Standard Model Precision Physics

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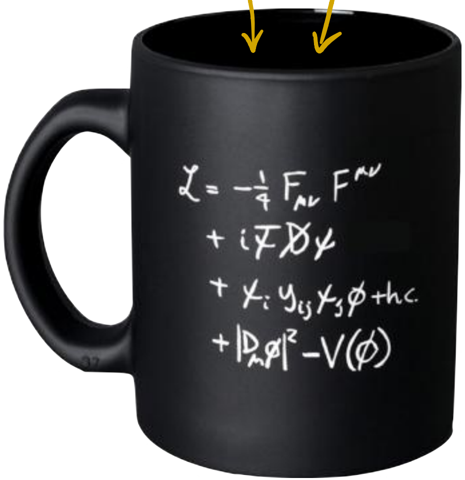
$SU(3)_c \times SU(2)_L \times U(1)_Y$



framework of QFT

$\hat{=}$ QM $\otimes$ SR

THEORY

PHENOMENOLOGY

- precision $\leftrightarrow$ Δ_{exp}
- making predictions
- learn about $\mathcal{L}_{TH}$?

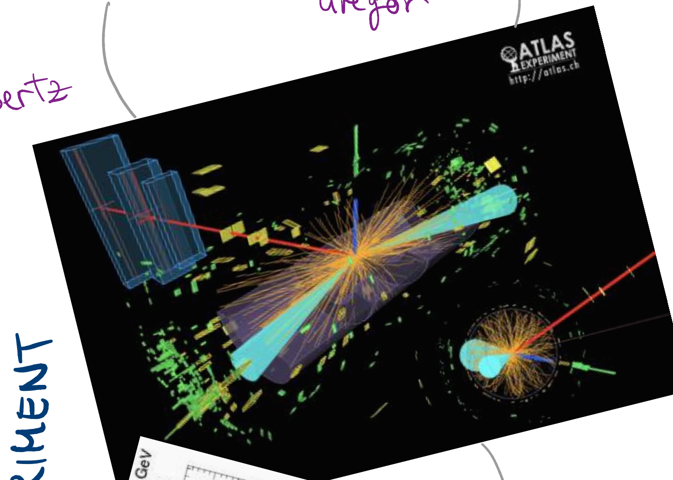
EXPERIMENT

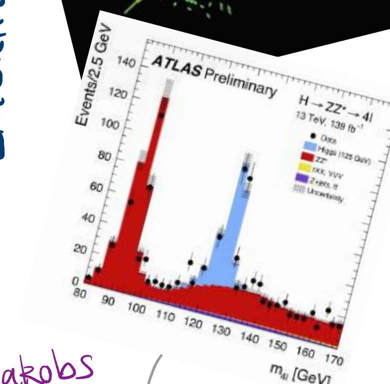
accelerators

detectors

Gregor

Rabbertz





data acquisition/ analysis

Jakobs

statistics

Aarrestad

The Plan

1. A brief recap of Quantum Field Theories
   * why QFTs? * perturbation theory & Feynman diagrams
2. The main construction Principles for QFTs
   * from gauge freedom to local symmetries
   * Yang-Mills theories
   * Spontaneous Symmetry Breaking
3. Strong Interactions
4. The electroweak Standard Model
5. LHC Phenomenology

Ø Conventions & Source files

* natural units: $\hbar = c = 1$
  - $[\hbar] = [Et] \Rightarrow [E] = [t]^{-1}$
  - $[c] = [x/t] \Rightarrow [x] = [t]$

  particle physics: $[E] = eV$

  $[E] = [p] = [m] = eV$

  $[t] = [x] = eV^{-1}$

* four-vectors: $x^\mu = (t, x, y, z)^T$ $\quad \partial_\mu = (\partial_t, \vec{\nabla})$
  - scalar product: $a \cdot b = a^\mu b_\mu = a^\mu g_{\mu\nu} b^\nu = a^0 b^0 - \vec{a} \cdot \vec{b}$ $\quad g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$
  - d'Alembert operator: $\Box \equiv \partial_\mu \partial^\mu = \partial_t^2 - \Delta$
  - energy-momentum conservation: $\delta^{(4)}(p_1 + p_2 - (p_a + p_b))$ $\quad a + b \to 1 + 2$
* Dirac algebra: $\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu = 2 g^{\mu\nu} \mathbb{1}_{4\times4}$
  - $\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = -i \gamma_0 \gamma_1 \gamma_2 \gamma_3$
  - Dirac "slash": $\not{a} = a_\mu \gamma^\mu$
  - conjugate spinor: $\bar{\psi} = \psi^\dagger \gamma^0$

org notebooks (mainly python)

* source files: github.com/aykhuss/Lectures-MariaLaach-SMprec

1 Quantum Field Theory — Why?

high energy physics (hep): study fundamental particles & interactions

$\Rightarrow$ resolve small distances through highly energetic collisions

Quantum Mechanics

$i\hbar\partial_t\psi = [-\frac{\hbar^2}{2m}\Delta + V]\psi$ Schrödinger Eqn

⊕ description of microscopic nature

⊖ not relativistic

⊖ fixed particle #

Special Relativity

$x^\mu \rightarrow \Lambda^\mu{}_\nu x^\nu$ (rotn & boosts)

• principle of relativity

⊕ processes @ high energy/velocity ($v \sim c$)

QM ⊗ SR

= QFT

1 Quantum Field Theory — How?

* particle X $\longleftrightarrow$ associated field $\Phi_X\underbrace{(t,\vec{x})}_{x^\mu}$ & elementary excitations

  (separate field $\forall$ species)

* $E=mc^2$ & QM $\Rightarrow$ particle creation / annihilation

* Quantization $\rightsquigarrow$ path-integral

$$\int \mathcal{D}[\Phi]\; e^{i\int d^4x\, \mathcal{L}(\partial_\mu\Phi,\Phi)}$$

↪ in principle, can compute anything from it

↪ full information encapsulated in $\mathcal{L}$ ← Lagrangian density

↪ extremely difficult integral to solve

1 Quantum Field Theory — How?

* particle X $\longleftrightarrow$ associated field $\Phi_X\overbrace{(t,\vec{x})}^{x^\mu}$ & elementary excitations

  (separate field $\forall$ species)

* $E=mc^2$ & QM $\Rightarrow$ particle creation / annihilation

* Quantization $\rightsquigarrow$ path-integral

  create/annihilate particle @ x_i

$$\mathcal{A}_{\Phi_1\cdots\Phi_n} \longleftrightarrow \int \mathcal{D}[\Phi]\, \Phi(x_1)\cdots\Phi(x_n)\, e^{i\int d^4x\, \mathcal{L}(\partial_\mu\Phi,\,\Phi)}$$

(amplitude)

↪ in principle, can compute <u>anything</u> from it

↪ full information encapsulated in $\mathcal{L}$

Lagrangian density

↪ <u>extremely</u> difficult integral to solve

... except a free theory (quadratic in Φ $\Rightarrow$ "Gauss" Integral)

Free Theory $\longleftrightarrow$ only monomials quadratic in the fields

Spin $\emptyset$: Klein-Gordon Eqn

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

$$\hookrightarrow (\Box + m^2)\,\phi(x) = \emptyset$$

Euler-Lagrange: $\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = \frac{\partial \mathcal{L}}{\partial \phi}$

Spin $\frac{1}{2}$: Dirac Eqn

$$\mathcal{L}_\psi = \bar{\psi}(i\not{\partial} - m)\psi$$

$$\hookrightarrow (i\not{\partial} - m)\,\psi(x) = \emptyset$$

Spin 1: Maxwell Eqn $\rightsquigarrow$ gauge freedom: $A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \theta(x)$

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu)$$

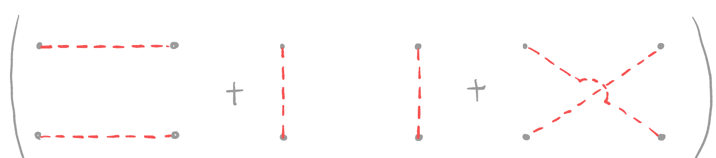
$$\hookrightarrow \partial_\mu F^{\mu\nu} = \emptyset$$

Free Theory — Feynman Rules

	incoming		outgoing		propagator	
Spin 0:	ϕ	1	ϕ	1		$\dfrac{i}{p^2-m^2}$
Spin ½:	f	$u_\alpha(p,\sigma)$	f	$\bar{u}_\alpha(p,\sigma)$	β, α	$\dfrac{i(\not{p}+m)_{\alpha\beta}}{p^2-m^2}$
	$\bar{f}$	$\bar{v}_\alpha(p,\sigma)$	$\bar{f}$	$v_\alpha(p,\sigma)$		
Spin 1:	A_μ	$\varepsilon^*_\mu(p,\lambda)$	A_μ	$\varepsilon_\mu(p,\lambda)$	μ, ν	$\dfrac{-i\,g_{\mu\nu}}{p^2}$

* extremely boring theory

$\phi + \phi \to \phi + \phi$



no scattering, just free propagation

Perturbation Theory

* too difficult to solve full interacting theory <u>exactly</u>

  ⇒ do it approximately (& systematically improvable)

* perturbation theory → interactions as perturbation around free theory

  $\alpha_{em} \sim 1/137$, $\alpha_{weak} \sim 1/30$, $\alpha_{strong} \sim 0.118$

$$\Rightarrow \mathcal{O} = \mathcal{O}^{(0)} + \alpha\, \mathcal{O}^{(1)} + \alpha^2 \mathcal{O}^{(2)} + \ldots$$

* example electron g-2: $a_e = \dfrac{g_e - 2}{2}$

$$a_{e^-}^{exp} = 1\,159\,652\,180.73(.28) \times 10^{-12} \quad (0.24 \text{ ppb})$$

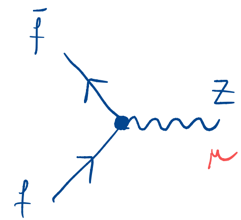
$$a_e^{\text{th}} = 1\,159\,652\,182.032(13)(12)(720) \times 10^{-12}$$

contribution	value in units of 10^{-12}
$A_1^{(2)}(\alpha/\pi)$	$1\,161\,409\,733.640 \pm 0.720$
$A_1^{(4)}(\alpha/\pi)^2$	$-1\,772\,305.065 \pm 0.003$
$A_1^{(6)}(\alpha/\pi)^3$	$14\,804.203$
$A_1^{(8)}(\alpha/\pi)^4$	-55.667
$A_1^{(10)}(\alpha/\pi)^5$	0.451 ± 0.013
$A_2^{(4)}(m_e/m_\mu)(\alpha/\pi)^2$	2.804
$A_2^{(6)}(m_e/m_\mu)(\alpha/\pi)^3$	-0.092
$A_2^{(8)}(m_e/m_\mu)(\alpha/\pi)^4$	0.026
$A_2^{(10)}(m_e/m_\mu)(\alpha/\pi)^5$	-0.0002
$A_2^{(4)}(m_e/m_\tau)(\alpha/\pi)^2$	0.010
$A_2^{(6)}(m_e/m_\tau)(\alpha/\pi)^3$	-0.0008
a_e(hadronic v.p.)	1.8490 ± 0.0108
a_e(hadronic v.p.,NLO)	-0.2213 ± 0.0012
a_e(hadronic v.p.,NNLO)	0.0280 ± 0.0002
a_e(hadronic l-l)	0.0370 ± 0.0050
a_e(weak)	0.03053 ± 0.00023

Feynman Rules with interactions

* from free theory $\rightarrow$ external states & propagators (# fields ≤ 2)
* interactions (# fields ≥ 3) $\rightarrow$ direct correspondence: $i\mathcal{L} \longleftrightarrow$ vertices

$$-e\, Z_\mu \bar{\psi}_f \gamma^\mu (v_f - a_f \gamma^5) \psi_f \quad \rightsquigarrow$$

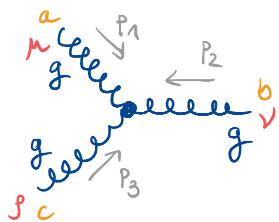


$$= -ie\gamma^\mu (v_f - a_f \gamma^5)$$

$\hookrightarrow$ sometimes more subtle (derivatives, identical fields)

$$\frac{i g_s}{2} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) f^{abc} A^{b,\mu} A^{b,\nu}$$

$\rightsquigarrow$



$$= -g_s f^{abc} \Big[g_{\mu\nu} (p_1 - p_2)_\rho + g_{\nu\rho} (p_2 - p_3)_\mu + g_{\rho\mu} (p_3 - p_1)_\nu \Big]$$

2 How to construct $\mathcal{L}$?

guiding principles

$*$ symmetries :

Poincaré invariance

gauge invariance

$*$ causality :

local monomials

$\Leftrightarrow$ $\phi_1(x)\,\phi_2(x)\,\ldots \in \mathcal{L}$

same x !

$*$ renormalizability

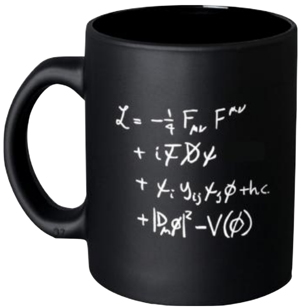
monomials w/ field dim ≤ 4

$\Leftrightarrow$ $[\phi] = [A] = 1$; $[\psi] = 3/2$

$\Rightarrow$ only 3- & 4- vertices

$*$ unitarity (probability ≤ 1)

$*$ minimality (for the SM)



$$
\begin{aligned}
&\textbf{1}\quad -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
&\tfrac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
&\textbf{2}\quad M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H - \\
&\tfrac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \tfrac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\tfrac{2M^2}{g^2} + \\
&\tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - i g c_w [\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
&W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
&W^-_\nu \partial_\nu W^+_\mu)] - i g s_w [\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
&W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
&\tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
&g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
&W^+_\nu W^-_\mu) - 2 A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha [H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
&\tfrac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - \\
&g M W^+_\mu W^-_\mu H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \tfrac{1}{2} i g [W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
&W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \tfrac{1}{2} g [W^+_\mu (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W^-_\mu (H \partial_\mu \phi^+ - \\
&\phi^+ \partial_\mu H)] + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - i g \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + \\
&i g s_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - i g \tfrac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
&i g s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \tfrac{1}{4} g^2 W^+_\mu W^-_\mu [H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \\
&\tfrac{1}{4} g^2 \tfrac{1}{c_w^2} Z^0_\mu Z^0_\mu [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-] - \tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- + \\
&W^-_\mu \phi^+) - \tfrac{1}{2} i g^2 \tfrac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- + \\
&W^-_\mu \phi^+) + \tfrac{1}{2} i g^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+ \phi^- - \\
&g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma\partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma\partial \nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + m_u^\lambda) u^\lambda_j - \\
&\textbf{3}\quad \bar{d}^\lambda_j (\gamma\partial + m_d^\lambda) d^\lambda_j + i g s_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j)] + \\
&\tfrac{ig}{4c_w} Z^0_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}^\lambda_j \gamma^\mu (\tfrac{4}{3}s_w^2 - \\
&1 - \gamma^5) u^\lambda_j) + (\bar{d}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3}s_w^2 - \gamma^5) d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} W^+_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) e^\lambda) + \\
&(\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}} W^-_\mu [(\bar{e}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa} \gamma^\mu (1 + \\
&\gamma^5) u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} \tfrac{m_e^\lambda}{M} [-\phi^+ (\bar{\nu}^\lambda (1-\gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1+\gamma^5) \nu^\lambda)] - \\
&\textbf{4}\quad \tfrac{g}{2} \tfrac{m_e^\lambda}{M} [H (\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\kappa (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) + \\
&m_u^\lambda (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m_u^\kappa (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1 - \\
&\gamma^5) u^\kappa_j] - \tfrac{g}{2} \tfrac{m_u^\lambda}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2} \tfrac{m_d^\lambda}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2} \tfrac{m_u^\lambda}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) - \\
&\tfrac{ig}{2} \tfrac{m_d^\lambda}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \\
&\textbf{5}\quad \tfrac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + i g c_w W^+_\mu (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + i g s_w W^+_\mu (\partial_\mu \bar{Y} X^- - \\
&\partial_\mu \bar{X}^+ Y) + i g c_w W^-_\mu (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + i g s_w W^-_\mu (\partial_\mu \bar{X}^- Y - \\
&\partial_\mu \bar{Y} X^+) + i g c_w Z^0_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + i g s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
&\partial_\mu \bar{X}^- X^-) - \tfrac{1}{2} g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2} \bar{X}^0 X^0 H] + \\
&\tfrac{1-2c_w^2}{2c_w} i g M [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \tfrac{1}{2c_w} i g M [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \\
&i g M s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \tfrac{1}{2} i g M [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
\end{aligned}
$$

From gauge freedom to local symmetries — QED

* The free Dirac Lagrangian has a global $U(1)$ symmetry

$$\mathcal{L}_f = \bar{\psi}(i\partial\!\!\!/ - m)\psi \quad ; \quad \psi \to e^{-i\alpha}\psi \quad (\bar{\psi} = \psi^\dagger\gamma^0 \longrightarrow e^{+i\alpha}\bar{\psi})$$

* Noether: continuous symmetry $\Rightarrow$ conserved current

$$\partial_\mu j^\mu = 0 \quad ; \quad j^\mu \propto \bar{\psi}\gamma^\mu\psi \quad (\text{opposite "charge" } j^0 \text{ for (anti-) particles})$$

* couple to EM interactions $j^\mu A_\mu$ ← EM vector potential: $(\varphi, \vec{A})$

$$\mathcal{L}_{EM} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free Maxwell}} - Q_f e(\bar{\psi}\gamma^\mu\psi)A_\mu \quad \leadsto \text{Euler-Lagrange: } \partial_\mu F^{\mu\nu} = j^\nu$$

$$\Rightarrow \boxed{\mathcal{L}_{QED} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - Qe(\bar{\psi}\gamma^\mu\psi)A_\mu}$$

From gauge freedom to local symmetries — QED

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$$\Rightarrow \boxed{\mathcal{L}_{QED} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - Qe(\bar{\psi}\gamma^\mu\psi)A_\mu}$$

depends on x!

* The symmetry of $\mathcal{L}_{QED}$ is actually local $U(1)$ $\leadsto$ $\psi(x) \longrightarrow e^{-i\alpha(x)}\psi(x)$
  - problematic term: $\bar{\psi}\, i\partial\!\!\!/\, \psi \longrightarrow \bar{\psi}\, i\partial\!\!\!/\, \psi + (\bar{\psi}\gamma^\mu\psi)(\partial_\mu\alpha)$
  - but we have gauge freedom: $A_\mu \longmapsto A_\mu + \frac{1}{eQ_f}(\partial_\mu\alpha)$

$\Bigg\}$ $\mathcal{L}$ invariant

The gauge paradigm — QED revisited

* turn the local (gauge) symmetry into the construction principle

  0. free Lagrangian of matter fields & global $U(1)$ symmetry $\quad \psi \longmapsto e^{-iQe\theta}\,\psi$

  1. promote to a local symmetry $\rightsquigarrow$ "minimal substitution" $\partial_\mu \rightsquigarrow D_\mu = \partial_\mu + iQeA_\mu$

     ↪ covariant derivative: $\quad D_\mu\psi \longmapsto e^{-iQe\theta(x)}\,D_\mu\psi$

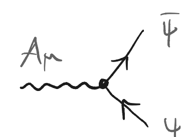
     ↪ gauge field $\quad A_\mu \longmapsto A_\mu + \partial_\mu\theta$

  2. dynamics for the gauge field

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$\Rightarrow \boxed{\mathcal{L}_{\text{QED}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - Qe(\bar{\psi}\gamma^\mu\psi)A_\mu}$$

interactions!



The gauge paradigm — non-Abelian Groups

* go beyond QED ⇝ apply gauge principle* to more complex groups G
* multiplet of matter fields & global "internal" symmetry

$$\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} \quad ; \quad \Phi \longrightarrow U(\vec{\theta})\,\Phi \quad ; \quad U(\vec{\theta}) = \exp\{-ig\,T^a\theta^a\}$$

matrix ↗ U; vector ↙ Φ

↪ $a = 1, \ldots, N = \dim(G)$; $\theta^a \leftrightarrow$ parameters of transformation

↪ $T^a \leftrightarrow$ generators of G (representation: $n\times n$ matrix)

properties: $[T^a, T^b] = i f^{abc}\, T^c$; $[T^a, T^b] = \frac{1}{2}\delta^{ab}$

structure constants

* important groups for the SM ($SU \rightarrow U^\dagger U = UU^\dagger = 1$ & $\det(U) = 1$ ⇝ N^2-1 generators)

↪ $SU(2)$: fundamental repⁿ $T_F^a = I^i = \sigma^i/2$ (Pauli matrices: $\sigma^{i=1,2,3}$; $f^{abc} = \varepsilon_{ijk}$)

↪ $SU(3)$: fundamental repⁿ $T_F^a = t^a = \lambda^a/2$ (Gell-Mann matrices: $\lambda^{a=1,\ldots,8}$)

↪ adjoint repⁿ $(T_A)_{ab} = -i f^{abc}$

*This is the only way we know how to consistently implement interacting spin-1

The gauge paradigm — Yang Mills Theories

* apply the same steps as we did for QED before

Ø. Lagrangian of matter fields & global symmetry G: $U(\vec{\theta}) = \exp\{-i g T^a \theta^a\}$

⇒ e.g. $\mathcal{L}_\Phi = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$

1. promote to a local ($\theta \rightsquigarrow \theta(x)$) symmetry

⇒ "minimal substitution" $\partial_\mu \rightsquigarrow D_\mu \equiv \partial_\mu + i g T^a A^a_\mu$

matrix

one gauge field for each generator

$\leftrightarrow$ adjoint rep$^{\underline{n}}$

covariant derivative

gauge coupling

⇒ transformations: $D_\mu \to U D_\mu U^{-1}$; $T^a A^a_\mu \longmapsto U\, T^a A^a_\mu U^\dagger - \frac{i}{g} U (\partial_\mu U^\dagger)$

2. dynamics for the gauge fields

⇒ covariant definition of field strength $[D_\mu, D_\nu] = i g T^a F^a_{\mu\nu}$

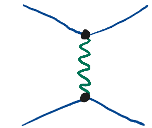
non-Abelian!

$\mathcal{L}_{gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$; explicit form: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$

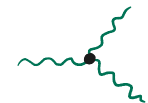
$$\Rightarrow \boxed{\mathcal{L}_{YM} = \mathcal{L}_\Phi \Big|_{\partial_\mu \to D_\mu} - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}}$$

Yang-Mills Theories — General Remarks

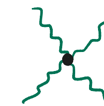
* similarly to QED: interaction between matter fields
  $\longleftrightarrow$ mediated through gauge-boson exchange



* in contrast to QED: gauge bosons self interact
  $\Rightarrow$ "F^2" part contains terms $(\partial A)A^2$ & A^4



(cubic)



(quartic)

* more features from non-Abelian nature
  $\Rightarrow$ single gauge coupling g for boson-boson & boson-matter ($\forall$ matter types)
  $\rightarrow$ unification
  $\Rightarrow$ charges are quantized owing to $[T^a, T^b] = i f^{abc} T^c$
  $\rightarrow$ c.f. w/ QED: all charges Q_x are arbitrary
* gauge symmetry (Abelian or not) forbids naive mass terms
  $\Rightarrow$ $M^2 (A^a_\mu A^{a\mu})$ breaks gauge invariance of $\mathcal{L}_{gauge}$

↯ $W^\pm$ & Z bosons ...

Spontaneous Symmetry Breaking

problem: naive mass term breaks gauge invariance:

$$M^2 A_\mu A^\mu \longrightarrow M^2 A_\mu A^\mu + 2M^2(\partial_\mu\theta)A^\mu + M^2(\partial_\mu\theta)(\partial^\mu\theta)$$

idea: retain gauge symmetry @ Lagrangian level

⇒ preserve all properties (conserved charges, unitarity, renormalizability)

give up the symmetry for the particle spectrum (in particular, the vacuum)

⇒ "hidden symmetry"

* spontaneous symmetry breaking [Brout, Englert, Higgs '64]

⇒ introduce a scalar field ← why?

with non-vanishing vacuum expectation value (vev)

that couples to the gauge boson(s)

Abelian Higgs Model

* $U(1)$ gauge theory with one complex scalar field $\phi(x) \in \mathbb{C}$

  ↳ most general form

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi)$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \longleftarrow \text{gauge inv.}$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$\phi \to e^{ie\theta(x)} \phi$$

$$A_\mu \to A_\mu + \partial_\mu \theta$$

* can also parametrize in terms of real components $(U(1) \simeq SO(2))$

  ↳ $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ $\quad \phi_i \in \mathbb{R}$ $\quad \Rightarrow \phi^\dagger \phi = \phi^* \phi = |\phi|^2 = \frac{1}{2}(\phi_1^2 + \phi_2^2)$

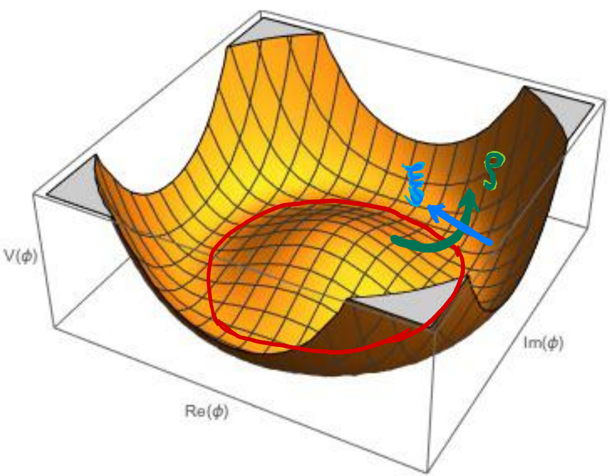
* $\lambda > 0$ potential must be bounded from below (stable)

* freedom: sign of μ^2 term

  ↳ $\mu^2 > 0$ boring $\longleftrightarrow$ simple mass term $m_\phi = \mu$

  ↳ $\mu^2 < 0$ degenerate minimum for $V(\phi)$

Abelian Higgs Model — The Potential



* minimum of the potential ($\leftrightarrow$ vacuum)

$$|\phi|_{vacuum} = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} = |\phi_0| \qquad \left(\phi_0 = \langle 0|\phi|0\rangle\right)$$

$\hookrightarrow$ infinitely many equivalent configurations

$\hookrightarrow$ physics does not depend on it, but we have to make a choice

$\longrightarrow$ "spontaneous" $\quad Re[\phi_0] = \frac{v}{\sqrt{2}}$, $Im[\phi_0] = 0$

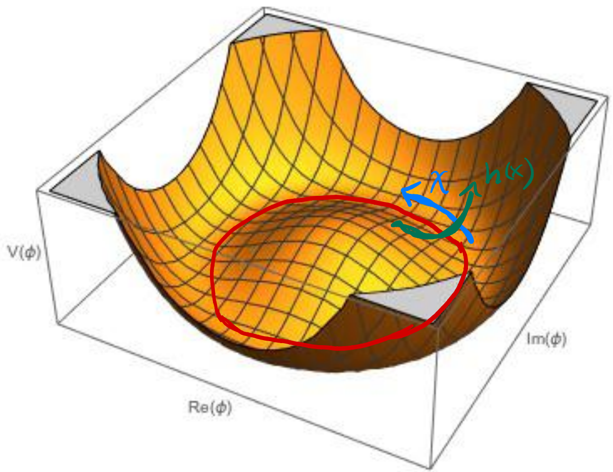
* parametrize deviations from the vacuum ("cartesian")

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x) + i\,\xi(x)\right)$$

$\hookrightarrow$ $V(\phi) = -\frac{\mu^4}{4\lambda} + \mu^2\rho^2 + (\text{cubic, quartic})$ $\rightsquigarrow$ $\begin{cases} \text{massive } \rho & m_\rho^2 = -2\mu^2 \\ \text{massless } \xi & \text{"Goldstone boson"} \end{cases}$

$\hookrightarrow$ $(D_\mu\phi)^\dagger(D^\mu\phi)$ $\rightsquigarrow$ mixing $\quad g A_\mu(\partial^\mu\xi)$ $\rightsquigarrow$ physical d.o.f. with the "unitary gauge"

Abelian Higgs Model — The Potential



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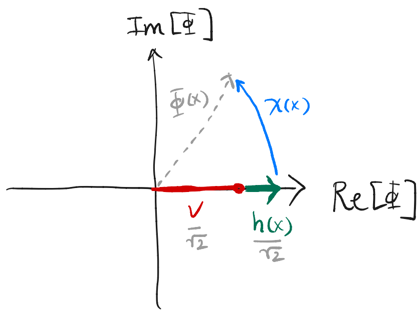
↪ infinitely many equivalent configurations

↪ physics does <u>not</u> depend on it, but we have to make a choice

⟶ "spontaneous" $\mathrm{Re}[\phi_0] = \frac{v}{\sqrt{2}}$, $\mathrm{Im}[\phi_0] = 0$

* parametrize deviations from the vacuum ("polar")



$$\phi(x) = \frac{1}{\sqrt{2}} e^{i\chi(x)} (v + h(x))$$

⇝ χ field can be eliminated by a gauge trafo!

Abelian Higgs Model — The Physical Spectrum

* $\phi(x) = \frac{1}{\sqrt{2}} e^{i\chi(x)} (v + h(x))$ has 2 degrees of freedom: $\chi(x)$ & $h(x)$ $\in \mathbb{R}$
  - $\chi(x)$ is unphysical ⇝ can be gauged away
  - unitary gauge ⇝ $\phi \to e^{-i\chi}\phi$
* read off physical content

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_\mu h)^2 + \underbrace{\frac{1}{2} g^2 v^2 A_\mu A^\mu}_{\text{mass term}} + \mu^2 h^2 \quad + (\text{interactions})$$

  - h (Higgs boson) with mass $M_h^2 = -2\mu^2$
  - the photon acquired a mass $M_A^2 = g^2 v^2$ ("ate" the Goldstone)
    - → massless photon : 2 transverse pol^n
    - massive photon : 2 transv. ⊕ 1 long. pol^n
* $\mathcal{L}$ still has full $U(1)$ gauge symmetry ("hidden")

3 Strong Interactions

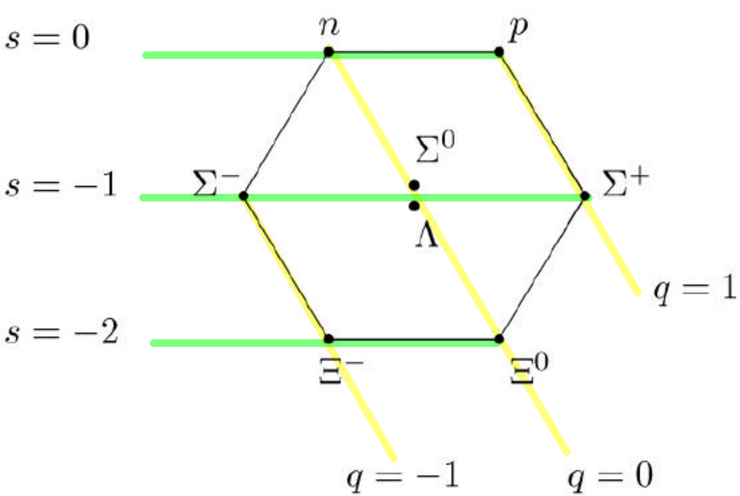
* Large Hadron Collider (LHC) & HL-LHC upgrade
  - ↪ will drive particle physics in the years to come
  - ↪ proton-proton collider ⇒ cannot avoid strong interactions (QCD)
* very rich phenomenology
  - ↪ low energies: non-perturbative bound states: hadrons
  - ↪ high energies: behaves like collection of free constituents: partons

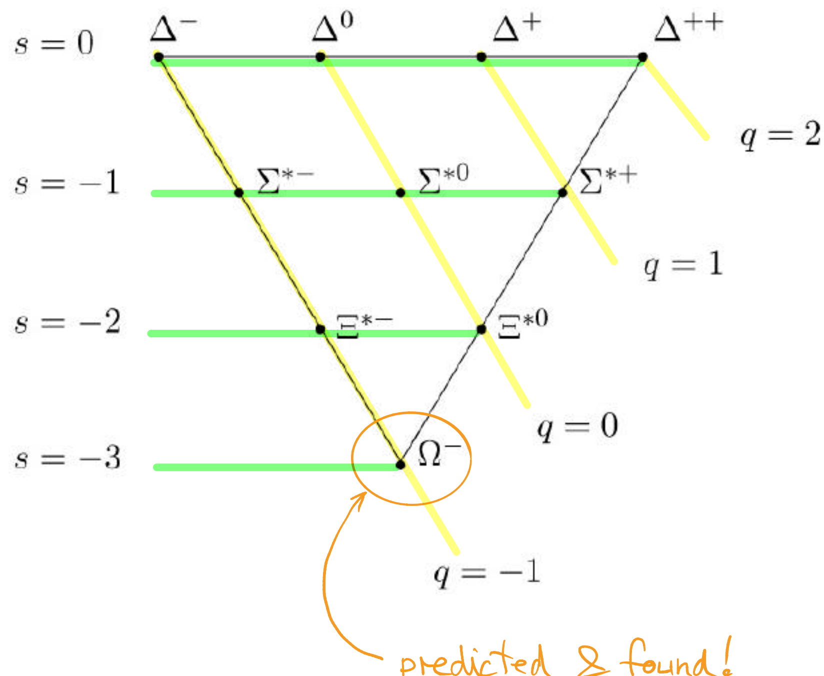
Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex. But not always.

Frank Wilczek

The Eightfold Way [Gell-Mann '61]

- hadron spectrum exhibits a pattern (s = "strangeness")





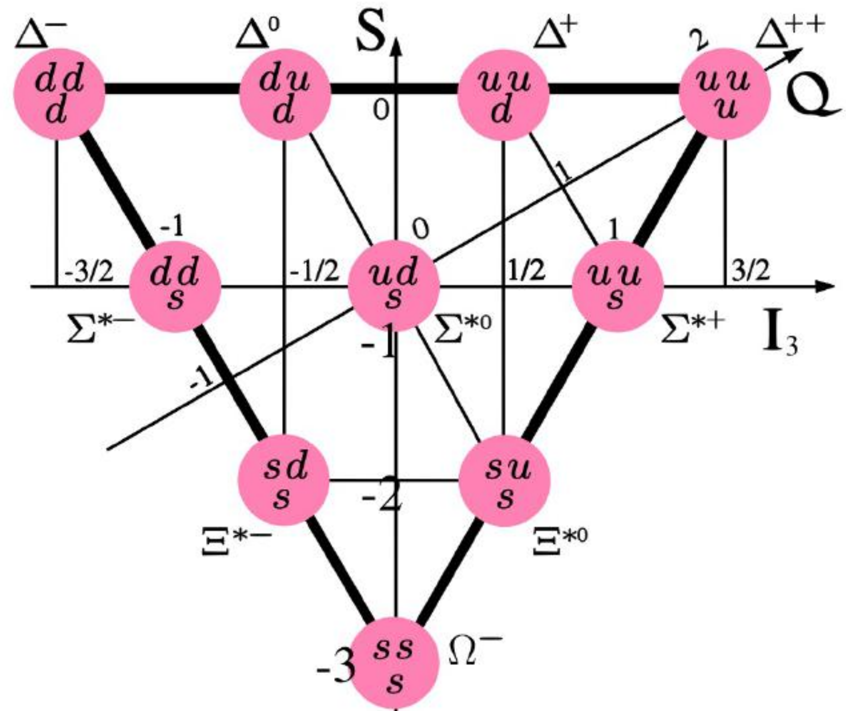
↳ What is the reason for this pattern?

The Quark Model [Gell-Mann, Zweig '64]

- proposal: spin-$\frac{1}{2}$ constituents (quarks: $\underbrace{u, d, s}_{}$)
  with fractal charges
  SU(3) flavour symmetry

u	$m_u \sim 4$ MeV	($Q = +2/3$)
d	$m_d \sim 7$ MeV	($Q = -1/3$)
S	$m_s \sim 135$ MeV	($Q = -1/3$)

proton: stable



⇒ "Explains" the pattern

but no free quarks can be seen!

The Spin-Statistics Issue

Δ^{++} is a state with

- spin $3/2$: $|\uparrow\uparrow\uparrow\rangle$
- $3\times$ up: $|uuu\rangle$ $(Q=+2)$

↯ Pauli's exclusion principle

⇒ Solution: new quantum number

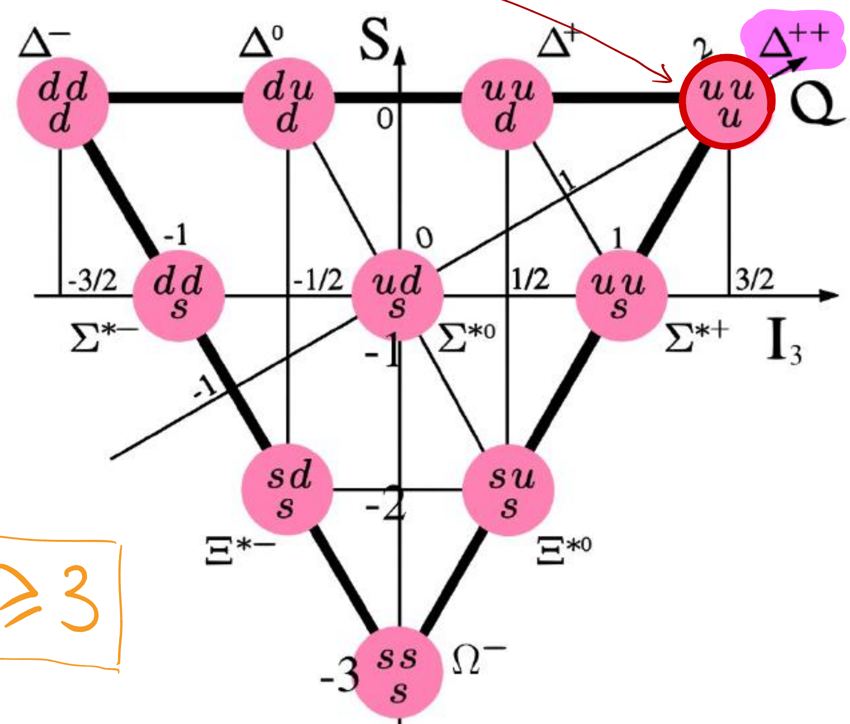
red green blue ...

u^1 u^2 u^3 ? } COLOUR

$\Delta^{++} \sim \epsilon_{ijk}\, u^i u^j u^k$

→ fully anti-symmetric

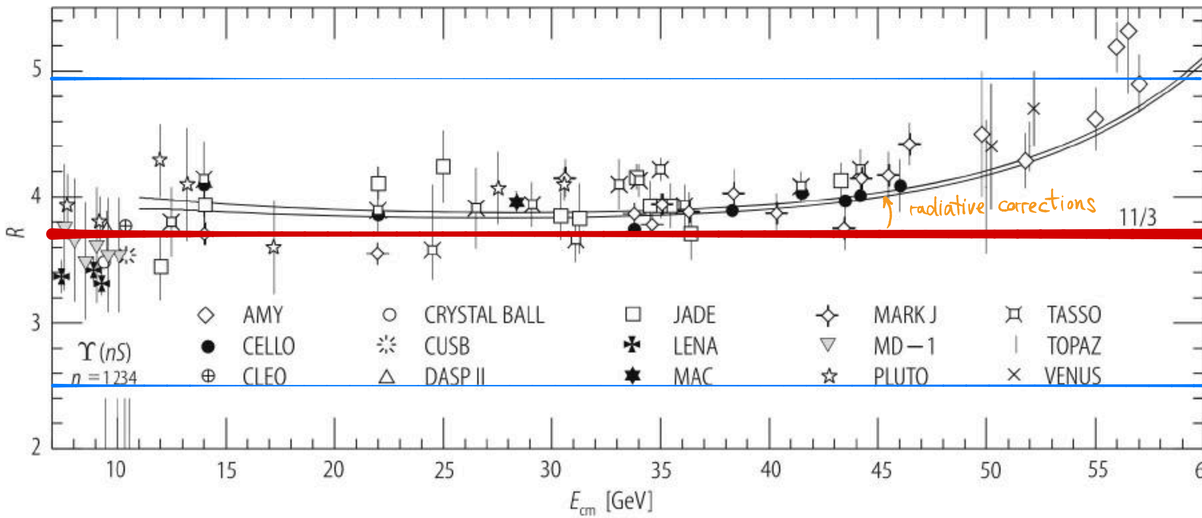
$\boxed{N_c \geq 3}$



Evidence for Colour

* The R-ratio: $R \equiv \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \propto N_c \sum_q Q_q^2$

lowest-order prediction $\quad R = N_c \frac{11}{9}$



$N_c = 4$

$N_c = 3$

$N_c = 2$

* pion decay: $\Gamma(\pi \to \gamma\gamma) \propto N_c^2$
* ABJ anomaly cancellation
* much much more ... (BR of W, τ decays)

$$\boxed{N_c = 3}$$

QCD & Colour Confinement

* QCD has an exact $SU(N_c)$ symmetry:

why not $SO(N_c)$?

$$UU^\dagger = U^\dagger U = \mathbb{1}\,, \qquad \det(U) = 1$$

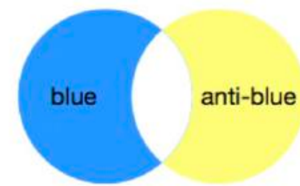
* no isolated colour charges $\left(V_{q\bar q}(r) \simeq C_F \left[\frac{\alpha_s(r)}{r} + \dots + \sigma r \right] \right)$

⇒ only colour singlet particles ⇝ hadrons have integer electr. charge

① MESONS (bosons: $\pi, \rho, \dots$)

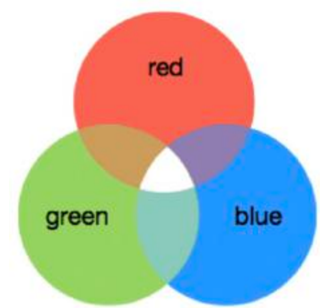
$$\bar q^i q^i \longrightarrow U^*_{ij}\, \bar q^j\, U_{ik}\, q^k = \bar q^i q^i$$

$$(U^\dagger)_{ji}\, U_{ik} = \delta_{jk}$$



② BARYONS (fermions: $p, n, \dots$)

$$\epsilon_{ijk}\, q^i q^j q^k \longrightarrow \underbrace{\epsilon_{ijk}\, U_{ii'} U_{jj'} U_{kk'}}_{\det(U)\,\epsilon_{i'j'k'}}\, q^{i'} q^{j'} q^{k'} = \epsilon_{ijk}\, q^i q^j q^k$$

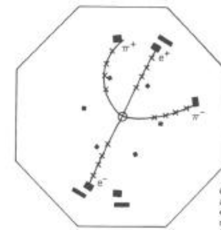


The Rest of the Family

eightfold way

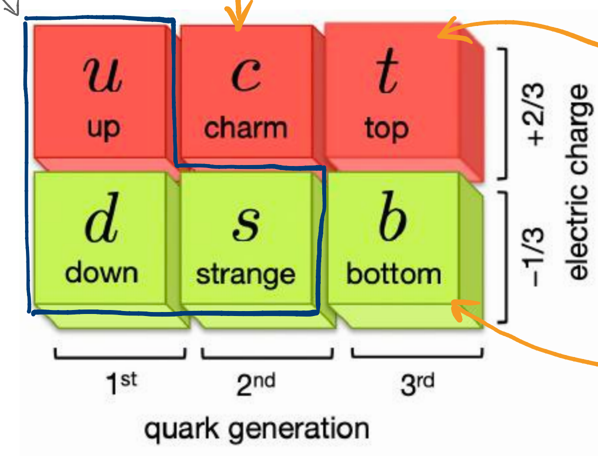
[postulated '70
GIM mechanism]

discovery '74



Computer reconstruction of a ψ' decay in the Mark I detector at SLAC, making a near-perfect image of the Greek letter ψ

$J/\psi \hat{=} (c\bar{c})$
"charmonium"



['87 $m_t > 50$ GeV (B-oscillation)
until '94 $m_t \in [145, 185]$ GeV
(EW precision data)
discovery '95 $m_t = 173$ GeV]

[postulated '73
CP violation
Kobayashi & Maskawa]
↓
discovery '77 (Υ)

Quantum Chromodynamics

* gauge principle $SU(3) \longleftrightarrow \psi_q = (\psi_q^r, \psi_q^g, \psi_q^b)^T$

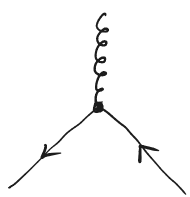
$$\mathcal{L}_{QCD} = \bar{\psi}_q (i\not{\partial} - m_q)\psi_q - \frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{a,\nu} - \partial^\nu A^{a,\mu}) \quad \longleftrightarrow \quad \text{"free"}$$

$$- g_s t^a A_\mu^a \bar{\psi}_q \gamma^\mu \psi_q \quad \longleftrightarrow \quad \bar{q}qg$$

$$+ \frac{g_s}{2} f^{abc} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) A^{b,\mu} A^{c,\nu} \quad \longleftrightarrow \quad g^3$$

$$- \frac{g_s^2}{4} f^{abc} f^{ade} A_\mu^b A_\nu^c A^{d,\mu} A^{e,\nu} \quad \longleftrightarrow \quad g^4$$

* pictorial repn of quark-gluon interaction

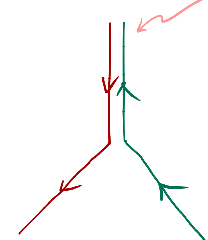


$\bar{\psi}^i \quad t_{ij}^a \quad \psi^j$

$$(1, 0, 0) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \longleftrightarrow$$

$\bar{\psi}^r \quad t^1 \quad \psi^g$

gluons carry colour & anti-colour



↪ "repaint" the quarks

Asymptotic Freedom

[Gross, Wilczek, Politzer '73]

* coupling strength $\alpha_s = \frac{g_s^2}{4\pi}$ depends on the scale Q^2 at which it is probed
* dependence is predicted (up to 5-loops)

$$\frac{d \ln \alpha_s(\mu^2)}{d \ln \mu^2} = \beta(\alpha_s) = -\frac{\alpha_s}{2\pi}\left[\beta_0 + \frac{\alpha_s}{2\pi}\beta_1 + \ldots\right]$$

$$\beta_0 = \frac{11}{6} C_A - \frac{N_f}{3}$$

anti-screening

screening

$\alpha_s(M_Z^2) = 0.1179 \pm 0.0010\,.$