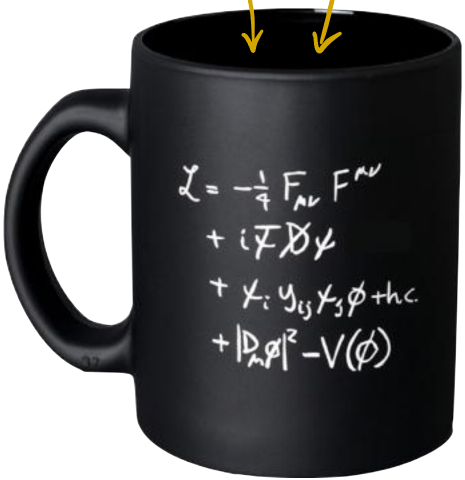
Standard Model Precision Physics

Alexander Huss

[alexander.huss@cern.ch]

54. Herbstschule für Hochenergiephysik — Maria Laach — 2023

$SU(3)_c \times SU(2)_L \times U(1)_Y$



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \psi_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

THEORY

framework of QFT

$\hat{=}$ QM $\otimes$ SR

PHENOMENOLOGY

- precision $\leftrightarrow$ Δ_{exp}
- making predictions
- learn about $\mathcal{L}_{TH}$?

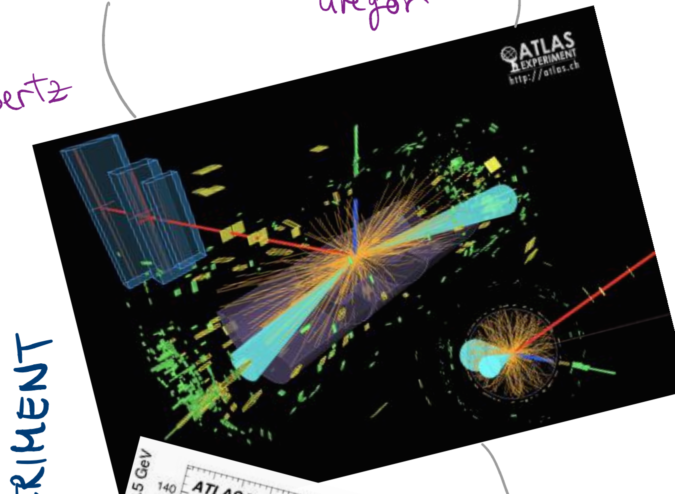
EXPERIMENT

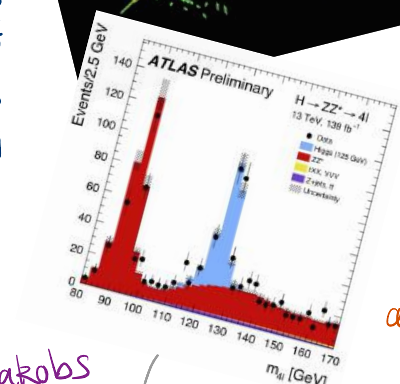
accelerators

detectors

Gregor

Rabbertz





data acquisition/analysis

statistics

Jakobs

Aarrestad

The Plan

1. A brief recap of Quantum Field Theories
   * why QFTs? * perturbation theory & Feynman diagrams
2. The main construction Principles for QFTs
   * from gauge freedom to local symmetries
   * Yang-Mills theories
   * Spontaneous Symmetry Breaking
3. Strong Interactions
4. The electroweak Standard Model
5. LHC Phenomenology

0 Conventions & Source files

* natural units: $\hbar = c = 1$
  - $[\hbar] = [Et] \Rightarrow [E] = [t]^{-1}$
  - $[c] = [x/t] \Rightarrow [x] = [t]$

  particle physics: $[E] = eV$

  $[E] = [p] = [m] = eV$

  $[t] = [x] = eV^{-1}$

* four-vectors: $x^\mu = (t, x, y, z)^T$ $\quad \partial_\mu = (\partial_t, \vec{\nabla})$
  - scalar product: $a \cdot b = a^\mu b_\mu = a^\mu g_{\mu\nu} b^\nu = a^0 b^0 - \vec{a} \cdot \vec{b}$ $\quad g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$
  - d'Alembert operator: $\Box \equiv \partial_\mu \partial^\mu = \partial_t^2 - \Delta$
  - energy-momentum conservation: $\delta^{(4)}(P_1 + P_2 - (P_a + P_b))$ $\quad a + b \to 1 + 2$

* Dirac algebra: $\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu = 2 g^{\mu\nu} \mathbb{1}_{4\times4}$
  - $\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = -i \gamma_0 \gamma_1 \gamma_2 \gamma_3$
  - Dirac "slash": $\not{a} = a_\mu \gamma^\mu$
  - conjugate spinor: $\bar{\psi} = \psi^\dagger \gamma^0$

* source files: github.com/aykhuss/Lectures-MariaLaach-SMprec

org notebooks (mainly python)

1 Quantum Field Theory — Why?

high energy physics (hep): study fundamental particles & interactions

↪ resolve small distances through highly energetic collisions

Quantum Mechanics

$i\hbar \partial_t \psi = \left[-\frac{\hbar^2}{2m} \Delta + V \right] \psi$ Schrödinger Eqn

⊕ description of microscopic nature

⊖ not relativistic

⊖ fixed particle #

Special Relativity

$x^\mu \longrightarrow \Lambda^\mu{}_\nu \, x^\nu$ (rotn & boosts)

• principle of relativity

⊕ processes @ high energy/velocity ($v \sim c$)

QM ∞ SR

= QFT

1 Quantum Field Theory — How?

* particle X $\longleftrightarrow$ associated field $\Phi_X\underbrace{(t,\vec{x})}_{x^\mu}$ & elementary excitations

(separate field $\forall$ species)

* $E=mc^2$ & QM $\Rightarrow$ particle creation / annihilation

* Quantization $\rightsquigarrow$ path-integral

$$\int \mathcal{D}[\Phi]\; e^{i\int d^4x\, \mathcal{L}(\partial_\mu \Phi, \Phi)}$$

↪ in principle, can compute <u>anything</u> from it

↪ full information encapsulated in $\mathcal{L}$ ← Lagrangian density

↪ <u>extremely</u> difficult integral to solve

1 Quantum Field Theory — How?

* particle X $\longleftrightarrow$ associated field $\Phi_X(\underbrace{t,\vec{x}}_{x^\mu})$ & elementary excitations

  (separate field $\forall$ species)

* $E=mc^2$ & QM $\Rightarrow$ particle creation / annihilation

* Quantization $\rightsquigarrow$ path-integral

create / annihilate particle @ x_i

$$\mathcal{A}_{\Phi_1 \ldots \Phi_n} \longleftrightarrow \int \mathcal{D}[\Phi]\, \Phi(x_1) \ldots \Phi(x_n)\, e^{i\int d^4x\, \mathcal{L}(\partial_\mu \Phi, \Phi)}$$

(amplitude)

↪ in principle, can compute <u>anything</u> from it

↪ full information encapsulated in $\mathcal{L}$

Lagrangian density

↪ <u>extremely</u> difficult integral to solve

... except a free theory (quadratic in Φ $\Rightarrow$ "Gauss" Integral)

Free Theory $\longleftrightarrow$ only monomials quadratic in the fields

Spin 0: Klein-Gordon Eqn

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

$$\hookrightarrow (\Box + m^2)\,\phi(x) = 0$$

Euler-Lagrange: $\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = \frac{\partial \mathcal{L}}{\partial \phi}$

Spin 1/2: Dirac Eqn

$$\mathcal{L}_\psi = \bar{\psi}\,(i\not{\partial} - m)\,\psi$$

$$\hookrightarrow (i\not{\partial} - m)\,\psi(x) = 0$$

Spin 1: Maxwell Eqn $\rightsquigarrow$ gauge freedom: $A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \theta(x)$

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad (F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu)$$

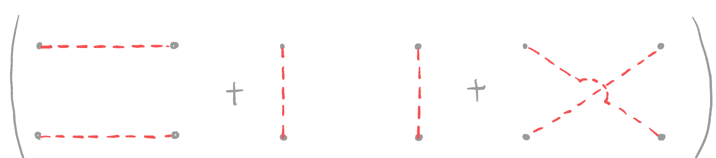
$$\hookrightarrow \partial_\mu F^{\mu\nu} = 0$$

Free Theory — Feynman Rules

	incoming	outgoing	propagator
Spin 0:	ϕ $\xrightarrow{p}$ 1	$\xrightarrow{p}$ ϕ 1	$\xrightarrow{p}$ $\dfrac{i}{p^2-m^2}$
Spin ½:	f $\xrightarrow{p}$ $u_\alpha(p,\sigma)$	$\xrightarrow{p}$ f $\bar{u}_\alpha(p,\sigma)$	$\beta \xrightarrow{p} \alpha$ $\dfrac{i(\not{p}+m)_{\alpha\beta}}{p^2-m^2}$
	$\bar{f}$ $\xrightarrow{p}$ $\bar{v}_\alpha(p,\sigma)$	$\xrightarrow{p}$ $\bar{f}$ $v_\alpha(p,\sigma)$	
Spin 1:	A_μ $\xrightarrow{p}$ $\varepsilon^*_\mu(p,\lambda)$	$\xrightarrow{p}$ A_μ $\varepsilon_\mu(p,\lambda)$	$\mu \xrightarrow{p} \nu$ $\dfrac{-i\,g_{\mu\nu}}{p^2}$

* extremely <u>boring</u> theory

$\phi + \phi \rightarrow \phi + \phi$



no scattering, just free propagation

Perturbation Theory

* too difficult to solve full interacting theory exactly
  → do it approximately (& systematically improvable)
* perturbation theory → interactions as perturbation around free theory
  $\alpha_{em} \sim 1/137$, $\alpha_{weak} \sim 1/30$, $\alpha_{strong} \sim 0.118$

$$\Rightarrow \quad \mathcal{O} = \mathcal{O}^{(0)} + \alpha\, \mathcal{O}^{(1)} + \alpha^2 \mathcal{O}^{(2)} + \dots$$

* example electron g-2: $a_e = \frac{g_e - 2}{2}$

$$a_{e^-}^{exp} = 1\,159\,652\,180.73(.28) \times 10^{-12} \quad (0.24 \text{ ppb})$$

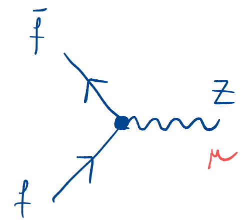
$$a_e^{\text{th}} = 1\,159\,652\,182.032(13)(12)(720) \times 10^{-12}$$

contribution	value in units of 10^{-12}
$A_1^{(2)}(\alpha/\pi)$	$1\,161\,409\,733.640 \pm 0.720$
$A_1^{(4)}(\alpha/\pi)^2$	$-1\,772\,305.065 \pm 0.003$
$A_1^{(6)}(\alpha/\pi)^3$	$14\,804.203$
$A_1^{(8)}(\alpha/\pi)^4$	-55.667
$A_1^{(10)}(\alpha/\pi)^5$	0.451 ± 0.013
$A_2^{(4)}(m_e/m_\mu)(\alpha/\pi)^2$	2.804
$A_2^{(6)}(m_e/m_\mu)(\alpha/\pi)^3$	-0.092
$A_2^{(8)}(m_e/m_\mu)(\alpha/\pi)^4$	0.026
$A_2^{(10)}(m_e/m_\mu)(\alpha/\pi)^5$	-0.0002
$A_2^{(4)}(m_e/m_\tau)(\alpha/\pi)^2$	0.010
$A_2^{(6)}(m_e/m_\tau)(\alpha/\pi)^3$	-0.0008
a_e(hadronic v.p.)	1.8490 ± 0.0108
a_e(hadronic v.p.,NLO)	-0.2213 ± 0.0012
a_e(hadronic v.p.,NNLO)	0.0280 ± 0.0002
a_e(hadronic l-l)	0.0370 ± 0.0050
a_e(weak)	0.03053 ± 0.00023

Feynman Rules with interactions

* from free theory → external states & propagators (# fields ≤ 2)
* interactions (# fields ≥ 3) → direct correspondence: $i\mathcal{L} \longleftrightarrow$ vertices

$$-e\, Z_\mu \bar{\psi}_f \gamma^\mu (v_f - a_f \gamma^5) \psi_f \quad \rightsquigarrow$$

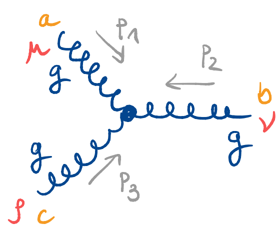


$$= -ie\,\gamma^\mu (v_f - a_f \gamma^5)$$

↪ sometimes more subtle (derivatives, identical fields)

$$\frac{i g_s}{2} (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) f^{abc} A^{b,\mu} A^{b,\nu}$$

$\rightsquigarrow$

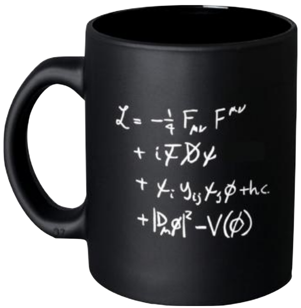


$$= -g_s f^{abc} \Big[g_{\mu\nu} (p_1 - p_2)_\rho + g_{\nu\rho} (p_2 - p_3)_\mu + g_{\rho\mu} (p_3 - p_1)_\nu \Big]$$

2 How to construct $\mathcal{L}$?

guiding principles

* symmetries :
  - Poincaré invariance
  - gauge invariance
* causality :
  - local monomials
  - $\Leftrightarrow \phi_1(x)\,\phi_2(x)\ldots \in \mathcal{L}$
  - same x!
* renormalizability
  - monomials w/ field dim ≤ 4
  - $\Leftrightarrow [\phi] = [A] = 1 \; ; \; [\psi] = 3/2$
  - $\Rightarrow$ only 3- & 4-vertices
* unitarity (probability ≤ 1)
* minimality (for the SM)



1

$$-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$$

$$\tfrac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$

2

$$M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H -$$

$$\tfrac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \tfrac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\tfrac{2M^2}{g^2} +$$

$$\tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - i g c_w [\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu -$$

$$W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu -$$

$$W^-_\nu \partial_\nu W^+_\mu)] - i g s_w [\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu -$$

$$W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$

$$\tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$

$$g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu -$$

$$W^+_\nu W^-_\mu) - 2 A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha [H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$$

$$\tfrac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] -$$

$$g M W^+_\mu W^-_\mu H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \tfrac{1}{2} i g [W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) -$$

$$W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \tfrac{1}{2} g [W^+_\mu (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W^-_\mu (H \partial_\mu \phi^+ -$$

$$\phi^+ \partial_\mu H)] + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - i g \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) +$$

$$i g s_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - i g \tfrac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) +$$

$$i g s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \tfrac{1}{4} g^2 W^+_\mu W^-_\mu [H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$$

$$\tfrac{1}{4} g^2 \tfrac{1}{c_w^2} Z^0_\mu Z^0_\mu [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-] - \tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- +$$

$$W^-_\mu \phi^+) - \tfrac{1}{2} i g^2 \tfrac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- +$$

$$W^-_\mu \phi^+) + \tfrac{1}{2} i g^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+ \phi^- -$$

$$g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma\partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma\partial \nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + m_u^\lambda) u^\lambda_j -$$

3

$$\bar{d}^\lambda_j (\gamma\partial + m_d^\lambda) d^\lambda_j + i g s_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j)] +$$

$$\tfrac{ig}{4c_w} Z^0_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}^\lambda_j \gamma^\mu (\tfrac{4}{3} s_w^2 -$$

$$1 - \gamma^5) u^\lambda_j) + (\bar{d}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3} s_w^2 - \gamma^5) d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} W^+_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) e^\lambda) +$$

$$(\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}} W^-_\mu [(\bar{e}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa} \gamma^\mu (1 +$$

$$\gamma^5) u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} \tfrac{m_e^\lambda}{M} [-\phi^+ (\bar{\nu}^\lambda (1-\gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1+\gamma^5) \nu^\lambda)] -$$

4

$$\tfrac{g}{2} \tfrac{m_e^\lambda}{M} [H (\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\kappa (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) +$$

$$m_u^\lambda (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m_u^\kappa (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1 -$$

$$\gamma^5) u^\kappa_j] - \tfrac{g}{2} \tfrac{m_u^\lambda}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2} \tfrac{m_d^\lambda}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2} \tfrac{m_u^\lambda}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) -$$

$$\tfrac{ig}{2} \tfrac{m_d^\lambda}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 -$$

5

$$\tfrac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + i g c_w W^+_\mu (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + i g s_w W^+_\mu (\partial_\mu \bar{Y} X^- -$$

$$\partial_\mu \bar{X}^+ Y) + i g c_w W^-_\mu (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + i g s_w W^-_\mu (\partial_\mu \bar{X}^- Y -$$

$$\partial_\mu \bar{Y} X^+) + i g c_w Z^0_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + i g s_w A_\mu (\partial_\mu \bar{X}^+ X^+ -$$

$$\partial_\mu \bar{X}^- X^-) - \tfrac{1}{2} g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2} \bar{X}^0 X^0 H] +$$

$$\tfrac{1-2c_w^2}{2c_w} i g M [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \tfrac{1}{2c_w} i g M [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] +$$

$$i g M s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \tfrac{1}{2} i g M [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]$$

From gauge freedom to local symmetries — QED

* The free Dirac Lagrangian has a global $U(1)$ symmetry

$$\mathcal{L}_f = \bar{\psi}(i\gamma - m)\psi \quad ; \quad \psi \to e^{-i\alpha}\psi \qquad (\bar{\psi} = \psi^\dagger\gamma^0 \longrightarrow e^{+i\alpha}\bar{\psi})$$

* Noether: continuous symmetry $\Rightarrow$ conserved current

$$\partial_\mu j^\mu = \emptyset \quad ; \quad j^\mu \propto \bar{\psi}\gamma^\mu\psi$$

(opposite "charge" j^0 for (anti-) particles)

* couple to EM interactions $j^\mu A_\mu$ ⟵ EM vector potential : $(\varphi, \vec{A})$

$$\mathcal{L}_{EM} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free Maxwell}} - Q_f e(\bar{\psi}\gamma^\mu\psi)A_\mu \quad \leadsto \quad \text{Euler-Lagrange: } \partial_\mu F^{\mu\nu} = j^\nu$$

$$\Rightarrow \boxed{\mathcal{L}_{QED} = \bar{\psi}(i\gamma - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - Qe(\bar{\psi}\gamma^\mu\psi)A_\mu}$$

From gauge freedom to local symmetries — QED

* The free Dirac Lagrangian has a global U(1) symmetry

$$\mathcal{L}_f = \bar{\psi}(i\not{\partial} - m)\psi \quad ; \quad \psi \to e^{-i\alpha}\psi \qquad (\bar{\psi} = \psi^\dagger\gamma^0 \longrightarrow e^{+i\alpha}\bar{\psi})$$

* Noether: continuous symmetry $\Rightarrow$ conserved current

$$\partial_\mu j^\mu = \emptyset \quad ; \quad j^\mu \propto \bar{\psi}\gamma^\mu\psi \qquad (\text{opposite "charge" } j^0 \text{ for (anti-) particles})$$

* couple to EM interactions $j^\mu A_\mu$ ← EM vector potential : $(\varphi, \vec{A})$

$$\mathcal{L}_{EM} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free Maxwell}} - Q_f e(\bar{\psi}\gamma^\mu\psi)A_\mu \quad \rightsquigarrow \text{Euler-Lagrange: } \partial_\mu F^{\mu\nu} = j^\nu$$

$$\Rightarrow \boxed{\mathcal{L}_{QED} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - Qe(\bar{\psi}\gamma^\mu\psi)A_\mu}$$

depends on x!

* The symmetry of $\mathcal{L}_{QED}$ is actually local U(1) $\rightsquigarrow \psi(x) \longrightarrow e^{-i\alpha(x)}\psi(x)$
  - problematic term: $\bar{\psi}\, i\not{\partial}\psi \longrightarrow \bar{\psi}\, i\not{\partial}\psi + (\bar{\psi}\gamma^\mu\psi)(\partial_\mu\alpha)$
  - but we have gauge freedom: $A_\mu \longmapsto A_\mu + \frac{1}{eQ_f}(\partial_\mu\alpha)$

  } $\mathcal{L}$ invariant

The gauge paradigm — QED revisited

* turn the local (gauge) symmetry into the construction principle

  0. free Lagrangian of matter fields & global $U(1)$ symmetry $\quad \psi \longmapsto e^{-iQe\theta}\,\psi$

  1. promote to a local symmetry $\rightsquigarrow$ "minimal substitution" $\partial_\mu \rightsquigarrow D_\mu = \partial_\mu + iQeA_\mu$

     ↪ covariant derivative: $\quad D_\mu\psi \longmapsto e^{-iQe\theta(x)}\,D_\mu\psi$

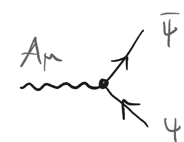
     ↪ gauge field $\quad A_\mu \longmapsto A_\mu + \partial_\mu\theta$

  2. dynamics for the gauge field

     $$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$\Rightarrow \boxed{\mathcal{L}_{QED} = \bar\psi(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - Qe(\bar\psi\gamma^\mu\psi)A_\mu}$$

interactions!



The gauge paradigm — non-Abelian Groups

* go beyond QED ⇝ apply gauge principle* to more complex groups G
* multiplet of matter fields & global "internal" symmetry

$$\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} \quad ; \quad \Phi \longrightarrow \underbrace{U(\vec{\theta})}_{\text{matrix}} \overbrace{\Phi}^{\text{vector}} \quad ; \quad U(\vec{\theta}) = \exp\{-i g\, T^a \theta^a\}$$

⇒ $a = 1, \dots, N = \dim(G)$; $\theta^a \leftrightarrow$ parameters of transformation

⇒ $T^a \leftrightarrow$ generators of G (representation: $n \times n$ matrix)

properties: $[T^a, T^b] = i f^{abc} T^c$; $\mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta^{ab}$

structure constants

* important groups for the SM ($SU \rightarrow U^\dagger U = U U^\dagger = 1$ & $\det(U) = 1$ ⇝ N^2-1 generators)

⇒ $SU(2)$: fundamental repⁿ $T_F^a = I^i = \sigma^i/2$ (Pauli matrices: $\sigma^{i=1,2,3}$; $f^{abc} = \varepsilon_{ijk}$)

⇒ $SU(3)$: fundamental repⁿ $T_F^a = t^a = \lambda^a/2$ (Gell-Mann matrices: $\lambda^{a=1,\dots,8}$)

⇒ adjoint repⁿ $(T_A)_{ab} = -i f^{abc}$

*This is the only way we know how to consistently implement interacting spin-1

The gauge paradigm — Yang Mills Theories

* apply the same steps as we did for QED before

0. Lagrangian of matter fields & global symmetry G: $U(\vec{\theta}) = \exp\{-i g T^a \theta^a\}$

   ↪ e.g. $\mathcal{L}_\Phi = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$

1. promote to a local ($\theta \rightsquigarrow \theta(x)$) symmetry

   matrix

   one gauge field for each generator ↔ adjoint rep$^{\underline{n}}$

   ↪ "minimal substitution" $\partial_\mu \rightsquigarrow D_\mu \equiv \partial_\mu + i g T^a A^a_\mu$

   covariant derivative

   gauge coupling

   ↪ transformations: $D_\mu \to U D_\mu U^{-1}$; $T^a A^a_\mu \longmapsto U T^a A^a_\mu U^\dagger - \frac{i}{g} U (\partial_\mu U^\dagger)$

2. dynamics for the gauge fields

   ↪ covariant definition of field strength $[D_\mu, D_\nu] = i g T^a F^a_{\mu\nu}$

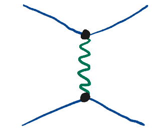
   $\mathcal{L}_{gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$; explicit form: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$

   non-Abelian!

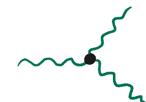
$$\Rightarrow \boxed{\mathcal{L}_{YM} = \mathcal{L}_\Phi \Big|_{\partial_\mu \to D_\mu} - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}}$$

Yang-Mills Theories — General Remarks

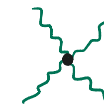
* similarly to QED: interaction between matter fields
  $\longleftrightarrow$ mediated through gauge-boson exchange



* in contrast to QED: gauge bosons self interact
  ⇨ "F^2" part contains terms $(\partial A)A^2$ & A^4



(cubic)



(quartic)

* more features from non-Abelian nature
  ⇨ single gauge coupling g for boson-boson & boson-matter ($\forall$ matter types)
  → unification
  ⇨ charges are quantized owing to $[T^a, T^b] = i f^{abc}\, T^c$
  → c.f. w/ QED: all charges Q_x are arbitrary

* gauge symmetry (Abelian or not) forbids naive mass terms
  ⇨ $M^2 (A^a_\mu A^{a\mu})$ breaks gauge invariance of $\mathcal{L}_{gauge}$

  ↯ $W^\pm$ & Z bosons ...

Spontaneous Symmetry Breaking

problem: naive mass term breaks gauge invariance:

$$M^2 A_\mu A^\mu \longrightarrow M^2 A_\mu A^\mu + 2M^2 (\partial_\mu \theta) A^\mu + M^2 (\partial_\mu \theta)(\partial^\mu \theta)$$

idea: retain gauge symmetry @ Lagrangian level

⇒ preserve all properties (conserved charges, unitarity, renormalizability)

give up the symmetry for the particle spectrum (in particular, the vacuum)

⇒ "hidden symmetry"

* spontaneous symmetry breaking [Brout, Englert, Higgs '64]

⇒ introduce a scalar field ⟵ why?

with non-vanishing vacuum expectation value (vev)

that couples to the gauge boson(s)

Abelian Higgs Model

* $U(1)$ gauge theory with one complex scalar field $\phi(x) \in \mathbb{C}$

  ↪ most general form

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi)$$

$$D_\mu = \partial_\mu - ie A_\mu$$

$$\phi \to e^{ie\theta(x)} \phi$$

$$A_\mu \to A_\mu + \partial_\mu \theta$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \leftarrow \text{gauge inv.}$$

* can also parametrize in terms of real components ($U(1) \simeq SO(2)$)

  ↪ $\phi = \frac{1}{\sqrt{2}} (\phi_1 + i \phi_2)$ $\quad \phi_i \in \mathbb{R}$ $\quad \Rightarrow \phi^\dagger \phi = \phi^* \phi = |\phi|^2 = \frac{1}{2} (\phi_1^2 + \phi_2^2)$

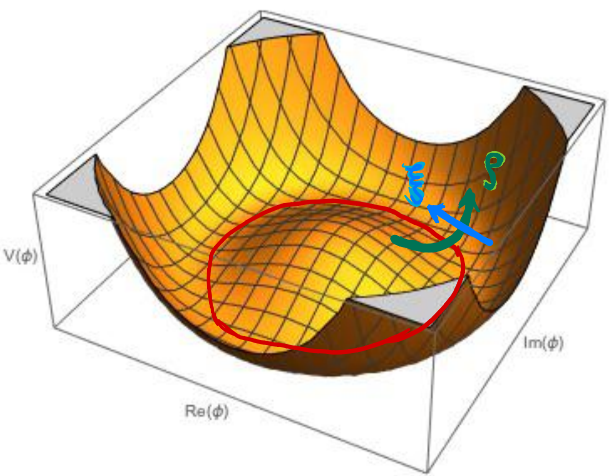
* $\lambda > 0$ potential must be bounded from below (stable)

* freedom: sign of μ^2 term

  ↪ $\mu^2 > 0$ boring $\leftrightarrow$ simple mass term $m_\phi = \mu$

  ↪ $\mu^2 < 0$ degenerate minimum for $V(\phi)$

Abelian Higgs Model – The Potential



* minimum of the potential ($\leftrightarrow$ vacuum)

$$|\phi|_{vacuum} = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} = |\phi_0| \qquad (\phi_0 = \langle 0|\phi|0\rangle)$$

$\hookrightarrow$ infinitely many equivalent configurations

$\hookrightarrow$ physics does not depend on it, but we have to make a choice

$\longrightarrow$ "spontaneous" $\quad Re[\phi_0] = \frac{v}{\sqrt{2}}, \; Im[\phi_0] = \emptyset$

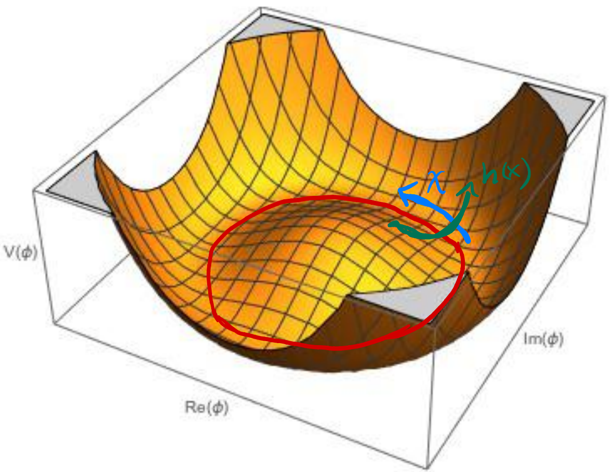
* parametrize deviations from the vacuum ("cartesian")

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \rho(x) + i\,\xi(x)\right)$$

$\hookrightarrow$ $V(\phi) = -\frac{\mu^4}{4\lambda} + \mu^2\rho^2 + (\text{cubic, quartic})$ $\rightsquigarrow$ $\begin{cases} \text{massive } \rho & m_\rho^2 = -2\mu^2 \\ \text{massless } \xi & \text{"Goldstone boson"} \end{cases}$

$\hookrightarrow$ $(D_\mu\phi)^\dagger (D^\mu\phi)$ $\rightsquigarrow$ mixing $\; g A_\mu (\partial^\mu \xi)$ $\rightsquigarrow$ physical d.o.f. with the "unitary gauge"

Abelian Higgs Model — The Potential



* minimum of the potential ($\leftrightarrow$ vacuum)

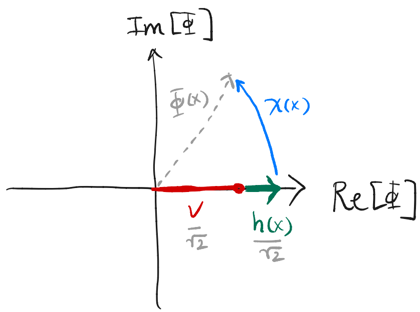
$$|\phi|_{vacuum} = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} = |\phi_0| \qquad (\phi_0 = \langle 0|\phi|0\rangle)$$

↪ infinitely many equivalent configurations

↪ physics does not depend on it,
but we have to make a choice

→ "spontaneous" $\quad Re[\phi_0] = \frac{v}{\sqrt{2}}, \; Im[\phi_0] = 0$

* parametrize deviations from the vacuum ("polar")



$$\phi(x) = \frac{1}{\sqrt{2}} e^{i\chi(x)} (v + h(x))$$

⇝ χ field can be eliminated by a gauge trafo!

Abelian Higgs Model — The Physical Spectrum

* $\phi(x) = \frac{1}{\sqrt{2}} e^{i\chi(x)}(v + h(x))$ has 2 degrees of freedom: $\chi(x)$ & $h(x)$ $\in \mathbb{R}$
  - $\chi(x)$ is unphysical ⇝ can be gauged away
  - unitary gauge ⇝ $\phi \to e^{-i\chi}\phi$
* read off physical content

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu h)^2 + \underbrace{\frac{1}{2}g^2v^2 A_\mu A^\mu}_{\text{mass term}} + \mu^2 h^2 \quad + (\text{interactions})$$

  - h (Higgs boson) with mass $M_h^2 = -2\mu^2$
  - the photon acquired a mass $M_A^2 = g^2v^2$ ("ate" the Goldstone)
    - → massless photon: 2 transverse pol^{n}
    - massive photon: 2 transv. ⊕ 1 long. pol^{n}
* $\mathcal{L}$ still has full $U(1)$ gauge symmetry ("hidden")

3 Strong Interactions

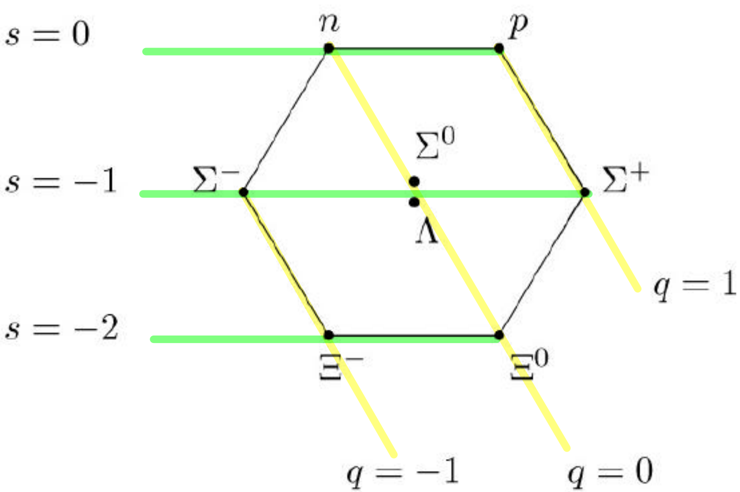
* Large Hadron Collider (LHC) & HL-LHC upgrade
  - ↪ will drive particle physics in the years to come
  - ↪ proton-proton collider ⇒ cannot avoid strong interactions (QCD)
* very rich phenomenology
  - ↪ low energies: non-perturbative bound states: hadrons
  - ↪ high energies: behaves like collection of free constituents: partons

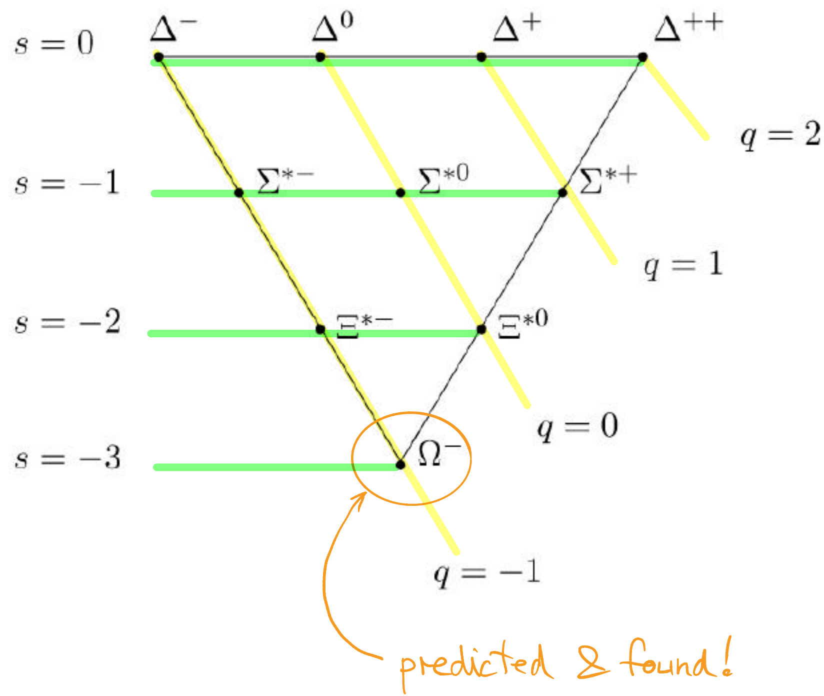
Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex. But not always.

Frank Wilczek

The Eightfold Way [Gell-Mann '61]

- hadron spectrum exhibits a pattern (s = "strangeness")





↳ What is the reason for this pattern?

The Quark Model [Gell-Mann, Zweig '64]

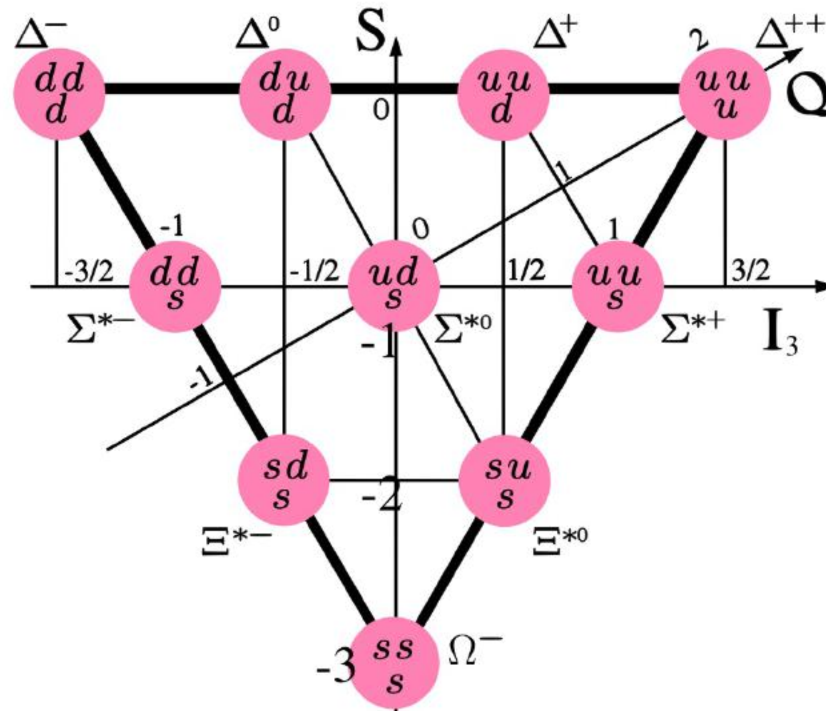
- proposal: spin-$\frac{1}{2}$ constituents (quarks: $\underbrace{u, d, s}_{\text{SU(3) flavour symmetry}}$)
  with fractal charges

u $m_u \sim 4$ MeV ($Q = +2/3$)

proton: stable

d $m_d \sim 7$ MeV ($Q = -1/3$)

s $m_s \sim 135$ MeV ($Q = -1/3$)



⇒ "Explains" the pattern

but no free quarks can be seen!

The Spin-Statistics Issue

Δ^{++} is a state with

- spin $3/2$: $|\uparrow\uparrow\uparrow\rangle$
- $3 \times$ up: $|uuu\rangle$ $(Q=+2)$

↯ Pauli's exclusion principle

⇒ Solution: new quantum number

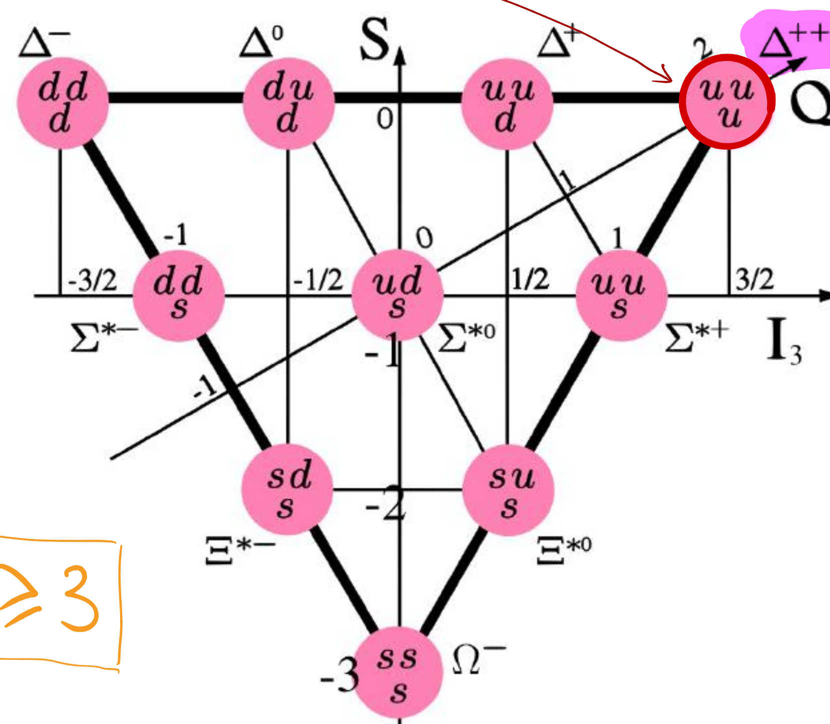
red green blue ...

u^1 u^2 u^3 ? } COLOUR

$\Delta^{++} \sim \epsilon_{ijk}\, u^i u^j u^k$

→ fully anti-symmetric

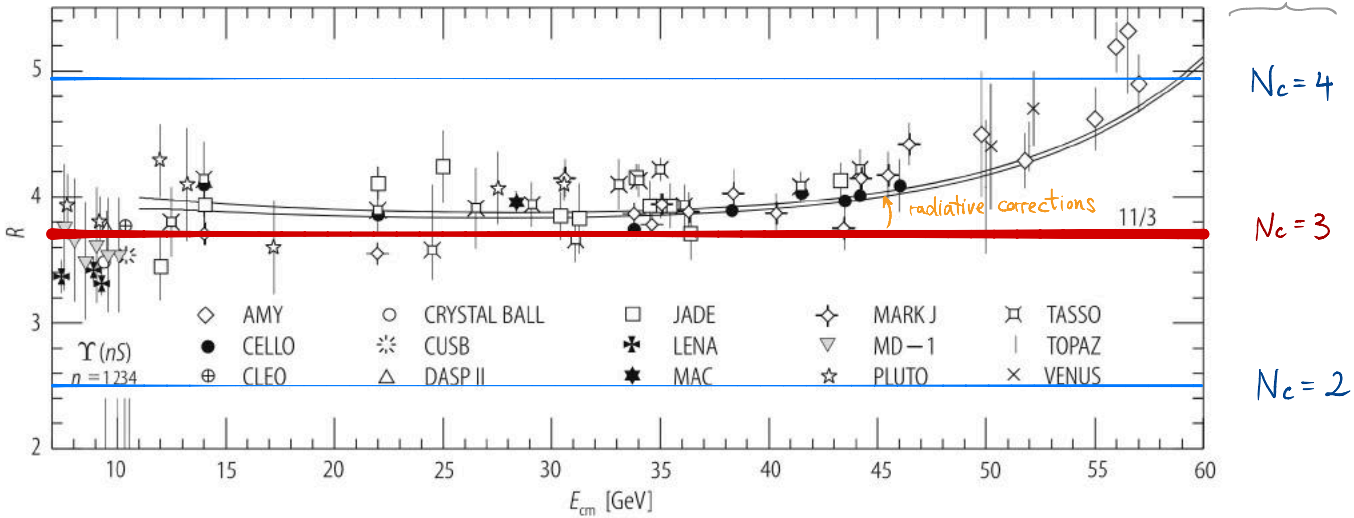
$\boxed{N_c \geq 3}$



Evidence for Colour

* The R-ratio: $R \equiv \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \propto N_c \sum_q Q_q^2$

$\underbrace{\text{lowest-order prediction}}$ $R = N_c \frac{11}{9}$



* pion decay: $\Gamma(\pi \to \gamma\gamma) \propto N_c^2$
* ABJ anomaly cancellation
* much much more ... (BR of W, τ decays)

$\boxed{N_c = 3}$

QCD & Colour Confinement

* QCD has an exact $SU(N_c)$ symmetry: why not $SO(N_c)$?

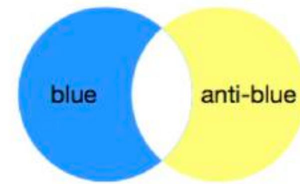
$$UU^\dagger = U^\dagger U = \mathbb{1}, \qquad \det(U) = 1$$

* no isolated colour charges $\left(V_{q\bar{q}}(r) \simeq C_F \left[\frac{\alpha_s(r)}{r} + \dots + \sigma r \right] \right)$

$\Rightarrow$ only colour singlet particles $\rightsquigarrow$ hadrons have integer electr. charge

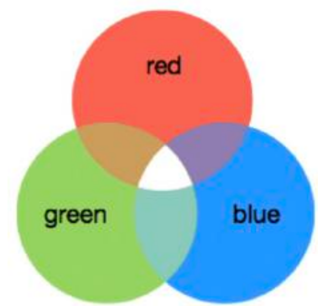
① MESONS (bosons: $\pi, \rho, \dots$)

$$\bar{q}^i q^i \longrightarrow \underbrace{U^*_{ij} \bar{q}^j U_{ik}}_{(U^\dagger)_{ji} U_{ik} = \delta_{jk}} q^k = \bar{q}^i q^i$$



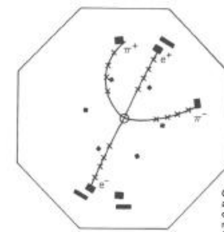
② BARYONS (fermions: $p, n, \dots$)

$$\epsilon_{ijk}\, q^i q^j q^k \longrightarrow \underbrace{\epsilon_{ijk} U_{ii'} U_{jj'} U_{kk'}}_{\det(U)\, \epsilon_{i'j'k'}} q^{i'} q^{j'} q^{k'} = \epsilon_{ijk}\, q^i q^j q^k$$



The Rest of the Family

discovery '74

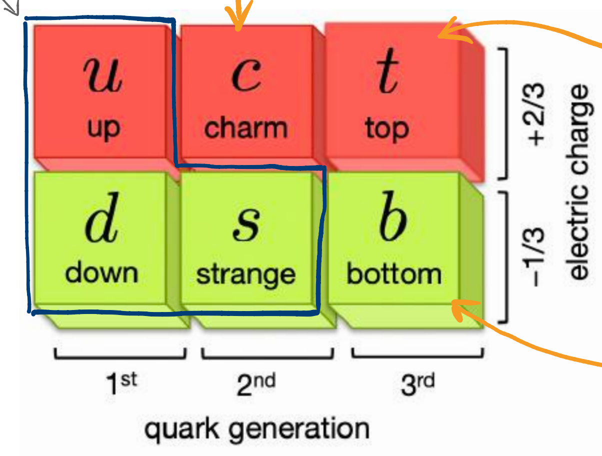


Computer reconstruction of a ψ' decay in the Mark I detector at SLAC, making a near-perfect image of the Greek letter ψ

$J/\psi \hat{=} (c\bar{c})$
"charmonium"

[postulated '70
GIM mechanism]

eightfold way



['87 $m_t > 50\,\mathrm{GeV}$ (B-oscillation)
until '94 $m_t \in [145, 185]\,\mathrm{GeV}$
(EW precision data)
discovery '95 $m_t = 173\,\mathrm{GeV}$]

[postulated '73
CP violation
Kobayashi & Maskawa]

discovery '77 (Υ)

Quantum Chromodynamics

* gauge principle $SU(3)$ $\longleftrightarrow$ $\psi_q = (\psi_q^r, \psi_q^g, \psi_q^b)^T$

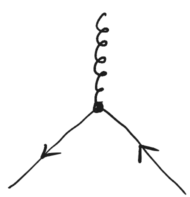
$$\mathcal{L}_{QCD} = \bar{\psi}_q (i\not{\partial} - m_q)\psi_q - \frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{a,\nu} - \partial^\nu A^{a,\mu}) \quad \longleftrightarrow \quad \text{"free"}$$

$$- g_s t^a A_\mu^a \bar{\psi}_q \gamma^\mu \psi_q \quad \longleftrightarrow \quad \bar{q} q g$$

$$+ \frac{g_s}{2} f^{abc} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) A^{b,\mu} A^{c,\nu} \quad \longleftrightarrow \quad g^3$$

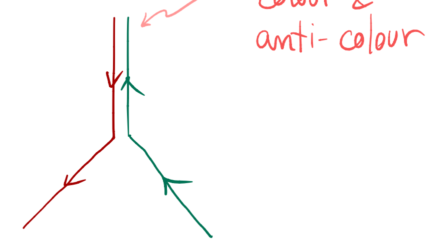
$$- \frac{g_s^2}{4} f^{abc} f^{ade} A_\mu^b A_\nu^c A^{d,\mu} A^{e,\nu} \quad \longleftrightarrow \quad g^4$$

* pictorial rep$^{\underline{n}}$ of quark-gluon interaction



$\bar{\psi}^i \quad t^a_{ij} \quad \psi^j$

$$\underbrace{(1,0,0)}_{\bar{\psi}^r} \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{t^1} \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{\psi^g} \longleftrightarrow$$



↪ "repaint" the quarks

Asymptotic Freedom

[Gross, Wilczek, Politzer '73]

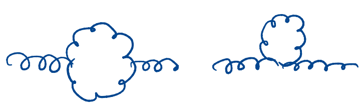
* coupling strength $\alpha_s = \frac{g_s^2}{4\pi}$ depends on the scale Q^2 at which it is probed
* dependence is predicted (up to 5-loops)

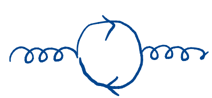
$$\frac{d \ln \alpha_s(\mu^2)}{d \ln \mu^2} = \beta(\alpha_s) = -\frac{\alpha_s}{2\pi}\left[\beta_0 + \frac{\alpha_s}{2\pi}\beta_1 + \dots\right]$$

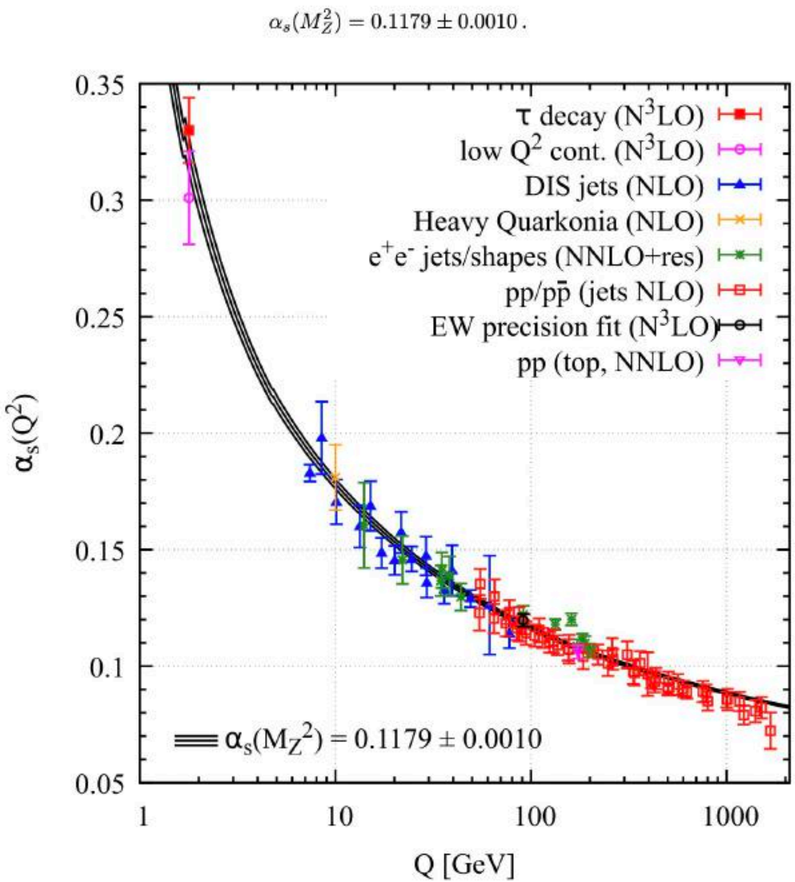
$$\beta_0 = \frac{11}{6} C_A - \frac{N_f}{3}$$

anti-screening

screening







The Plan

1. A brief recap of Quantum Field Theories
2. The main construction Principles for QFTs
   * gauge symmetries * Yang-Mills * SSB
3. Strong Interactions
4. The electroweak Standard Model
   * bottom-up construction
     ↪ Fermi model ⇝ gauge symmetry ⇝ SM
   * Predictions at Lepton colliders
     ↪ EW precision tests
5. LHC Phenomenology

4. The Electroweak Standard Model

* late 1940's Success of QED (rad. corrections) ⇒ boom in elementary particle theory

* 1950's confusion & frustration ⇝ give up QFT?
    - ↳ strong interactions
      perturbation theory useless? → S-Matrix program
    - ↳ weak interactions
      4-fermion theory plagued by infinities

* "During this time of confusion and frustration [...] there emerged three good ideas" [Weinberg]

(1) quark model
(2) gauge (local) symmetries
(3) spontaneous symmetry breaking

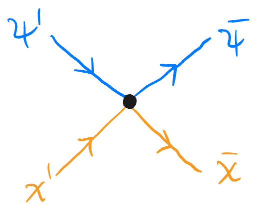
} last lecture

Weak Interactions — Fermi model

* discovery in β-decay $\quad n \to p + e^- + \bar{\nu}_e$, $\quad p \to n + e^+ + \nu_e$ (not for free p)

  more: $\quad \mu$-decay $\quad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, ...

* modeled as 4-fermion interaction



$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}} \left[\bar{\psi}\, \Gamma_X\, \psi'\right] \left[\bar{\chi}\, \Gamma^X\, \chi'\right]$$

↪ Dirac bilinear $\quad \Gamma_X \in \{ \underset{\text{scalar}}{\mathbb{1}},\ \overset{\text{pseudo-scalar}}{\gamma^5},\ \underset{\text{vector}}{\gamma_\mu},\ \overset{\text{pseudo-vector}}{\gamma_\mu\gamma_5},\ \underset{\text{tensor}}{\sigma_{\mu\nu}} \}$

↪ experiment ⇒ parity violation [Wu et al. '57]

⇒ "V–A" $\quad \Gamma_X = \gamma_\mu(\mathbb{1} - \gamma_5)$

[$\omega_\pm = \frac{1}{2}(1 \pm \gamma_5)$ projectors

↪ left-/right-handed fermions $\psi_{L/R} = \omega_\mp \psi$

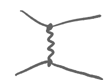
⇒ only left-handed participate

$$\boxed{\mathcal{L}_{\text{Fermi}} = -\frac{4 G_F}{\sqrt{2}} \left[\bar{\psi}_L \gamma_\mu \psi'_L\right]\left[\bar{\chi}_L \gamma^\mu \chi'_L\right]}$$

Weak Interactions — Fermi model

* universal coupling $G_F = 1.16639 \cdot 10^{-5}\ \mathrm{GeV}^{-2}$

* unitarity violation $\sigma(\nu_\mu e \to \nu_\mu e)$ etc. grow $\propto E^2$
* not renormalizable $\rightarrow$ no radiative corrections
* try to formulate as a gauge theory

  0. Identify the multiplets
  1. Introduce mediator bosons 
  2. Identify group & complete
  3. Link to the real world (EW & SSB)

Towards a Gauge Theory of EW Interactions

0. Look at muon decay as an explicit example $\left(\mu^\pm \to e^\pm + \overset{\leftrightarrow}{\nu}_e + \overset{\leftrightarrow}{\nu}_\mu\right)$

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left(\left[\bar{\psi}_{L,\mu}\gamma_\rho\psi_{L,\nu_\mu}\right]\left[\bar{\psi}_{L,e}\gamma^\rho\psi_{L,\nu_e}\right] + \left[\bar{\psi}_{L,\nu_\mu}\gamma_\rho\psi_{L,\mu}\right]\left[\bar{\psi}_{L,\nu_e}\gamma^\rho\psi_{L,e}\right]\right)$$

↪ doublet structure $L_\ell = \begin{pmatrix}\psi_{L,\nu_\ell}\\ \psi_{L,\ell}\end{pmatrix}$

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}}\left[\left(\bar{\psi}_{L,\nu_\mu},\bar{\psi}_{L,\mu}\right)\gamma_\rho\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}\psi_{L,\nu_\mu}\\ \psi_{L,\mu}\end{pmatrix}\right]\left[\left(\bar{\psi}_{L,\nu_e},\bar{\psi}_{L,e}\right)\gamma^\rho\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}\psi_{L,\nu_e}\\ \psi_{L,e}\end{pmatrix}\right] + \left[\underset{\sigma_+}{\begin{pmatrix}0&1\\0&0\end{pmatrix}} \leftrightarrow \underset{\sigma_-}{\begin{pmatrix}0&0\\1&0\end{pmatrix}}\right]$$

$$= -\frac{4G_F}{\sqrt{2}}\sum_{a=\pm}\left[\bar{L}_\mu\gamma_\rho\sigma_a L_\mu\right]\left[\bar{L}_e\gamma^\rho\sigma_a L_e\right] \quad , \qquad \sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$$

Towards a Gauge Theory of EW Interactions

1. In gauge theories, interactions mediated through gauge-boson exchange

↳ structure $\mathcal{L}_{int} = g\,[\bar{\psi}\gamma^\mu T^a \psi]\,W^a_\mu$

↳ compare with Fermi $\propto [\bar{L}\gamma^\mu \sigma_\pm L]$

⇒ introduce two charged gauge bosons $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$

$$\mathcal{L}_{IVB} = \frac{g}{2\sqrt{2}} \sum_\ell \left([\bar{L}_\ell \gamma^\mu \sigma_+ L_\ell]\, W^+_\mu + [\bar{L}_\ell \gamma^\mu \sigma_- L_\ell]\, W^-_\mu \right)$$

$$= \frac{g}{2} \sum_\ell \sum_{i=1,2} [\bar{L}_\ell \gamma^\mu \sigma_i L_\ell]\, W^i_\mu$$

* stop here & introduce massive $W^\pm$ "intermediate-vector-boson model"

↳ $\sqrt{2}\, G_F = \frac{g^2}{4 M_W^2}$ ⇝ $M_W = \mathcal{O}(100\ \text{GeV})$ ⟷ but <u>not</u> a gauge theory!

Towards a Gauge Theory of EW Interactions

2. We identified two generators $\sigma_\pm$ (or σ_1, σ_2)

   - if we want a group → better close $[\sigma_+, \sigma_-] = \sigma_3$
   - need to include $\sigma_3 = \begin{pmatrix} 1 & \emptyset \\ \emptyset & -1 \end{pmatrix}$

     it is diagonal ($\ell^\pm \to \ell^\pm$; $\overset{(-)}{\nu}_\ell \to \overset{(-)}{\nu}_e$) $\Rightarrow$ a neutral current W_μ^3

* We already know a neutral boson

   - Could this be the photon $W_\mu^3 = A_\mu$?

Towards a Gauge Theory of EW Interactions

2. We identified two generators $\sigma_\pm$ (or σ_1, σ_2)
   - if we want a <u>group</u> → better close $[\sigma_+, \sigma_-] = \sigma_3$
   - need to include $\sigma_3 = \begin{pmatrix} 1 & \emptyset \\ \emptyset & -1 \end{pmatrix}$

     it is diagonal $(\ell^\pm \to \ell^\pm ; \overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e)$ ⇒ a neutral current W_μ^3

* We already know a neutral boson
   - Could this be the photon $W_\mu^3 = A_\mu$?

No
- eigenvalues of $\hat{Q} \neq \hat{I}_W^3 = \frac{\sigma^3}{2}$
- $\hat{Q}$ also acts on right-handed fields!

Towards a Gauge Theory of EW Interactions

2. We identified two generators $\sigma_\pm$ (or σ_1, σ_2)
   - if we want a group → better close $[\sigma_+, \sigma_-] = \sigma_3$
   - need to include $\sigma_3 = \begin{pmatrix} 1 & \emptyset \\ \emptyset & -1 \end{pmatrix}$

     it is diagonal ($\ell^\pm \to \ell^\pm$; $\overset{(-)}{\nu}_\ell \to \overset{(-)}{\nu}_e$) ⇒ a neutral current W^3_μ

* Rewrite the charge operator $\hat{Q} = \hat{I}^3_W + \frac{\hat{Y}}{2}$ ← hyper charge

- a second neutral current B_μ
- non-trivial inclusion of $U(1)_{QED}$

  ⇒ $SU(2)_W \times U(1)_Y$ EW unification!

- $$\boxed{D_\mu = \partial_\mu - i g \hat{I}^i_W W^i_\mu + i g' \frac{\hat{Y}}{2} B_\mu}$$

		generation			representation	charges		
		1st	2nd	3rd		I^3_w	Y	Q
leptons	$\Psi_L^{L_i'}$	$\begin{pmatrix}\nu_e' \\ e'\end{pmatrix}_L$	$\begin{pmatrix}\nu_\mu' \\ \mu'\end{pmatrix}_L$	$\begin{pmatrix}\nu_\tau' \\ \tau'\end{pmatrix}_L$	$(\mathbf{1},\mathbf{2})_{-1}$	$\frac{1}{2}$ / $-\frac{1}{2}$	-1 / -1	0 / -1
	$\Psi_R^{\ell_i'}$	e_R'	μ_R'	τ_R'	$(\mathbf{1},\mathbf{1})_{-2}$	0	-2	-1
quarks	$\Psi_L^{Q_i'}$	$\begin{pmatrix}u' \\ d'\end{pmatrix}_L$	$\begin{pmatrix}c' \\ s'\end{pmatrix}_L$	$\begin{pmatrix}t' \\ b'\end{pmatrix}_L$	$(\mathbf{3},\mathbf{2})_{\frac{1}{3}}$	$\frac{1}{2}$ / $-\frac{1}{2}$	$\frac{1}{3}$ / $\frac{1}{3}$	$\frac{2}{3}$ / $-\frac{1}{3}$
	$\Psi_R^{u_i'}$	u_R'	c_R'	t_R'	$(\mathbf{3},\mathbf{1})_{\frac{4}{3}}$	0	$\frac{4}{3}$	$\frac{2}{3}$
	$\Psi_R^{d_i'}$	d_R'	s_R'	b_R'	$(\mathbf{3},\mathbf{1})_{-\frac{2}{3}}$	0	$-\frac{2}{3}$	$-\frac{1}{3}$

Towards a Gauge Theory of EW Interactions

3. generate gauge-boson masses through SSB

what about
$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \in \mathbb{R}^3$
in adjoint repⁿ
$SU(2) \simeq SO(3)$?
$Y_\Phi = \emptyset$ & $\Phi_0 = (0, 0, \frac{v}{\sqrt{2}})^T$

$\Rightarrow$ The SM introduces an SU(2) doublet

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad ; \quad \phi_i \in \mathbb{C}$$

- implicit choices & constraints

(a) in our repⁿ $\rightsquigarrow$ $\hat{Q}$ diagonal $\Rightarrow$ ϕ_i eigenstates of $\hat{Q}$

(b) Gell-Mann-Nishijima : $\hat{Q} = \hat{I}^3_W + \frac{Y}{2}$ $\rightsquigarrow$ $|Q_1 - Q_2| \overset{!}{=} 1$

(c) QED better remain unbroken (photon $\overset{!}{=}$ massless)

$\Rightarrow$ vev must have $\hat{Q} = \emptyset$

$\Rightarrow$ Only choices $Y_\Phi = \pm 1$ $\qquad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ or $\begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$

charge conj.

Towards a Gauge Theory of EW Interactions

3. The Higgs Potential

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 \qquad (\mu^2, \lambda > 0)$$

- minimum $\quad |\langle \Phi_0 \rangle| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$

- excitations from the vev & unitary gauge

$$\rightsquigarrow \quad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H \end{pmatrix} \qquad H \in \mathbb{R}$$

- gauge boson masses

$$(D_\mu \Phi)^\dagger (D^\mu \Phi)\Big|_{v^2} = \frac{v^2}{2}\left\{ \frac{g^2}{2} W_\mu^+ W^{-,\mu} + \frac{1}{4}\underbrace{\left[g W_\mu^3 + g' B_\mu\right]^2}_{\text{mixing}} \right\}$$

mixing $\Rightarrow$ diagonalize

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

& identify EM coupling $\quad e = \frac{g g'}{\sqrt{g^2 + g'^2}}$

$$\sin\theta_W \equiv s_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

weak mixing angle

Towards a Gauge Theory of EW Interactions

3. charge- & mass-eigenstates

↪ $$(D_\mu \Phi)^\dagger (D^\mu \Phi)\Big|_{v^2} = \frac{e^2 v^2}{4 s_W^2} W_\mu^+ W^{-,\mu} + \frac{e^2 v^2}{4 s_W^2 c_W^2} Z_\mu Z^\mu$$

$$\Rightarrow M_W = \frac{ev}{2 s_W} \,, \quad M_Z = \frac{ev}{2 s_W c_W} = \frac{M_W}{c_W} \,, \quad M_A \equiv 0$$

↪ couplings $\quad g = e/s_W \,, \quad g' = e/c_W$

↪ $$D_\mu = \partial_\mu + ie\hat{Q} A_\mu - i \frac{e}{s_W c_W}\left[\hat{I}_W^3 - s_W^2 \hat{Y}\right] Z_\mu - i \frac{e}{\sqrt{2}\, s_W}\left[\hat{I}_W^+ W_\mu^+ + \hat{I}_W^- W_\mu^-\right]$$

↪ $$M_H = \sqrt{2\mu^2}$$

Towards a Gauge Theory of EW Interactions

3. Fermion masses ⟷ SM distinguishes L/R fermions ("chiral")

↪ naive mass term $m_f \bar{\psi}_f \psi_f = m_f (\bar{\psi}_{L,f} \psi_{R,f} + \bar{\psi}_{R,f} \psi_{L,f})$ forbidden

↪ Higgs to the rescue (doublet w/ $Y_\Phi = \pm 1$) ⇒ Yukawa interactions

$$\mathcal{L}_{Yuk} = - \sum_{i,j=1}^{3} \left(\bar{L}_{i,L}\, G^{\ell}_{ij}\, \ell_{j,R}\, \Phi + \bar{Q}_{i,L}\, G^{u}_{ij}\, u_{j,R}\, \Phi^{c} + \bar{Q}_{i,L}\, G^{d}_{ij}\, d_{j,R}\, \Phi + h.c. \right)$$

generations complex (3x3)

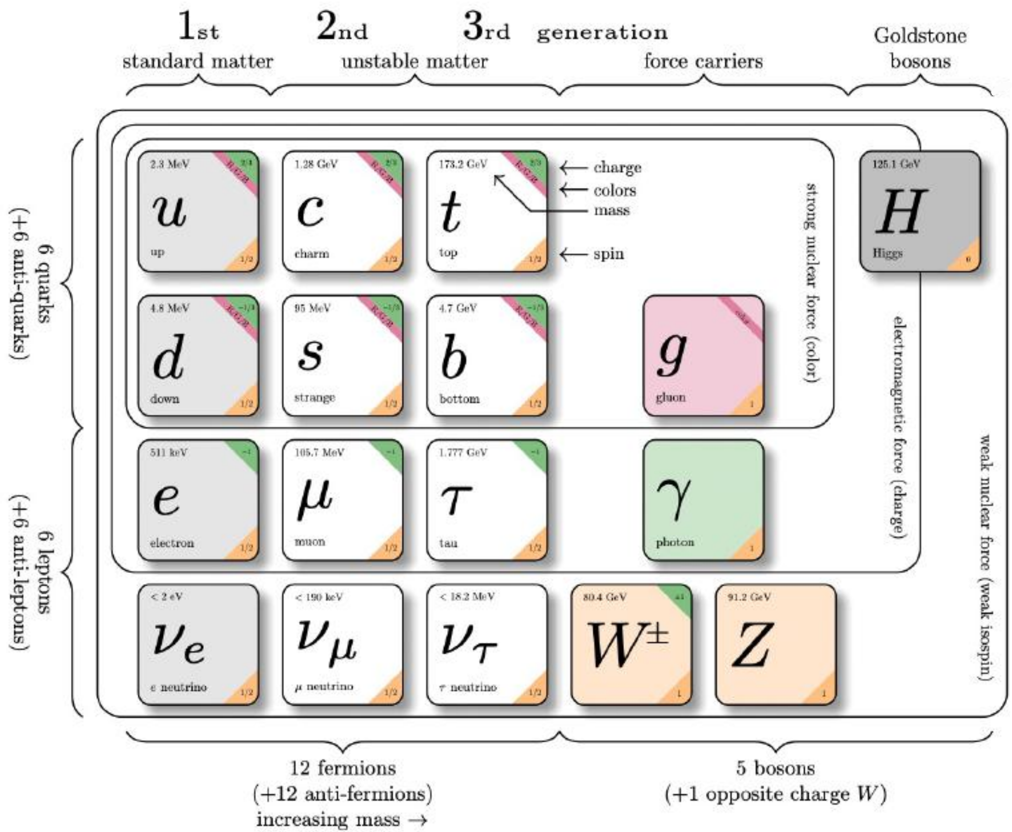
⇒ gauge inv. & vev generates mass matrix ($m_\nu = \emptyset$ assumed)

$$M^f_{ij} = \frac{v}{\sqrt{2}} G^f_{ij} \quad , \ f = \ell, u, d$$

- diagonalize ⇒ unitary transform $\frac{v}{\sqrt{2}} U^f_L\, G^f\, (U^f_R)^\dagger = \mathrm{diag}(m_f)$
- drops out everywhere except in charged-current-int$^{\underline{n}}$

⇒ $V_{CKM} = U^u_L (U^d_L)^\dagger$ ⇝ $\frac{e}{\sqrt{2}\, s_W} \bar{u}_L \gamma^\mu V_{CKM}\, d_L + h.c.$

When the dust settles ...



parameters

3 couplings

$(g, g', g_s) \leftrightarrow (e, s_W, g_s)$

2 Higgs potential

$(\mu^2, \lambda) \longleftrightarrow (M_H, \lambda)$

9_{+3} fermion masses

6 quarks + 3 leptons (ν)

4_{+4} CKM matrix (PMNS)

3 angles + 1 phase

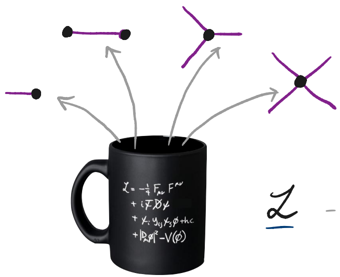
18_{+7}

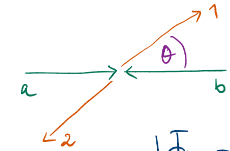
The Standard Model

[Glashow, Salam, Weinberg '67]

* particle content fully veryfied (last: Higgs 2012)
  ↪ consistent QFT (unitarity, renormalizable ⟷ precision!, anomaly free)
* full set of input parameters known
  ↪ independent predictions & self-consistency tests $\frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \rho \simeq 1$
* reference theory for particle physics
  ↪ refer to "Beyond the Standard Model"

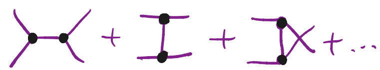
Making Predictions





$$d\Phi_2 = \frac{d\cos\theta}{16\pi}$$

$\mathcal{L}$ ⇢ scattering amplitudes

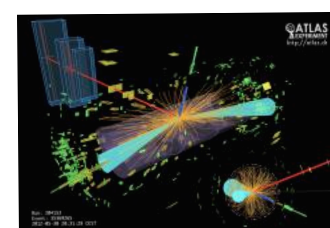


Feynman diagrams & rules

cross sections*

$$d\sigma_{2\to n} = \frac{1}{F} \langle |\mathcal{M}|^2 \rangle \, d\Phi_n$$

Event rates: $N = L\,\sigma$



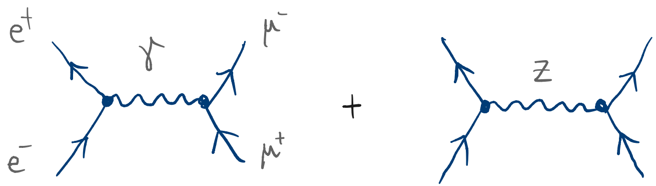
* decay rates $(\tau = 1/\Gamma)$

$$d\Gamma_{1\to n} = \frac{1}{2M} \langle |\mathcal{M}|^2 \rangle \, d\Phi_n$$

Some Fun at the Z pole

Consider the process $e^+ e^- \to \mu^+ \mu^-$

At lowest order (tree level) there are two diagrams



$$= i \mathcal{M}^{tree}_{e^+e^- \to \mu^+\mu^-}$$

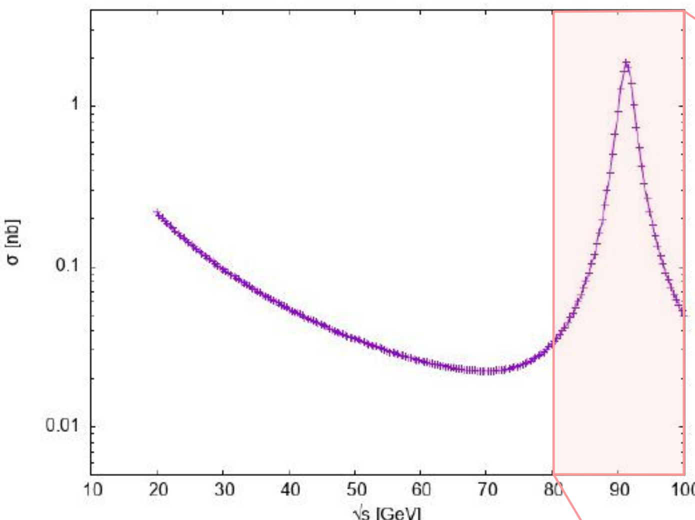
$\Rightarrow$ inserting into Fermi's golden rule $\left[s = E_{cm}^2 ; \quad P_a \cdot P_1 = p_a^\mu P_{1,\mu} = E_{cm}^2 (1 - \cos\theta) \right]$

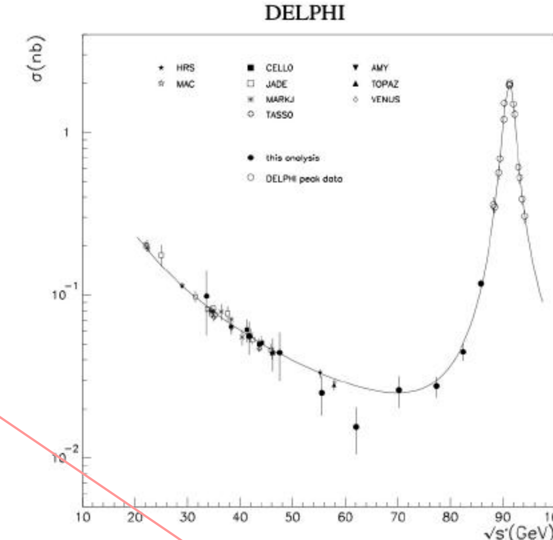
$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2 \pi}{2s} \left[(1 + \cos^2\theta) \, G_1(s) + 2 \cos\theta \, G_2(s) \right]$$

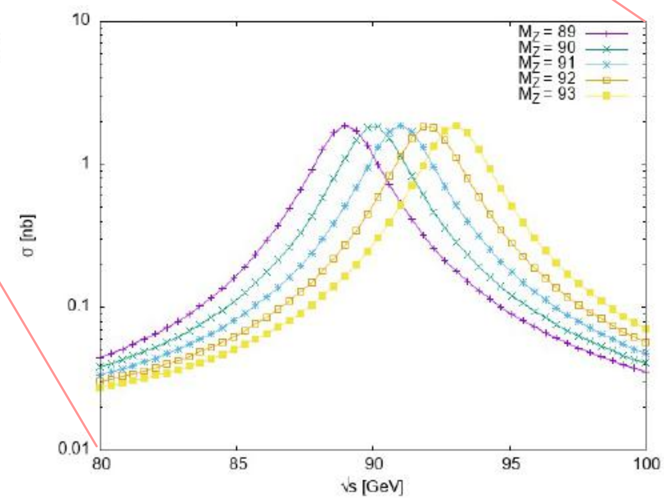
$$G_1(s) = 1 + 2 v_e^2 \, \mathrm{Re}\left\{ \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right\} + (v_e^2 + a_e^2)^2 \left| \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2$$

$$G_2(s) = \emptyset + 2 a_e^2 \, \mathrm{Re}\left\{ \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right\} + 4 v_e^2 \cdot a_e^2 \left| \frac{}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2$$

Theory vs. Data σ

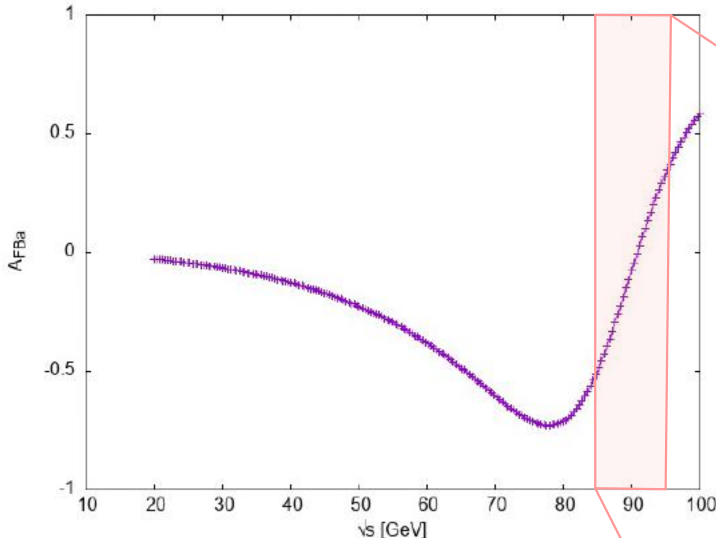


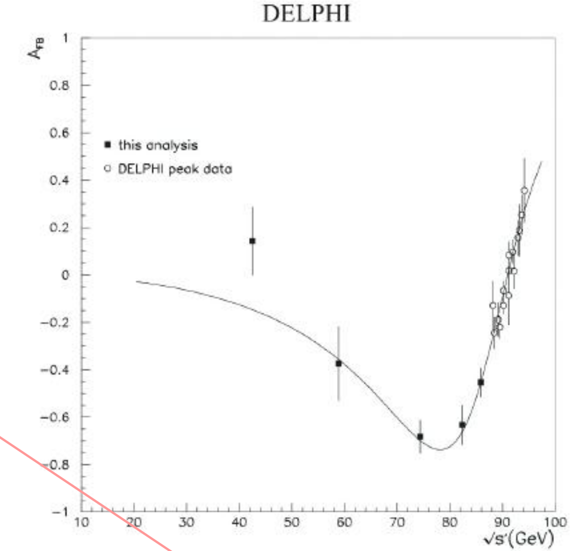


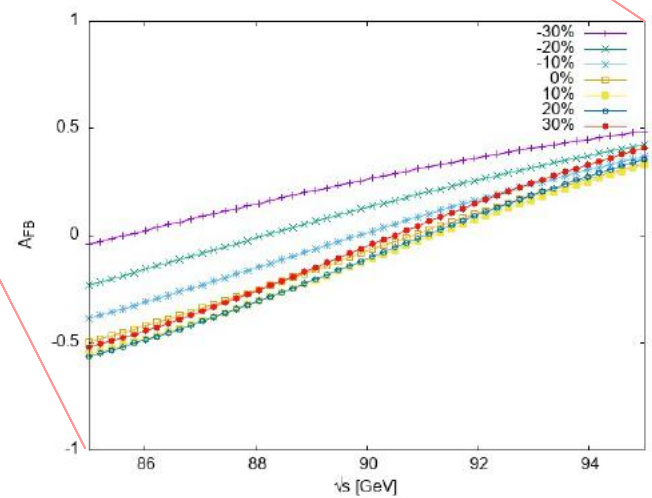


M_Z extraction

Theory vs. Data A_{FB}





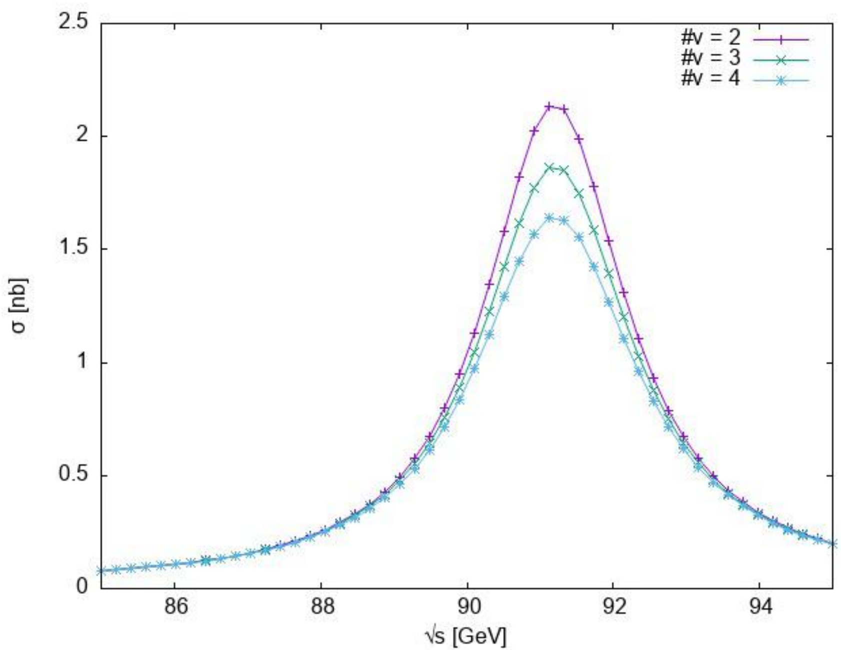


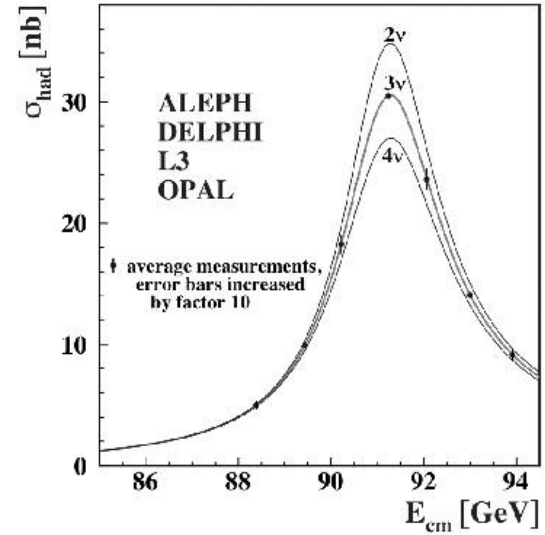
$\sin^2\theta_W$ extraction

test:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

Theory vs. Data σ_{had}





try to implement it yourself

$e^+e^- \to \mu^+\mu^-$

$\leadsto e^+e^- \to q\bar{q}$

generations = 3

(with light neutrinos)

Standard Model Precision Physics

github.com/aykhuss

1. A brief recap of Quantum Field Theories
2. The main construction Principles for QFTs
3. Strong Interactions
4. The electroweak Standard Model
   * bottom-up construction & Z-pole observables
   * EW precision tests & renormalization
5. LHC Phenomenology
   * Parton Distribution Functions
   * The Drell-Yan process & higher-order calculations
   * Transverse momentum resummation
   * Parton Showers & Event generators

The Standard Model @ Tree level

* our explorations so far were Born-level (LO, tree) predictions
  - Let us be more quantitative in the comparison

* Some SM predictions @ tree level ($s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$)

  - $\alpha^{(0)} = \dfrac{e^2}{4\pi}$

  - $G_F^{(0)} = \dfrac{\alpha\pi}{\sqrt{2}\, s_W^2 M_W^2} = \dfrac{1}{\sqrt{2}\, v^2}$

  - $M_Z^{(0)} = M_Z = \dfrac{e\,v}{2\, s_W c_W}$

  - $M_W^{(0)} = M_W = \dfrac{e v}{2\, s_W}$

  - $\left(s_{W,\mathrm{eff}}^{(0)}\right)^2 = s_W^2$

  - $\Gamma^{(0)}_{Z\to e^+e^-} = \dfrac{\alpha M_Z}{6\, s_W^2 c_W^2}\left[\left(-\frac{1}{2}+s_W^2\right)^2 + s_W^4\right] = \dfrac{e^3 v}{48\pi\, s_W^3 c_W^3}\left[\left(-\frac{1}{2}+s_W^2\right)^2 + s_W^4\right]$

all of them only depend on 3 EW Lagrangian parameters (@ tree level)

(e, v, s_W)

The Standard Model @ Tree level

... and they are among the most precisely measured

$\hookrightarrow$ $\hat{\alpha}_0 = (137.035\,999\,084(21))^{-1}$

$\hookrightarrow$ $\hat{G}_F = 1.166\,378\,8(6) \cdot 10^{-5}\ \mathrm{GeV}^{-2}$

$\hookrightarrow$ $\hat{M}_Z = 91.1876(21)\ \mathrm{GeV}$

① pick as <u>input</u> (most precise)

$\hookrightarrow$ $\hat{M}_W = 80.377(12)\ \mathrm{GeV}$

$\hookrightarrow$ $(\hat{s}_{W,\mathrm{eff}})^2 = 0.231\,53(4)$

② predict these

$\Rightarrow$ check for consistency

The Standard Model @ Tree level

① invert tree-level relations $\hat{O} = O^{(0)}$ to get Lagrange parameters

$\hookrightarrow$ $e^2 = 4\pi\hat{\alpha}$

$\hookrightarrow$ $v^2 = \dfrac{1}{\sqrt{2}\,\hat{G}_F}$

$\hookrightarrow$ $\dfrac{e^2 v^2}{4 s_W^2 c_W^2} = M_Z^2 \Rightarrow s_W^2 = \dfrac{1}{2}\left(1 - \sqrt{1 - \dfrac{4\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F \hat{M}_Z^2}}\right)$

② insert into predictions for remaining observables

$\hookrightarrow$ $M_W^{(0)} = \dfrac{e v}{2 s_W} = 80.9389 \longleftrightarrow \mathcal{O}(10\sigma)$

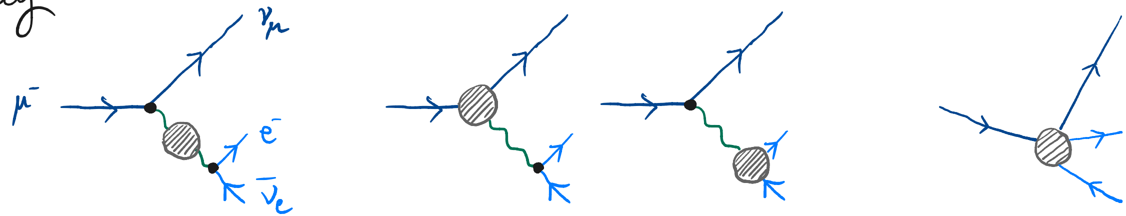
$\hookrightarrow$ $(s_{W,\text{eff}}^{(0)})^2 = s_W^2 = 0.212152 \longleftrightarrow \mathcal{O}(100\sigma)$

SM is excluded?!

Radiative corrections

* higher-order corrections require evaluation of loop diagrams

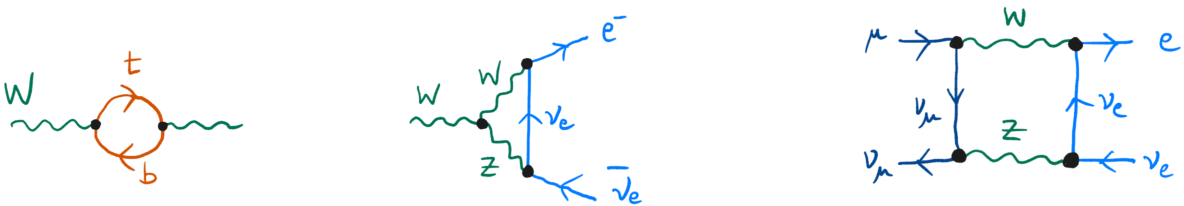
→ μ decay



self energies/ bubbles

vertices/ triangles

boxes

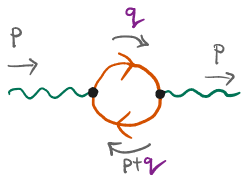


→ one unconstrained momentum q

$\Rightarrow \int \frac{d^4q}{(2\pi)^4}$ (QM: sum over all intermediate states)

Ultraviolet (UV) Divergences

* loop integrations can be divergent



$$\sim \int \frac{d^4q}{(2\pi)^4} \frac{\{1, q, q^2\}}{(q^2-m_1^2)((q+p)^2-m_2^2)} \xrightarrow{q \text{ large}} \int \frac{d^4q}{q^4}$$

"UV"

logarithmic divergence

* regularization ⇝ make expression finite & well defined (TH extension)
  - ⊳ most common dimensional regularization $D = 4-2\epsilon$ dimensions
    - ⟷ original (divergent) theory : $\epsilon \longmapsto 0$
    - ⟷ divergences appear as poles : $1/\epsilon^n$

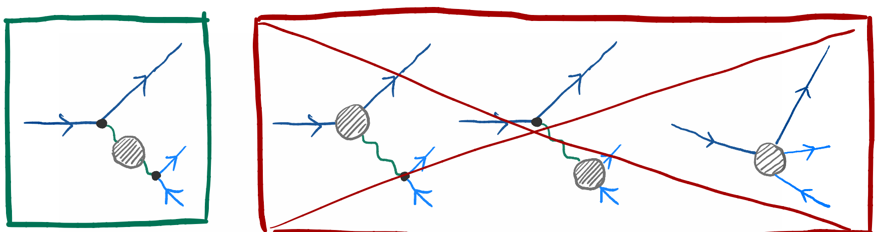
$$\int \frac{d^4q}{q^4} \to \int d\Omega_D \int_\delta^\infty dq\, \frac{q^{D-1}}{q^4} = \frac{(2\pi)^{2-\epsilon}}{\Gamma(2-\epsilon)} \frac{1}{2\epsilon} \frac{1}{\delta^{2\epsilon}} \propto \frac{1}{\epsilon} + \ln\delta + \mathcal{O}(\epsilon)$$

* renormalization ⇝ relations between physical quantities
  - ⟷ finite if theory is renormalizable

Oblique Corrections

* full renormalization of the SM beyond the scope here
  ↪ dedicated tutorials by Stefan Kallweit on radiative corr.

* we will instead use the set of EW precision observables from above to demonstrate UV cancellation explicitly
  ↪ @ tree level ⟷ exchange of gauge bosons
  ⇒ largest contributions from self energies "oblique corrections"



⟷ tractable subset
well defined (fermion loops)
largest effects from (t, b)

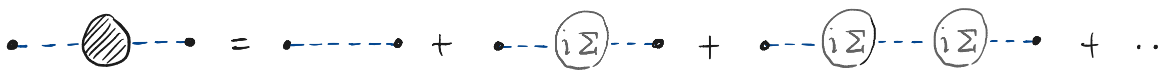
Particle Masses & Radiative Corrections

* How do we define a particle's mass in a QFT?

Particle Masses & Radiative Corrections

* How do we define a particle's mass in a QFT?
  - it is the location of the propagator pole $\frac{i}{p^2 - m^2}$
  - How do radiative corrections impact the pole position?

example for a scalar



$$= \frac{i}{p^2 - m^2} + \frac{i}{p^2 - m^2}\, i\Sigma_i\, \frac{i}{p^2 - m^2} + \dots = \frac{i}{p^2 - m^2} \sum_{n=0}^{\infty} \left(i\Sigma_i \frac{i}{p^2 - m^2} \right)^n$$

Dyson summation

$\sum a^n \rightarrow \frac{1}{1-a}$

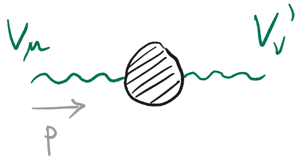
$$= \frac{i}{p^2 - m^2 + \Sigma_i(p^2)}$$

$\Rightarrow$ higher-order corrections impact the location & residue of propagator pole!

Gauge boson Self Energy & Corrections to the Mass

* gauge-boson self energy

$$-i\,\Sigma^{V_\mu V'_\nu}(p) =$$



$$= -i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Sigma_T^{VV'}(p^2) - i\,\frac{p_\mu p_\nu}{p^2}\,\Sigma_L^{VV'}(p^2)$$

⇾ In the following, we will only need $\Sigma_T^{VV'}$

- for mass definition, we project on physical states $\varepsilon^\mu(p)\,p_\mu \equiv 0$
- for our observables, we neglect fermion masses $[\bar\psi\gamma^\mu\psi]\,p_\mu \to 0$

* the on-shell mass

will drop it in the following

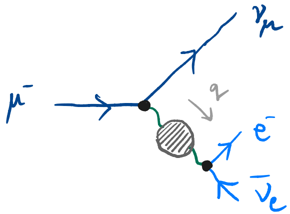
$$\mathrm{Re}\left[\text{"propagator denominator"}\right] \overset{!}{=} 0 = M^2_{V,os} - M^2_V + \mathrm{Re}\left[\Sigma_T^{VV}(M^2_{V,os})\right]$$

$$\Rightarrow \quad M^2_{V,os} = M^2_V - \Sigma_T^{VV}(M^2_{V,os})$$

parameter in $\mathcal{L}$

Oblique Corrections to G_F

* observable defined via μ-decay $\longleftrightarrow$ $\tau_\mu^{-1} = \Gamma_\mu$



use $\displaystyle \frac{-i g_{\mu\nu}}{q^2 - M_W^2 + \Sigma_T^{WW}(q^2)} \xrightarrow[q^2 \to 0]{m_\mu^2 \ll M_W^2} i g_{\mu\nu} \left[\frac{1}{M_W^2} + \frac{\Sigma_T^{WW}(0)}{M_W^4} + \dots \right]$

LO

$$\Rightarrow \quad G_F^{(1)} = G_F^{(0)} \left[1 + \frac{\Sigma_T^{WW}(0)}{M_W^2} \right]$$

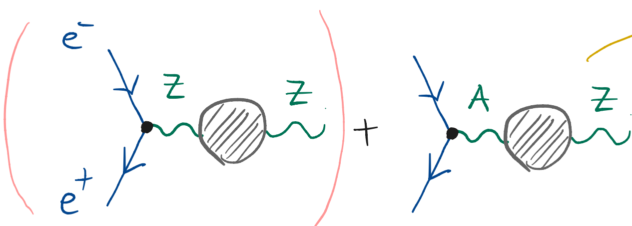
Oblique Corrections to $s^2_{w,eff}$

* the effective weak mixing angle can be defined via the Z-boson polarization asymmetry

$$A_e = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{\sigma(e^+_L e^-_L \to Z) - \sigma(e^+_R e^-_R \to Z)}{\sigma(e^+_L e^-_L \to Z) + \sigma(e^+_R e^-_R \to Z)}$$

define $$\hat{A}_e \equiv \frac{\left[\frac{1}{2} - (\hat{s}_{w,eff})^2\right]^2 - (\hat{s}_{w,eff})^4}{\left[\frac{1}{2} - (\hat{s}_{w,eff})^2\right]^2 + (\hat{s}_{w,eff})^4}$$ tree-level relation $s_w \longmapsto s_{w,eff}$

* higher-order corrections



gives a change to effective coupling

$$g_e^- = \frac{I_W^3 - s_w^2 Q_f}{s_w c_w} \longmapsto g_e^- - \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} Q_f$$

$$g_e^+ = -\frac{s_w}{c_w} Q_f \longmapsto g_e^+ - \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} Q_f$$

overall normalization to $\sigma_{L/R}$ $\Rightarrow$ cancels in ratio!

$\Leftrightarrow$ LSZ / WF norm. $\leftrightarrow$ $R_Z \propto \left.\frac{d\Sigma_T^{ZZ}(q^2)}{dq^2}\right|_{q^2=M_Z^2}$

Oblique Corrections to $s^2_{w,\mathrm{eff}}$

$$\frac{I_W^3 - s_W^2 Q_f}{s_W c_W} - \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} Q_f = \frac{1}{s_W c_W}\left[I_W^3 - \left(s_W^2 + s_W c_W \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} \right) Q_f \right]$$

$$- \frac{s_W}{c_W} Q_f - \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} Q_f = - \frac{Q_f}{s_W c_W} \left(s_W^2 + s_W c_W \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2} \right)$$

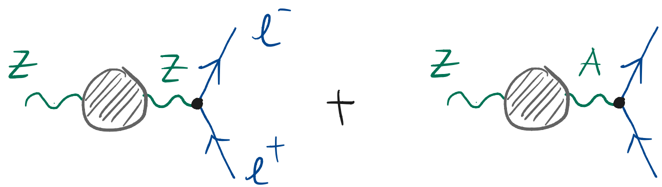
$\hookrightarrow$ looks like tree-level coupling after $s_W^2 \longmapsto \left(s_{W,\mathrm{eff}}^{(1)}\right)^2$ in numerator

$$\Rightarrow \quad \left(s_{W,\mathrm{eff}}^{(1)}\right)^2 = s_W^2 + s_W c_W \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2}$$

Oblique Corrections to $\Gamma_{Z\to e^+e^-}$

* Z-decay $\leftrightarrow$ no longer ratio $\Rightarrow$ normalization matters

  $\hookrightarrow$ 2nd diagram captured by $s_{W,\mathrm{eff}}$



$$\Rightarrow \Gamma^{(1)}_{Z\to e^+e^-} = \left(1 - \Sigma_T^{ZZ\,\prime}(M_Z^2)\right) \Gamma^{(0)}_{Z\to e^+e^-} \Bigg| \begin{array}{l} s_W \to s^{(1)}_{W,\mathrm{eff}} \text{ in numerator} \\ M_Z \to M^{(1)}_{Z,\mathrm{OS}} \end{array}$$

Oblique Corrections to α

* $\alpha_\emptyset \sim 1/137$ is not a suitable input if we're <u>not</u> dealing w/ ext. γ's
  $\longleftrightarrow \Pi(\emptyset) \equiv \left.\frac{\Sigma_T^{AA}(q^2)}{q^2}\right|_{q^2\to\emptyset}$ induces sensitivity to mass-singular terms $\sim \ln(m_f)$

* avoided with coupling defined at high scale $\rightsquigarrow \alpha(M_Z)$ (or α_{G_μ})

$$\Rightarrow \quad \alpha^{(1)}(M_Z) = \frac{e^2}{4\pi}\left[1 - \frac{\Sigma_T^{AA}(M_Z^2)}{M_Z^2}\right]$$

running from $Q^2=\emptyset \longrightarrow Q^2 = M_Z^2$

$\Delta\alpha(M_Z) = \Pi^{AA}(\emptyset) - \Pi^{AA}(M_Z^2) \sim 6\%$

* need to use corresponding input value
  $\hat{\alpha}(M_Z) = (127.951(9))^{-1}$

EPO @ 1-loop

* 1-loop predictions* in terms of (bare) Lagrange parameters

$$\hookrightarrow \alpha^{(1)}(M_Z) = \frac{e^2}{4\pi}\left[1 - \frac{\Sigma_{T}^{AA}(M_Z^2)}{M_Z^2}\right]$$

$$\hookrightarrow G_F^{(1)} = \frac{\alpha\pi}{\sqrt{2}\, s_W^2 M_W^2} = \frac{1}{\sqrt{2}\, v^2}\left[1 + \frac{\Sigma_{T}^{WW}(0)}{M_W^2}\right]$$

$$\hookrightarrow \left(M_{Z,os}^{(1)}\right)^2 = M_Z^2 - \Sigma_T^{ZZ}(M_Z^2)$$

① pick as input

$$\hookrightarrow \left(M_{W,os}^{(1)}\right)^2 = M_W^2 - \Sigma_T^{WW}(M_W^2)$$

$$\hookrightarrow \left(s_{W,eff}^{(1)}\right)^2 = s_W^2 + s_W c_W \frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2}$$

$$\hookrightarrow \Gamma^{(1)}_{Z\to e^+e^-} = \frac{\alpha M_{Z,os}}{6\, s_W^2 c_W^2}\left[\left(-\frac{1}{2} + s_{W,eff}^2\right)^2 + s_{W,eff}^4\right]$$

② predict these

beyond our control

* take Re[...] where appropriate ; note: $\Sigma(M_{os}^2) = \Sigma(M^2) + \mathcal{O}(\alpha^2)$
* remember: only oblique corrections

* $\Sigma \leftrightarrow$ UV divergent!

EPO @ 1-loop

① invert relations $\hat{O} = O^{(1)}$ to get Lagrange parameters*

$$\hookrightarrow \quad e^2 = 4\pi\hat{\alpha}(M_Z)\left[1 + \frac{\Sigma_T^{AA}(\hat{M}_Z^2)}{\hat{M}_Z^2}\right]$$

$$\hookrightarrow \quad v^2 = \frac{1}{\sqrt{2}\,\hat{G}_F}\left[1 + \frac{\Sigma_T^{WW}(0)}{\hat{M}_W^2}\right]$$

$$\hookrightarrow \quad \hat{M}_Z^2 = \underbrace{\frac{e^2 v^2}{4 s_W^2 c_W^2}}_{M_Z^2} - \Sigma_T^{ZZ}(\hat{M}_Z^2)$$

$$\delta s \equiv -\frac{\Sigma_T^{ZZ}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{AA}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{WW}(0)}{\hat{M}_W^2}$$

$$\Rightarrow s_W^2 = \underbrace{\frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\hat{\alpha}}{\sqrt{2}\,\hat{G}_F\hat{M}_Z^2}}\right)}_{\left(s_{W,\mathrm{eff}}^{(0)}\right)^2}\left(1 + \underbrace{\frac{c_W^2}{c_W^2 - s_W^2}}\delta s\right) = \hat{s}_W^{(0)\,2}\left(1 + \frac{\hat{c}_W^{(0)\,2}}{\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}}\delta s\right)$$

$\mathcal{O}(\alpha)$

$\Rightarrow$ can use tree-level relations

* UV divergent!

EPO @ 1-loop

② insert into predictions for remaining observables

$$\hookrightarrow \left(s_{W,\text{eff}}^{(1)}\right)^2 = \hat{s}_W^{(0)2} + \frac{\hat{s}_W^{(0)2}\,\hat{c}_W^{(0)2}}{\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}} \left(\delta s + \frac{\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}}{\hat{s}_W^{(0)}\,\hat{c}_W^{(0)}} \frac{\Sigma_T^{AZ}(\hat{M}_Z^2)}{\hat{M}_Z^2} \right)$$

$$\hookrightarrow \left(M_{W,\text{os}}^{(1)}\right)^2 = \frac{\pi\,\hat{\alpha}(M_Z)}{\sqrt{2}\,\hat{G}_F\,\hat{s}_W^{(0)2}} \left(1 + \frac{\Sigma_T^{AA}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{WW}(0)}{\hat{M}_W^2} - \frac{\hat{c}_W^{(0)2}}{\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}}\,\delta s - \frac{\Sigma_T^{WW}(\hat{M}_W^2)}{\hat{M}_W^2} \right)$$

$$\delta s = -\frac{\Sigma_T^{ZZ}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{AA}(\hat{M}_Z^2)}{\hat{M}_Z^2} + \frac{\Sigma_T^{WW}(0)}{\hat{M}_W^2}$$

* expressed everything in terms of measured quantities $\hat{\mathcal{O}}$

$\hookrightarrow$ Renormalizable theory $\Rightarrow$ all UV divergences cancel

* all we need to do is insert $\Sigma^{VV'}$ ...

EPO @ 1-loop

[Denner arXiv:0709.1075]

$$
\begin{aligned}
\Sigma_T^{AA}(k^2) = -\frac{\alpha}{4\pi}\Bigg\{&\frac{2}{3}\sum_{f,i} N_C^f 2Q_f^2\Big[-(k^2+2m_{f,i}^2)B_0(k^2,m_{f,i},m_{f,i}) \\
&+2m_{f,i}^2 B_0(0,m_{f,i},m_{f,i})+\frac{1}{3}k^2\Big] \\
&+\Big\{\Big[3k^2+4M_W^2\Big]B_0(k^2,M_W,M_W)-4M_W^2B_0(0,M_W,M_W)\Big\}\Bigg\},
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
\Sigma_T^{AZ}(k^2) = -\frac{\alpha}{4\pi}\Bigg\{&\frac{2}{3}\sum_{f,i} N_C^f(-Q_f)\big(g_f^+ + g_f^-\big)\Big[-(k^2+2m_{f,i}^2)B_0(k^2,m_{f,i},m_{f,i}) \\
&+2m_{f,i}^2B_0(0,m_{f,i},m_{f,i})+\frac{1}{3}k^2\Big] \\
&-\frac{1}{3s_Wc_W}\Big\{\Big[(9c_W^2+\frac{1}{2})k^2+(12c_W^2+4)M_W^2\Big]B_0(k^2,M_W,M_W) \\
&-(12c_W^2-2)M_W^2B_0(0,M_W,M_W)+\frac{1}{3}k^2\Big\}\Bigg\},
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
\Sigma_T^{ZZ}(k^2) = -\frac{\alpha}{4\pi}\Bigg\{&\frac{2}{3}\sum_{f,i} N_C^f\Big\{\big((g_f^+)^2+(g_f^-)^2\big)\Big[-(k^2+2m_{f,i}^2)B_0(k^2,m_{f,i},m_{f,i}) \\
&+2m_{f,i}^2B_0(0,m_{f,i},m_{f,i})+\frac{1}{3}k^2\Big]+\frac{3}{4s_W^2c_W^2}m_{f,i}^2B_0(k^2,m_{f,i},m_{f,i})\Big\} \\
&+\frac{1}{6s_W^2c_W^2}\Big\{\Big[(18c_W^4+2c_W^2-\frac{1}{2})k^2+(24c_W^4+16c_W^2-10)M_W^2\Big]B_0(k^2,M_W,M_W) \\
&-(24c_W^4-8c_W^2+2)M_W^2B_0(0,M_W,M_W)+(4c_W^2-1)\frac{1}{3}k^2\Big\} \\
&+\frac{1}{12s_W^2c_W^2}\Big\{\Big(2M_H^2-10M_Z^2-k^2\Big)B_0(k^2,M_Z,M_H) \\
&-2M_Z^2B_0(0,M_Z,M_Z)-2M_H^2B_0(0,M_H,M_H) \\
&-\frac{(M_Z^2-M_H^2)^2}{k^2}\Big(B_0(k^2,M_Z,M_H)-B_0(0,M_Z,M_H)\Big)-\frac{2}{3}k^2\Big\}\Bigg\},
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
\Sigma_T^{W}(k^2) = -\frac{\alpha}{4\pi}\Bigg\{&\frac{2}{3}\frac{1}{2s_W^2}\sum_i\Big[-\Big(k^2-\frac{m_{l,i}^2}{2}\Big)B_0(k^2,0,m_{l,i})+\frac{1}{3}k^2 \\
&+m_{l,i}^2B_0(0,m_{l,i},m_{l,i})+\frac{m_{l,i}^4}{2k^2}\Big(B_0(k^2,0,m_{l,i})-B_0(0,0,m_{l,i})\Big)\Big] \\
&+\frac{2}{3}\frac{1}{2s_W^2}3\sum_{i,j}|V_{ij}|^2\Big[-\Big(k^2-\frac{m_{u,i}^2+m_{d,j}^2}{2}\Big)B_0(k^2,m_{u,i},m_{d,j})+\frac{1}{3}k^2 \\
&+m_{u,i}^2B_0(0,m_{u,i},m_{u,i})+m_{d,j}^2B_0(0,m_{d,j},m_{d,j}) \\
&+\frac{(m_{u,i}^2-m_{d,j}^2)^2}{2k^2}\Big(B_0(k^2,m_{u,i},m_{d,j})-B_0(0,m_{u,i},m_{d,j})\Big)\Big] \\
&+\frac{2}{3}\Big\{\Big(2M_W^2+5k^2\Big)B_0(k^2,M_W,\lambda)-2M_W^2B_0(0,M_W,M_W) \\
&-\frac{M_W^4}{k^2}\Big(B_0(k^2,M_W,\lambda)-B_0(0,M_W,\lambda)\Big)+\frac{1}{3}k^2\Big\} \\
&+\frac{1}{12s_W^2}\Big\{\Big[(40c_W^2-1)k^2+(16c_W^2+54-10c_W^{-2})M_W^2\Big]B_0(k^2,M_W,M_Z) \\
&-(16c_W^2+2)\Big[M_W^2B_0(0,M_W,M_W)+M_Z^2B_0(0,M_Z,M_Z)\Big]+(4c_W^2-1)\frac{2}{3}k^2 \\
&-(8c_W^2+1)\frac{(M_W^2-M_Z^2)^2}{k^2}\Big(B_0(k^2,M_W,M_Z)-B_0(0,M_W,M_Z)\Big)\Big\} \\
&+\frac{1}{12s_W^2}\Big\{\Big(2M_H^2-10M_W^2-k^2\Big)B_0(k^2,M_W,M_H) \\
&-2M_W^2B_0(0,M_W,M_W)-2M_H^2B_0(0,M_H,M_H) \\
&-\frac{(M_W^2-M_H^2)^2}{k^2}\Big(B_0(k^2,M_W,M_H)-B_0(0,M_W,M_H)\Big)-\frac{2}{3}k^2\Big\}\Bigg\}.
\end{aligned}
\tag{B.4}
$$

exercise: isolate the fermion loops & check UV cancellation

$B_0(q^2, m_1, m_2)$ bubble integral

The two-point function is given by

$$
\begin{aligned}
B_0(p_{10},m_0,m_1) &= \Delta-\int_0^1 dx\log\frac{[p_{10}^2x^2-x(p_{10}^2-m_0^2+m_1^2)+m_1^2-i\varepsilon]}{\mu^2}+O(D-4) \\
&= \Delta+2-\log\frac{m_0m_1}{\mu^2}+\frac{m_0^2-m_1^2}{p_{10}^2}\log\frac{m_1}{m_0}-\frac{m_0m_1}{p_{10}^2}(\frac{1}{r}-r)\log r \\
&\quad +O(D-4),
\end{aligned}
\tag{4.23}
$$

where r and $\frac{1}{r}$ are determined from

$$
x^2+\frac{m_0^2+m_1^2-p_{10}^2-i\varepsilon}{m_0m_1}x+1=(x+r)(x+\frac{1}{r}).
\tag{4.24}
$$

EPO @ 1-loop

* only terms $\sim m_t^2$

$\Delta = \frac{1}{\epsilon} + \text{const.}$ (UV divergence)

$$\Sigma_T^{WW}(q^2)\Big|_{(t,b)} = -\frac{\alpha}{4\pi} N_c \frac{1}{2 s_W^2} m_t^2 \left[\Delta - \ln\left(\frac{m_t^2}{\mu^2}\right) + \frac{1}{2}\right] + \mathcal{O}(q^2)$$

$$\Sigma_T^{ZZ}(q^2)\Big|_{t} = -\frac{2\alpha}{\pi} N_c \frac{1}{16 s_W^2 c_W^2} m_t^2 \left[\Delta - \ln\left(\frac{m_t^2}{\mu^2}\right)\right] + \mathcal{O}(q^2)$$

$$\Rightarrow \quad \delta s\Big|_{m_t^2} = -\frac{\alpha}{4\pi} N_c \frac{m_t^2}{2 s_W^2} \left(\frac{1}{M_W^2}\left[\Delta \cancel{- \ln\left(\frac{m_t^2}{\mu^2}\right)} + \frac{1}{2}\right] - \overbrace{\frac{1}{c_W^2}\frac{1}{M_Z^2}}^{1/M_W^2}\left[\Delta \cancel{- \ln\left(\frac{m_t^2}{\mu^2}\right)}\right]\right)$$

$$= -\frac{\alpha}{16\pi s_W^2} N_c \frac{m_t^2}{M_W^2}$$

EPO @ 1-loop

improved agreement!

* predictions only including dominant m_t^2 corrections

↪ $$\left(s_{W,\mathrm{eff}}^{(1)}\right)^2 = \hat{s}_W^{(0)\,2}\left(1 - \frac{3\,\hat{\alpha}(M_Z)}{16\pi\,\hat{s}_W^{(0)}\left(\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}\right)}\,\frac{\hat{m}_t^2}{\hat{M}_Z^2}\right) \simeq 0.230423$$

↪ $$\left(M_{W,\mathrm{os}}^{(1)}\right)^2 = \hat{M}_W^2\left(1 + \frac{3\,\hat{\alpha}(M_Z)}{16\pi\,\hat{s}_W^{(0)}\left(\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}\right)}\,\frac{\hat{m}_t^2}{\hat{M}_Z^2}\right) \simeq 80.9045$$

* radiative corrections ⇝ sensitivity on top quark (Higgs)

↪ predict before discovery:

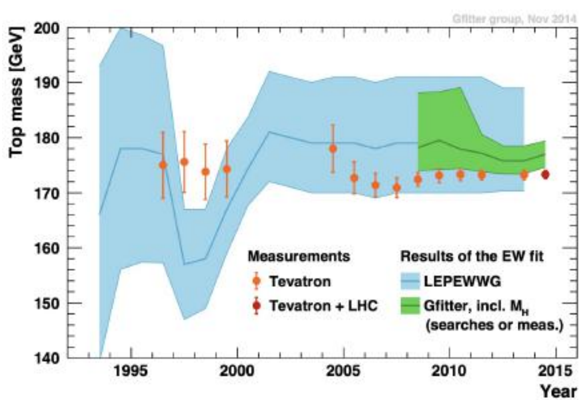
$$m_t^2 = \left(1 - \frac{\left(s_{W,\mathrm{eff}}^{(\mathrm{exp})}\right)^2}{\hat{s}_W^{(0)2}}\right)\frac{16\pi\,\hat{s}_W^{(0)2}\left(\hat{c}_W^{(0)2} - \hat{s}_W^{(0)2}\right)}{3\,\hat{\alpha}(M_Z)}\cdot\hat{M}_Z^2 \;\rightsquigarrow\; m_t \simeq 140\ \mathrm{GeV}$$

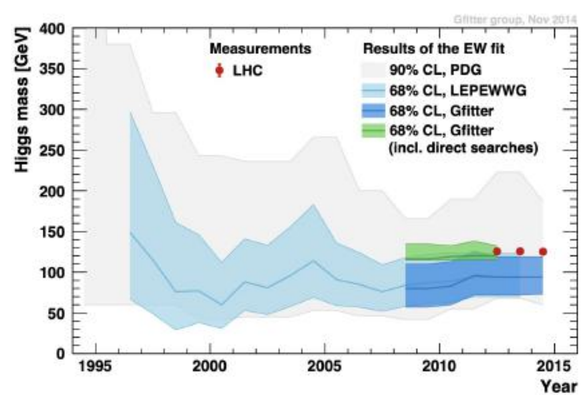
↪ Higgs mass only enters logarithmically (poorer constaint)

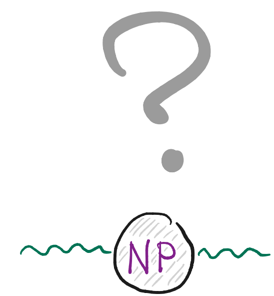
$$M_W^{(1)\,2} \longmapsto M_W^{(1)2} - \frac{5\,\hat{\alpha}}{24\pi}\,\frac{\hat{c}^2\,\hat{M}_Z^2}{\hat{c}^2 - \hat{s}^2}\ln\left(\hat{M}_H^2/\hat{M}_W^2\right) \quad ; \quad \left(s_{W,\mathrm{eff}}^{(1)}\right)^2 \longmapsto \left(s_{W,\mathrm{eff}}^{(1)}\right)^2 + \frac{\hat{\alpha}\,(1+9\,\hat{s}^2)}{48\pi\,(\hat{c}^2 - \hat{s}^2)}\ln\left(\hat{M}_H^2/\hat{M}_W^2\right)$$

Radiative Corrections – Intermediate Summary

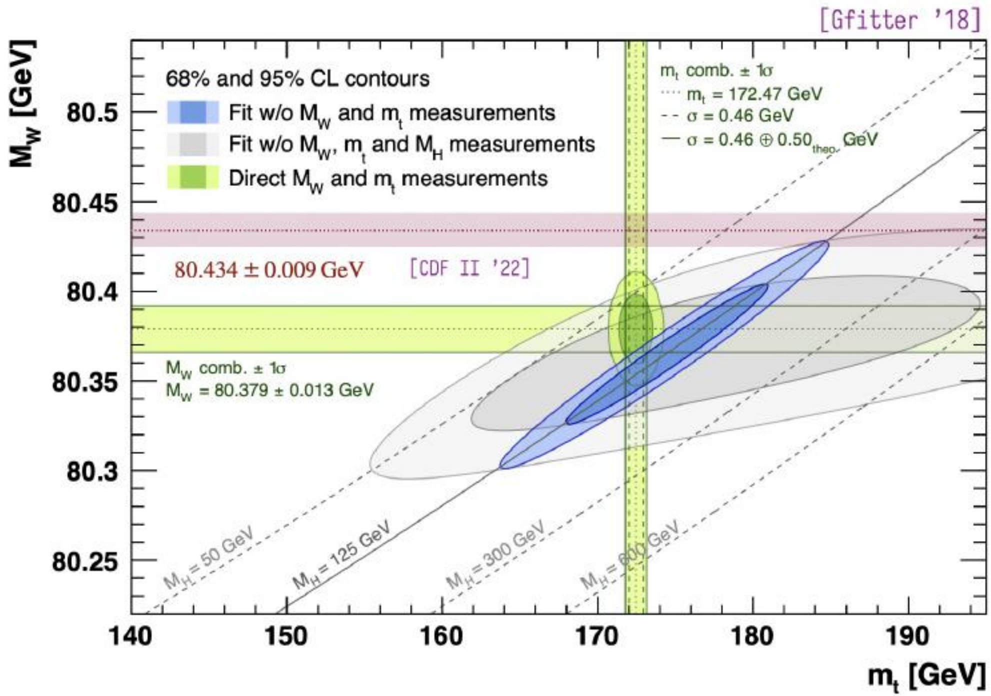
* We showed in a pedestrian (brute-force) way renormalization
  - ⇾ once physical (observable) quantities are expressed in terms of other physical quantities ⇒ UV singularities cancel
  - ⇾ more systematic approach through counterterms (tutorials)
* Radiative corrections can introduce sensitivity to states that are not directly accessible (yet)

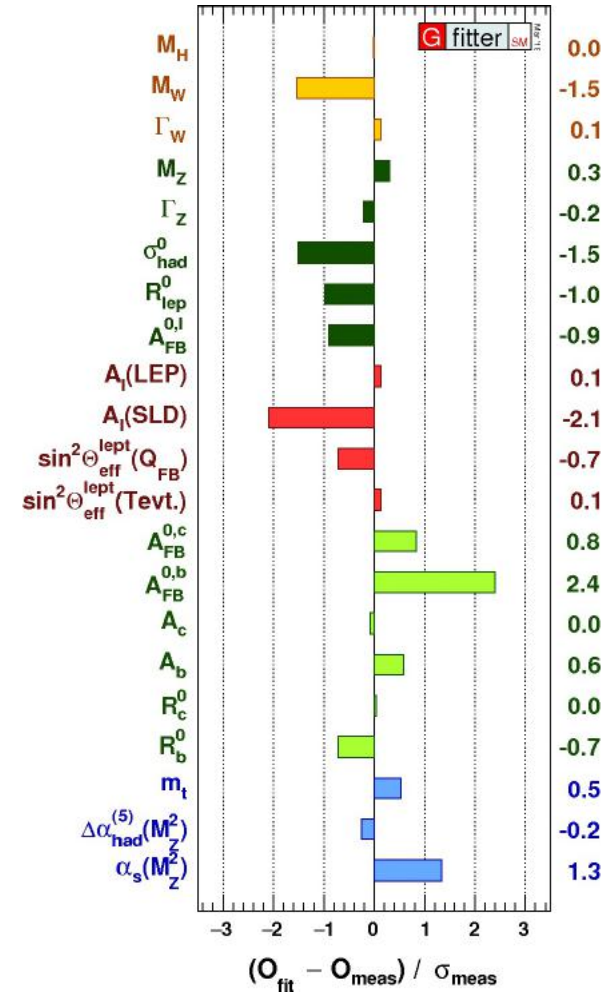






Global EW Fit





m/GeV	measured	fit value
m_t	172.47 ± 0.68	176.4 ± 2.1
M_H	125.1 ± 0.2	90^{+21}_{-18}
M_W	80.379 ± 0.013	80.354 ± 0.007

5. LHC Phenomenology

At the LHC we collide protons $\neq$ elementary

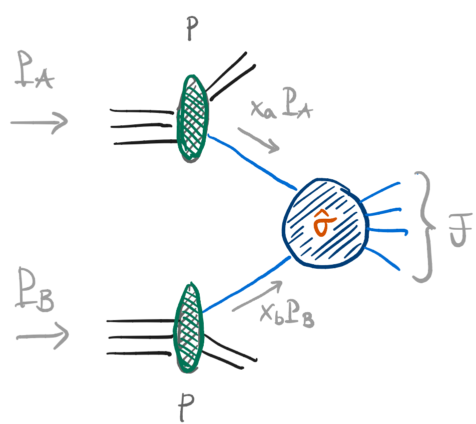
$\rightsquigarrow$ hadronic cross section = sum over scattering of partons (q & g)

fractional momentum

$$d\sigma_{A+B\to\mathcal{F}+X}(P_A, P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \; f_a(x_a)\, f_b(x_b)\, d\hat{\sigma}_{a+b\to\mathcal{F}+X}(x_a P_A, x_b P_B)$$

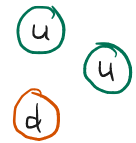
parton distribution function

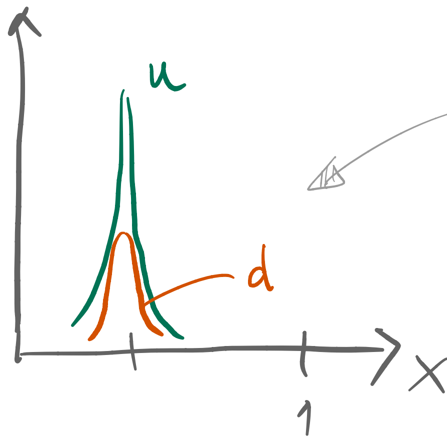
$\hat{=}$ number density for parton a to carry momentum fraction $[x, x+dx]$ of parent hadron



Parton Distribution Functions

* just free quarks? ($p = uud$)



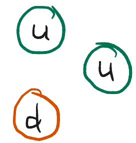


$$f_a(x) = \delta_{au}\, 2\, \delta(x - 1/3) + \delta_{ad}\, 1\, \delta(x - 1/3)$$

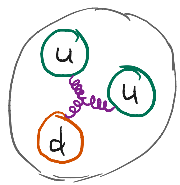
+ some smearing

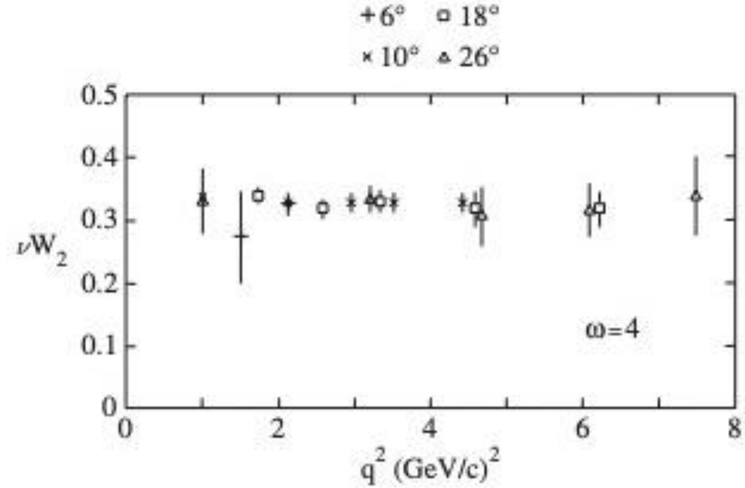
Parton Distribution Functions

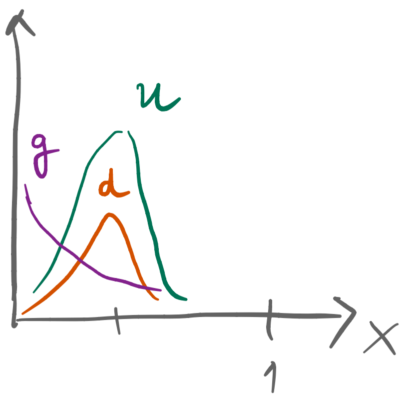
* just free quarks? ($p = uud$)



* bound by gluons?







naive parton model:

⇝ composition of point particles

⇝ zoom in ($Q^2 \nearrow$) ⟷ same composition

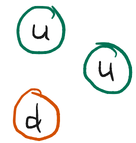
scaling (PDFs independent on scale at which it is probed, as long as $Q^2 \gg m_p^2$)

⇝ Callan-Gross: $F_2(x) = 2x F_1$

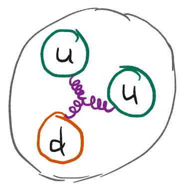
⇒ quarks are spin ½

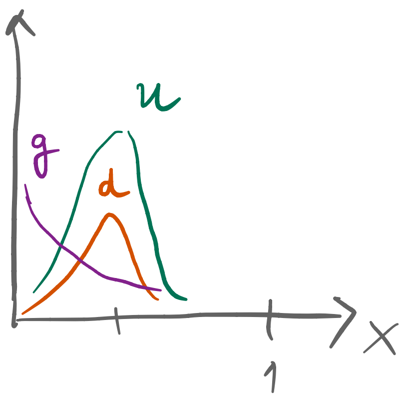
Parton Distribution Functions

* just free quarks? ($p = uud$)

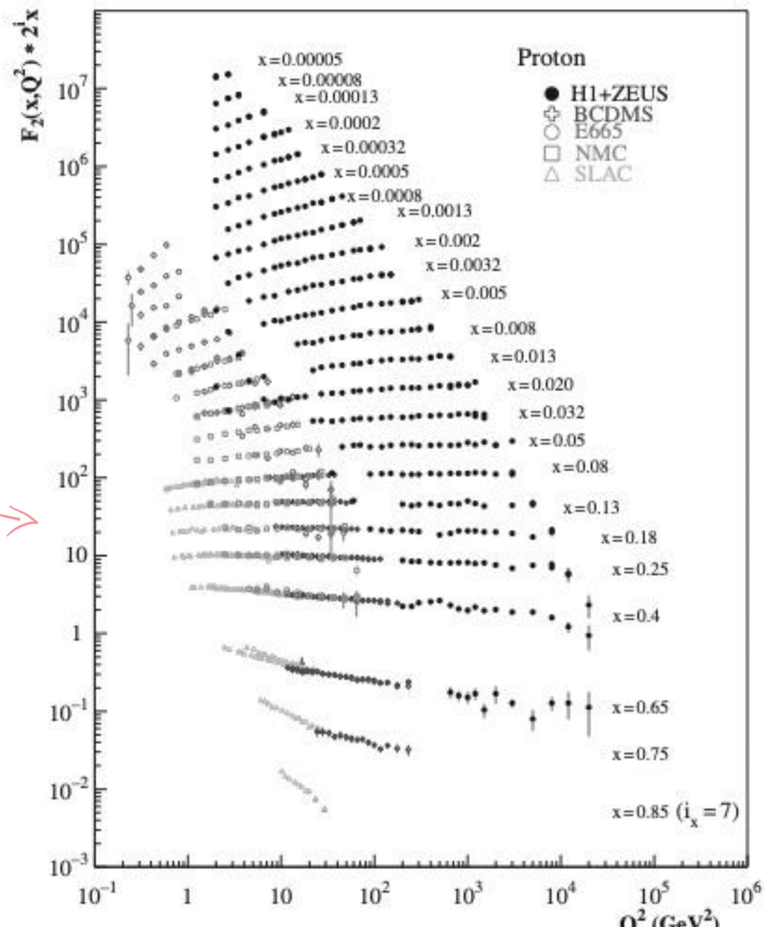


* bound by gluons?



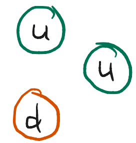


scaling violation

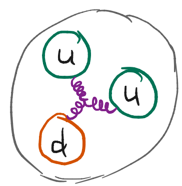


Parton Distribution Functions

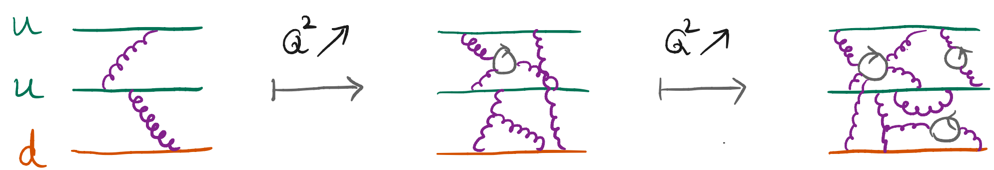
* just free quarks? ($p = uud$)



* bound by gluons?



* QCD-improved parton model



$\Rightarrow$ predominantly shifts partons from high-x to low-x

Parton Evolution

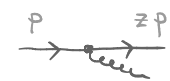
some work in lattice QCD more recently.

* PDFs are non-perturbative objects ⇝ can't "compute" them
* but their evolution (dependence on Q^2 or μ_F^2) is perturbatively calculable

↪ Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP) equations

$$\mu \frac{d}{d\mu} f_a(x,\mu) = \frac{\alpha_s}{\pi} \int_x^1 \frac{dz}{z} \underbrace{P_{ab}(z)} f_b\left(\frac{x}{z}, \mu_F\right)$$

→ splitting functions:

$P_{qq}(z) \hat{=}$ 

$P_{gq}(z) \hat{=}$ 

$P_{qg}(z) \hat{=}$ 

$P_{gg}(z) \hat{=}$ 

Parton Evolution

[org/pdf]

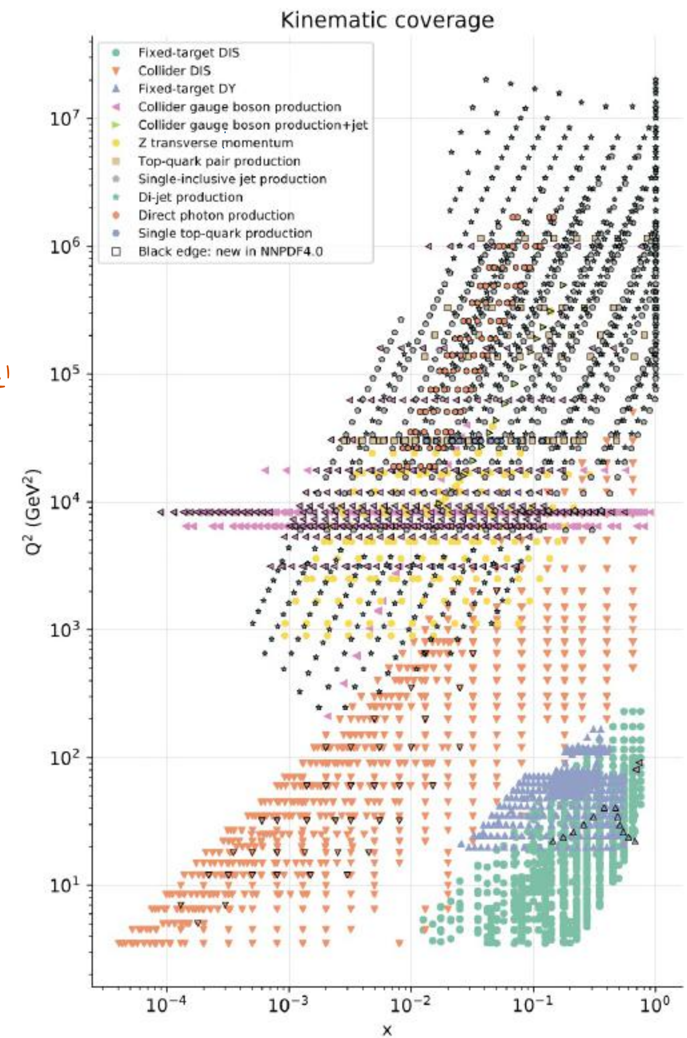
* PDFs are universal
  - pick a common "starting scale"

    $$\mu_0^2 = \mathcal{O}(1\,\mathrm{GeV}^2) \longleftrightarrow m_p,\ m_c$$

  - parametrise functions
    - $x f(x, \mu_0^2) = A x^B (1-x)^C \left[1 + Dx + Ex^2\right] - A' x^{B'} (1-x)^{C'}$

      (HERAPDF)
    - Neural Network (NNPDF)
    - ...
  - fit the data

⇒ use evolution for predictions



The Drell-Yan process

[org/dy]

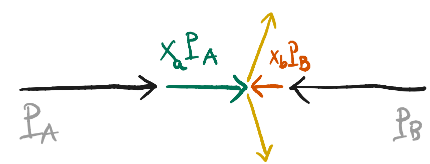
$$d\sigma_{DY} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_a(x_a)\, f_b(x_b)\, d\hat{\sigma}_{ab\to \ell^+\ell^-}$$

(only $(a,b) \in \{(q,\bar{q}),(\bar{q},q)\}$ @ LO)

* After integrating out $Z \to \ell^+\ell^-$ decay we'll look at the observables of the intermediate gauge boson $q^\mu = (p_1+p_2)^\mu = (p_a+p_b)^\mu$

$$M_{\ell\ell} = \sqrt{q^2} \quad ; \quad Y_{\ell\ell} = \frac{1}{2}\ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$$

rapidity: $Y \to Y + \frac{1}{2}\ln(x_1/x_2)$

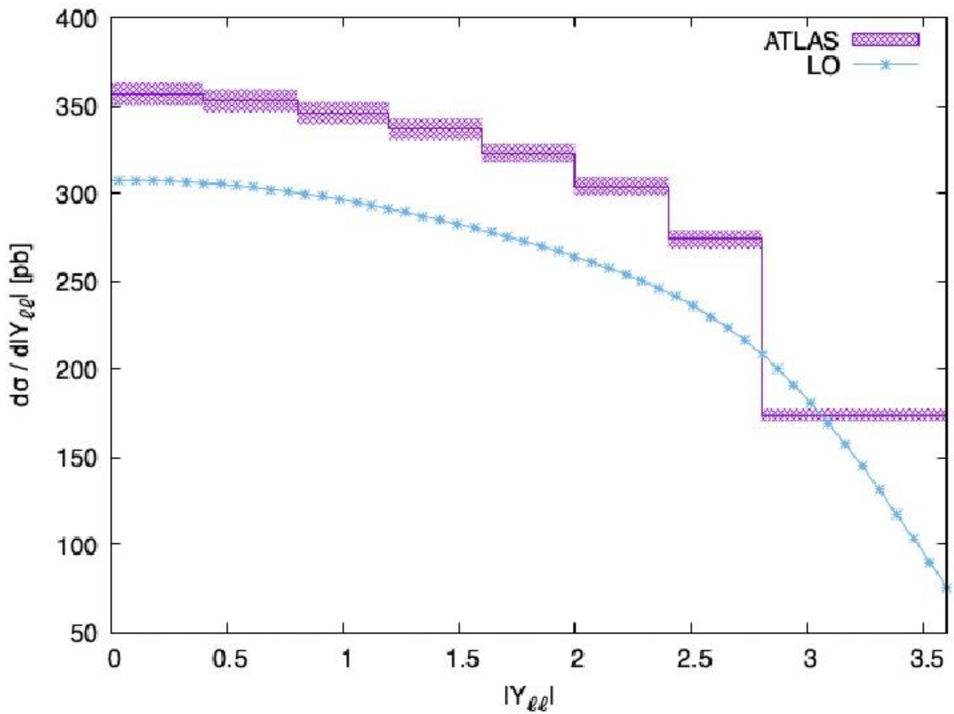


$$\Rightarrow \boxed{\frac{d^2\sigma_{DY}}{dM_{\ell\ell}\, dY_{\ell\ell}} = f_a(x_a)\, f_b(x_b)\, \frac{2M_{\ell\ell}}{E_{cm}^2}\, \hat{\sigma}_{ab} \quad \Bigg| \quad x_{a/b} = \frac{M_{\ell\ell}}{E_{cm}} e^{\pm Y_{\ell\ell}}}$$

The Drell-Yan process

ATLAS: $\sigma_Z = 1055.3 \pm 0.7\,(\text{stat.}) \pm 2.2\,(\text{syst.}) \pm 19.0\,(\text{lumi.})\,\text{pb}$

LO: `#total 897.1018242299992 pb`

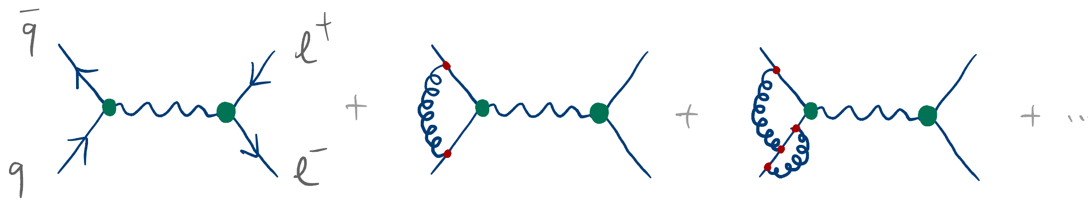


* right ballpark
  $\alpha_s \sim 0.1$
  ⇒ $\mathcal{O}(10\%)$ anticipated
  ⇒ often much worse!
  (extreme: $ggH \leftrightarrow 100\%$)
* @ LO only $q\bar{q}$ annihilation
  ⇒ 50% of protons are g!
* no error estimate here
  ⇒ no quantitative comp

⇒ need at least NLO

The Drell-Yan process

$\Rightarrow$ we want to go to the next order! $\rightsquigarrow$ diagrams with <u>loops</u>!

$\mathcal{M}_{2\to 2} =$ 

$\mathcal{O}(\alpha)\ \mathcal{M}^{(0)}_{2\to 2}$ $\qquad$ $\mathcal{O}(\alpha\alpha_s)\ \mathcal{M}^{(1)}_{2\to 2}$ $\qquad$ $\mathcal{O}(\alpha\alpha_s^2)\ \mathcal{M}^{(2)}_{2\to 2}$

$$\Rightarrow |\mathcal{M}_{2\to 2}|^2 = \underbrace{|\mathcal{M}^{(0)}_{2\to 2}|^2}_{\mathcal{O}(\alpha^2)} + \underbrace{2\,\mathrm{Re}\left\{\left(\mathcal{M}^{(0)}_{2\to 2}\right)^* \mathcal{M}^{(1)}_{2\to 2}\right\}}_{\mathcal{O}(\alpha^2\alpha_s)} + \underbrace{|\mathcal{M}^{(1)}_{2\to 2}|^2 + 2\,\mathrm{Re}\left\{\left(\mathcal{M}^{(0)}_{2\to 2}\right)^* \mathcal{M}^{(2)}_{2\to 2}\right\}}_{\mathcal{O}(\alpha^2\alpha_s^2)} + \dots$$

"virtual"

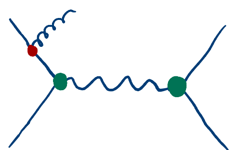
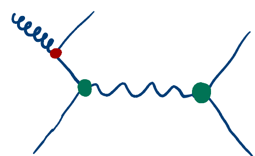
Divergences in Loop Diagrams $\rightsquigarrow \int \frac{d^4k}{(2\pi)^4}$

① ultraviolet (UV) $\leftrightsquigarrow$ large loop momentum

$\hookrightarrow$ treated by renormalization (last lecture)

② infrared (IR) $\leftrightsquigarrow$ soft and/or collinear

$\hookrightarrow$ requires real emission contribution & PDF renormalization

$\mathcal{M}_{2\to3} =$  $+ \ldots$;  $+ \ldots$

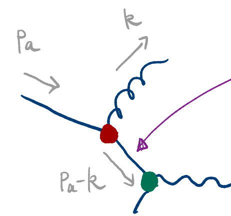
$\mathcal{O}(\alpha\sqrt{\alpha_s})\ \mathcal{M}^{(0)}_{2\to3}$

technically, a different process <u>but</u> necessary to include (at least when unresolved)

$$|\mathcal{M}_{2\to3}|^2 = \underbrace{|\mathcal{M}^{(0)}_{2\to3}|^2}_{\mathcal{O}(\alpha^2\alpha_s)\ \text{“real”}} + \ldots$$

IR Divergences

Let's look at this sub-diagram
(appears both in the virtual & real contribution)



this propagator contains

$$\frac{1}{(p_a-k)^2} = \frac{1}{E_a E\,(1-\cos\theta)}$$

$\Rightarrow$ potential divergence when
(a) gluon "soft": $E \to \emptyset$
(b) gluon collinear: $\vec{k} \parallel \vec{p}_a$

(a) the "virtual" (loop) corrections give

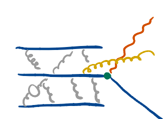
$$\hat{\sigma}_{LO}(p_a,p_b) \cdot \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \times \left\{ \frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} + \text{finite} \right\}$$

(b) the "real" corrections give

$$\hat{\sigma}_{LO}(p_a,p_b) \cdot \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \times \left\{ -\frac{1}{\epsilon^2} - \frac{3}{2}\frac{1}{\epsilon} + \text{finite} \right\}$$

$$+ \int_0^1 dz_a\, \hat{\sigma}_{LO}(z_a p_a, p_b) \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \times \left\{ \underbrace{-\frac{1}{\epsilon}}_{} - \ln\left(\mu_F^2/Q^2\right) \right\} P_{qq}(z_a) + \begin{pmatrix} z_a \leftrightarrow z_b \\ q_{in} \to g_{in} \end{pmatrix}$$

this is absorbed as part of the "NLO PDF"

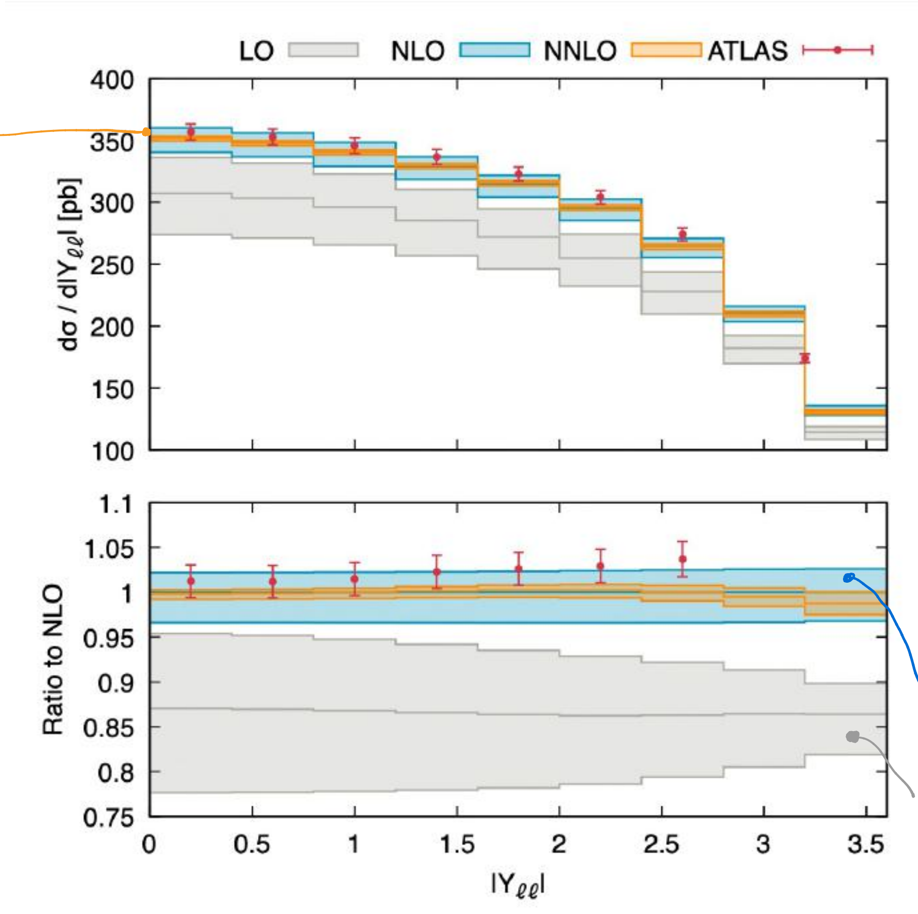


part of the proton if its scale $< \mu_F$ $\Big\}$ $f_a(x) \longmapsto f_a(x, \mu_F^2)$

UNIVERSAL $\leftrightarrow$ PDF evolution

The Drell-Yan process at higher orders

at least NNLO needed to match Δ_{EXP} for normalized spectra, even beyond $\rightsquigarrow$ N^{3}LO



on repository

the notebook

Missing Higher Orders — Uncertainties

* cross section for scale choice μ_0

$$\sigma(\alpha_s(\mu_0),\mu_0) = \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^n \underbrace{\sigma^{(0)}}_{\text{LO}} + \left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^{n+1} \underbrace{\sigma^{(1)}}_{\text{NLO}} + \ldots$$

scale variation @ lower orders generates higher-order terms

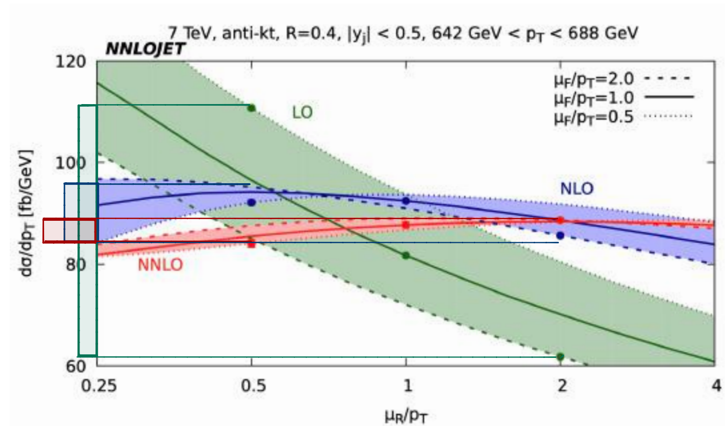
⟶ prediction for scale dependence

$$\sigma(\alpha_s(\mu),\mu) = \left(\frac{\alpha_s(\mu)}{2\pi}\right)^n \sigma^{(0)} + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^{n+1} \left[\sigma^{(1)} + \underbrace{n\,\beta_0 \ln\left(\frac{\mu^2}{\mu_0^2}\right)\sigma^{(0)}}\right] + \ldots$$

⇒ handle on missing terms from truncation of the perturbative series

"scale uncertainties"

(no statistical meaning!)



Factorization of IR Divergences

* IR singularities are universal & factorize

  ↳ soft limit: $\mathcal{M}_{n+1}(p_1, \ldots, p_n, g(k)) \xrightarrow{k \to 0} \varepsilon_\mu^a(k)^* \, g_s \, \hat{J}^{a,\mu} \otimes \mathcal{M}_n(p_1, \ldots, p_n)$

  - eikonal current $\hat{J}^{a,\mu} = \sum_{i=1}^{n} \frac{p_i^\mu}{(p_i \cdot k)} \hat{T}_i^a$

$$\hat{T}_i^a = \begin{cases} t^a & q(\text{in}), \bar{q}(\text{out}) \\ -t^a & q(\text{out}), \bar{q}(\text{in}) \\ T_A^a & g \end{cases}$$

  ↳ collinear limit:

$$|\mathcal{M}_{n+1}(p_1, \ldots, p_n, g(k))|^2 \xrightarrow{k \parallel p_i} \frac{g_s^2}{(p_i \cdot k)} P_{gi}(z) \, |\mathcal{M}_n(p_1, \ldots, (p_i + k), \ldots p_n)|^2$$

↳ splitting function

* The cancellation of IR divergences is dictated by the Kinoshita-Lee-Nauenberg (KLN) theorem

  ↳ When is it valid?

Infrared Safety

* In general, measurements involve non-trivial fiducial constraints

$$\int |\mathcal{M}_n|^2 d\Phi_n \rightsquigarrow \int |\mathcal{M}_n|^2 \underbrace{\mathcal{F}_n(p_1, \dots, p_n)}_{} d\Phi_n$$

→ measurement function

- fiducial cuts
- jet algorithm
- histograms
- isolation
- ...

* KLN cancellation only preserved if $\mathcal{F}_n$ sufficiently inclusive / IR safe

↪ soft safety

$$\mathcal{F}_n(p_1, \dots, p_{n+1}) \xrightarrow{p_i \to \emptyset} \mathcal{F}_{n-1}(p_1, \dots, \cancel{p_i}, \dots, p_n)$$

↪ collinear safety

$$\mathcal{F}_n(p_1, \dots, p_{n+1}) \xrightarrow{p_i \parallel p_j} \mathcal{F}_{n-1}(p_1, \dots, \cancel{p_i}, \dots, \cancel{p_j}, \dots, p_n, p_i + p_j)$$

Infrared Safety

* which observables are IR safe?

↪ # of final-state partons

↪ $H_T = \sum_{i=1}^{n} p_{T,i}$

↪ $\sum_{i=1}^{n} (p_{T,i})^2$

↪ thrust $\quad T = \max_{\vec{n}} \dfrac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$

↪ jets

↪ sphericity $\quad S^{\alpha\beta} = \dfrac{\sum_{i=1}^{n} p_i^{\alpha} p_j^{\beta}}{\sum_j |\vec{p}_j|^2}$

↪ spherocity $\quad S = \left(\dfrac{4}{\pi}\right)^2 \max_{\vec{n}} \left[\dfrac{\sum_{i=1}^{n} |\vec{p}_i \times \vec{n}|}{\sum_j |\vec{p}_j|} \right]^2$

Infrared Safety

* which observables are IR safe?

- \# of final-state partons — NO
- $H_T = \sum_{i=1}^{n} p_{T,i}$ — YES
- $\sum_{i=1}^{n} (p_{T,i})^2$ — NO
- thrust $\quad T = \max_{\vec{n}} \dfrac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$ — YES
- jets — DEPENDS (ask Klaus)
- sphericity $\quad S^{\alpha\beta} = \dfrac{\sum_{i=1}^{n} p_i^{\alpha} p_j^{\beta}}{\sum_j |\vec{p}_j|^2}$ — NO
- spherocity $\quad S = \left(\dfrac{4}{\pi}\right)^2 \max_{\vec{n}} \left[\dfrac{\sum_{i=1}^{n} |\vec{p}_i \times \vec{n}|}{\sum_j |\vec{p}_j|} \right]^2$ — YES

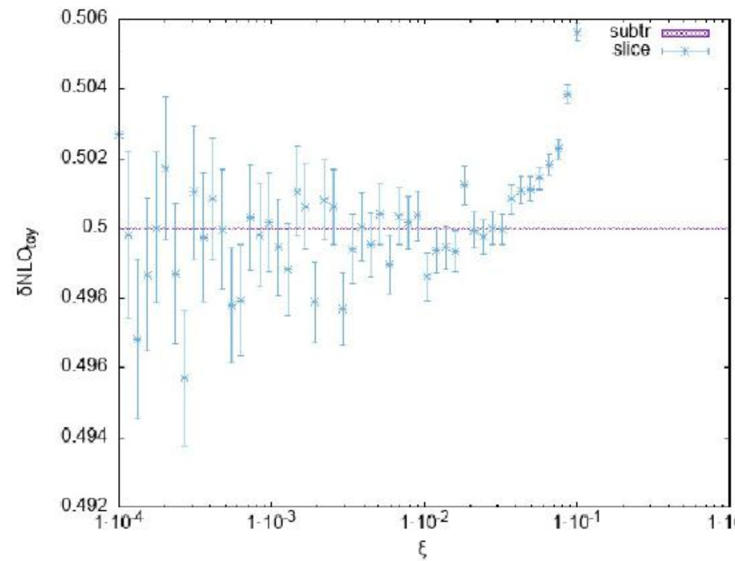
Infrared Subtraction

* Predictions that admit for arbitrary (IR safe) observables
  - ⇒ require to make the cancellation that happens inclusively manifest @ differential level
  - ⇝ "integrate without integrating" ?!
* Subtraction methods accomplish this by re-shuffling IR divergences
  - ↪ @ NLO solved problem [CS dipoles, FKS, ...]
  - ↪ @ NNLO tremendous progress in past decade $(2 \to 3)$
    [q_T, $\mathcal{T}_N$, antenna, stripper, Colorful, nested S-C, P2B, ...]
  - ↪ @ N^3LO only very special cases $(2 \to 1)$ [q_T, P2B]

Infrared Subtraction

* broadly, 2 approaches: local subtraction, slicing
* for the interested ⇝ [org/toy-nlo] (toy example to illustrate main ideas)

$$\left(\frac{1}{\epsilon}+a\right)\mathcal{J}(0) \qquad \int_0^1 \frac{1+bx}{x^{1+\epsilon}}\,\mathcal{J}(x)\,\mathrm{d}x$$



Accuracy of the calculation vs. Observables

* back to the Drell-Yan process @ NLO

$\Leftrightarrow$ What is the formal accuracy of associated observables?

$\hookrightarrow$ σ^{tot}

$\hookrightarrow$ $\dfrac{d\sigma}{dY_Z}$

$\hookrightarrow$ $\dfrac{d\sigma}{dY_{e^+}}$

$\hookrightarrow$ $\dfrac{d\sigma}{dp_{T,e^-}}$

$\hookrightarrow$ $\dfrac{d\sigma}{dp_{T,Z}}$

Accuracy of the calculation vs. Observables

* back to the Drell-Yan process @ NLO
  $\Leftrightarrow$ What is the formal accuracy of associated observables?

$\hookrightarrow$ σ^{tot} NLO

$\hookrightarrow$ $\frac{d\sigma}{dY_Z}$ NLO

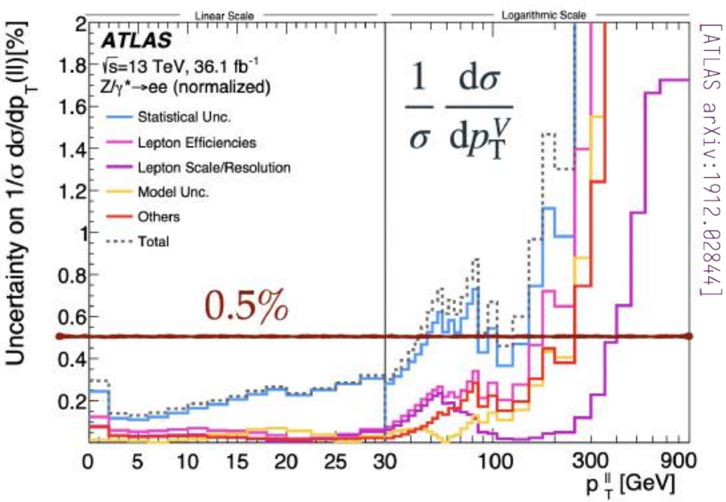
$\hookrightarrow$ $\frac{d\sigma}{dY_{e^+}}$ NLO

$\hookrightarrow$ $\frac{d\sigma}{dp_{T,e^-}}$ depends
NLO for $p_{T,e^+} \lesssim M_Z/2$
LO for $p_{T,e^+} \gtrsim M_Z/2$

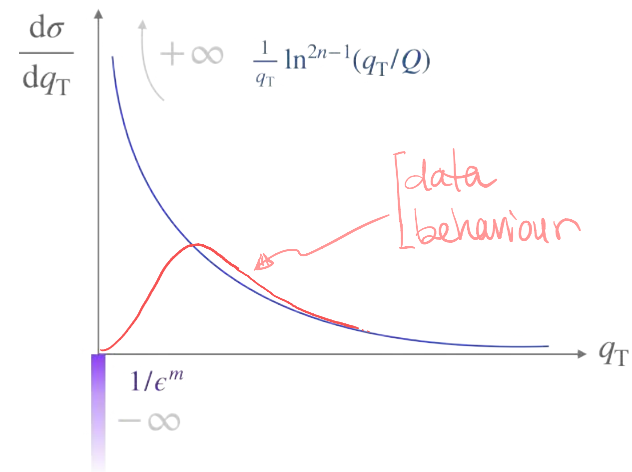
$\hookrightarrow$ $\frac{d\sigma}{dp_{T,Z}}$ LO $\Leftrightarrow$ accuracy one order lower b.c.
$p_{T,Z} > \emptyset$ requires at least one emission!

Transverse Momentum Distribution q_T

* extraordinary EXP precision



* fixed-order completely off



* incomplete cancellations between "virtual" & "real"

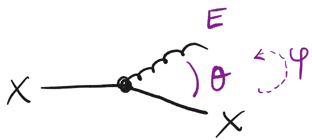
→ q_T probes small scales of a high-scale process $(Q^2 \sim M^2)$

⇝ sensitivity to large logs $\ln^{2n}(q_T/Q)$

Emission Probability — Soft && Collinear

* before: factorization individually in the soft and collinear limits
  - ⇒ focus on the most leading behaviour ⟷ [s & c] ⊕ phase-space

↳ emission probability $$d\omega_{X\to X+g} = \frac{\alpha_s}{\pi^2} C_X \frac{dE}{E} \frac{d\theta}{\theta} d\varphi$$



$$\hookrightarrow C_X = \begin{cases} C_F = 4/3 & q \\ C_A = 3 & g \end{cases}$$

* there are two logarithmic singularities ⇝ "double-logs"
  - ↳ change of variables & $\int dE\,(\ldots)$

$$\Rightarrow d\omega_{X\to X+g} = \frac{\alpha_s}{\pi^2} C_X \ln\left(Q^2/k_T^2\right) \frac{d\vec{k}_T^2}{k_T}$$

* single emission factorizes with the same (leading) log-divergence as the NLO predictions ($q_T = k_T$ @ NLO)

q_T Resummation

* Divergence $\longleftrightarrow$ $\alpha_s^n \ln^{2n}(Q^2/q_T^2) \sim 1$ terms at each truncated order

  $\hookrightarrow$ resum this behaviour to all orders

* n emissions in the S & C approx:

$\vec{q}_T$ is a vectorial quantity

not factorized!

$$\frac{1}{\sigma_0}\frac{d\sigma_n}{d^2\vec{q}_T} = \frac{1}{n!}\prod_{i=\phi}^{n}\left[\int \frac{\alpha_s}{\pi^2} C_X \ln\left(Q^2/k_{T,i}^2\right)\frac{d^2\vec{k}_{T,i}}{k_{T,i}}\right]\delta^{(2)}\left(\vec{q}_T - \sum_{i=1}^{n}\vec{k}_{T,i}\right)$$

$$\delta^{(2)}\left(\vec{q}_T - \sum_{i=1}^{n}\vec{k}_{T,i}\right) = \frac{1}{(2\pi)^2}\int d^2\vec{b}\; e^{-i\vec{b}\cdot\left(\vec{q}_T - \sum_i \vec{k}_{T,i}\right)}$$

(Fourier transform)

$$= \frac{1}{n!}\int \frac{d^2\vec{b}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{q}_T}\left[\int \frac{\alpha_s}{\pi^2} C_X \ln\left(Q^2/k_T^2\right)\frac{d^2\vec{k}_T}{k_T}\, e^{i\vec{b}\vec{k}_T}\right]^n$$

fully factorizes in impact parameter space!

q_T Resummation

* integrating $\int d^2\vec{k}_T$ & $\sum_{n=0}^{\infty}$ emissions exponentiate

$$\frac{1}{\sigma_0}\frac{d^2\sigma_{res}}{d^2\vec{q}_T} = \int \frac{d^2\vec{b}}{(2\pi)^2}\, e^{-i\vec{b}\vec{q}_T} \exp\left[-\frac{\alpha_s}{2\pi} C_X \ln^2(Q^2b^2)\right]$$

$\int d\varphi$

$$\frac{1}{\sigma_0}\frac{d\sigma_{res}}{dq_T^2} = \int_0^\infty db\, \frac{b}{2} J_0(q_T b) \exp\left[-\frac{\alpha_s}{2\pi} C_X \ln^2(Q^2b^2)\right]$$

* we worked in the leading double-log approx

$$\alpha_s \ln^2\left(Q^2/q_T^2\right) \sim 1 \gg \alpha_s \ln\left(Q^2/q_T\right),\ \alpha_s$$

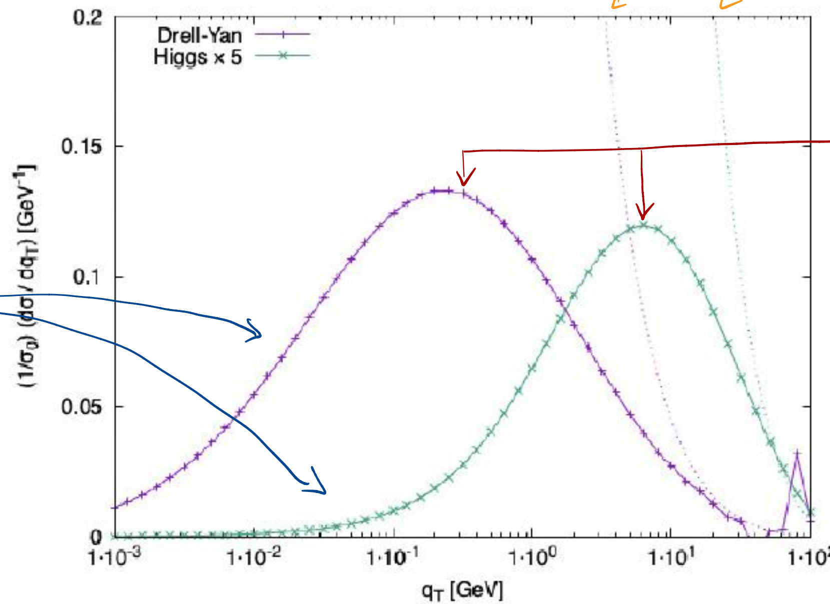
$\hookrightarrow$ very rough but OK to give an idea

proper treatment by [Collins, Soper, Sterman '85] $\left(\mu^2 \simeq q_T^2\right)$

q_T Resummation [org/res]

divergent fixed order

resummed



"Sudakov peak"

@ larger value for $gg \to H$ because

$C_A = 3 > 4/3 = C_F$

larger colour factor

$*$ would we have expanded further $\rightarrow$ different behaviour

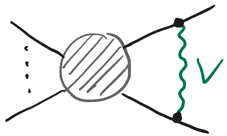
$\hookrightarrow$ naive leading ($k_{T,i} \to \emptyset \;\forall i$) vanishes exponentially

instead small q_T dominated by $\sum_i \vec{k}_{T,i}$ balancing out

Electroweak Corrections

[Denner, Dittmaier 1912.06823]

* Sudakov Logarithms



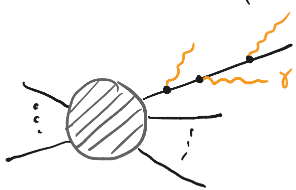
$\sim \ln^2\left(\frac{s_{ij}}{M_V^2}\right)$ + sub-leading Logs

Sudakov regime

$(s_{ij} \gg M_V^2)$

→ typical size in $\mathcal{O}(\text{TeV})$ regimes -10 to -20%

* Final-state photon radiation



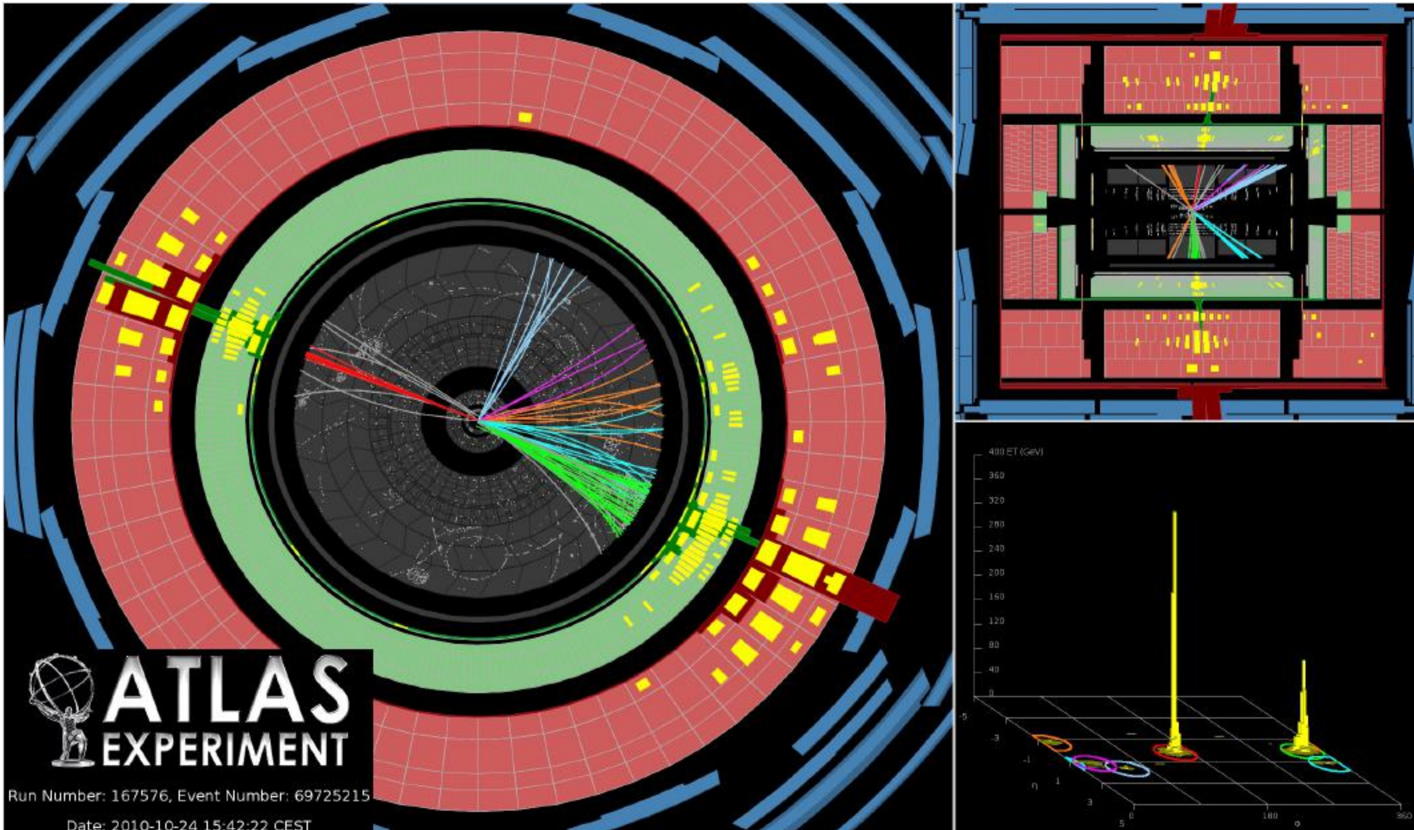
$\rightarrow \alpha^n \ln^2\left(Q^2/m_e^2\right)$

→ for "bare leptons" (coll. unsafe) → $\mathcal{O}(100\%)$

"dressed leptons" → $\mathcal{O}(50\%)$

→ large where σ varies strongly: resonances, shoulders, ...

Events at hadron colliders look complex



why? Any chance to compute this with what we did so far?

Nope ... [org/diags]

```
2->2 gluon scattering has 4 diagrams
2->3 gluon scattering has 25 diagrams
2->4 gluon scattering has 220 diagrams
2->5 gluon scattering has 2485 diagrams
2->6 gluon scattering has 34300 diagrams
2->7 gluon scattering has 559405 diagrams
2->8 gluon scattering has 10525900 diagrams
2->9 gluon scattering has 224449225 diagrams
2->10 gluon scattering has 5348843500 diagrams
2->11 gluon scattering has 140880765025 diagrams
```

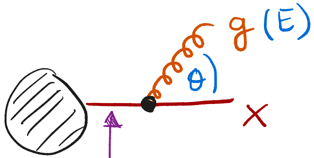
$\leftrightarrow 10^{11}$!

* even with more efficient recursive approaches not the multiplicities you'd want to tackle. $(\#\dim(\Phi_n) = 3n-4)$

$\hookrightarrow$ map back to fewer initiating objects — Jets

$\hookrightarrow$ model full complexity approximately — Parton Showers

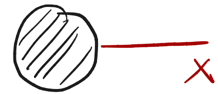
The QCD emission pattern



gluon

soft & collinear

enhanced (divergent) $\sim \dfrac{1}{E_x\, E\,(1-\cos\theta)}$

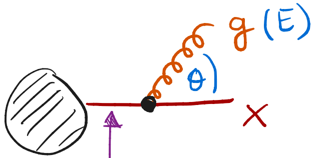


$\times\; d\omega_{x \to x+g}$

$$\boxed{2\,\frac{\alpha_s}{\pi}\, C_x\, \frac{dE}{E}\, \frac{d\theta}{\theta}}$$

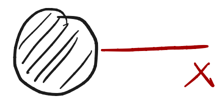
$$C_x \begin{cases} = C_F = 4/3 & \text{if } x \equiv q \\ = C_A = 3 & \text{if } x = g \end{cases}$$

The QCD emission pattern



gluon $\longmapsto$ $\left\{ \begin{array}{l} \text{soft \&} \\ \text{collinear} \end{array} \right.$

enhanced (divergent) $\sim \dfrac{1}{E_x E (1-\cos\theta)}$



$\times \; d\omega_{x \to x+g}$

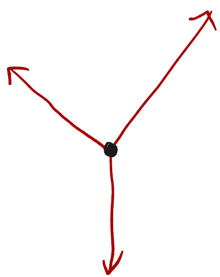
$$\boxed{2 \frac{\alpha_s}{\pi} C_x \frac{dE}{E} \frac{d\theta}{\theta}}$$

$$C_x \begin{cases} = C_F = 4/3 & \text{if } x = q \\ = C_A = 3 & \text{if } x = g \end{cases}$$

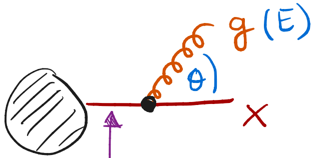
⇒ jets are an emergent feature of QCD

① high energetic partons

↪ hard scattering

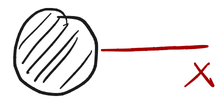


The QCD emission pattern



gluon: soft & collinear

enhanced (divergent) $\sim \dfrac{1}{E_x E(1-\cos\theta)}$



$\times\ d\omega_{x\to x+g}$

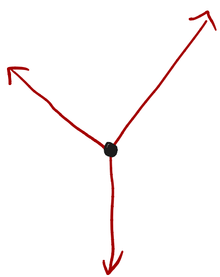
$$\boxed{2\,\frac{\alpha_s}{\pi}\,C_x\,\frac{dE}{E}\,\frac{d\theta}{\theta}}$$

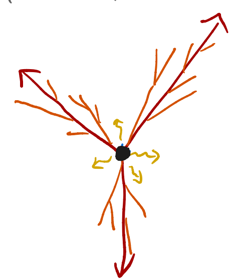
$$C_x \begin{cases} = C_F = 4/3 & \text{if } x=q \\ = C_A = 3 & \text{if } x=g \end{cases}$$

⇒ <u>jets are an emergent feature of QCD</u>

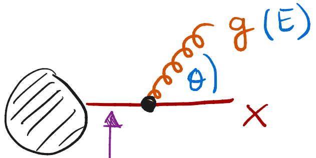
① high energetic partons
↪ hard scattering

② asymptotic freedom & $d\omega$
↪ pert. parton shower

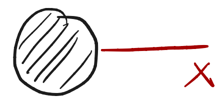




The QCD emission pattern



$\xrightarrow{\text{gluon}}$ { soft & collinear



$\times\; d\omega_{x\to x+g}$

$$\boxed{2\,\frac{\alpha_s}{\pi}\,C_x\,\frac{dE}{E}\,\frac{d\theta}{\theta}}$$

$$C_x \begin{cases} = C_F = 4/3 & \text{if } x \equiv q \\ = C_A = 3 & \text{if } x = g \end{cases}$$

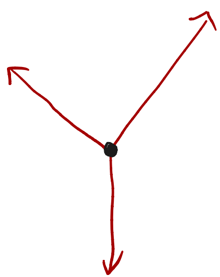
enhanced (divergent) $\sim \dfrac{1}{E_x\, E\,(1-\cos\theta)}$

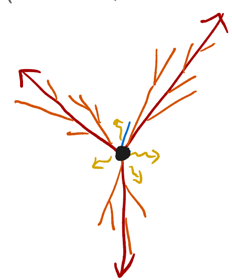
⇒ jets are an emergent feature of QCD

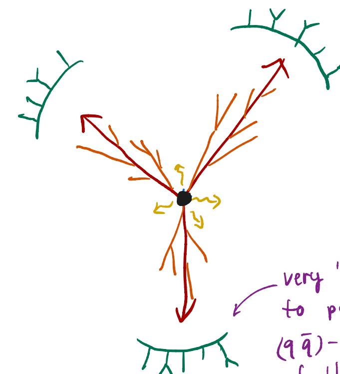
① high energetic partons ↪ hard scattering

② asymptotic freedom & $d\omega$ ↪ pert. parton shower

③ hadronization

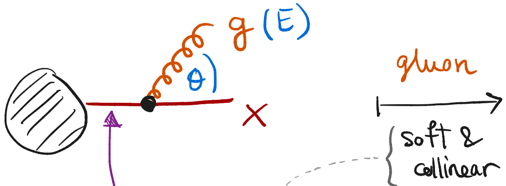




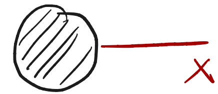


very "cheap" to pop a $(q\bar{q})$-pair out of the vacuum
$m_{u,d} \ll \Lambda_{QCD}$

The QCD emission pattern



enhanced (divergent) $\sim \dfrac{1}{E_x E(1-\cos\theta)}$



$\times\ d\omega_{x \to x+g}$

$$\boxed{2\,\frac{\alpha_s}{\pi}\,C_x\,\frac{dE}{E}\,\frac{d\theta}{\theta}}$$

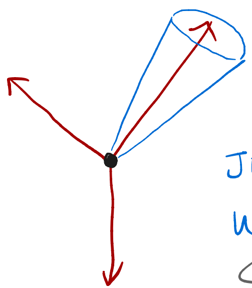
$$C_x \begin{cases} = C_F = 4/3 & \text{if } x = q \\ = C_A = 3 & \text{if } x = g \end{cases}$$

⇒ jets are an emergent feature of QCD

① high energetic partons ↪ hard scattering

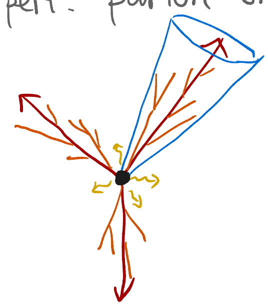
② asymptotic freedom & $d\omega$ ↪ pert. parton shower

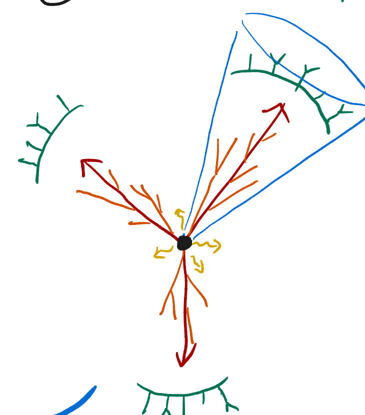
③ hadronization



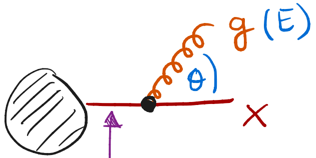
JETS "map back" to underlying hard partons

↪ precision fixed order



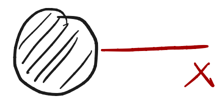


The QCD emission pattern



gluon $\longmapsto$ $\left\{ \begin{array}{l} \text{soft \&} \\ \text{collinear} \end{array} \right.$

enhanced (divergent) $\sim \dfrac{1}{E_x E (1-\cos\theta)}$



$\times \; d\omega_{x \to x+g}$

$$\boxed{2 \, \frac{\alpha_s}{\pi} C_x \frac{dE}{E} \frac{d\theta}{\theta}}$$

$$C_x \begin{cases} = C_F = 4/3 & \text{if } x \equiv q \\ = C_A = 3 & \text{if } x = g \end{cases}$$

$\Rightarrow$ emission factorizes!

Integral over E & θ diverges $\rightsquigarrow$ introduce a scale $q^2 > Q_0^2$

$\leftrightarrow$ emission "resolved"

$$\Rightarrow \quad P_x \simeq \frac{\alpha_s C_x}{2\pi} \underbrace{\ln^2\left(Q^2/Q_0^2\right)} + \mathcal{O}\left(\alpha_s \ln Q^2, \alpha_s^2\right)$$

probability to emit a resolved gluon

potentially a very large log $\longrightarrow$ will want to resum these to all orders

$$\left(\left. \begin{array}{l} Q_0 = \Lambda_{QCD} = 0.2 \text{ GeV} \\ Q = 100 \text{ GeV} \end{array} \right\} \Rightarrow \ln(\ldots) = \mathcal{O}(10) \right)$$

Parton Showers

* We wish to account for an arbitrary number of emissions ordered in our resolution variable $Q^2 > q_1^2 > q_2^2 > \ldots > Q_0^2$ (strong ordering)
* current scale q_n^2 $\rightsquigarrow$ probability to have next emission @ q_{n+1}^2 ?

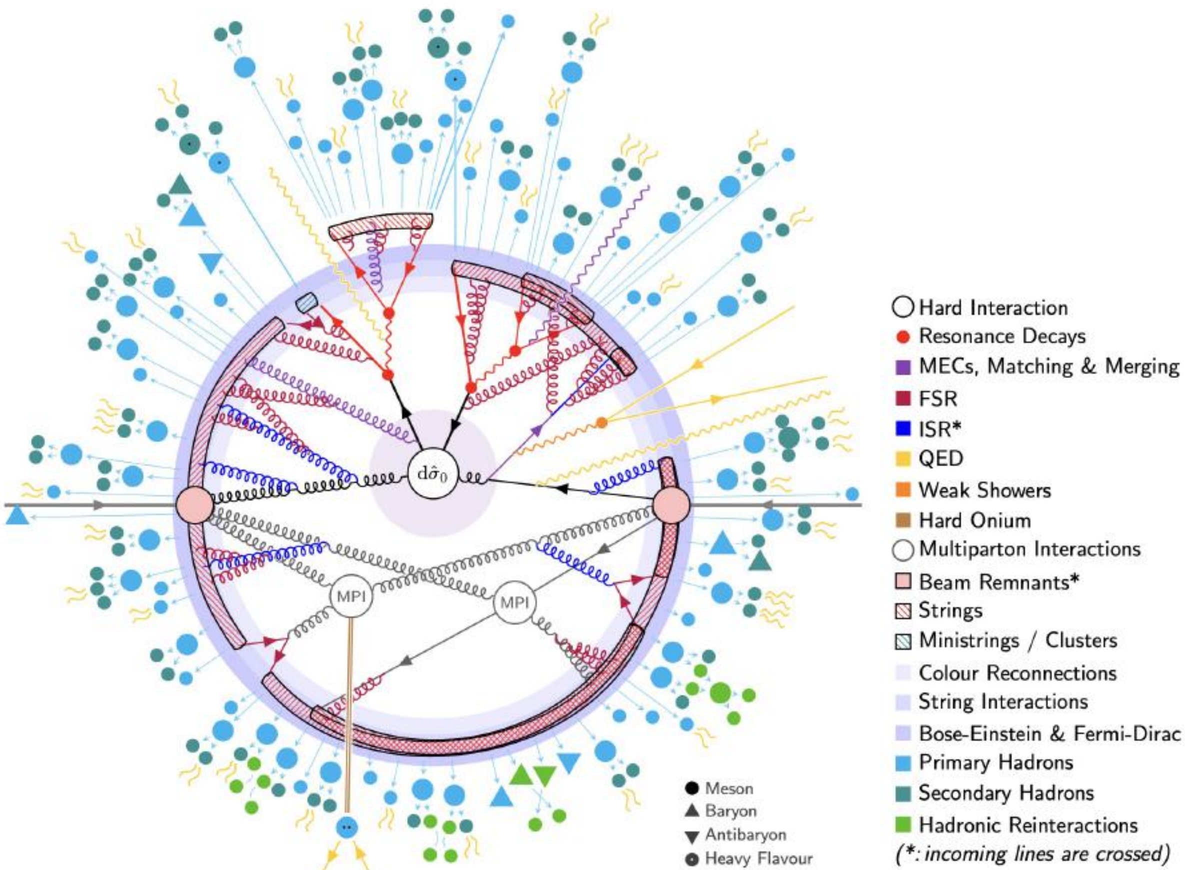
$$\longleftrightarrow \underbrace{\begin{pmatrix} \text{pobability of having} \\ \underline{\text{no}} \text{ emissions } q_n^2 \to q_{n+1}^2 \end{pmatrix}}_{\substack{\text{Sudakov from factor} \\ \Delta(q_n^2, q_{n+1}^2)}} \times \underbrace{\begin{pmatrix} \text{emission} \\ @ \; q_{n+1}^2 \end{pmatrix}}_{\left.\frac{d\omega_{X\to X+g}}{dq^2}\right|_{q^2=q_{n+1}^2}}$$

$$\Leftrightarrow \frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\frac{d\omega}{dq^2}$$

$$\left(\Delta(Q^2, q^2-dq^2) = \Delta(Q^2,q^2)\underbrace{\Delta(q^2,q^2-dq^2)}_{\left(1-\frac{d\omega}{dq^2}\right)}\right)$$

[org/toy-shower]

Full event generator



Summary & Conclusions

* Standard Model in excellent agreement with experiments so far

→ New Physics likely hiding in more subtle effects

⇒ Precision is key in the interpretation of the data

* exciting times to do precision phenomenology

& the future is bright!