The Plan

1. A brief recap of Quantum Field Theories
2. The main construction Principles for QFTs
   * gauge symmetries * Yang-Mills * SSB
3. Strong Interactions
4. The electroweak Standard Model
   * bottom-up construction
     ↪ Fermi model ⇝ gauge symmetry ⇝ SM
   * Predictions at Lepton colliders
     ↪ EW precision tests
5. LHC Phenomenology

4. The Electroweak Standard Model

* late 1940's Success of QED (rad. corrections) ⇒ boom in elementary particle theory

* 1950's confusion & frustration ⤳ give up QFT?
  - ↳ strong interactions
    perturbation theory useless? → S-Matrix program
  - ↳ weak interactions
    4-fermion theory plagued by infinities

* "During this time of confusion and frustration [...] there emerged three good ideas" [Weinberg]

(1) quark model
(2) gauge (local) symmetries
(3) spontaneous symmetry breaking

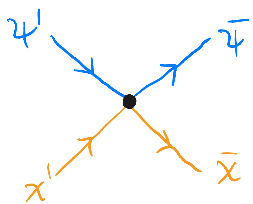
} last lecture

Weak Interactions — Fermi model

* discovery in β-decay $n \to p + e^- + \bar{\nu}_e$, $p \to n + e^+ + \nu_e$ (not for free p)

  more: μ-decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, ...

* modeled as 4-fermion interaction



$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}} \left[\bar{\psi}\,\Gamma_X\,\psi'\right]\left[\bar{\chi}\,\Gamma^X\,\chi'\right]$$

↳ Dirac bilinear $\Gamma_X \in \{\underset{\text{scalar}}{\mathbb{1}}, \overset{\text{pseudo-scalar}}{\gamma^5}, \underset{\text{vector}}{\gamma_\mu}, \overset{\text{pseudo-vector}}{\gamma_\mu\gamma_5}, \underset{\text{tensor}}{\sigma_{\mu\nu}}\}$

↳ experiment ⇒ parity violation [Wu et al. '57]

⇒ "V−A" $\Gamma_X = \gamma_\mu(\mathbb{1} - \gamma_5)$

[$\omega_\pm = \frac{1}{2}(1 \pm \gamma_5)$ projectors

↳ left-/right-handed fermions $\psi_{L/R} = \omega_\mp \psi$

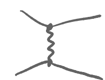
⇒ only left-handed participate

$$\boxed{\mathcal{L}_{\text{Fermi}} = -\frac{4G_F}{\sqrt{2}}\left[\bar{\psi}_L\gamma_\mu\psi'_L\right]\left[\bar{\chi}_L\gamma^\mu\chi'_L\right]}$$

Weak Interactions — Fermi model

* universal coupling $\quad G_F = 1.16639 \cdot 10^{-5}\ \mathrm{GeV}^{-2}$

* unitarity violation $\quad \sigma(\nu_\mu e \to \nu_\mu e)$ etc. grow $\propto E^2$
* not renormalizable $\quad \to$ no radiative corrections
* try to formulate as a gauge theory

0. Identify the multiplets
1. Introduce mediator bosons 
2. Identify group & complete
3. Link to the real world (EW & SSB)

Towards a Gauge Theory of EW Interactions

0. Look at muon decay as an explicit example $(\mu^{\pm} \to e^{\pm} + \overset{\leftrightarrow}{\nu}_e + \overset{\leftrightarrow}{\nu}_\mu)$

$$\mathcal{L} = -\frac{4 G_F}{\sqrt{2}} \left([\bar{\psi}_{L,\mu} \gamma_\rho \psi_{L,\nu_\mu}][\bar{\psi}_{L,e} \gamma^\rho \psi_{L,\nu_e}] + [\bar{\psi}_{L,\nu_\mu} \gamma_\rho \psi_{L,\mu}][\bar{\psi}_{L,\nu_e} \gamma^\rho \psi_{L,e}] \right)$$

$\hookrightarrow$ doublet structure $\quad L_\ell = \begin{pmatrix} \psi_{L,\nu_\ell} \\ \psi_{L,\ell} \end{pmatrix}$

$$\mathcal{L} = -\frac{4 G_F}{\sqrt{2}} \left[(\bar{\psi}_{L,\nu_\mu}, \bar{\psi}_{L,\mu}) \gamma_\rho \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_{L,\nu_\mu} \\ \psi_{L,\mu} \end{pmatrix} \right] \left[(\bar{\psi}_{L,\nu_e}, \bar{\psi}_{L,e}) \gamma^\rho \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_{L,\nu_e} \\ \psi_{L,e} \end{pmatrix} \right] + \left[\underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{\sigma_+} \leftrightarrow \underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{\sigma_-} \right]$$

$$= -\frac{4 G_F}{\sqrt{2}} \sum_{a=\pm} [\bar{L}_\mu \gamma_\rho \sigma_a L_\mu][\bar{L}_e \gamma^\rho \sigma_a L_e] \quad , \quad \sigma_\pm = \frac{1}{2}(\sigma_1 \pm i \sigma_2)$$

Towards a Gauge Theory of EW Interactions

1. In gauge theories, interactions mediated through gauge-boson exchange

↪ structure $\mathcal{L}_{int} = g\,[\bar{\psi}\gamma^\mu T^a \psi]\, W^a_\mu$

↪ compare with Fermi $\propto [\bar{L}\gamma^\mu \sigma_{\pm} L]$

⇒ introduce two charged gauge bosons $W^{\pm}_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$

$$\mathcal{L}_{IVB} = \frac{g}{2\sqrt{2}} \sum_\ell \left([\bar{L}_\ell \gamma^\mu \sigma_+ L_\ell]\, W^+_\mu + [\bar{L}_\ell \gamma^\mu \sigma_- L_\ell]\, W^-_\mu \right)$$

$$= \frac{g}{2} \sum_\ell \sum_{i=1,2} [\bar{L}_\ell \gamma^\mu \sigma_i L_\ell]\, W^i_\mu$$

* stop here & introduce massive $W^\pm$ "intermediate-vector-boson model"

↪ $\sqrt{2}\, G_F = \frac{g^2}{4 M_W^2}$ ⇝ $M_W = \mathcal{O}(100\ \text{GeV})$ ⟷ but not a gauge theory!

Towards a Gauge Theory of EW Interactions

2. We identified two generators $\sigma_\pm$ (or σ_1, σ_2)
   - if we want a group → better close $[\sigma_+, \sigma_-] = \sigma_3$
   - need to include $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

     it is diagonal ($\ell^\pm \to \ell^\pm$; $\overset{(-)}{\nu}_\ell \to \overset{(-)}{\nu}_e$) ⇒ a neutral current W^3_μ

* We already know a neutral boson
   - Could this be the photon $W^3_\mu = A_\mu$?

Towards a Gauge Theory of EW Interactions

2. We identified two generators $\sigma_\pm$ (or σ_1, σ_2)
   - if we want a group → better close $[\sigma_+, \sigma_-] = \sigma_3$
   - need to include $\sigma_3 = \begin{pmatrix} 1 & \emptyset \\ \emptyset & -1 \end{pmatrix}$

     it is diagonal ($\ell^\pm \to \ell^\pm$; $\overset{(-)}{\nu}_\ell \to \overset{(-)}{\nu}_e$) $\Rightarrow$ a neutral current W_μ^3

* We already know a neutral boson
   - Could this be the photon $W_\mu^3 = A_\mu$?

No
- eigenvalues of $\hat{Q} \neq \hat{I}_W^3 = \frac{\sigma^3}{2}$
- $\hat{Q}$ also acts on right-handed fields!

Towards a Gauge Theory of EW Interactions

2. We identified two generators $\sigma_\pm$ (or σ_1, σ_2)

↳ if we want a group → better close $[\sigma_+, \sigma_-] = \sigma_3$

↳ need to include $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

it is diagonal $(\ell^\pm \to \ell^\pm \, ; \, \overset{(-)}{\nu}_\ell \to \overset{(-)}{\nu}_e)$ ⇒ a neutral current W^3_μ

* Rewrite the charge operator $\hat{Q} = \hat{I}^3_W + \frac{\hat{Y}}{2}$ ← hyper charge

↳ a second neutral current B_μ

↳ non-trivial inclusion of $U(1)_{QED}$

⇒ $SU(2)_W \times U(1)_Y$ EW unification!

↳ $$\boxed{D_\mu = \partial_\mu - i g \hat{I}^i_W W^i_\mu + i g' \frac{\hat{Y}}{2} B_\mu}$$

		generation			represen-tation	charges		
		1st	2nd	3rd		I^3_w	Y	Q
leptons	$\Psi^{L'_i}_L$	$\begin{pmatrix} \nu_e' \\ e' \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu' \\ \mu' \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau' \\ \tau' \end{pmatrix}_L$	$(\mathbf{1}, \mathbf{2})_{-1}$	$\frac{1}{2}$ $-\frac{1}{2}$	-1 -1	0 -1
	$\Psi^{\ell'_i}_R$	e'_R	μ'_R	τ'_R	$(\mathbf{1}, \mathbf{1})_{-2}$	0	-2	-1
quarks	$\Psi^{Q'_i}_L$	$\begin{pmatrix} u' \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c' \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t' \\ b' \end{pmatrix}_L$	$(\mathbf{3}, \mathbf{2})_{\frac{1}{3}}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{3}$ $\frac{1}{3}$	$\frac{2}{3}$ $-\frac{1}{3}$
	$\Psi^{u'_i}_R$	u'_R	c'_R	t'_R	$(\mathbf{3}, \mathbf{1})_{\frac{4}{3}}$	0	$\frac{4}{3}$	$\frac{2}{3}$
	$\Psi^{d'_i}_R$	d'_R	s'_R	b'_R	$(\mathbf{3}, \mathbf{1})_{-\frac{2}{3}}$	0	$-\frac{2}{3}$	$-\frac{1}{3}$

Towards a Gauge Theory of EW Interactions

3. generate gauge-boson masses through SSB

what about $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \in \mathbb{R}^3$ in adjoint repⁿ $SU(2) \simeq SO(3)$? $Y_\Phi = \emptyset$ & $\Phi_0 = (0,0,\frac{v}{\sqrt{2}})^T$

$\Rightarrow$ The SM introduces an SU(2) doublet

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad ; \quad \phi_i \in \mathbb{C}$$

- implicit choices & constraints

(a) in our repⁿ $\rightsquigarrow$ $\hat{Q}$ diagonal $\Rightarrow$ ϕ_i eigenstates of $\hat{Q}$

(b) Gell-Mann-Nishijima : $\hat{Q} = \hat{I}^3_W + \frac{Y}{2}$ $\rightsquigarrow$ $|Q_1 - Q_2| \overset{!}{=} 1$

(c) QED better remain unbroken (photon $\overset{!}{=}$ massless)

$\Rightarrow$ vev must have $\hat{Q} = \emptyset$

$\Rightarrow$ Only choices $Y_\Phi = \pm 1$ $\qquad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ or $\begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$ charge conj.

Towards a Gauge Theory of EW Interactions

3. The Higgs Potential $V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2$ $(\mu^2, \lambda > 0)$

- minimum $|\langle \Phi_0 \rangle| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$

- excitations from the vev & unitary gauge

  $\rightsquigarrow$ $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$ $H \in \mathbb{R}$

- gauge boson masses

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) \Big|_{v^2} = \frac{v^2}{2} \left\{ \frac{g^2}{2} W^+_\mu W^{-,\mu} + \frac{1}{4} \underbrace{\left[g W^3_\mu + g' B_\mu \right]^2}_{\text{mixing} \Rightarrow \text{diagonalize}} \right\}$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}$$

& identify EM coupling $e = \frac{g g'}{\sqrt{g^2 + g'^2}}$

$\sin\theta_W \equiv s_W = \frac{g}{\sqrt{g^2 + g'^2}}$

weak mixing angle

Towards a Gauge Theory of EW Interactions

3. charge- & mass-eigenstates

↪ $(D_\mu \Phi)^\dagger (D^\mu \Phi)\Big|_{v^2} = \frac{e^2 v^2}{4 s_W^2} W_\mu^+ W^{-,\mu} + \frac{e^2 v^2}{4 s_W^2 c_W^2} Z_\mu Z^\mu$

$\Rightarrow M_W = \frac{ev}{2 s_W}$, $M_Z = \frac{ev}{2 s_W c_W} = \frac{M_W}{c_W}$, $M_A \equiv 0$

↪ couplings $g = e/s_W$, $g' = e/c_W$

↪ $D_\mu = \partial_\mu + ie\hat{Q} A_\mu - i \frac{e}{s_W c_W} [\hat{I}_W^3 - s_W^2 \hat{Y}] Z_\mu - i \frac{e}{\sqrt{2} s_W} [\hat{I}_W^+ W_\mu^+ + \hat{I}_W^- W_\mu^-]$

↪ $M_H = \sqrt{2\mu^2}$

Towards a Gauge Theory of EW Interactions

3. Fermion masses $\leftrightarrow$ SM distinguishes L/R fermions ("chiral")

↪ naive mass term $m_f \bar{\psi}_f \psi_f = m_f (\bar{\psi}_{L,f} \psi_{R,f} + \bar{\psi}_{R,f} \psi_{L,f})$ forbidden

↪ Higgs to the rescue (doublet w/ $Y_\Phi = \pm 1$) $\Rightarrow$ Yukawa interactions

$$\mathcal{L}_{Yuk} = - \sum_{i,j=1}^{3} \left(\bar{L}_{i,L} G^{\ell}_{ij} \ell_{j,R} \Phi + \bar{Q}_{i,L} G^{u}_{ij} u_{j,R} \Phi^c + \bar{Q}_{i,L} G^{d}_{ij} d_{j,R} \Phi + h.c. \right)$$

generations

complex (3x3)

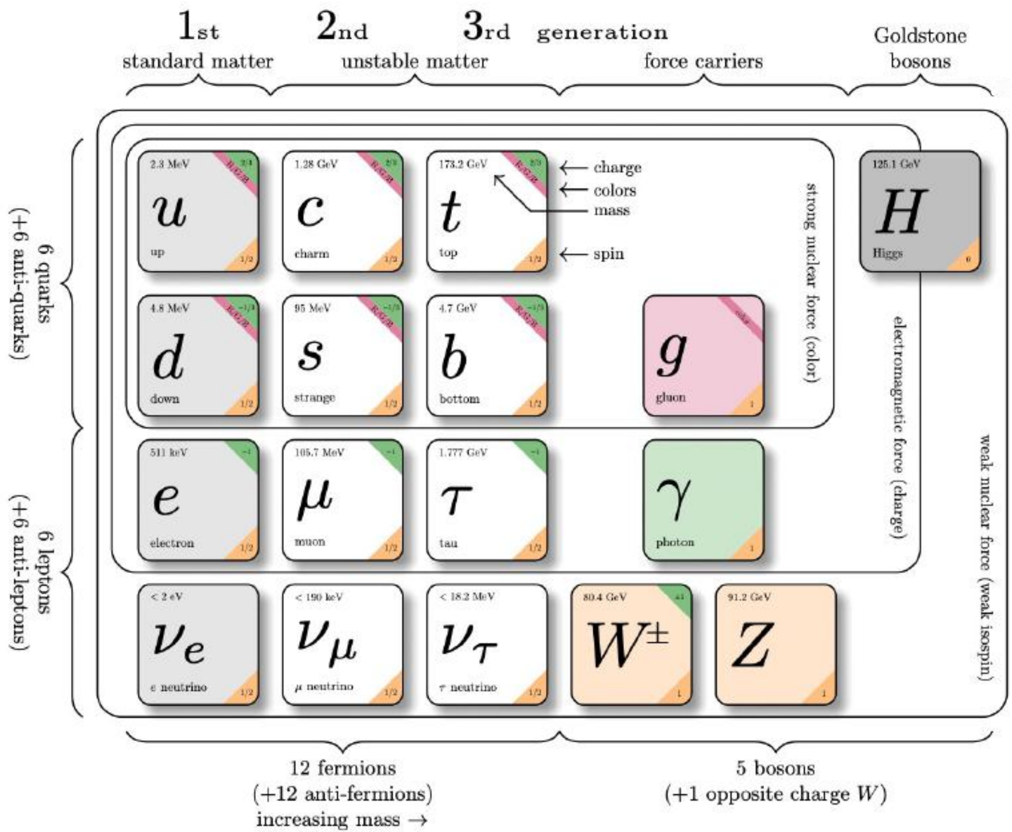
$\Rightarrow$ gauge inv. & vev generates mass matrix ($m_\nu = \emptyset$ assumed)

$$M^f_{ij} = \frac{v}{\sqrt{2}} G^f_{ij} \quad , \; f = \ell, u, d$$

- diagonalize $\Rightarrow$ unitary transform $\frac{v}{\sqrt{2}} U^f_L G^f (U^f_R)^\dagger = \mathrm{diag}(m_f)$
- drops out everywhere except in charged-current-int$^{\underline{n}}$

$\Rightarrow$ $V_{CKM} = U^u_L (U^d_L)^\dagger \rightsquigarrow \frac{e}{\sqrt{2} s_W} \bar{u}_L \gamma^\mu V_{CKM} d_L + h.c.$

When the dust settles ...



parameters

3 couplings

$(g, g', g_s) \leftrightarrow (e, s_W, g_s)$

2 Higgs potential

$(\mu^2, \lambda) \leftrightarrow (M_H, \lambda)$

9_{+3} fermion masses

6 quarks + 3 leptons (ν)

4_{+4} CKM matrix (PMNS)

3 angles + 1 phase

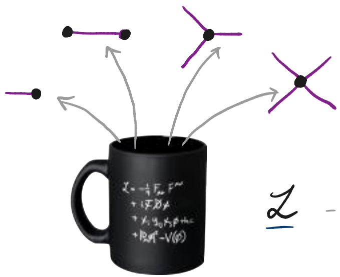
18_{+7}

The Standard Model

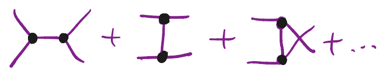
[Glashaw, Salam, Weinberg '67]

* particle content fully veryfied (last: Higgs 2012)
  - ↪ consistent QFT (unitarity, renormalizable ↔ precision!, anomaly free)
* full set of input parameters known
  - ↪ independent predictions & self-consistency tests $\frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \rho \simeq 1$
* reference theory for particle physics
  - ↪ refer to "Beyond the Standard Model"

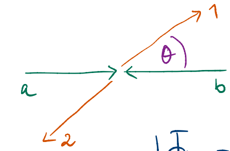
Making Predictions



$\mathcal{L}$ - - - → scattering amplitudes



Feynman diagrams
& rules

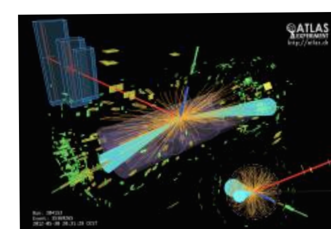


$$d\Phi_2 = \frac{d\cos\theta}{16\pi}$$

cross sections*

$$d\sigma_{2\to n} = \frac{1}{F}\langle|\mathcal{M}|^2\rangle\, d\Phi_n$$

Event rates: $N = L\,\sigma$



* decay rates $(\tau = 1/\Gamma)$

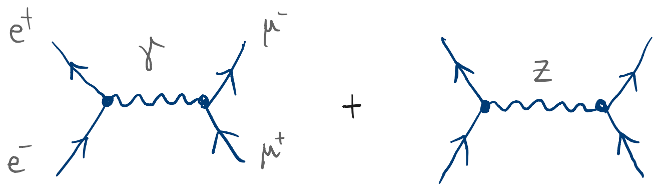
$$d\Gamma_{1\to n} = \frac{1}{2M}\langle|\mathcal{M}|^2\rangle\, d\Phi_n$$

Some Fun at the Z pole

[org/epem]

Consider the process $e^+ e^- \to \mu^+ \mu^-$

At lowest order (tree level) there are two diagrams



$$= i\mathcal{M}^{\text{tree}}_{e^+e^-\to\mu^+\mu^-}$$

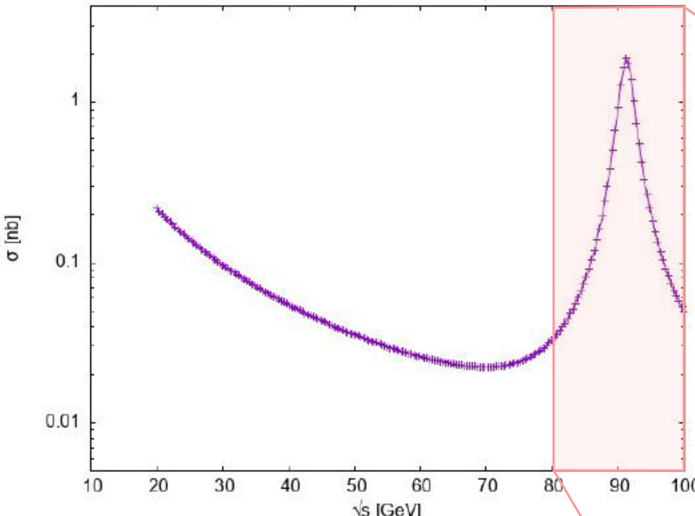
$\Rightarrow$ inserting into Fermi's golden rule $\left[s = E_{cm}^2; \quad P_a \cdot P_1 = p_a^\mu P_{1,\mu} = E_{cm}^2 (1-\cos\theta) \right]$

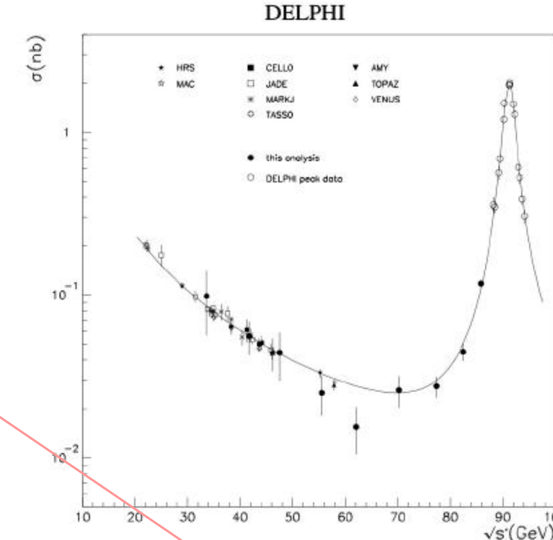
$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2 \pi}{2s} \left[(1+\cos^2\theta)\, G_1(s) + 2\cos\theta\, G_2(s) \right]$$

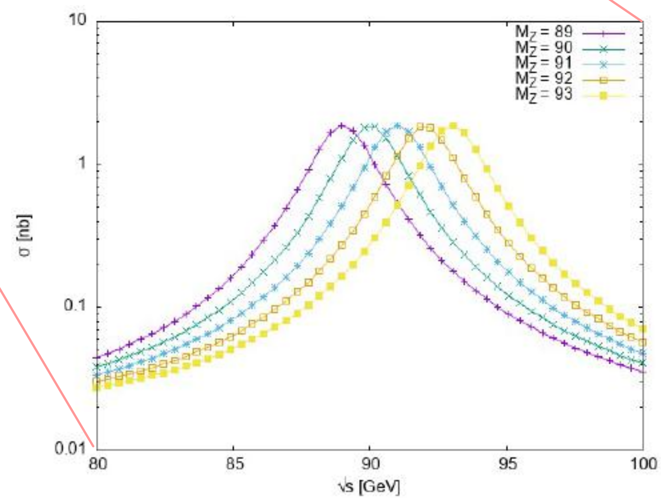
$$G_1(s) = 1 + 2 V_e^2 \,\mathrm{Re}\left\{ \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right\} + (V_e^2 + a_e^2)^2 \left| \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2$$

$$G_2(s) = \emptyset + 2 a_e^2 \,\mathrm{Re}\left\{ \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right\} + 4 V_e^2 \cdot a_e^2 \left| \frac{}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2$$

Theory vs. Data σ

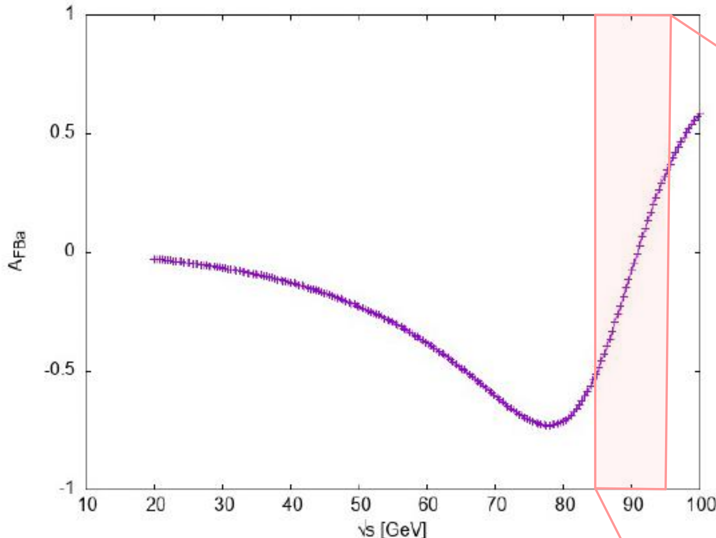


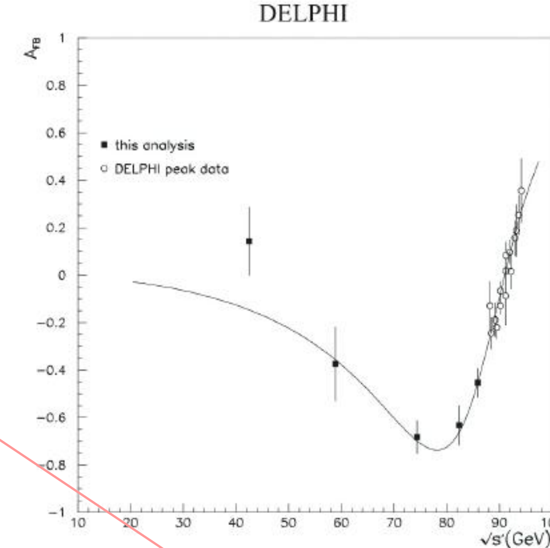


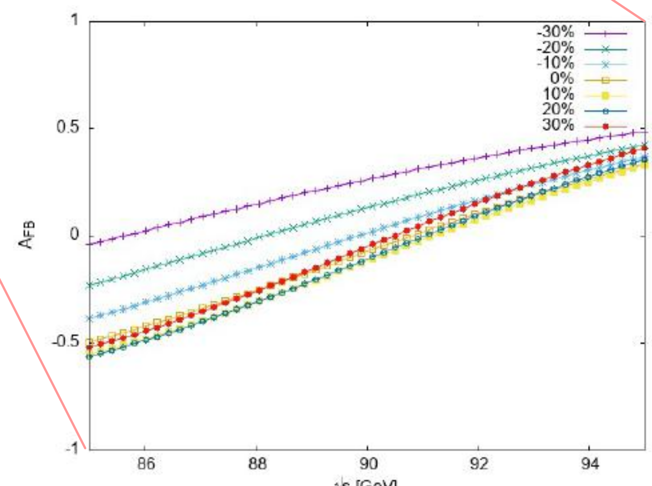


M_Z
extraction

Theory vs. Data A_{FB}





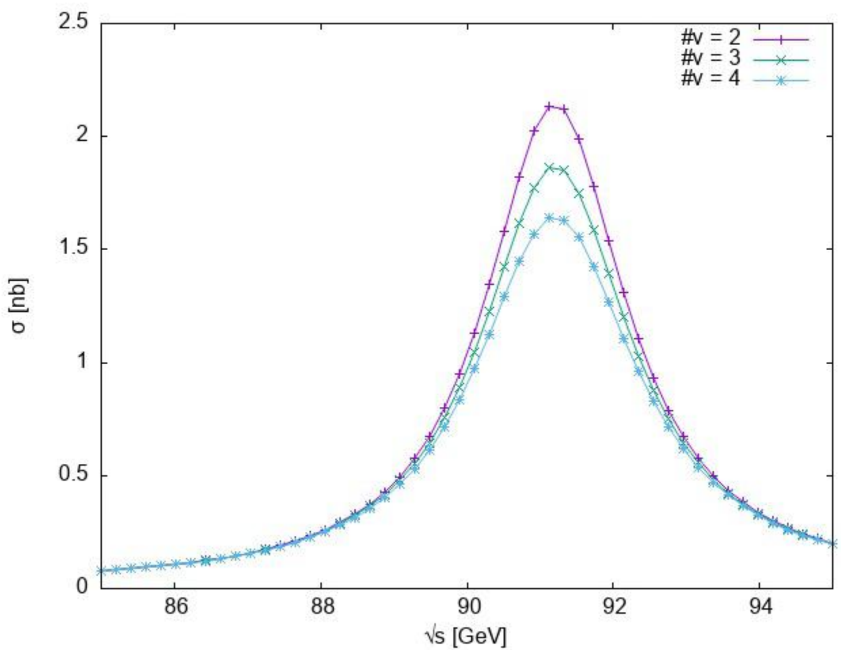


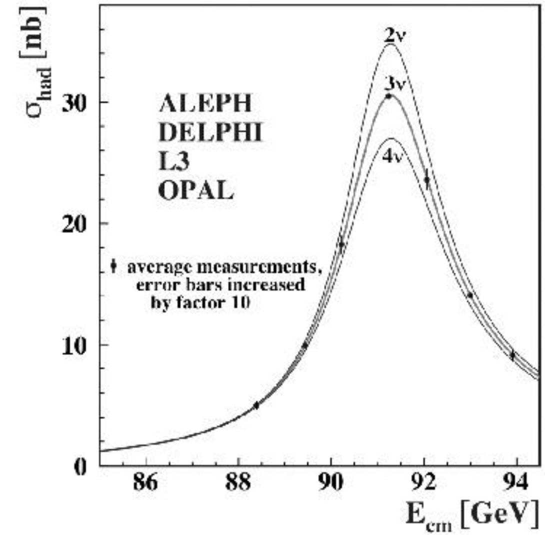
$\sin^2\theta_W$ extraction

test:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

Theory vs. Data σ_{had}





try to implement it yourself

$e^+e^- \to \mu^+\mu^-$

$\rightsquigarrow e^+e^- \to q\bar{q}$

generations = 3

(with light neutrinos)

The Standard Model @ Tree level

* our explorations so far were Born-level (LO, tree) predictions

  ⇾ Let us be more quantitative in the comparison

* Some SM predictions @ tree level ($s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$)

  ⇾ $\alpha^{(0)} = \frac{e^2}{4\pi}$

  ⇾ $G_F^{(0)} = \frac{\alpha\pi}{\sqrt{2}\, s_W^2 M_W^2} = \frac{1}{\sqrt{2}\, v^2}$

  ⇾ $M_Z^{(0)} = M_Z = \frac{e v}{2 s_W c_W}$

  ⇾ $M_W^{(0)} = M_W = \frac{e v}{2 s_W}$

  ⇾ $(s_W^{(0)})^2 = s_W^2$

  ⇾ $\Gamma^{(0)}_{Z\to e^+e^-} = \frac{\alpha M_Z}{6 s_W^2 c_W^2}\left[\left(-\frac{1}{2}+s_W^2\right)^2 + s_W^4\right] = \frac{e^3 v}{48\pi s_W^3 c_W^3}\left[\left(-\frac{1}{2}+s_W^2\right)^2 + s_W^4\right]$

all of them only depend on 3 EW Lagrangian parameters (@ tree level)

The Standard Model @ Tree level

... and they are among the most precisely measured

- $\hat{\alpha}_0 = (137.035\,999\,084(21))^{-1}$
- $\hat{G}_F = 1.166\,378\,8(6) \cdot 10^{-5}\ \text{GeV}^{-2}$
- $\hat{M}_Z = 91.1876(21)\ \text{GeV}$

① pick as <u>input</u> (most precise)

- $\hat{M}_W = 80.377(12)\ \text{GeV}$
- $(\hat{s}_W)^2 = 0.231\,53(4)$

② predict these

$\Rightarrow$ check for consistency

The Standard Model @ Tree level

① invert tree-level relations $\hat{O} = O^{(0)}$ to get Lagrange parameters

$$\hookrightarrow \quad e^2 = 4\pi\hat{\alpha}$$

$$\hookrightarrow \quad v^2 = \frac{1}{\sqrt{2}\,\hat{G}_F}$$

$$\hookrightarrow \quad \frac{e^2 v^2}{4 s_W^2 c_W^2} = M_Z^2 \Rightarrow s_W^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F \hat{M}_Z^2}}\right)$$

② insert into predictions for remaining observables

$$\hookrightarrow \quad M_W^{(0)} = \frac{e v}{2 s_W} = 80.9389 \longleftrightarrow \mathcal{O}(10\sigma)$$

$$\hookrightarrow \quad (s_W^{(0)})^2 = s_W^2 = 0.212152 \longleftrightarrow \mathcal{O}(100\sigma)$$

SM is excluded?!