F$, we also see how a gluon generates more emissions than quarks. This property can be exploited to try and discriminate between "quark jets" and "gluon jets".





- To increase the amount of emissions, try out setting the strong coupling to $\alpha_s = 0.5$. How does the picture change?