Standard Model Precision Physics

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1. A brief recap of Quantum Field Theories
2. The main construction Principles for QFTs
3. Strong Interactions
4. The electroweak Standard Model
   * bottom-up construction & Z-pole observables
   * EW precision tests & renormalization
5. LHC Phenomenology
   * Parton Distribution Functions
   * The Drell-Yan process & higher-order calculations
   * Transverse momentum resummation
   * Parton Showers & Event generators

The Standard Model @ Tree level

* our explorations so far were Born-level (LO, tree) predictions
  - ↪ Let us be more quantitative in the comparison
* Some SM predictions @ tree level ($s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$)
  - ↪ $\alpha^{(0)} = \dfrac{e^2}{4\pi}$
  - ↪ $G_F^{(0)} = \dfrac{\alpha\pi}{\sqrt{2}\, s_W^2 M_W^2} = \dfrac{1}{\sqrt{2}\, v^2}$
  - ↪ $M_Z^{(0)} = M_Z = \dfrac{e\, v}{2\, s_W c_W}$
  - ↪ $M_W^{(0)} = M_W = \dfrac{e v}{2\, s_W}$
  - ↪ $(s_{W,\mathrm{eff}}^{(0)})^2 = s_W^2$
  - ↪ $\Gamma^{(0)}_{Z\to e^+e^-} = \dfrac{\alpha M_Z}{6\, s_W^2 c_W^2}\left[\left(-\tfrac{1}{2}+s_W^2\right)^2 + s_W^4\right] = \dfrac{e^3 v}{48\pi\, s_W^3 c_W^3}\left[\left(-\tfrac{1}{2}+s_W^2\right)^2 + s_W^4\right]$

all of them only depend on 3 EW Lagrangian parameters (@ tree level)

(e, v, s_W)

The Standard Model @ Tree level

... and they are among the most precisely measured

$$\hat{\alpha}_0 = (137.035\,999\,084(21))^{-1}$$

$$\hat{G}_F = 1.166\,378\,8(6) \cdot 10^{-5}\ \text{GeV}^{-2}$$

$$\hat{M}_Z = 91.1876(21)\ \text{GeV}$$

① pick as input (most precise)

$$\hat{M}_W = 80.377(12)\ \text{GeV}$$

$$(\hat{s}_{W,\text{eff}})^2 = 0.231\,53(4)$$

② predict these

$\Rightarrow$ check for consistency

The Standard Model @ Tree level

① invert tree-level relations $\hat{O} = O^{(0)}$ to get Lagrange parameters

↪ $e^2 = 4\pi\hat{\alpha}$

↪ $v^2 = \dfrac{1}{\sqrt{2}\,\hat{G}_F}$

↪ $\dfrac{e^2 v^2}{4 s_W^2 c_W^2} = M_Z^2 \Rightarrow s_W^2 = \dfrac{1}{2}\left(1 - \sqrt{1 - \dfrac{4\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F \hat{M}_Z^2}}\right)$

② insert into predictions for remaining observables

↪ $M_W^{(0)} = \dfrac{e v}{2 s_W} = 80.9389 \longleftrightarrow \mathcal{O}(10\sigma)$

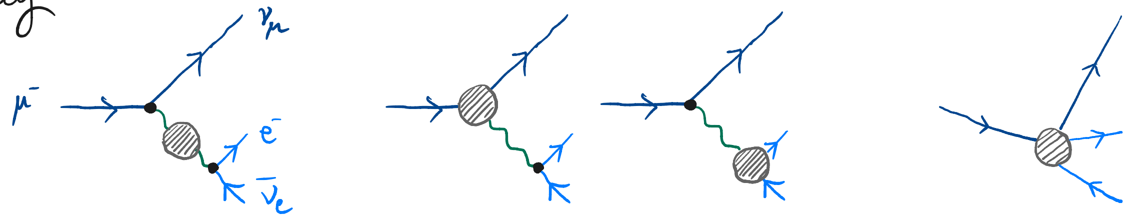
↪ $(s_{W,\mathrm{eff}}^{(0)})^2 = s_W^2 = 0.212152 \longleftrightarrow \mathcal{O}(100\sigma)$

SM is excluded?!

Radiative corrections

* higher-order corrections require evaluation of loop diagrams

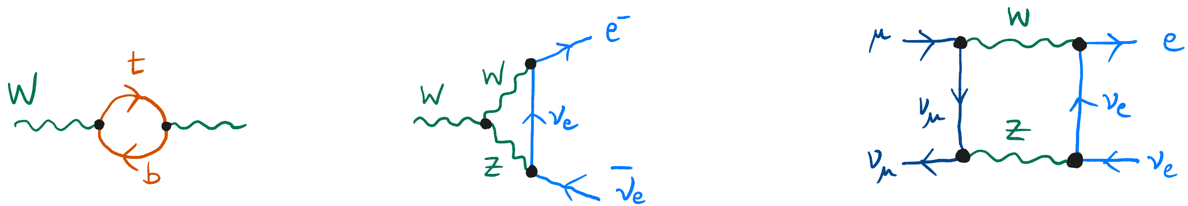
→ μ decay



self energies/ bubbles

vertices/ triangles

boxes

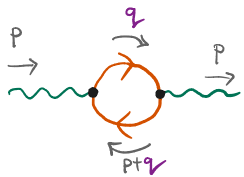


→ one unconstrained momentum q

$$\Rightarrow \int \frac{d^4q}{(2\pi)^4}$$ (QM: sum over all intermediate states)

Ultraviolet (UV) Divergences

* loop integrations can be divergent



$$\sim \int \frac{d^4q}{(2\pi)^4} \frac{\{1, q, q^2\}}{(q^2-m_1^2)\,((q+p)^2-m_2^2)} \xrightarrow{q \text{ large}} \int \frac{d^4q}{q^4}$$

"UV"

logarithmic divergence

* regularization ⇝ make expression finite & well defined (TH extension)

↳ most common dimensional regularization $D = 4-2\epsilon$ dimensions

⟷ original (divergent) theory : $\epsilon \mapsto 0$

⟷ divergences appear as poles : $1/\epsilon^n$

$$\int \frac{d^4q}{q^4} \to \int d\Omega_D \int_\delta^\infty dq \, \frac{q^{D-1}}{q^4} = \frac{(2\pi)^{2-\epsilon}}{\Gamma(2-\epsilon)} \frac{1}{2\epsilon} \frac{1}{\delta^{2\epsilon}} \propto \frac{1}{\epsilon} + \ln\delta + \mathcal{O}(\epsilon)$$

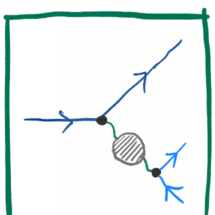
* renormalization ⇝ relations between physical quantities

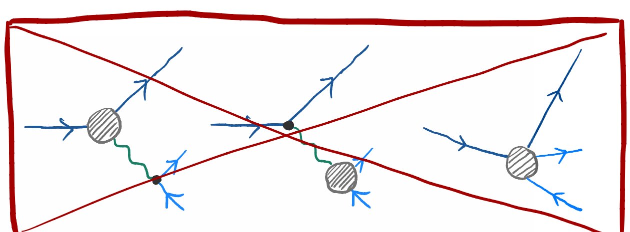
⟷ finite if theory is renormalizable

Oblique Corrections

* full renormalization of the SM beyond the scope here
  ↪ dedicated tutorials by Stefan Kallweit on radiative corr.

* we will instead use the set of EW precision observables from above to demonstrate UV cancellation explicitly
  ↪ @ tree level ⟷ exchange of gauge bosons
  ⇒ largest contributions from self energies "oblique corrections"





⟷ tractable subset
well defined (fermion loops)
largest effects from (t, b)

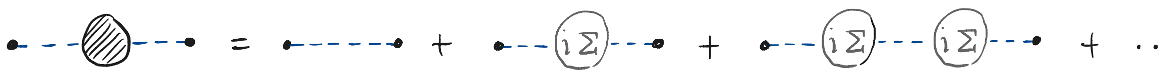
Particle Masses & Radiative Corrections

* How do we define a particle's mass in a QFT?

Particle Masses & Radiative Corrections

* How do we define a particle's mass in a QFT?
  - it is the location of the propagator pole $\frac{i}{p^2-m^2}$
  - How do radiative corrections impact the pole position?

example for a scalar



$$= \frac{i}{p^2-m^2} + \frac{i}{p^2-m^2}\, i\Sigma_i\, \frac{i}{p^2-m^2} + \dots = \frac{i}{p^2-m^2} \sum_{n=0}^{\infty} \left(i\Sigma_i \frac{i}{p^2-m^2} \right)^n$$

Dyson summation

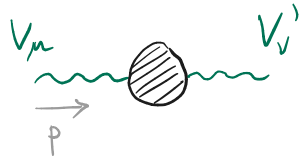
$\Sigma a^n \to \frac{1}{1-a}$

$$= \frac{i}{p^2-m^2+\Sigma_i(p^2)}$$

$\Rightarrow$ higher-order corrections impact the location & residue of propagator pole!

Gauge boson Self Energy & Corrections to the Mass

* gauge-boson self energy

$$-i\,\Sigma^{V_\mu V'_\nu}(p) = $$  $$= -i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Sigma_T^{VV'}(p^2) - i\,\frac{p_\mu p_\nu}{p^2}\,\Sigma_L^{VV'}(p^2)$$

→ In the following, we will only need $\Sigma_T^{VV'}$

- for mass definition, we project on physical states $\varepsilon^\mu(p)\,p_\mu \equiv 0$
- for our observables, we neglect fermion masses $[\bar{\psi}\gamma^\mu\psi]\,p_\mu \to 0$

* the on-shell mass

will drop it in the following

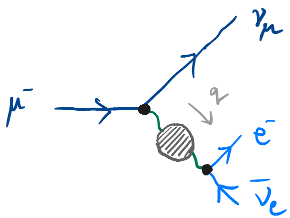
$$\mathrm{Re}[\text{"propagator denominator"}] \stackrel{!}{=} 0 = M_{V,os}^2 - M_V^2 + \mathrm{Re}\left[\Sigma_T^{VV}(M_{V,os}^2)\right]$$

$$\Rightarrow \quad M_{V,os}^2 = M_V^2 - \Sigma_T^{VV}(M_{V,os}^2)$$

parameter in $\mathcal{L}$

Oblique Corrections to G_F

* observable defined via μ-decay $\longleftrightarrow$ $\tau_\mu^{-1} = \Gamma_\mu$



use $\dfrac{-i g_{\mu\nu}}{q^2 - M_W^2 + \Sigma_T^{WW}(q^2)} \xrightarrow[q^2 \to 0]{m_\mu^2 \ll M_W^2} i g_{\mu\nu} \left[\underbrace{\frac{1}{M_W^2}}_{\text{LO}} + \frac{\Sigma_T^{WW}(0)}{M_W^4} + \dots \right]$

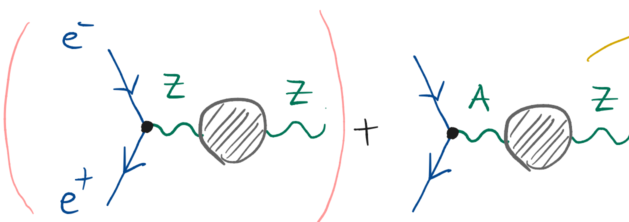
$$\Rightarrow \quad G_F^{(1)} = G_F^{(0)} \left[1 + \frac{\Sigma_T^{WW}(0)}{M_W^2} \right]$$

Oblique Corrections to $s^2_{w,\text{eff}}$

* the effective weak mixing angle can be defined via the Z-boson polarization asymmetry $A_e = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{\sigma(e^+_L e^-_L \to Z) - \sigma(e^+_R e^-_R \to Z)}{\sigma(e^+_L e^-_L \to Z) + \sigma(e^+_R e^-_R \to Z)}$

define $\hat{A}_e \equiv \frac{\left[\frac{1}{2} - (\hat{s}_{w,\text{eff}})^2\right]^2 - (\hat{s}_{w,\text{eff}})^4}{\left[\frac{1}{2} - (\hat{s}_{w,\text{eff}})^2\right]^2 + (\hat{s}_{w,\text{eff}})^4}$ ⟵ tree-level relation $s_w \longmapsto s_{w,\text{eff}}$

* higher-order corrections



gives a change to effective coupling

$$g_e^- = \frac{I_W^3 - s_W^2 Q_f}{s_W c_W} \longmapsto g_e^- - \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} Q_f$$

$$g_e^+ = -\frac{s_W}{c_W} Q_f \longmapsto g_e^+ - \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} Q_f$$

overall normalization to $\sigma_{L/R}$ ⇒ cancels in ratio!

⟺ LSZ / WF norm. ⟷ $R_Z \propto \left.\frac{d\Sigma_T^{ZZ}(q^2)}{dq^2}\right|_{q^2 = M_Z^2}$

Oblique Corrections to $s^2_{W,\mathrm{eff}}$

$$\frac{I_W^3 - s_W^2 Q_f}{s_W c_W} - \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} Q_f = \frac{1}{s_W c_W}\left[I_W^3 - \left(s_W^2 + s_W c_W \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} \right) Q_f \right]$$

$$- \frac{s_W}{c_W} Q_f - \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} Q_f = - \frac{Q_f}{s_W c_W} \left(s_W^2 + s_W c_W \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} \right)$$

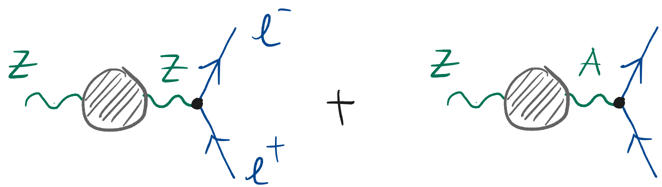
$\hookrightarrow$ looks like tree-level coupling after $s_W^2 \longmapsto \left(s_{W,\mathrm{eff}}^{(1)}\right)^2$ in numerator

$$\Rightarrow \left(s_{W,\mathrm{eff}}^{(1)}\right)^2 = s_W^2 + s_W c_W \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2}$$

Oblique Corrections to $\Gamma_{Z\to e^+e^-}$

* Z-decay $\leftrightarrow$ no longer ratio $\Rightarrow$ normalization matters

$\hookrightarrow$ 2nd diagram captured by $s_{W,\text{eff}}$



$$\Rightarrow \quad \Gamma^{(1)}_{Z\to e^+e^-} = \left(1 - \Sigma_T^{ZZ\,\prime}(M_Z^2)\right) \Gamma^{(0)}_{Z\to e^+e^-}$$

$s_W \to s^{(1)}_{W,\text{eff}}$ in numerator

$M_Z \to M^{(1)}_{Z,\text{os}}$

Oblique Corrections to α

* $\alpha_\emptyset \sim 1/137$ is not a suitable input if we're not dealing w/ ext. γ's
  $\leftrightarrow \Pi(\emptyset) \equiv \left.\frac{\Sigma_T^{AA}(q^2)}{q^2}\right|_{q^2\to\emptyset}$ induces sensitivity to mass-singular terms $\sim \ln(m_f)$

* avoided with coupling defined at high scale $\rightsquigarrow \alpha(M_Z)$ (or α_{G_μ})

$$\Rightarrow \quad \alpha^{(1)}(M_Z) = \frac{e^2}{4\pi}\left[1 - \frac{\Sigma_T^{AA}(M_Z^2)}{M_Z^2}\right]$$

running from $Q^2=\emptyset \longrightarrow Q^2 = M_Z^2$

$\Delta\alpha(M_Z) = \Pi^{AA}(\emptyset) - \Pi^{AA}(M_Z^2) \sim 6\%$

* need to use corresponding input value
  $\hat{\alpha}(M_Z) = (127.951(9))^{-1}$

EPO @ 1-loop

* 1-loop predictions* in terms of (bare) Lagrange parameters

$$\hookrightarrow \alpha^{(1)}(M_Z) = \frac{e^2}{4\pi}\left[1 - \frac{\Sigma_T^{AA}(M_Z^2)}{M_Z^2}\right]$$

$$\hookrightarrow G_F^{(1)} = \frac{\alpha\pi}{\sqrt{2}\, s_W^2 M_W^2} = \frac{1}{\sqrt{2}\, v^2}\left[1 + \frac{\Sigma_T^{WW}(\emptyset)}{M_W^2}\right]$$

$$\hookrightarrow \left(M_{Z,os}^{(1)}\right)^2 = M_Z^2 - \Sigma_T^{ZZ}(M_Z^2)$$

① pick as input

$$\hookrightarrow \left(M_{W,os}^{(1)}\right)^2 = M_W^2 - \Sigma_T^{WW}(M_W^2)$$

$$\hookrightarrow \left(s_{W,eff}^{(1)}\right)^2 = s_W^2 + s_W c_W \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2}$$

$$\hookrightarrow \Gamma^{(1)}_{Z\to e^+e^-} = \frac{\alpha M_{Z,os}}{6\, s_W^2 c_W^2}\left[\left(-\frac{1}{2} + s_{W,eff}^2\right)^2 + s_{W,eff}^4\right]$$

② predict these

beyond our control

* take Re[...] where appropriate ; note: $\Sigma(M_{os}^2) = \Sigma(M^2) + \mathcal{O}(\alpha^2)$
* remember: only oblique corrections

* $\Sigma \leftrightarrow$ UV divergent!

EPO @ 1-loop

① invert relations $\hat{O} = O^{(1)}$ to get Lagrange parameters*

$$e^2 = 4\pi\hat{\alpha}(M_Z)\left[1 + \frac{\Sigma_T^{AA}(\hat{M}_Z^2)}{\hat{M}_Z^2}\right]$$

$$v^2 = \frac{1}{\sqrt{2}\,\hat{G}_F}\left[1 + \frac{\Sigma_T^{WW}(0)}{\hat{M}_W^2}\right]$$

$$\hat{M}_Z^2 = \underbrace{\frac{e^2 v^2}{4 s_W^2 c_W^2}}_{M_Z^2} - \Sigma_T^{ZZ}(\hat{M}_Z^2)$$

$$\delta_S \equiv -\frac{\Sigma_T^{ZZ}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{AA}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{WW}(0)}{\hat{M}_W^2}$$

$$\Rightarrow s_W^2 = \underbrace{\frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F\hat{M}_Z^2}}\right)}_{(s_{W,\mathrm{eff}}^{(0)})^2}\left(1 + \underbrace{\frac{c_W^2}{c_W^2 - s_W^2}}\delta_S\right) = \hat{s}_W^{(0)\,2}\left(1 + \frac{\hat{c}_W^{(0)\,2}}{\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}}\delta_S\right)$$

$O(\alpha)$

$\Rightarrow$ can use tree-level relations

* UV divergent!

EPO @ 1-loop

② insert into predictions for remaining observables

$$\Rightarrow \quad \left(s_{W,\mathrm{eff}}^{(1)}\right)^2 = \hat{s}_W^{(0)2} + \frac{\hat{s}_W^{(0)2}\,\hat{c}_W^{(0)2}}{\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}} \left(\delta_S + \frac{\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}}{\hat{s}_W^{(0)}\,\hat{c}_W^{(0)}} \, \frac{\Sigma_T^{AZ}(\hat{M}_Z^2)}{\hat{M}_Z^2} \right)$$

$$\Rightarrow \quad \left(M_{W,\mathrm{os}}^{(1)}\right)^2 = \frac{\pi\,\hat{\alpha}(M_Z)}{\sqrt{2}\,\hat{G}_F\,\hat{s}_W^{(0)2}} \left(1 + \frac{\Sigma_T^{AA}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{WW}(0)}{\hat{M}_W^2} - \frac{\hat{c}_W^{(0)2}}{\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}}\,\delta_S - \frac{\Sigma_T^{WW}(\hat{M}_W^2)}{\hat{M}_W^2} \right)$$

$$\delta_S = -\frac{\Sigma_T^{ZZ}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{AA}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{WW}(0)}{\hat{M}_W^2}$$

* expressed everything in terms of measured quantities $\hat{\mathcal{O}}$

  $\Rightarrow$ Renormalizable theory $\Rightarrow$ all UV divergences cancel

* all we need to do is insert $\Sigma^{VV'}$ ...

EPO @ 1-loop

[Denner arXiv:0709.1075]

$$
\begin{aligned}
\Sigma_T^{AA}(k^2) = -\frac{\alpha}{4\pi}\bigg\{&\frac{2}{3}\sum_{f,i} N_C^f 2Q_f^2\Big[-(k^2+2m_{f,i}^2)B_0(k^2,m_{f,i},m_{f,i}) \\
&+ 2m_{f,i}^2 B_0(0,m_{f,i},m_{f,i}) + \frac{1}{3}k^2\Big] \\
&+ \Big\{\Big[3k^2+4M_W^2\Big]B_0(k^2,M_W,M_W) - 4M_W^2 B_0(0,M_W,M_W)\Big\}\bigg\},
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
\Sigma_T^{AZ}(k^2) = -\frac{\alpha}{4\pi}\bigg\{&\frac{2}{3}\sum_{f,i} N_C^f(-Q_f)\big(g_f^+ + g_f^-\big)\Big[-(k^2+2m_{f,i}^2)B_0(k^2,m_{f,i},m_{f,i}) \\
&+ 2m_{f,i}^2 B_0(0,m_{f,i},m_{f,i}) + \frac{1}{3}k^2\Big] \\
&- \frac{1}{3s_W c_W}\Big\{\Big[(9c_W^2+\frac{1}{2})k^2 + (12c_W^2+4)M_W^2\Big]B_0(k^2,M_W,M_W) \\
&- (12c_W^2-2)M_W^2 B_0(0,M_W,M_W) + \frac{1}{3}k^2\Big\}\bigg\},
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
\Sigma_T^{ZZ}(k^2) = -\frac{\alpha}{4\pi}\bigg\{&\frac{2}{3}\sum_{f,i} N_C^f\Big\{\big((g_f^+)^2+(g_f^-)^2\big)\Big[-(k^2+2m_{f,i}^2)B_0(k^2,m_{f,i},m_{f,i}) \\
&+ 2m_{f,i}^2 B_0(0,m_{f,i},m_{f,i}) + \frac{1}{3}k^2\Big] + \frac{3}{4s_W^2c_W^2}m_{f,i}^2 B_0(k^2,m_{f,i},m_{f,i})\Big\} \\
&+ \frac{1}{6s_W^2c_W^2}\Big\{\Big[(18c_W^4+2c_W^2-\frac{1}{2})k^2 + (24c_W^4+16c_W^2-10)M_W^2\Big]B_0(k^2,M_W,M_W) \\
&- (24c_W^4-8c_W^2+2)M_W^2 B_0(0,M_W,M_W) + (4c_W^2-1)\frac{1}{3}k^2\Big\} \\
&+ \frac{1}{12s_W^2c_W^2}\Big\{\Big(2M_H^2-10M_Z^2-k^2\Big)B_0(k^2,M_Z,M_H) \\
&- 2M_Z^2 B_0(0,M_Z,M_Z) - 2M_H^2 B_0(0,M_H,M_H) \\
&- \frac{(M_Z^2-M_H^2)^2}{k^2}\Big(B_0(k^2,M_Z,M_H)-B_0(0,M_Z,M_H)\Big) - \frac{2}{3}k^2\Big\}\bigg\},
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
\Sigma_T^{W}(k^2) = -\frac{\alpha}{4\pi}\bigg\{&\frac{2}{3}\frac{1}{2s_W^2}\sum_i\Big[-\Big(k^2-\frac{m_{l,i}^2}{2}\Big)B_0(k^2,0,m_{l,i}) + \frac{1}{3}k^2 \\
&+ m_{l,i}^2 B_0(0,m_{l,i},m_{l,i}) + \frac{m_{l,i}^4}{2k^2}\Big(B_0(k^2,0,m_{l,i}) - B_0(0,0,m_{l,i})\Big)\Big] \\
&+ \frac{2}{3}\frac{1}{2s_W^2}3\sum_{i,j}|V_{ij}|^2\Big[-\Big(k^2-\frac{m_{u,i}^2+m_{d,j}^2}{2}\Big)B_0(k^2,m_{u,i},m_{d,j}) + \frac{1}{3}k^2 \\
&+ m_{u,i}^2 B_0(0,m_{u,i},m_{u,i}) + m_{d,j}^2 B_0(0,m_{d,j},m_{d,j}) \\
&+ \frac{(m_{u,i}^2-m_{d,j}^2)^2}{2k^2}\Big(B_0(k^2,m_{u,i},m_{d,j}) - B_0(0,m_{u,i},m_{d,j})\Big)\Big] \\
&+ \frac{2}{3}\Big\{\Big(2M_W^2+5k^2\Big)B_0(k^2,M_W,\lambda) - 2M_W^2 B_0(0,M_W,M_W) \\
&- \frac{M_W^4}{k^2}\Big(B_0(k^2,M_W,\lambda) - B_0(0,M_W,\lambda)\Big) + \frac{1}{3}k^2\Big\} \\
&+ \frac{1}{12s_W^2}\Big\{\Big[(40c_W^2-1)k^2 + (16c_W^2+54-10c_W^{-2})M_W^2\Big]B_0(k^2,M_W,M_Z) \\
&- (16c_W^2+2)\Big[M_W^2B_0(0,M_W,M_W) + M_Z^2B_0(0,M_Z,M_Z)\Big] + (4c_W^2-1)\frac{2}{3}k^2 \\
&- (8c_W^2+1)\frac{(M_W^2-M_Z^2)^2}{k^2}\Big(B_0(k^2,M_W,M_Z) - B_0(0,M_W,M_Z)\Big)\Big\} \\
&+ \frac{1}{12s_W^2}\Big\{\Big(2M_H^2-10M_W^2-k^2\Big)B_0(k^2,M_W,M_H) \\
&- 2M_W^2B_0(0,M_W,M_W) - 2M_H^2B_0(0,M_H,M_H) \\
&- \frac{(M_W^2-M_H^2)^2}{k^2}\Big(B_0(k^2,M_W,M_H) - B_0(0,M_W,M_H)\Big) - \frac{2}{3}k^2\Big\}\bigg\}.
\end{aligned}
\tag{B.4}
$$

exercise: isolate the fermion loops & check UV cancellation

$B_0(q^2,m_1,m_2)$ bubble integral

The two-point function is given by

$$
\begin{aligned}
B_0(p_{10},m_0,m_1) &= \Delta - \int_0^1 dx \log\frac{[p_{10}^2x^2 - x(p_{10}^2-m_0^2+m_1^2)+m_1^2-i\varepsilon]}{\mu^2} + O(D-4) \\
&= \Delta + 2 - \log\frac{m_0m_1}{\mu^2} + \frac{m_0^2-m_1^2}{p_{10}^2}\log\frac{m_1}{m_0} - \frac{m_0m_1}{p_{10}^2}\Big(\frac{1}{r}-r\Big)\log r \\
&\quad + O(D-4),
\end{aligned}
\tag{4.23}
$$

where r and $\frac{1}{r}$ are determined from

$$
x^2 + \frac{m_0^2+m_1^2-p_{10}^2-i\varepsilon}{m_0m_1}x + 1 = (x+r)\Big(x+\frac{1}{r}\Big).
\tag{4.24}
$$

EPO @ 1-loop

$*$ only terms $\sim m_t^2$

$\Delta = \frac{1}{\epsilon} + \text{const.}$ (UV divergence)

$$\Sigma_T^{WW}(q^2)\Big|_{(t,b)} = -\frac{\alpha}{4\pi} N_c \frac{1}{2 s_W^2} m_t^2 \left[\Delta - \ln\left(\frac{m_t^2}{\mu^2}\right) + \frac{1}{2}\right] + \mathcal{O}(q^2)$$

$$\Sigma_T^{ZZ}(q^2)\Big|_{t} = -\frac{2\alpha}{\pi} N_c \frac{1}{16 s_W^2 c_W^2} m_t^2 \left[\Delta - \ln\left(\frac{m_t^2}{\mu^2}\right)\right] + \mathcal{O}(q^2)$$

$$\Rightarrow \quad \delta_S\Big|_{m_t^2} = -\frac{\alpha}{4\pi} N_c \frac{m_t^2}{2 s_W^2} \left(\frac{1}{M_W^2}\left[\cancel{\Delta - \ln\left(\frac{m_t^2}{\mu^2}\right)} + \frac{1}{2}\right] - \underbrace{\frac{1}{c_W^2}\frac{1}{M_Z^2}}^{1/M_W^2}\left[\cancel{\Delta - \ln\left(\frac{m_t^2}{\mu^2}\right)}\right] \right)$$

$$= -\frac{\alpha}{16\pi s_W^2} N_c \frac{m_t^2}{M_W^2}$$

EPO @ 1-loop

improved agreement!

* predictions only including dominant m_t^2 corrections

$$\hookrightarrow \quad \left(s_{W,\mathrm{eff}}^{(1)}\right)^2 = \hat{s}_W^{(0)\,2}\left(1 - \frac{3\,\hat{\alpha}(M_Z)}{16\pi\,\hat{s}_W^{(0)}\left(\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}\right)}\,\frac{\hat{m}_t^2}{\hat{M}_Z^2}\right) \simeq 0.230423$$

$$\hookrightarrow \quad \left(M_{W,\mathrm{os}}^{(1)}\right)^2 = \hat{M}_W^2\left(1 + \frac{3\,\hat{\alpha}(M_Z)}{16\pi\,\hat{s}_W^{(0)}\left(\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}\right)}\,\frac{\hat{m}_t^2}{\hat{M}_Z^2}\right) \simeq 80.9045$$

* radiative corrections $\rightsquigarrow$ sensitivity on top quark (Higgs)

$\hookrightarrow$ predict before discovery:

$$m_t^2 = \left(1 - \frac{\left(s_{W,\mathrm{eff}}^{(\mathrm{exp})}\right)^2}{\hat{s}_W^{(0)\,2}}\right)\frac{16\pi\,\hat{s}_W^{(0)2}\left(\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}\right)}{3\,\hat{\alpha}(M_Z)}\cdot\hat{M}_Z^2 \rightsquigarrow m_t \simeq 140\ \mathrm{GeV}$$

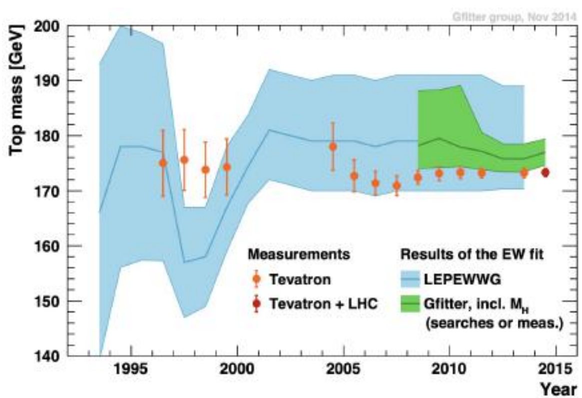
$\hookrightarrow$ Higgs mass only enters logarithmically (poorer constaint)

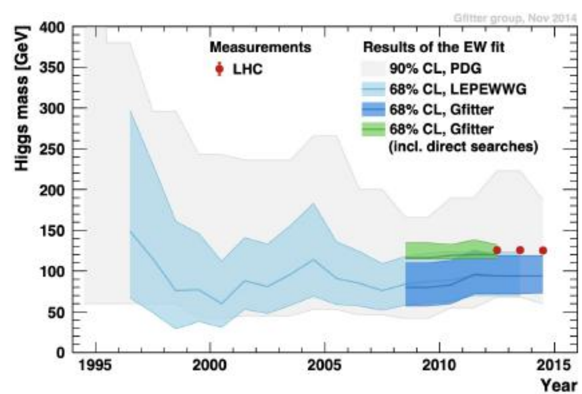
$$M_W^{(1)\,2} \longmapsto M_W^{(1)\,2} - \frac{5\,\hat{\alpha}}{24\pi}\,\frac{\hat{c}^2\,\hat{M}_Z^2}{\hat{c}^2 - \hat{s}^2}\ln\left(\hat{M}_H^2/\hat{M}_W^2\right) \quad ; \quad \left(s_{W,\mathrm{eff}}^{(1)}\right)^2 \longmapsto \left(s_{W,\mathrm{eff}}^{(1)}\right)^2 + \frac{\hat{\alpha}\,(1 + 9\,\hat{s}^2)}{48\pi\,(\hat{c}^2 - \hat{s}^2)}\ln\left(\hat{M}_H^2/\hat{M}_W^2\right)$$

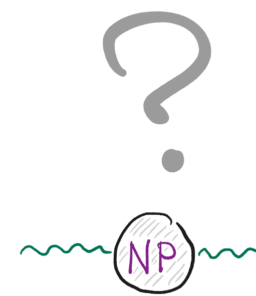
Radiative Corrections – Intermediate Summary

* We showed in a pedestrian (brute-force) way renormalization
  - ⇨ once physical (observable) quantities are expressed in terms of other physical quantities ⇒ UV singularities cancel
  - ⇨ more systematic approach through counterterms (tutorials)

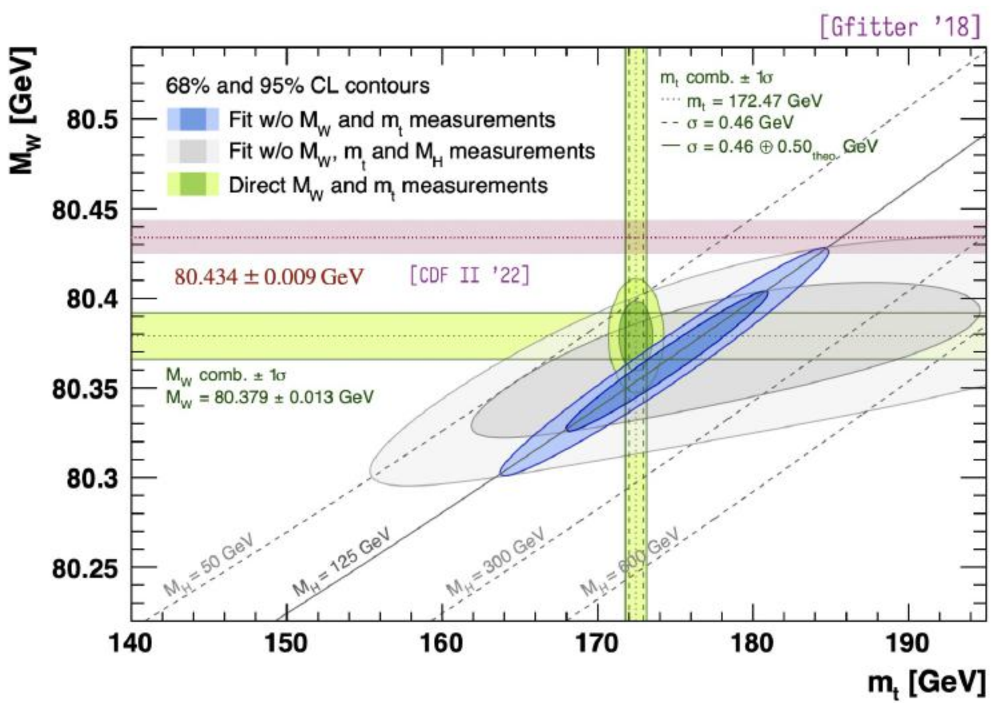
* Radiative corrections can introduce sensitivity to states that are not directly accessible (yet)

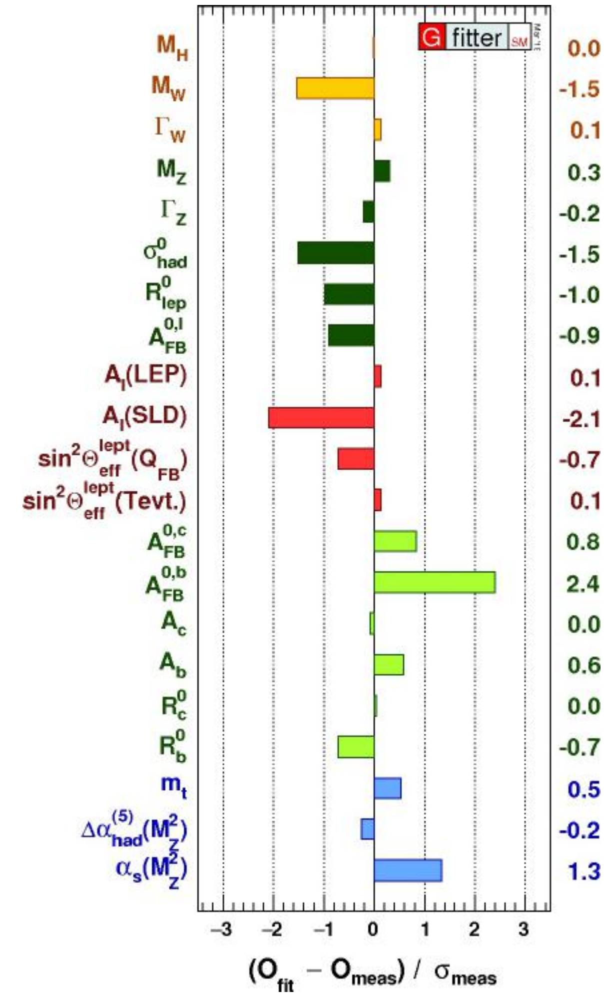






Global EW Fit





m/GeV	measured	fit value
m_t	172.47 ± 0.68	176.4 ± 2.1
M_H	125.1 ± 0.2	90^{+21}_{-18}
M_W	80.379 ± 0.013	80.354 ± 0.007